

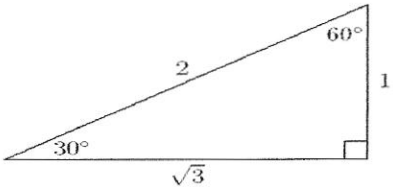
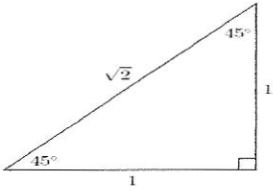
Workshop Solutions to Section 2.6

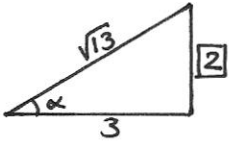
<p>1) The inverse of the function $f = \{(0,3), (-2,1), (3,4), (5,-2), (1,7)\}$ is $f^{-1} = \{(3,0), (1,-2), (4,3), (-2,5), (7,1)\}$</p>	<p>2) Find the inverse of the function $f(x) = 2x + 3$. <u>Solution:</u> Let $y = 2x + 3$ $2x = y - 3$ $x = \frac{y-3}{2}$ Now, change x with y ($x \Leftrightarrow y$) $y = \frac{x-3}{2}$ $\therefore f^{-1}(x) = \frac{x-3}{2}$</p>
<p>3) Find the inverse of the function $f(x) = 3 - 2x$. <u>Solution:</u> Let $y = 3 - 2x$ $2x = 3 - y$ $x = \frac{3-y}{2}$ Now, change x with y ($x \Leftrightarrow y$) $y = \frac{3-x}{2}$ $\therefore f^{-1}(x) = \frac{3-x}{2}$</p>	<p>4) Find the inverse of the function $f(x) = 3 - \frac{x}{2}$. <u>Solution:</u> Let $y = 3 - \frac{x}{2}$ $2y = 6 - x$ $x = 6 - 2y$ Now, change x with y ($x \Leftrightarrow y$) $y = 6 - 2x$ $\therefore f^{-1}(x) = 6 - 2x$</p>
<p>5) Find the inverse of the function $f(x) = \sqrt{2x - 3}$. <u>Solution:</u> Let $y = \sqrt{2x - 3}$ by squaring both sides $y^2 = 2x - 3$ $2x = y^2 + 3$ $x = \frac{y^2+3}{2}$ Now, change x with y ($x \Leftrightarrow y$) $y = \frac{x^2+3}{2}$ $\therefore f^{-1}(x) = \frac{x^2+3}{2}$</p>	<p>6) Find the inverse of the function $f(x) = \sqrt[3]{3 - 2x}$. <u>Solution:</u> Let $y = \sqrt[3]{3 - 2x}$ by cubing both sides $y^3 = 3 - 2x$ $2x = 3 - y^3$ $x = \frac{3-y^3}{2}$ Now, change x with y ($x \Leftrightarrow y$) $y = \frac{3-x^3}{2}$ $\therefore f^{-1}(x) = \frac{3-x^3}{2}$</p>
<p>7) Find the inverse of the function $f(x) = (2x + 3)^2, x \in [0, \infty)$. <u>Solution:</u> Let $y = (2x + 3)^2$ Take the square root for both sides $\sqrt{y} = 2x + 3$ $2x = \sqrt{y} - 3$ $x = \frac{\sqrt{y}-3}{2}$ Now, change x with y ($x \Leftrightarrow y$) $y = \frac{\sqrt{x}-3}{2}$ $\therefore f^{-1}(x) = \frac{\sqrt{x}-3}{2}$</p>	<p>8) Find the inverse of the function $f(x) = -(x - 3)^3$. <u>Solution:</u> Let $y = -(x - 3)^3$ $-y = (x - 3)^3$ Take the cubic root for both sides $\sqrt[3]{-y} = x - 3$ $x = \sqrt[3]{-y} + 3$ Now, change x with y ($x \Leftrightarrow y$) $y = \sqrt[3]{-x} + 3$ $\therefore f^{-1}(x) = \sqrt[3]{-x} + 3$</p>
<p>9) Find the inverse of the function $f(x) = \frac{x}{x-3}$. <u>Solution:</u> Let $y = \frac{x}{x-3}$ $y(x-3) = x$ $xy - 3y = x$ $xy - x = 3y$ $x(y-1) = 3y$ $x = \frac{3y}{y-1}$ Now, change x with y ($x \Leftrightarrow y$) $y = \frac{3x}{x-1}$ $\therefore f^{-1}(x) = \frac{3x}{x-1}$</p>	<p>10) Find the inverse of the function $f(x) = \frac{x-3}{x}$. <u>Solution:</u> Let $y = \frac{x-3}{x}$ $xy = x - 3$ $xy - x = -3$ $x(y-1) = -3$ $x = \frac{-3}{y-1} = -\frac{3}{y-1} = \frac{3}{-(y-1)} = \frac{3}{1-y}$ Now, change x with y ($x \Leftrightarrow y$) $y = \frac{3}{1-x}$ $\therefore f^{-1}(x) = \frac{3}{1-x}$</p>

<p>11) Find the inverse of the function $f(x) = \frac{x+2}{x-3}$.</p> <p>Solution: Let $y = \frac{x+2}{x-3}$ $y(x-3) = x+2$ $xy - 3y = x+2$ $xy - x = 3y+2$ $x(y-1) = 3y+2$ $x = \frac{3y+2}{y-1}$ Now, change x with y ($x \Leftrightarrow y$) $y = \frac{3x+2}{x-1}$ $\therefore f^{-1}(x) = \frac{3x+2}{x-1}$</p>	<p>12) Find the inverse of the function $f(x) = \sqrt{x} + 5$.</p> <p>Solution: Let $y = \sqrt{x} + 5$ $\sqrt{x} = y - 5$ by squaring both sides $x = (y-5)^2$ Now, change x with y ($x \Leftrightarrow y$) $y = (x-5)^2$ $\therefore f^{-1}(x) = (x-5)^2$</p>
<p>13) Find the inverse of the function $f(x) = \sqrt[3]{x^5}$.</p> <p>Solution: Let $y = \sqrt[3]{x^5}$ $y = x^{\frac{5}{3}}$ $y^{\frac{3}{5}} = (x^{\frac{5}{3}})^{\frac{3}{5}}$ $x = \sqrt[5]{y^3}$ Now, change x with y ($x \Leftrightarrow y$) $y = \sqrt[5]{x^3}$ $\therefore f^{-1}(x) = \sqrt[5]{x^3}$</p>	<p>14) Find the inverse of the function $f(x) = 2x^3 - 5$.</p> <p>Solution: Let $y = 2x^3 - 5$ $2x^3 = y + 5$ $x^3 = \frac{y+5}{2}$ take the cubic root for both sides $x = \sqrt[3]{\frac{y+5}{2}}$ Now, change x with y ($x \Leftrightarrow y$) $y = \sqrt[3]{\frac{x+5}{2}}$ $\therefore f^{-1}(x) = \sqrt[3]{\frac{x+5}{2}}$</p>
<p>15) Find the inverse of the function $f(x) = \sqrt[3]{\frac{x+2}{5}}$.</p> <p>Solution: Let $y = \sqrt[3]{\frac{x+2}{5}}$ by cubing both sides $y^3 = \frac{x+2}{5}$ $5y^3 = x+2$ $x = 5y^3 - 2$ Now, change x with y ($x \Leftrightarrow y$) $y = 5x^3 - 2$ $\therefore f^{-1}(x) = 5x^3 - 2$</p>	<p>16) Evaluate $2^{\log_2(5x+3)}$.</p> <p>Solution: $2^{\log_2(5x+3)} = 5x+3$</p>
<p>18) $\log_2 64 - \log_2 32 + \log_2 2 = \log_2 \frac{64 \times 2}{32}$ $= \log_2 4 = \log_2 2^2$ $= 2 \log_2 2$ $= 2 \times 1 = 2$</p> <p>OR $\log_2 64 - \log_2 32 + \log_2 2 = \log_2 2^6 - \log_2 2^5 + \log_2 2$ $= 6 - 5 + 1 = 2$</p>	<p>17) Evaluate $\log_2 2^{(5x+3)}$.</p> <p>Solution: $\log_2 2^{(5x+3)} = 5x+3$</p>
<p>20) $\log_3 54 - \log_3 2 = \log_3 \frac{54}{2}$ $= \log_3 27 = \log_3 3^3 = 3$</p>	<p>19) $\log_3 27 - \log_3 81 + 5 \log_3 3 = \log_3 \frac{27 \times 3^5}{81}$ $= \log_3 81 = \log_3 3^4$ $= 4 \log_3 3$ $= 4 \times 1 = 4$</p> <p>OR $\log_3 27 - \log_3 81 + 5 \log_3 3 = \log_3 3^3 - \log_3 3^4 + 5 \times 1$ $= 3 - 4 + 5 = 4$</p>
<p>22) If $\ln(x+3) = 5$, then $x =$</p> <p>Solution: $\ln(x+3) = 5$ $e^{\ln(x+3)} = e^5$ $x+3 = e^5$ $x = e^5 - 3$</p>	<p>21) If $\log_2(6+2x) = 1$, then $x =$</p> <p>Solution: $\log_2(6+2x) = 1$ $2^{\log_2(6+2x)} = 2^1$ $6+2x = 2$ $2x = 2-6 = -4$ $x = -2$</p>
<p>22) If $\ln(x+3) = 5$, then $x =$</p> <p>Solution: $\ln(x+3) = 5$ $e^{\ln(x+3)} = e^5$ $x+3 = e^5$ $x = e^5 - 3$</p>	<p>23) If $\ln(x) = 5$, then $x =$</p> <p>Solution: $\ln(x) = 5$ $e^{\ln(x)} = e^5$ $x = e^5$</p>

<p>24) If $e^{(2x-3)} = 5$, then $x =$</p> <p><u>Solution:</u></p> $e^{(2x-3)} = 5$ $\ln e^{(2x-3)} = \ln 5$ $2x - 3 = \ln 5$ $2x = \ln 5 + 3$ $x = \frac{\ln 5 + 3}{2}$	<p>25) $\log_3 2 = \frac{\ln 2}{\ln 3}$</p>
<p>27) $\log_3 18 - \log_3 6 = \log_3 \frac{18}{6}$</p> $= \log_3 3$ $= 1$	<p>26) $\log 25 + \log 4 = \log(25 \times 4)$</p> $= \log 100 = \log 10^2$ $= 2$
<p>29) $e^{3 \ln 2} = e^{\ln 2^3} = 2^3 = 8$</p>	<p>28) $\log_2 6 - \log_2 15 + \log_2 20 = \log_2 \frac{6 \times 20}{15}$</p> $= \log_2 8 = \log_2 2^3$ $= 3$
<p>30) If $3^{2-x} = 6$, then $x =$</p> <p><u>Solution:</u></p> $3^{2-x} = 6$ $\log_3 3^{2-x} = \log_3 6$ $2 - x = \log_3 6$ $x = 2 - \log_3 6 = 2 - \log_3 (3 \times 2)$ $= 2 - (\log_3 3 + \log_3 2) = 2 - (1 + \log_3 2)$ $= 2 - 1 - \log_3 2$ $= 1 - \log_3 2$	<p>31) Find the inverse of the function $f(x) = 5 + \ln x$.</p> <p><u>Solution:</u></p> <p>Let $y = 5 + \ln x$</p> $\ln x = y - 5$ $e^{\ln x} = e^{y-5}$ $x = e^{y-5}$ <p>Now, change x with y ($x \leftrightarrow y$)</p> $y = e^{x-5}$ $\therefore f^{-1}(x) = e^{x-5}$
<p>32) Find the domain of the function</p> $f(x) = \sin^{-1}(3x + 5).$ <p><u>Solution:</u></p> <p>We know that the domain of $\sin^{-1}(x)$ is $[-1, 1]$. So,</p> $-1 \leq 3x + 5 \leq 1$ $-6 \leq 3x \leq -4$ $-2 \leq x \leq -\frac{4}{3}$ $\therefore D_f = \left[-2, -\frac{4}{3}\right]$	<p>33) Find the domain of the function</p> $f(x) = \cos^{-1}(3x - 5).$ <p><u>Solution:</u></p> <p>We know that the domain of $\cos^{-1}(x)$ is $[-1, 1]$. So,</p> $-1 \leq 3x - 5 \leq 1$ $4 \leq 3x \leq 6$ $\frac{4}{3} \leq x \leq 2$ $\therefore D_f = \left[\frac{4}{3}, 2\right]$
<p>34) Find the domain of the function</p> $f(x) = 2\sin^{-1}(x) + 1.$ <p><u>Solution:</u></p> <p>We know that the domain of $\sin^{-1}(x)$ is $[-1, 1]$. So,</p> $\therefore D_f = [-1, 1]$	

Before proceeding to the questions 35-55, we should be aware of the following well-known right triangles:

30° – 60° Right Triangle	30° – 60° Right Triangle																		
																			
<p>We know that $30^\circ = \frac{\pi}{6}$ and $60^\circ = \frac{\pi}{3}$, so</p>	<p>We know that $45^\circ = \frac{\pi}{4}$, so</p>																		
<table border="0"> <tr> <td>$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$</td> <td>$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$</td> </tr> <tr> <td>$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$</td> <td>$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$</td> </tr> <tr> <td>$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$</td> <td>$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$</td> </tr> <tr> <td>$\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$</td> <td>$\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$</td> </tr> <tr> <td>$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$</td> <td>$\sec\left(\frac{\pi}{3}\right) = 2$</td> </tr> <tr> <td>$\csc\left(\frac{\pi}{6}\right) = 2$</td> <td>$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$</td> </tr> </table>	$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$	$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$	$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$	$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$	$\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$	$\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$	$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$	$\sec\left(\frac{\pi}{3}\right) = 2$	$\csc\left(\frac{\pi}{6}\right) = 2$	$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$	<table border="0"> <tr> <td>$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$</td> </tr> <tr> <td>$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$</td> </tr> <tr> <td>$\tan\left(\frac{\pi}{4}\right) = 1$</td> </tr> <tr> <td>$\cot\left(\frac{\pi}{4}\right) = 1$</td> </tr> <tr> <td>$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$</td> </tr> <tr> <td>$\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$</td> </tr> </table>	$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$	$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$	$\tan\left(\frac{\pi}{4}\right) = 1$	$\cot\left(\frac{\pi}{4}\right) = 1$	$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$	$\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$
$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$	$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$																		
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$\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$	$\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$																		
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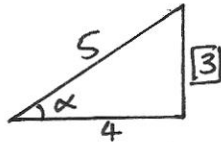
<p>35) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ Solution: Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\sin \theta = \frac{\sqrt{3}}{2}$ Use the 30° – 60° right triangle to find θ. Thus, $\theta = \frac{\pi}{3}$</p>	<p>36) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ Solution: Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\sin \theta = \frac{\sqrt{3}}{2}$ Use the 30° – 60° right triangle to find θ. Thus, $\theta = \frac{\pi}{3}$</p>
<p>37) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$ Solution: Let $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $\tan \theta = \frac{1}{\sqrt{3}}$ Use the 30° – 60° right triangle to find θ. Thus, $\theta = \frac{\pi}{6}$</p>	<p>38) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) =$ Solution: Let $\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $\sin \theta = \frac{1}{\sqrt{2}}$ Use the 45° – 45° right triangle to find θ. Thus, $\theta = \frac{\pi}{4}$</p>
<p>39) If $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$, then $\tan \alpha =$ Solution: $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$ $\cos \alpha = \frac{3}{\sqrt{13}} = \frac{\text{adj}}{\text{hyp}}$  Now, we should find the length of the opposite side using the Pythagorean Theorem, so $\text{opposite} = \sqrt{(\sqrt{13})^2 - 3^2} = \sqrt{13 - 9} = \sqrt{4} = 2$ $\therefore \tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{2}{3}$</p>	<p>40) If $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$, then $\csc \alpha =$ Solution: $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$ $\cos \alpha = \frac{3}{\sqrt{13}} = \frac{\text{adj}}{\text{hyp}}$ Now, we should find the length of the opposite side using the Pythagorean Theorem, so $\text{opposite} = \sqrt{(\sqrt{13})^2 - 3^2} = \sqrt{13 - 9} = \sqrt{4} = 2$ $\therefore \csc \alpha = \frac{1}{\sin \alpha} = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{13}}{2}$</p>

41) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\csc \alpha =$

Solution:

$$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\cos \alpha = \frac{4}{5} = \frac{\text{adj}}{\text{hyp}}$$



Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$$

$$\therefore \csc \alpha = \frac{1}{\sin \alpha} = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$

42) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\cot \alpha =$

Solution:

$$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\cos \alpha = \frac{4}{5} = \frac{\text{adj}}{\text{hyp}}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$$

$$\therefore \cot \alpha = \frac{1}{\tan \alpha} = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$$

43) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\tan \alpha =$

Solution:

$$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\cos \alpha = \frac{4}{5} = \frac{\text{adj}}{\text{hyp}}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$$

$$\therefore \tan \alpha = \frac{1}{\cot \alpha} = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

44) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\sin \alpha =$

Solution:

$$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\cos \alpha = \frac{4}{5} = \frac{\text{adj}}{\text{hyp}}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$$

$$\therefore \sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$$

45) $\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right) =$

Solution:

$$\text{Let } \alpha = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\cos \alpha = \frac{4}{5} = \frac{\text{adj}}{\text{hyp}}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$$

$$\therefore \sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \sin(\alpha) = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$$

46) $\tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right) =$

Solution:

$$\text{Let } \alpha = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\cos \alpha = \frac{4}{5} = \frac{\text{adj}}{\text{hyp}}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$$

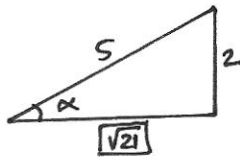
$$\therefore \tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \tan(\alpha) = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

47) $\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) =$

Solution:

$$\text{Let } \alpha = \sin^{-1}\left(\frac{2}{5}\right)$$

$$\sin \alpha = \frac{2}{5} = \frac{\text{opp}}{\text{hyp}}$$



Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$$|\text{adjacent}| = \sqrt{5^2 - 2^2} = \sqrt{25 - 4} = \sqrt{21}$$

$$\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin(2\alpha)$$

Now, use the identity $\sin(2x) = 2 \sin x \cdot \cos x$. Thus,

$$\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha)$$

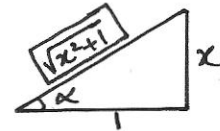
$$= 2 \times \frac{2}{5} \times \frac{\sqrt{21}}{5} = \frac{4\sqrt{21}}{25}$$

48) $\cos(\tan^{-1} x) =$

Solution:

$$\text{Let } \alpha = \tan^{-1} x$$

$$\tan \alpha = x = \frac{\text{opp}}{\text{adj}}$$



Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$$

$$\cos(\tan^{-1} x) = \cos(\alpha) = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{x^2 + 1}}$$

49) $\sin(\tan^{-1} x) =$

Solution:

$$\text{Let } \alpha = \tan^{-1} x$$

$$\tan \alpha = x = \frac{\text{opp}}{\text{adj}}$$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$$

$$\sin(\tan^{-1} x) = \sin(\alpha) = \frac{\text{opp}}{\text{hyp}} = \frac{x}{\sqrt{x^2 + 1}}$$

50) $\csc(\tan^{-1} x) =$

Solution:

$$\text{Let } \alpha = \tan^{-1} x$$

$$\tan \alpha = x = \frac{\text{opp}}{\text{adj}}$$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$$

$$\csc(\tan^{-1} x) = \csc(\alpha) = \frac{1}{\sin \alpha} = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{x^2 + 1}}{x}$$

$$51) \sec(\tan^{-1} x) =$$

Solution:

$$\text{Let } \alpha = \tan^{-1} x$$

$$\tan \alpha = x = \frac{\text{opp}}{\text{adj}}$$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$$

$$\sec(\tan^{-1} x) = \sec(\alpha) = \frac{1}{\cos \alpha} = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{x^2 + 1}}{1} = \sqrt{x^2 + 1}$$

$$52) \sec\left(\sin^{-1} \frac{x}{3}\right) =$$

Solution:

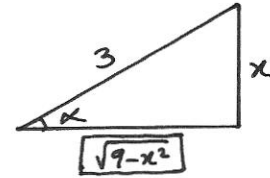
$$\text{Let } \alpha = \sin^{-1} \frac{x}{3}$$

$$\sin \alpha = \frac{x}{3} = \frac{\text{opp}}{\text{hyp}}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$$

$$\sec\left(\sin^{-1} \frac{x}{3}\right) = \sec(\alpha) = \frac{1}{\cos \alpha} = \frac{\text{hyp}}{\text{adj}} = \frac{3}{\sqrt{9 - x^2}}$$



$$53) \cot\left(\sin^{-1} \frac{x}{3}\right) =$$

Solution:

$$\text{Let } \alpha = \sin^{-1} \frac{x}{3}$$

$$\sin \alpha = \frac{x}{3} = \frac{\text{opp}}{\text{hyp}}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$$

$$\cot\left(\sin^{-1} \frac{x}{3}\right) = \cot(\alpha) = \frac{1}{\tan \alpha} = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{9 - x^2}}{x}$$

$$54) \tan\left(\sin^{-1} \frac{x}{3}\right) =$$

Solution:

$$\text{Let } \alpha = \sin^{-1} \frac{x}{3}$$

$$\sin \alpha = \frac{x}{3} = \frac{\text{opp}}{\text{hyp}}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$$

$$\tan\left(\sin^{-1} \frac{x}{3}\right) = \tan(\alpha) = \frac{1}{\cot \alpha} = \frac{\text{opp}}{\text{adj}} = \frac{x}{\sqrt{9 - x^2}}$$

$$55) \cos\left(\sin^{-1} \frac{x}{3}\right) =$$

Solution:

$$\text{Let } \alpha = \sin^{-1} \frac{x}{3}$$

$$\sin \alpha = \frac{x}{3} = \frac{\text{opp}}{\text{hyp}}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$$

$$\cos\left(\sin^{-1} \frac{x}{3}\right) = \cos(\alpha) = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{9 - x^2}}{3}$$