



KING SAUD UNIVERSITY  
PREPARATORY YEAR DEANSHIP  
BASIC SCIENCE DEPARTMENT



MATH 150

FINAL EXAM / WINTER 2014-2015

DATE: 04/01/2015

INSTRUCTOR: ..... SECTION: ..... ST. NAME: .....

TIME ALLOWED: 3 Hours ST. ID: .....

\* This exam consists of 9 essay questions pointed in two pages for a total of 50 marks.

QUESTION ONE: Find the limit if exists.

(A)  $\lim_{x \rightarrow 1} \frac{2x^2 + 5}{x^3 + 2}$

(B)  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 4x + 3}$

(C)  $\lim_{x \rightarrow 0} \frac{\tan(9x)}{\sin(6x)}$

(D)  $\lim_{x \rightarrow 3^-} \frac{x + 2}{x - 3}$

(E)  $\lim_{x \rightarrow 0^+} x^2 \sin\left(\frac{1}{x}\right)$  (Use the Squeeze Theorem).

(8 Marks: <sup>a</sup>1 + <sup>b</sup>2 + <sup>c</sup>1 + <sup>d</sup>2 + <sup>e</sup>2)

QUESTION TWO:

(A) Find the horizontal asymptotes for the function  $f(x) = \frac{x - 4x^2}{2x^2 - 1}$ .

(B) Find the value of the constant  $c$  such that the function

$$f(x) = \begin{cases} cx^2, & \text{if } x < 2 \\ 5cx - 2, & \text{if } x \geq 2 \end{cases}$$

is continuous on  $(-\infty, \infty)$ .

(5 Marks: 2 + 3)

QUESTION THREE: Find the first derivative  $\frac{dy}{dx}$  for the following functions:

①  $y = x^7 - 4x^{-3}$

②  $y = \sqrt{x^3 - x + 5}$

③  $y = \cot^2(7x)$

④  $y = e^{x - \cos x}$

⑤  $y = \log_2(x^3 + 3x - 4)$

⑥  $y^2 + xy = x^3 + 2$

(11 Marks: ①1 + ②1 + ③2 + ④2 + ⑤2 + ⑥3)

QUESTION FOUR:

(A) Let  $y = t^2 - t$  and  $t = x^2$ . Find  $\frac{dy}{dx}$ .

(B) Find the equation of the tangent line to the curve of  $f(x) = x^3 - 2x + 1$  at the point  $x = 1$ .

(5 Marks: 2 + 3)

QUESTION FIVE: Verify that the function  $f(x) = x^2 - 4x + 3$  satisfies the three hypotheses of Rolle's Theorem on the interval  $[1, 3]$ . Then find all possible values of  $c$  that satisfy the conclusion of the theorem.

(3 Marks)

QUESTION SIX: Evaluate the following limits using L'Hopital's rule:

②  $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x}$

②  $\lim_{x \rightarrow \infty} x \tan\left(\frac{1}{x}\right)$

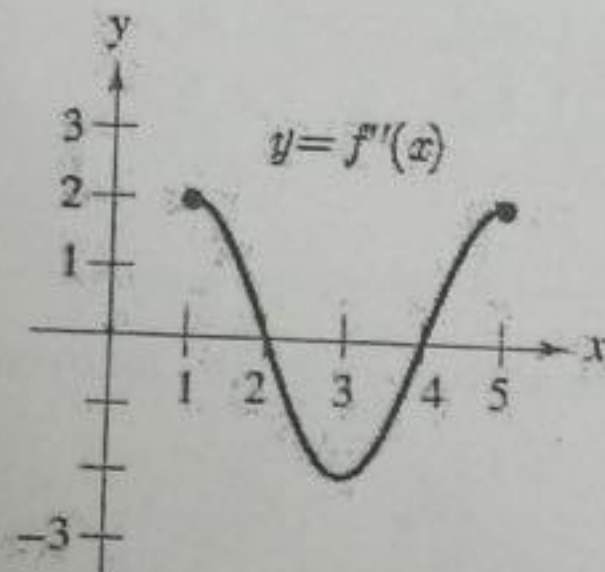
(5 Marks: 2 + 3)

QUESTION SEVEN: Given  $f(x) = x^3 - 12x$ . Find:

- ① The critical numbers of  $f$ .
- ② The intervals on which  $f$  is increasing.
- ③ The relative extrema of  $f$ .
- ④ The intervals on which  $f$  is concave down.
- ⑤ The  $x$ -coordinates of inflection points of  $f$ .

(7 Marks: 2 + 1 + 1 + 2 + 1)

QUESTION EIGHT: Use the graph of  $y = f''(x)$  below to find the following:



- ① The intervals on which  $f$  is concave up.
- ② The  $x$ -coordinates of inflection points of  $f$ .

(3 Marks: 1 + 2)

QUESTION NINE: Find two nonnegative real numbers whose sum is 12 with the property that the product of them is largest.

(3 Marks)

GOOD LUCK

Page 2 of 2