



هل اختبار الهيد الأول (رياض 101) الفصل الأول (1441)

Question 1:

A) Classify each of the following numbers into rational and irrational

$$\left\{ \sqrt{2}, \frac{7}{\sqrt{9}}, 2.\overline{76}, \frac{3\pi}{2} \right\}$$

$$\text{rational} \Rightarrow \left\{ \frac{7}{\sqrt{9}} = \frac{7}{3}, 2.\overline{76} \right\}$$

$$\text{Irrational} = \left\{ \sqrt{2}, \frac{3\pi}{2} \right\}$$

B) Solve the following inequalities, and write your answer in interval notation:

1) $3x + 5 \leq -10$

2) $0 < |2x - 3| < 6$

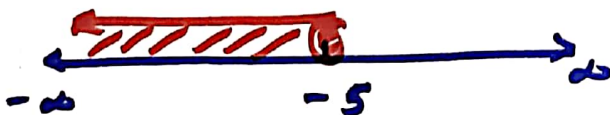
① $3x + 5 \leq -10$

$3x \leq -10 - 5$

$\frac{3x}{3} \leq \frac{-15}{3}$

$x \leq -5$

S.S = $(-\infty, -5]$



② $0 < |2x - 3| < 6$

$|2x - 3| > 0$ and $|2x - 3| < 6$

$2x - 3 = 0$

$\frac{2x}{2} = \frac{3}{2}$
 $x = \frac{3}{2}$

S.S = $R - \left\{ \frac{3}{2} \right\}$

$-6 < 2x - 3 < 6$
 $+3$ $+3$

$-\frac{3}{2} < \frac{2x}{2} < \frac{9}{2}$

$-\frac{3}{2} < x < \frac{9}{2}$

S.S = $\left(-\frac{3}{2}, \frac{9}{2} \right)$

∩

$\therefore \text{S.S} = R - \left\{ \frac{3}{2} \right\} \cap \left(-\frac{3}{2}, \frac{9}{2} \right)$

$= \left(-\frac{3}{2}, \frac{9}{2} \right) - \left\{ \frac{3}{2} \right\}$

or $\text{S.S} = \left(-\frac{3}{2}, \frac{3}{2} \right) \cup \left(\frac{3}{2}, \frac{9}{2} \right)$



Question 3:

Consider the function $f(x) = \frac{3x-2}{2x+5}$

- A) Show that $f(x)$ is a one-to-one function.
 B) Find $f^{-1}(x)$.
 C) Find the range of $f(x)$.

(A) $D_f \Rightarrow 2x+5 \neq 0 \Rightarrow x \neq -\frac{5}{2} \Rightarrow D_f = \mathbb{R} - \{-\frac{5}{2}\}$

Let $x_1, x_2 \in D_f$ such that

$$f(x_1) = f(x_2)$$

$$\frac{3x_1-2}{2x_1+5} = \frac{3x_2-2}{2x_2+5}$$

$$6x_1x_2 + 15x_1 - 4x_2 - 10 = 6x_1x_2 + 15x_2 - 4x_1 - 10$$

$$15x_1 + 4x_1 = 15x_2 + 4x_2$$

$$\frac{19x_1}{19} = \frac{19x_2}{19} \quad (\div 19)$$

$$\therefore x_1 = x_2 \Rightarrow f \text{ is (1-1) function}$$

(B) $y = \frac{3x-2}{2x+5}$

$$x = \frac{3y-2}{2y+5}$$

$$2xy + 5x = 3y - 2$$

$$2xy - 3y = -5x - 2$$

$$y(2x-3) = -5x-2$$

$$y = \frac{-5x-2}{2x-3}$$

$$f^{-1}(x) = \frac{-5x-2}{2x-3}$$

(C) Range of $f(x)$
 = Domain of $f^{-1}(x)$

to find $D_{f^{-1}}$

$$2x-3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$D_{f^{-1}} = \mathbb{R} - \{\frac{3}{2}\}$$

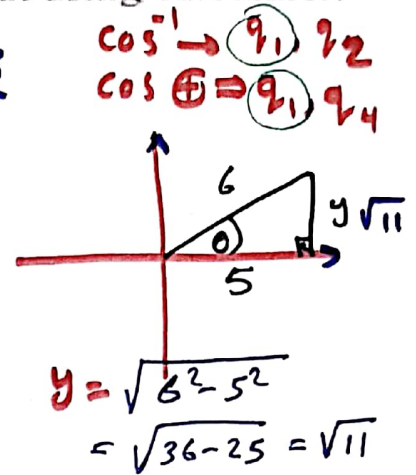
$$\therefore \text{Range of } f(x) = \mathbb{R} - \{\frac{3}{2}\}$$

Question 4:

A) Find the exact value of $\tan\left(\cos^{-1}\left(\frac{5}{6}\right)\right)$, without using calculator.

$$\text{Let } \cos^{-1}\left(\frac{5}{6}\right) = \theta \Rightarrow \cos \theta = \frac{5}{6}$$

$$\Rightarrow \tan\left(\cos^{-1}\left(\frac{5}{6}\right)\right) = \tan \theta = \frac{\sqrt{11}}{5}$$



B) Verify the identity:

$$\text{L.H.S} = 2 \sin x \cos x \cdot \frac{\sin x}{\cos x}$$

$$= 2 \sin^2 x = \frac{2}{\csc^2 x} = \text{R.H.S}$$

القوانين المتربة

$$\sin(2x) \tan x = \frac{2}{\csc^2 x}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin x = \frac{1}{\csc x}$$

C) Solve the following equation for x in $[0, 2\pi]$.

$$2 \cos^2 x - 1 = \cos x$$

$$2 \cos^2 x - \cos x - 1 = 0$$

$$(2 \cos x + 1)(\cos x - 1) = 0$$

$$2 \cos x + 1 = 0$$

$$2 \cos x = -1$$

$$\cos x = -\frac{1}{2}$$

$$\cos x - 1 = 0$$

$$\cos x = 1 \Rightarrow x = 0, 360$$

$$x = 0, 2\pi$$

$$x = 180 - 60 = 120$$

$$x = 120 = \frac{2\pi}{3}$$

$$x = 180 + 60$$

$$x = 240 = \frac{4\pi}{3}$$

$$\text{S.S} = \left\{ 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi \right\}$$

مع اطيب تمنياتي للجميع بالتوفيق والنجاح

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