3.1 - Derivatives of Polynomials and Exponential Functions

1) Derivative of a Constant function

$$\frac{d}{dx}[c] = 0 \quad \text{for all } c \in \mathbb{R}$$

Example:-

$\frac{d}{dx}[\pi^2] = 0$ $\quad$ $\frac{d}{dx}[5^c] = 0$ $\quad$ $\frac{d}{dy}[18.5] = 0$

$\frac{d}{dx}[\sqrt{30}] = 0$ $\quad$ $\frac{d}{dx}[\ln(9)] = 0$

$\frac{d}{dx}[\sin(\frac{\pi}{2})] = 0$ $\quad$ $\frac{d}{dx}[\cos^2(5)] = 0$

if $f(x) = \sqrt{4+c^2}$ then $f'(x) = 0$

2) if $\underline{f(x) = ax}$ for all $a \in \mathbb{R}$ then $\underline{f'(x) = a}$

Example:

$\frac{d}{dx}[10x] = 10$

if $f(x) = \frac{-3}{4}x$ then $f'(x) = \frac{-3}{4}$

if $f(x) = -x$ then $f'(x) = -1$

$\frac{d}{dt}[2t] = 2$

if $f(\emptyset) = 18.5\emptyset$ then $f'(\emptyset) = 18.5$

3) if $f(x) = x^n$ then $f'(x) = n x^{n-1}$

Example:

$\frac{d}{dx}[x^2] = 2x \quad \frac{d}{dx}[x^3] = 3x^2 \quad \frac{d}{dx}[x^4] = 4x^3$

$$\frac{d}{dx}\left[\frac{1}{x^5}\right] = \frac{d}{dx}[x^{-5}]$$

$$= -5x^{-5-1}$$

$$= -5x^{-6}$$

$$= \frac{-5}{x^6}$$

$$\frac{d}{dx}\left[\sqrt[3]{x^2}\right] = \frac{d}{dx}\left[(x^2)^{1/3}\right]$$

$$= \frac{d}{dx}\left[x^{2/3}\right]$$

$$= \frac{2}{3}x^{2/3-1}$$

$$= \frac{2}{3}x^{-\frac{1}{3}}$$

$$= \frac{2}{3x^{1/3}} = \frac{2}{3\sqrt[3]{x}}$$

$$\frac{d}{dx}\left[x^2\sqrt{x}\right] = \frac{d}{dx}\left[x^2 . x^{1/2}\right]$$

$$= \frac{d}{dx}\left[x^{2+\frac{1}{2}}\right]$$

$$= \frac{d}{dx}\left[x^{5/2}\right]$$

$$= \frac{5}{2}x^{5/2-1}$$

$$= \frac{5}{2}x^{3/2}$$

$$= \frac{5}{2}\sqrt{x^3}$$

4] $\frac{d}{dx}[cf(x)] = c.\frac{d}{dx}[f(x)]$

$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

Example:

a) $\frac{d}{dx}[x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + \sqrt{\frac{2}{5}}]$

$8x^7 + 12(5)x^4 - 4(4)x^3 + 10(3)x^2 - 6 + 0$

$8x^7 + 60x^4 - 16x^3 + 30x^2 - 6$

$8x^7 + 60x^4 - 16x^3 + 30x^2 - 6$

b) If $f(x) = (3x-2)^2$ then $f'(x) = \text{----}$

$f(x) = 9x^2 - 2(3x)(2) + 4$

$= 9x^2 - 12x + 4$

$f'(x) = 18x - 12$

c) $\frac{d}{dx}[x^2(1-2x)] = \frac{d}{dx}[x^2 - 2x^3]$

$= 2x - 6x^2$

d) $\frac{d}{dt}[\sqrt{t}(t-1)] = \frac{d}{dt}[t^{1/2}(t-1)]$

$= \frac{d}{dt}[t^{1/2}.t^1 - t^{1/2}] = \frac{d}{dt}[t^{3/2} - t^{1/2}]$

$= \frac{3}{2}t^{3/2-1} - \frac{1}{2}t^{1/2-1}$

$= \frac{3}{2}t^{1/2} - \frac{1}{2}t^{-1/2} = \frac{3}{2}\sqrt{t} - \frac{1}{2t^{1/2}}$

$= \frac{3}{2}\sqrt{t} - \frac{1}{2\sqrt{t}} = \frac{3\sqrt{t}.\sqrt{t} - 1}{2\sqrt{t}}$

$= \frac{3t-1}{2\sqrt{t}}$

e) $\frac{d}{dx}\left[(2x+3)(4x-5)\right]$

$\frac{d}{dx}\left[2x(4x-5)+3(4x-5)\right]$

$\frac{d}{dx}\left[8x^2-10x+12x-15\right]$

$\frac{d}{dx}\left[8x^2+2x-15\right] = 16x+2$

f) $\frac{d}{dx}\left[(x-2)^3\right] = \frac{d}{dx}\left[x^3-3(2)x^2+3(4)x-2^3\right]$

$= \frac{d}{dx}\left[x^3-6x^2+12x-8\right]$

$= 3x^2 - \cancel{2}(6)x + 12$

$= 3x^2-12x+12$

g) $\frac{d}{dx}\left[x(2x+3)^2\right] = \frac{d}{dx}\left[x(4x^2+12x+9)\right]$

$= \frac{d}{dx}\left[4x^3+12x^2+9x\right]$

$= 12x^2+24x+9$

h) $f(t) = (3x^2+2)(x^3-5)$

$f'(t) =$ H.W

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if $G(x) = \frac{5x^2+4x+3}{x^2}$ then $G'(x) = \text{——}$

$$G(x) = \frac{5x^2}{x^2} + \frac{4x}{x^2} + \frac{3}{x^2}$$

$$= 5 + \frac{4}{x} + \frac{3}{x^2}$$

$$= 5 + 4x^{-1} + 3x^{-2}$$

$$G'(x) = 0 + 4(-1)x^{-1-1} + 3(-2)x^{-2-1}$$

$$= -4x^{-2} - 6x^{-3}$$

$$= \frac{-4}{x^2} - \frac{6}{x^3}$$

$$= \frac{-4x}{x^2 \cdot x} - \frac{6}{x^3}$$

$$= \frac{-4x}{x^3} - \frac{6}{x^3}$$

$$= \frac{-4x-6}{x^3}$$

if $y = \frac{\sqrt{x}+x}{x^2}$ then $y' = \text{---}$

$$y = \frac{x^{1/2}+x^1}{x^2} = \frac{x^{1/2}}{x^2} + \frac{x^1}{x^2} = x^{1/2-2} + x^{1-2}$$

$$= x^{-3/2} + x^{-1}$$

$$y' = \frac{-3}{2}x^{-\frac{3}{2}-1} - x^{-1-1} = \frac{-3}{2}x^{-5/2} - x^{-2} = \frac{-3}{2x^{5/2}} - \frac{1}{x^2} = \frac{-3}{2\sqrt{x^5}} - \frac{1}{x^2}$$

5] $\frac{d}{dx}[a^x] = a^x . \ln a$

$\frac{d}{dx}[e^x] = e^x$

Example

$\frac{d}{dx}[\pi^x] = \pi^x . \ln\pi = \ln(\pi) . (\pi)^x$

$\frac{d}{dx}[\sqrt{2^x}] = \frac{d}{dx}[(\sqrt{2})^x]$

$= (\sqrt{2})^x . \ln\sqrt{2}$

$= (\sqrt{2})^x . \ln 2^{1/2}$

$= (\sqrt{2})^x . \frac{1}{2}\ln 2$

$= \frac{1}{2}\ln 2 . (\sqrt{2})^x$

$\frac{d}{dx}[3^x + x^3] = \frac{d}{dx}[3^x] + \frac{d}{dx}[x^3]$

$= 3^x . \ln(3) + 3x^2$

$\frac{d}{dx}[e^x - x^e] = \frac{d}{dx}[e^x] - \frac{d}{dx}[x^e]$

$= e^x - e x^{e-1}$

$= e(e^{x-1} - x^{e-1})$

if $y = e^{x+1} + x^2$ then find ① y''' or $\frac{d^3y}{dx^3}$

② $y^{(100)}$

$y' = e^{x+1} + 2x$

$y'' = e^{x+1} + 2$

$y''' = e^{x+1}$

$y^{(4)} = e^{x+1}$

$y^{(5)} = e^{x+1}$

$\vdots$

$y^{(100)} = e^{x+1}$