GRE MATH REVIEW #5

Exponents and Radicals

Many numbers can be expressed as the product of a number multiplied by itself a number of times. For example, 16 can be expressed as $2 \cdot 2 \cdot 2 \cdot 2$. Another way to write this is 2^4. The 4 is called the **exponent** and the 2 is called the **base**. The expression 2^4 is read "2 to the fourth power".

Here is **a list of some definitions** you need to know in order to follow this discussion of algebra. However, you will not need to know the definitions on the GRE.

1. **Variable:** A letter that represents an unknown number.

2. **Term:** A product of any combination of variables and numbers. For example, 2x, $3xy^2$, $10y^3$, etc. are all terms.

3. **Coefficient:** A number or variable in a term. For example, in the term $2xy^2$, 2 is a coefficient of xy^2, x is a coefficient of $2y^2$, y^2 is a coefficient of 2x, etc.

4. **Expression:** Any number of terms combined by addition or subtraction signs. For example, $3xyz + 6x^2y – 10z$ is an algebraic expression.

5. **Equation:** Two expressions or terms set equal to each other. Do not get an expression and an equation confused. An equation has an equal sign in it; and an expression does not.

An easy way to remember the definitions of term, equation, and expression is to think of an equation as the algebraic equivalent of a sentence and an expression as the algebraic equivalent of a phrase. Then a term is just one word in the sentence or phrase.

When **multiplying terms with the same base**, just add the exponents. For example, $x^5 \bullet x^3 = x^8$. However, $x^5 \bullet y^3 \neq (xy)^8$ because x^5 and y^3 do not have the same base. Do not make the mistake of adding the exponents when multiplying numbers with different bases. Another common error is adding the exponents when adding two terms with like bases. This rule does not apply to addition. For example, $2^4 + 2^2 \neq 2^6$.

When **dividing terms with the same base,** just subtract the exponents. For example, $3^5 \div 3^3 = 3^2$. Again, this rule does not apply to division of terms with different bases or to subtraction of terms with like bases. For example, $x^5 \div y^3 \neq (x/y)^2$and $2^4 – 2^2 \neq 2^2$.

When **adding or subtracting** two or more terms with exponents the terms must have like bases and like exponents. If the bases and exponents are just alike, simply add the numerical coefficients. Remember that if there is no numerical coefficient, it is understood to be 1. For example, $x^2 + 5x^2 – 2x^2 = 4x^2$.

When **raising an exponent to another exponent,** simply multiply the exponents. For example, $(x^5)^2 = x^{10}$. If there is a term with several coefficients in the parentheses, you must distribute the exponent to every coefficient. For example, $(2xy^2)^3 = 8x^3y^6$ and $(3/2)^2 = 9/4$. This rule does not apply if there is an addition or subtraction sign inside the parentheses. For example, $(2x + 3y)^2 \neq 4x^2 + 9y^2$, this is a very common mistake, so be careful not to make it!

The following is **a list of characteristics of exponents** that you should commit to memory for the GRE:

1. Any number raised to 0 is always 1. For example, $5^0 = 1$, $x^0 = 1$, etc.

2. Any number without an exponent is understood to have an exponent of 1. For example, $x = x^1$, $2 = 2^1$, etc.

3. Raising a number greater than 1 to a power greater than 1 results in a bigger number. For example, $2^2 = 4$.

4. Raising a fraction between 0 and 1to a power greater than 1 results in a smaller fraction. For example, $(1/2)^2 = 1/4$. Recall from our discussion on fractions that we said multiplying a fraction by another fraction results in a smaller fraction. Raising a number to a power is equivalent to multiplying that number by itself.

5. A negative number raised to an even power results in a positive number. For example, $(-3)^2 = 9$.

6. A negative number raised to an odd power results in a negative number. For example, $(-2)^3 = -8$.

Although **negative exponents,** such as 2^{-3}, are a very important concept in your algebra class, you will NOT see them on the GRE, so don’t worry about them.

You should have a feel for **the relative size of exponents.** Remember that they are just shorthand notation for multiplication. So, 2^5 is twice as large as 2^4. And 2^{10} is more than 10 times as large as 10^2 (Why?).

The **radical** sign $\sqrt{\ }$ indicates the square root of the number under the radical. Similarly, the sign $\sqrt[3]{\ }$ indicates the cube root of the number under the radical. If $x^2 = 16$, then x = $\pm$ 4 and $\pm$ 4 are called the positive and negative square roots of the number 16. However, $\sqrt{16}$ = +4. In other words, the radical sign only refers to the positive root of the number under the radical. Hence, if the GRE asks for $\sqrt{25}$, the answer is +5, not –5.

Unlike for an algebra class, there are only **two radical rules you need to know for the GRE:**

1. $\sqrt{x}\,\sqrt{y} = \sqrt{xy}$

   For example, $\sqrt{2}\,\sqrt{3} = \sqrt{6}$ and $\sqrt{32} = \sqrt{16}\,\sqrt{2} = 4\sqrt{2}$

2. $\sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}}$

   For example, $\sqrt{\dfrac{5}{16}} = \dfrac{\sqrt{5}}{\sqrt{16}} = \dfrac{\sqrt{5}}{4}$ and $\dfrac{\sqrt{32}}{\sqrt{2}} = \sqrt{\dfrac{32}{2}} = \sqrt{16} = 4$

The following values should be committed to memory for the GRE in order to be able to work radical problems more quickly:

$\sqrt{1}$	=	1
$\sqrt{2}$	=	1.4
$\sqrt{3}$	=	1.7
$\sqrt{4}$	=	2
$\sqrt{5}$	=	2.2

Algebra

You are required to know very little real algebra for the GRE. Algebra methods learned in your algebra classes will often mislead you and will usually take up way too much time. For instance, never try to set up an algebraic and work through to an answer. The GRE only cares which space you blacken on the answer sheet, not how you work an algebra problem. In this section, we will discuss the small subset of algebra rules that you actually need to know to do well on the GRE.

There are a few more definitions that you will need to know in order to follow this section. However, again, you do not need to know these for the GRE.

1. **Binomial:** An algebraic expression containing 2 terms.

2. **Trinomial:** An algebraic expression containing 3 terms.

3. **Polynomial:** A binomial, trinomial, or any other algebraic expression containing two or more terms.

The following are some **helpful hints** to remember for the GRE. See Review #1 if you need to review factoring and "unfactoring" (i.e., the distributive laws).

1. When you encounter a problem containing an expression that can be factored, you should always factor that expression. For example, if you see an expression such as $4x + 4y$, you should immediately factor it into $4(x + y)$.

2. Similarly, whenever you see an expression that has been factored, you should immediately "unfactor" it, i.e. "multiply it out". For example, if a problem contains the expression $4(x + y)$, multiply it out, to get $4x + 4y$.

When **multiplying polynomials,** remember the distributive law and multiply every term in the first polynomial by every term in the second polynomial (FOIL). For example,

$$\begin{aligned}(x + 4)(2x - 1) &= x(2x) + x(-1) + 4(2x) + 4(-1)\\ &= 2x^2 + (-x) + 8x + (-4)\\ &= 2x^2 + 7x - 4\end{aligned}$$

There are **three expressions that you need to commit to memory** for the GRE in both their **factored and unfactored forms.** They are:

1. $x^2 - y^2 = (x + y)(x - y)$
   $x^2 - y^2$ is the unfactored form; $(x + y)(x - y)$ is the factored form.

2. $x^2 + 2xy + y^2 = (x + y)(x + y) = (x + y)^2$
   $x^2 + 2xy + y^2$ is the unfactored form; $(x + y)(x + y)$ and $(x + y)^2$ are equivalent factored forms.
3. $x^2 - 2xy + y^2 = (x - y)(x - y) = (x - y)^2$
   $x^2 - 2xy + y^2$ is the unfactored form; $(x - y)(x - y)$ and $(x - y)^2$ are equivalent factored forms.

Here is an example of a problem in which you need to factor and know one of these expressions. **Example:** Simplify the following expression:

$$\underline{4x^2 - 4}$$

$x - 1$

Your should immediately recognize that you need to factor the numerator which results in the expression $4(x^2 - 1)$ for the numerator. Next, you should recognize that $x^2 - 1$ is of the form $x^2 - y^2$ and therefore can be written in its factored form $(x - 1)(x + 1)$. Hence, the original fraction can be written as

$$\frac{4(x + 1)(x - 1)}{(x - 1)}$$

Now you should recognize that the fraction can be reduced by canceling out the common factor $(x - 1)$ in the numerator and the denominator. The final simplified, factored form is $4(x + 1)$.

Whenever you see a complicated-looking algebraic expression, simplify it if possible by **combining similar terms,** i.e. terms with like bases and like exponents. For example,

$$\begin{aligned} & (4x^2 + 4x + 2) + (3 - 7x) - (5 - 3x) \\ &= 4x^2 + 4x + 2 + 3 - 7x - 5 + 3x \\ &= 4x^2 + (4x - 7x + 3x) + (2 + 3 - 5) \\ &= 4x^2 \end{aligned}$$

When given two **simultaneous equations** on the GRE and asked to solve them, don't solve them using the techniques you learned in algebra class. Instead, look for shortcuts which usually involve adding or subtracting the two equations. And you'll never have to worry about solving a system with more than two equations.

Let's see an **example.** If $5x + 4y = 6$ and $4x + 3y = 5$, then $x + y = ?$ Instead of using substitution or elimination like you learned in algebra, just add or subtract the two equations. First, let's add them and see what we get. Remember, we are looking for $x + y$, so we really don't need to know what x and y are individually.

$$\begin{array}{r} 5x + 4y = 6 \\ \underline{+\ 4x + 3y = 5} \\ 9x + 7y = 11 \end{array}$$

Obviously, that didn't help us much. So let's subtract them.

$$\begin{array}{r} 5x + 4y = 6 \\ \underline{-(4x + 3y = 5)} \\ x + y = 1 \end{array}$$

Since we're looking for the value of the expression $x + y$, the answer would be 1. We never even had to find out the values of x and y. Whenever you encounter simultaneous equations, try adding or subtracting them, factoring something out, or multiplying by something. You will never need to use the methods you learned in your algebra classes to solve simultaneous equations on the GRE.

The following is an example of the type of problem on the GRE that involves factoring and the distributive law. **Example:** If y + 3 = 2x, then 3y – 6x =

(a) –9 (b) –3 (c) 0 (d) 3 (e) 9

Instead of solving for y and plugging –3 + 2x in for y in the second expression, just factor out a three in the second expression: 3(y – 2x). This is just the distributive law working in reverse. Notice that the first equation can be written y – 2x = -3 (why?). By substituting –3 in for y – 2x, we get 3(-3) = -9, so the answer is (a). Always be on the lookout for chances to factor and use the distributive laws.

The GRE loves **equations set equal to zero** because of the unique properties of 0. One of the most important properties of 0, which was mentioned in Review #1, is the fact that the product of anything and 0 is 0. Hence, if a product is equal to 0, one of the factors in that product must be 0. In other words, if ab = 0, then either a or b must be 0 or both are 0. This fact can be used to solve some equations on the GRE.

Here's an **example.** What are all the values of y for which y(y + 5) = 0? In order for the product y(y +5) to equal 0, either y must be 0 or y + 5 must be 0 or both of them must be 0. In order for y +5 to be 0, y would have to be –5. Hence, the values of y for which y(y + 5) is 0 are y = 0 and y = -5.

In an equation, one expression is equal to another. In an **inequality**, one expression is not equal to another expression. (See Review #1, page 3, for a list of symbols and their meanings.) However, inequalities are solved just like equations. You can factor, unfactor, simplify, multiply/divide both sides by a constant, add/subtract terms from both sides, etc. The one primary difference is that **if you multiply or divide both sides of an inequality by a negative number, you must reverse the direction of the inequality symbol.** It's easy to see why. For instance, we know that 2 < 4. Multiplying both sides by –2 results in –4 on the left side and –8 on the right side. Clearly, -4 is greater than –8, so the inequality symbol must be reversed: -4 > -8.

From your algebra classes, you probably remember hearing about **functions** even if didn't you understand them. Usually, the symbol f(x), read "f of x", was used to represent a function. The GRE contains function problems, but instead of using the f(x), they are disguised by funny-looking symbols such as #, *, @, etc. If you remember how functions work, just think of functions when you work these problems and perform the operations. However, you can still work these problems even if you don't remember functions. Think of a funny-looking symbol as representing a set of operations or instructions. Here's an **example:** If x @ y = (x – y)/2, what is the value of 3 @ 5? To find the answer, just substitute 3 in for x and 5 in for y. Since x @ y = (x – y)/2, then 3 @ 5 = (3 – 5)/2 = -1.

The GRE also contains your favorite type of problems: **word problems.** You have to learn how to translate them into mathematical equations. The following is list of words found in word problems and their mathematical translations. These are the same translations that are used in percentage problems (see Review #4).

Word	**Symbol**
is	=
of, times, product	×
what (or the unknown value)	any variable
more, sum	+
less, difference	-
ratio, quotient	÷

The following **formulas** frequently appear in GRE word problems, so you should commit them to memory:

1. distance = (rate)(time)
   This formula can also be expressed as rate = distance/time or time = distance/rate.

2. total price = (number of items)(cost per item)

3. sale price = (original price) – (% discount)(original price)

Exercise 5

1. $3^4 \bullet 3^2 = ?$

2. If x = 3, what is $(2x)^3$?

3. If x = 4, what is $(x^2)^3$?

4. If $4^2 + 3^2 = x^2$, what is x?

5. Approximate $3\sqrt{3}$.

6. Simplify: (a) $\sqrt{56}$ (b) $\sqrt{14/98}$

7. $(3x + 4)(8x - 3) = ?$

8. Factor or unfactor the following expressions:

   (a) $4x^2 - 9y^2$
   (b) $(2x + 3y)^2$
   (c) $16x^2 - 24xy + 9y^2$
   (d) $(4x + y)(4x - y)$
   (e) $x^2 - 10x + 9$

9. Simplify the following expression by combining like terms:

   $(6x^2 + 7x - 7) - 2x(3x^2y + 3x + 2) + 9$

10. If $x - y = 7$ and $-x + 2y = 3$, then y = ?

11. If 4ab = 0 and a > 1, then b = ?

12. If $x + 3 < 2x + 4$, then x > ?

13. If x & y = $x^2 + 3y$, what is the value of x^2 & 3y?

EXERCISE 5 SOLUTIONS

1. $3^6 = 729$

2. $6^3 = 216$

3. $16^3 = 4096$

4. +5 or –5

5. Approximately 5

6. (a) $2\sqrt{14}$ (b) $\sqrt{7}/7$

7. $24x^2 + 23x - 12$

8. (a) $(2x + 3y)(2x - 3y)$
   (b) $4x^2 + 12xy + 9y^2$
   (c) $(4x - 3y)(4x - 3y)$
   (d) $16x^2 - y^2$
   (e) $(x - 9)(x - 1)$

9. $6x^2 + 7x - 7 - 6x^3y - 6x^2 - 4x + 9 = -6x^3y + 3x + 2$

10. 10

11. 0

12. –1

13. $x^4 + 9y$

Compiled by Robyn Wright, 1992
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Reference:
Robinson, Adam, and John Katzman. The Princeton Review – Cracking the System: The GRE 1992 Edition. New York: Villard, 1991.105 – 201.