GRE MATH REVIEW #4

Percentages

A **percent** is just a shorthand way of expressing a fraction whose denominator is 100. Percent means "per 100", "out of 100", or "divided by 100". For example, 25% = 25/100 = 0.25 and 0.3% = 0.3/100 = 0.003. In terms of money, 50 cents out of a dollar is 50 cents out of 100, which is 50/100 of a dollar or 50% of a dollar.

To find a percentage of something, the percents must be converted to decimals and then multiplied by some number. Never directly add and/or subtract percents; you must first multiply them by something. When finding percentages, convert the sentence into a mathematical equation. For example, "5 is what percent of 100" can be converted into "5 = x(100)" where x is the percent you are looking for.

When working with percents, it is usually necessary to **convert percents to decimals** before performing computations with them. Since a percent is just a fraction with denominator 100, you convert a percent to a decimal by moving the decimal point two places from right to left. For example, 6% is equivalent to (.06). In the following example, it is necessary to convert the percent to a decimal.

Example 1: What is 30 percent of 200?

To find 30% of 200, convert 30% to .30. Then multiply 200 by .30, which results in 60. Hence, 60 is 30% of 200.

To **convert a decimal to a percent**, move the decimal point two places from left to right and add a % sign. For example, 0.8 = 80% and 0.02 = 2%.

To **convert a percent to a fraction**, just make the percent the numerator of a fraction with denominator 100 and reduce the fraction. For example, 40% = 40/100 = 4/10 = 2/5.

To **convert a fraction to a percent,** divide the numerator by the denominator and move the decimal point two places to the right.

Example 2: Express 4/5 as a percent.

To do so, use long division and move the decimal point. 5 divided into 4 is 0.8 which is equivalent to 80%. Hence, 4/5 = 0.8 = 80%.

The following is a **list of percents and their fraction and decimal equivalents** which should be committed to memory:

0.01=	1/100=	1%
0.1=	1/10=	10%
0.2=	1/5=	20%
0.25=	1/4=	25%

0.5=	1/2=	50%
0.75=	3/4=	75%

Ratios

Ratios, like fractions, decimals, and percents, are just another way of expressing division. Every fraction is a ratio and every ratio is a fraction. A fraction is just the ratio of the numerator to the denominator. The ratio 1: 2 (read "1 to 2") is equivalent to the fraction 1/2 or the decimal 0.5 or 50% or just 1 divided by 2. On the GRE, ratios may be expressed in any of the following ways:

(1) x/y
(2) the ratio of x to y
(3) x is to y
(4) x : y

Anytime you see a ratio, **treat it just like a fraction**. Anything you can do to a fraction, you can also do to a ratio, including cross-multiplying, reducing, finding common denominators, etc.

Here are some examples of how to work ratio problems on the GRE.

Example 3: If you have 3 coins and the ratio of pennies to nickels is 2 : 1, how many pennies and how many nickels are there?

Clearly, there are 2 pennies and 1 nickel since you only have 3 coins altogether and there are 3 parts in the ratio (2 and 1 = 3).

Example 4: If you have 24 coins and the ratio of pennies to nickels is 2 : 1, how many of each type of coin do you have?

Since the ratio again contains 3 parts (2 and 1), divide 24 by 3 to get the number of coins in each part; then multiply each part of the ratio by the result . Dividing 24 by 3 yields 8 which means each of the 3 parts in the ratio consists of 8 coins. Two of the parts are pennies at 8 coins per part, so there are 16 pennies. One part is nickels which makes 8 nickels.

Example 5: At a camp for boys and girls, the ratio of girls to boys is 5 : 3. If the camp's enrollment is 160, how many of the children are boys?

(a) 20 (b) 36 (c) 45 (d) 60 (e) 100

A ratio of 5 : 3 means 8 total parts. To find out how many children are in each part, divide the total enrollment by the total number of parts. (160 divided by 8 is 20); that means each part is 20 children. Three parts are boys and 3 multiplied by 20 is 60 which is the answer (d). Note that there is no need to find out how many girls there are. Also, be careful and use the correct part of the ratio that answers the question.

Proportions

The GRE often contains questions in which you must compare two ratios which are proportional. These questions take a given ratio, or relationship, and project it onto a larger or smaller scale while leaving out one piece of information.

Example 6: If 10 baskets contain a total of 50 eggs, how many eggs would 7 baskets contain?

(a) 10 (b) 17 (c) 35 (d) 40 (e) 50

To solve a problem like this, set up the two proportional ratios, one of which will have a missing piece of information. Think of the ratios like this: 10 baskets is to 50 eggs as 7 baskets is to *x* eggs. Then set up these ratios as proportional fractions and cross-multiply:

$$\frac{10}{50} \quad \frac{7}{\times}$$

$10x = 350$

$x = 35$

OR

$10 : 50$

$7 : x$

$10x = 350$

$x = 35$

Hence, the answer is (c). Note that we could have reduced the fraction 10/50 to 1/5 and made the cross-multiplication easier.

Averages

The average, or arithmetic mean, of a set of numbers is the sum of all the numbers in the set divided by the total number of numbers in the set. The formula to remember is

$$\text{average} = \frac{\text{the sum of the numbers being averaged}}{\text{the number of elements}}$$

For example, the average of the numbers 1, 2, 3, 4, 5, 6, 7 is 28 divided by 7 which is 4. The GRE always refers to an average as an "average (arithmetic mean)". Just ignore the parenthetical remark so it doesn't confuse you.

In an averaging problem, you may be asked to **find the total first**. For example, suppose a problem states that the average of 4 test scores is 80 find the sum of the tests. Recall the formula above. In this case we already know the average and the number of elements; we need to find the sum of the tests. Hence, just cross-multiply and the total of the test scores is the product of the two: $80 \times 4 = 320$.

Suppose you are told that two of these scores are 90 and 95 and you want to find the average of the other two scores. The sum of 90 and 95 is 185. So, the total of the other two scores is 320 – 185 = 135. Hence, the average of the remaining two scores is 135 divided by 2 which is (67.5).

Do not get confused **if two or more of the elements being averaged are the same.** For example, the average of 5, 5, 5, and 20 is 5 + 5 + 5 + 20 divided by 4 which is 35 divided by 4 which is (8.75). You do not add 5 and 20 and divide by 2, nor do you add 5 and 20 and divide by 4.

Another situation which may confuse you is **when a new element is added to a set that has already been averaged.** Suppose you take two tests and earn scores of 70 and 80. The average of your two tests is 75. Now, suppose you take a third test and earn another score of 70. Does your average remain 75? No. Your new average is (70 + 80 + 70)/3 = 73.33. Note that your new average is NOT (75 + 70)/2 = 72.5. Suppose your third test score was 75. Then your average over all three tests is still 75 (Why?).

Another common error occurs **when averaging a set of numbers that includes 0.** For example, what is the average of 0, 0, 0, and 4? A careless person would say 4, but the answer is 1! An easy averaging problem can be made difficult on the GRE if certain information is left out.

Example 7: The average test score earned by a group of students is 80. If 40% of the students have an average score of 70, what is the average score of the remaining 60%?

(a) 70.33 (b) 80 (c) 86.67 (d) 90 (e) 95

An important piece of information is missing from this question. Do not try to solve this problem by setting up a complex algebraic equation. Since the problem is dealing with percentages, the actual number in the group is irrelevant. Just pick a number that's easy to work with. Since all of our percents are multiples of 10, choose 10 as the number of students in the group. 40%, or 4, of these students have an average score of 70. We want to know the average score of the remaining 6 (60% of 10).

Now that we've gotten rid of the percents, it is just an averaging problem. So first find the totals. If the average score of 10 students is 80, then the total sum of their test scores is 800. Just remember the averaging formula and cross-multiply the average and the number of elements. Since 4 of the students have an average score of 70, their total score is 280. Again, just cross-multiply. To find the total score of the remaining 6 students, just subtract: 800 – 280 = 520. Hence, the average score must be 520 divided by 6 which is 86.67 or choice (c).

EXERCISE 4

1. What is 40% of 350?
2. 2 is what percent of 16?
3. 2.2 is 20% of what number?
4. 10% of 24 equals 20% of what number?
5. 20% of 25% of x is 10. What is x?
6. The following chart illustrates fraction, decimal, and percent equivalents. Fill in the blanks.

Fraction	Decimal	Percent	Ratio
1/2	0.5	50%	1 : 2
1/3			
			2 : 3
		25%	
	0.75		
1/5			
		40%	
	0.6		
4/5			
			1 : 6
		12.5%	

7. What is the average of the numbers 24, 24, 26, 28, and 40?
8. If the average of 5 numbers is 20, what is their total?
9. If the average of 5 numbers is 20, what is the largest that any of the numbers could be?
10. If the average of 11, 17, 15, 28, and x is 19.6, what is the value of x?
11. Jim's average score on 4 math tests was 80 out of a possible 100. If his scores on 2 of the tests were 65 and 70, what is the lowest that either of his other scores could have been?

EXERCISE 4 SOLUTIONS

1. 140

2. 12.5%

3. 11

4. 12

5. 200

7. 28.4

8. 100

9. 100

10. 27

11. 85

6.

Fraction	Decimal	Percent	Ratio
1/2	0.5	50%	1 : 2
1/3	0.3333	33.33%	1 : 3
2/3	0.6666	66.66%	2 : 3
1/4	0.25	25%	1 : 4
3/4	0.75	75%	3 : 4
1/5	0.2	20%	1 : 5
2/5	0.4	40%	2 : 5
3/5	0.6	60%	3 : 5
4/5	0.8	80%	4 : 5
1/6	0.1666	16.67%	1 : 6
1/8	0.125	12.5%	1 : 8

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Reference:
Robinson, Adam and John Katzman. The Princeton Review – Cracking the System: The GRE 1992 Edition. New York: Villard, 1991. 105 – 201.