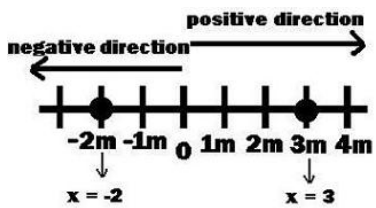
# CHAPTER(2)

## *Motion along a Straight Line*

## الحركة في خط مستقيم( على مستوى واحد)

**Position:**

هو موضع الجسم بالنسبة لنقطة الأصل ويعبر عنه بدلالة المحور $x$ والتي يمكن أن تكون إما موجبه أو سالبه.



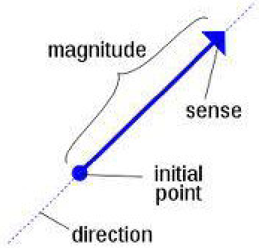

أهم الكميات الفيزيائية التي تصف الحركة هي:

1- الأزاحة displacement (كميه متجه لها مقدار واتجاه)

2- السرعة velocity (كميه متجه لها مقدار واتجاه)

3- التسارع acceleration (كميه متجه لها مقدار واتجاه)

## Displacement :




$$\Delta x = \text{change } position = x_{final} - x_{initial}$$

| |
|---|
| Displacement is a vector quantity it has both magnitude and direction<br>الإزاحة كميه متجه لها مقدار واتجاه |

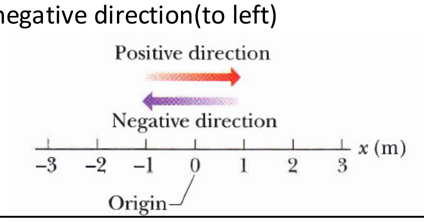
$$\Delta x = \pm \; no.$$

direction

يحدد الاتجاه من نقطة البداية الى نقطة النهاية ويمثل الاتجاه بالإشارة إذا كانت الحركة على محور واحد

**+**➔positive direction(to right)

**-**➔negative direction(to left)




magnitude

المسافة بين نقطة البداية والنهاية

| | | | |
|---|---|---|---|
| **Position** | $x$ | | $x$ |
| **Displacement** | $\Delta x = x_f - x_i$ | | |
| **Average Velocity** | $v_{avg} = \dfrac{\Delta x}{\Delta t}$ | **Velocity** | $v = \dfrac{dx}{dt}$<br>التفاضل الأول لدالة $x$ |
| **Average acceleration** | $a_{avg} = \dfrac{\Delta v}{\Delta t}$ | **Acceleration** | $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$<br>التفاضل الأول للسرعة $v$ أو التفاضل الثاني لدالة $x$ |

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# Velocity (Unit: $\frac{length}{time}$, m/s, km/h)

## Average السرعة المتوسطة

### Average Velocity

تحدد بمقدار واتجاه

$$\upsilon_{avg} = \frac{change\ position}{time\ interval}$$

$$\upsilon_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

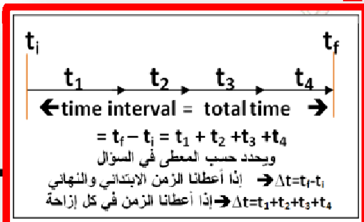
$\upsilon_{avg} = \pm$ (direction) no (magnitude)

### Average Speed

تحدد بمقدار فقط

$$S_{avg} == \frac{\text{Total distance}}{total\ time}$$

$$S_{avg} = \frac{x_{tot}}{\Delta t} = \frac{x_1 + x_2 + x_3 \ldots.}{t_1 + t_2 + t_3 \ldots.}$$




## Instantaneous السرعة اللحظية

### Instantaneous Velocity

(or Velocity) تحدد بمقدار واتجاه

$$\upsilon = \frac{dx}{dt}$$

$\upsilon = \pm$ (direction) no (magnitude)

اشارة الناتج تحدد الاتجاه (+إلى اليمين, - إلى اليسار)

### Instantaneous Speed

(or Speed) تحدد بمقدار فقط

$$S = |\upsilon|$$

القيمة المطلقة للسرعة $\upsilon$

# Acceleration (Unit: $\frac{length}{time^2}$, m/s$^2$, km/h$^2$)

### Average acceleration

(تحدد بمقدار واتجاه)

$$a_{avg} = \frac{change\ velocity}{time\ interval}$$

$$a_{avg} = \frac{\Delta\upsilon}{\Delta t} = \frac{\upsilon_f - \upsilon_i}{t_f - t_i}$$

$a_{avg} = \pm$ (direction) no (magnitude)

### Instantaneous acceleration (or acceleration)

(تحدد بمقدار واتجاه)

$$a = \frac{d\upsilon}{dt} = \frac{d^2x}{dt^2}$$

$a = \pm$ (direction) no (magnitude)

إشارة الناتج تحدد الاتجاه (+إلى اليمين, - إلى اليسار)

If $\upsilon, a$ have the same sign (+/+ or -/-)→speed increase

إذا كانت إشارة السرعة والتسارع متشابهه فإن السرعة تتزايد

If $\upsilon, a$ have different sign (+/- or -/+)→speed decrease

إذا كانت إشارة السرعة والتسارع مختلفة فإن السرعة تتناقص

# الحركة في خط مستقيم (بتسارع ثابت)

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## Constant Acceleration
الحركة الأفقية بتسارع ثابت

1- $\upsilon = \upsilon_0 + a\,t$

2- $x - x_0 = \upsilon_0 t + \frac{1}{2} a t^2$

3- $\upsilon^2 = \upsilon_0^2 + 2\,a(x - x_0)$

4- $x - x_0 = \frac{1}{2}\,(\upsilon + \upsilon_0)\,t$

5- $x - x_0 = \upsilon t - \frac{1}{2} a t^2$

(السرعة الابتدائية) $\upsilon_0$ is the initial velocity
(السرعة النهائية) $\upsilon$ is the final velocity
(التسارع) $a$ is the acceleration
(الإزاحة) $x$ is displacement
(الزمن) $t$ is time

## Free-Fall Acceleration
الحركة العمودية بتسارع ثابت( السقوط الحر)
عندما يتحرك أي جسم عموديا للأعلى أو للأسفل

لكتابة معادلات الحركة العمودية بتكرار نفس قوانين الحركة الأفقيه مع مراعاة تغير المعادلات كالتالي:

1 – استبدال كل $x$ بـ $y$

2 - استبدال التسارع الافقي $a$ بتسارع الجاذبية $-g$

1- $\upsilon = \upsilon_0 - g\,t$

2- $y - y_0 = \upsilon_0 t - \frac{1}{2} g t^2$

3- $\upsilon^2 = \upsilon_0^2 - 2\,g(y - y_0)$

4- $y - y_0 = \frac{1}{2}\,(\upsilon + \upsilon_0)\,t$

5- $y - y_0 = \upsilon t + \frac{1}{2} g t^2$

ملاحظة: عند التعويض عن قيمة $g$ في القانون عند حل المسائل فإننا نضع القيمة بدون الإشارة

$g = 9.8\ \text{m/s}^2$

ملاحظة :عند حل مثل هذه المسائل فأفضل طريقة أن تحلها باستخدام المعطيات فقط ولا تستخدم القيم التي حسبتها في فقرات سابقة وذلك لتفادي الاخطاء المتكررة

ناء فرحان

**Ascent** الحركة لأعلى

(السرعة النهائية) $v = 0$

$y \rightarrow (+)$

$a = -g = -9.8$

**Upward (+)**

(السرعة الابتدائية) $v_0 = +No.$

$+y$

$-y$

Maximum height

$a = -9.8$

Drop, Fall يسقط

Throw يقذف
Rise يرتفع

**Descent** الحركة لأسفل

$v_0 = 0$ (السرعة الابتدائية)

$+y$

$-y$

$y \rightarrow (-)$

$a = -g = -9.8$

**Downward (-)**

$v = -No.$ (السرعة النهائية)

$a = -g = -9.8$ always at any point above the ground
التسارع دائما قيمة ثابتة (a=-9.8) عند أي نقطة فوق سطح الأرض

How Long . زمن
How high ارتفاع
How far بعد
How fast سرعة

Stop➔ v=0

If the particle starts its motion from the rest, that means the initial speed is zero
إذا بدء الجسم حركته من السكون (the rest) ، فإن سرعته الابتدائية تساوي صفر

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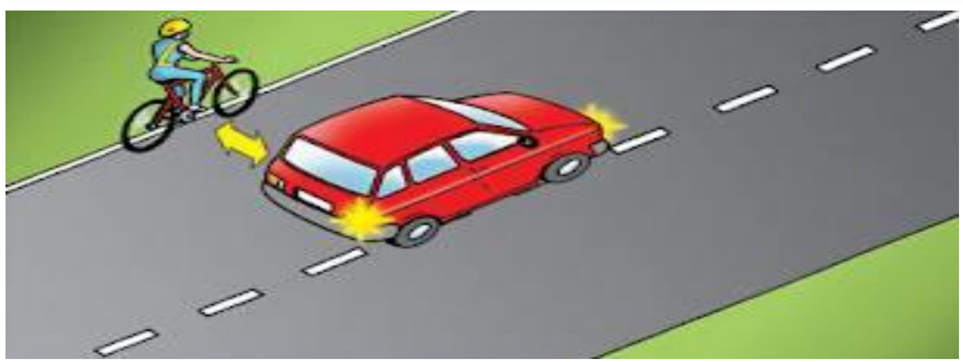
# Motion Along a Straight Line 2



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# Learning Outcomes

***By the end of the chapter student should be able:***

- to locate the position of the particle with respect to the origin in one dimension (x or y).
- to identify the positive direction along x-axis using different word such as (right/east ), and negative direction by using words such as (left/west).
- to identify the positive direction along y-axis using different word such as (up/north ), and negative direction by using words such as (down/south).
- to calculate the displacement in magnitude and determine its direction.
- to differentiate between displacement and distance.
- to define velocity in general and to differentiate between velocity and Speed.
- to define the average velocity and average speed.
- to calculate the average velocity and its direction.
- to calculate the average speed.
- to differentiate between the average velocity and average speed.
- to define the instantaneous velocity and speed.
- to calculate the instantaneous velocity and speed.
- to differentiate between calculating the average velocity and instantaneous velocity from position function at certain time.

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# Learning Outcomes

***By the end of the chapter student should be able:***

- to differentiate between average and instantaneous velocity.
- to define the average acceleration.
- to calculate the average acceleration and determine its direction.
- to define the instantaneous acceleration.
- to calculate the instantaneous acceleration from position function or velocity function and determine its direction.
- to differentiate between average and instantaneous acceleration.
- to explain motion with constant acceleration.
- to apply the equations of motion with constant acceleration to solve problems.
- to define free- fall .
- to define the acceleration of free fall and its direction when the particle is moving upward or downward.
- to determine the sign of velocity and displacement of a particle in free fall moving downward and upward.
- to use the equations of motion with constant acceleration to find the equations of free fall.
- to apply the equations of free fall to solve problems.

# 2-1 WHAT IS PHYSICS?

In this chapter, we study the basic physics of motion where the object (race car, tectonic plate, blood cell, or any other object) moves along a single axis. Such motion is called *one-dimensional motion*.

## 2-2 Motion

The classification and comparison of motions (called **kinematics**)

In this chapter,

1. The motion is along a straight line only. The line may be vertical, horizontal, or slanted, but it must be straight.
2. Forces (pushes and pulls) cause motion but will not be discussed until Chapter 5. In this chapter we discuss only the motion itself and changes in the motion.
3. The moving object is either a **particle** (by which we mean a point-like object such as an electron) or an object that moves like a particle

# Physical Quantities

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## Physical Quantities

### Vector Quantities

magnitude + direction

Follow certain rules of addition and multiplication

### Scalar Quantities

+Ve Number

-Ve Number

Follow the rules of ordinary algebra

# 2-3 | Position and Displacement

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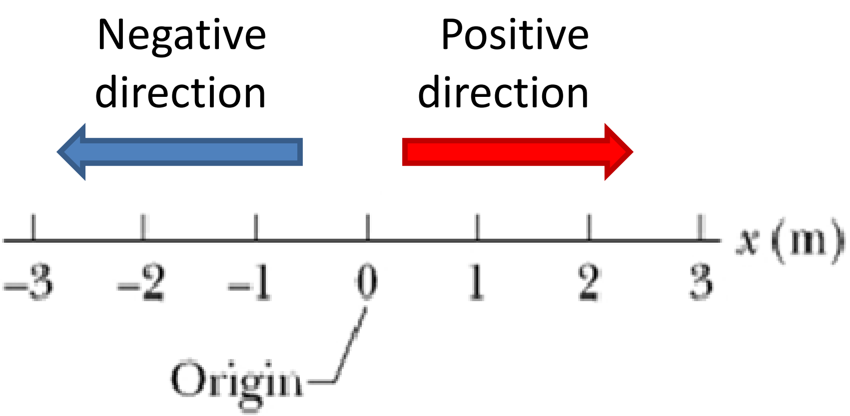
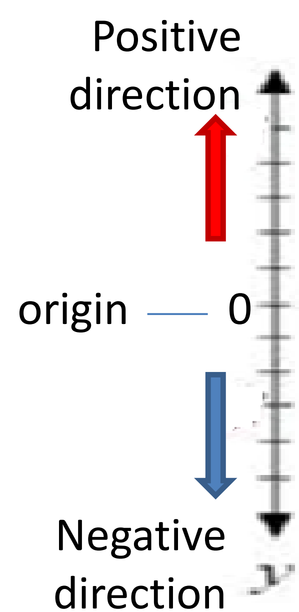
**To locate an object means to find it's position relative to reference point origin ( or zero point ) of an axis .**

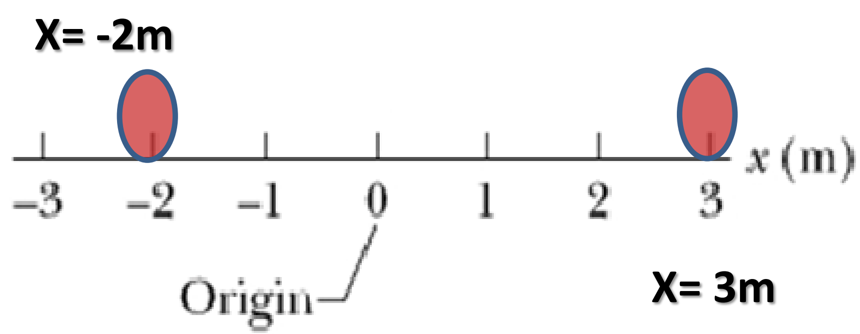
On the X- axis

On the Y- axis



## First: Position




- Position: x
- Unit: m.

# Second: Displacement

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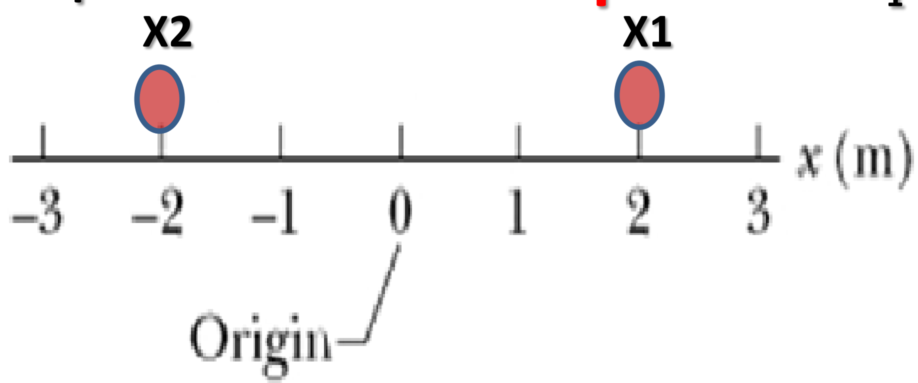
**If the particle move from the position $x_1$ to the position $x_2$**




$X=(-2)-(2)=-4m\Delta$

$\Delta x$ = 4m to the left ∴.

$\boxed{\textbf{Displacement} : \Delta x = x_2 - x_1}$

- **Unit: m.**
- **It is a vector quantity: has magnitude and direction.**
- **Direction: if $\Delta x$ is positive $\Rightarrow$ moving to the right**

  **if $\Delta x$ is negative $\Rightarrow$ moving to the left**

**Distance** : d

**It is a scalar quantity: has no direction.**

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# What is the difference between displacement and distance?

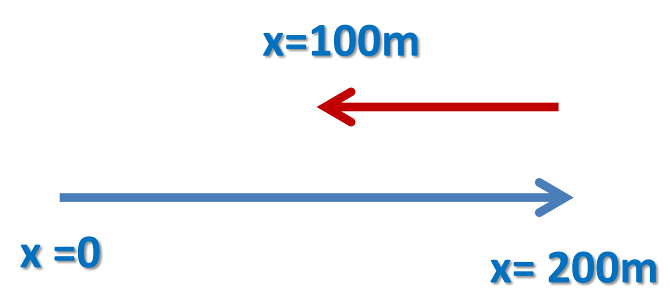
if a particle moves from x =0 m to x= 200m and then back to x=100m

**Displacement**

$\Delta x = 100 - 0 = 100$ m

**Distance**

$d = 200 + 100 = 300$ m



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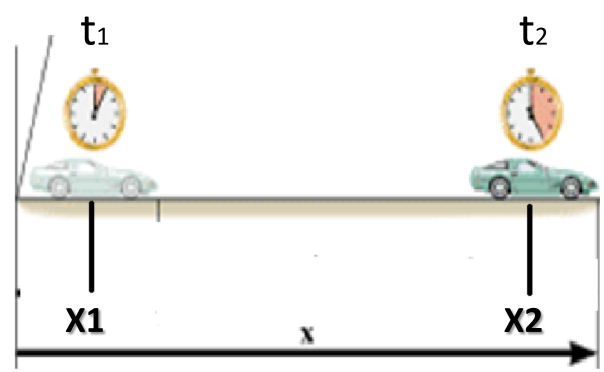
## Average Velocity:

- The ratio of displacement that occurs during a particular time interval to that interval.

$$\boxed{v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}}$$




- **Unit of is m/s.**
- $v_{avg}$ **is a vector quantity.**

  - **if it is positive $\Rightarrow$ moving to the right**
  - **if it is negative $\Rightarrow$ moving to the left**

## Average Speed:

- The ratio of total distance that occurs during a particular time interval to that interval

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}.$$

- Unit of $S_{avg}$ is m/s
  - $s_{avg}$ is a scalar quantity

## Sample Problem 2-1

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

(b) What is the time interval $\Delta t$ from the beginning of your drive to your arrival at the station?



(c) What is your average velocity $v_{avg}$ from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

# 2-5 | Instantaneous Velocity and Speed

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## Instantaneous Velocity ( or velocity)

The velocity at any instant is obtained from the average velocity by shrinking the time interval $\Delta t$ closer and closer to 0. As $\Delta t$ dwindles, the average velocity approaches a limiting value, which is the velocity at that instant:

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.}$$

Note that $v$ is the rate at which position $x$ is changing with time at a given instant; that is, $v$ is the derivative of $x$ with respect to $t$.

- **Unit of is m/s.**
- $v$ **is a vector quantity.**

  - **if it is positive $\Rightarrow$ moving to the right**
  - **if it is negative $\Rightarrow$ moving to the left**

# 2-5 | Instantaneous Velocity and Speed



## Speed:

**Speed** is the magnitude of velocity; that is, speed is velocity that has been stripped of any indication of direction, either in words or via an algebraic sign.

### Sample Problem 2-3

The position of a particle moving on an $x$ axis is given by

$$x = 7.8 + 9.2t - 2.1t^3, \qquad (2\text{-}5)$$

with $x$ in meters and $t$ in seconds. What is its velocity at $t = 3.5$ s? Is the velocity constant, or is it continuously changing?

# 2-6 | Acceleration



When a particle's velocity changes, the particle is said to undergo **acceleration** (or to accelerate).

## Acceleration

### Average Acceleration

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t},$$

### Instantaneous Acceleration (Or Acceleration)

$$a = \frac{dv}{dt}.$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

**Rem:** If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

If

$$v = +ve,\quad a = +ve \quad or \quad v = -ve,\quad a = -ve$$

Speed increase

$$v = +ve,\quad a = -ve \quad or \quad v = -ve,\quad a = +ve$$

Speed decrease

## Sample Problem 2-4 Build your skill

A particle's position on the $x$ axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with $x$ in meters and $t$ in seconds.

(a) Because position $x$ depends on time $t$, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

(b) Is there ever a time when $v = 0$?

# 2-7 | Constant Acceleration: A Special Case

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- **Constant acceleration does not mean the velocity is constant, it means the velocity changes with constant rate.**
- **Constant acceleration does not mean a=0. If a=0 $\Rightarrow$ v is constant.**

**TABLE 2-1**

## Equations for Motion with Constant Acceleration[a]

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| Equation | Missing Quantity |
|---|---|
| $v = v_0 + at$ | $x - x_0$ |
| $x - x_0 = v_0 t + \frac{1}{2}at^2$ | $v$ |
| $v^2 = v_0^2 + 2a(x - x_0)$ | $t$ |
| $x - x_0 = \frac{1}{2}(v_0 + v)t$ | $a$ |
| $x - x_0 = vt - \frac{1}{2}at^2$ | $v_0$ |

$x_0$ → Initial position

$x$ → final position

$x - x_0$ → displacment

$v_0$ → Initial velocity

$v$ → final velocity

$t$ → time

$a$ → Constant acceleration

## Rem:

- when the object starts from rest $\boxed{\Rightarrow v_0 = 0}$
- when the object stops $\boxed{\Rightarrow v = 0}$
- $x_0 = 0$ unless something else mentioned in the problem.

## Sample Problem 2-5

The head of a woodpecker is moving forward at a speed of 7.49 m/s when the beak makes first contact with a tree limb. The beak stops after penetrating the limb by 1.87 mm. Assuming the acceleration to be constant, find the acceleration magnitude in terms of $g$.


| Equation |
| --- |
| $v = v_0 + at$ |
| $x - x_0 = v_0 t + \frac{1}{2}at^2$ |
| $v^2 = v_0^2 + 2a(x - x_0)$ |
| $x - x_0 = \frac{1}{2}(v_0 + v)t$ |
| $x - x_0 = vt - \frac{1}{2}at^2$ |

# 2-9 | Free-Fall Acceleration

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- Free fall is the motion of an object under influence of Gravity and ignoring any other effects such as air resistance.

- All objects in free fall accelerate downward at the same rate and is independent of the object's mass, density or shape.

- This acceleration is called **the free-fall acceleration**.

$$\boxed{g = 9.8\, m/s^2 \quad downward}$$

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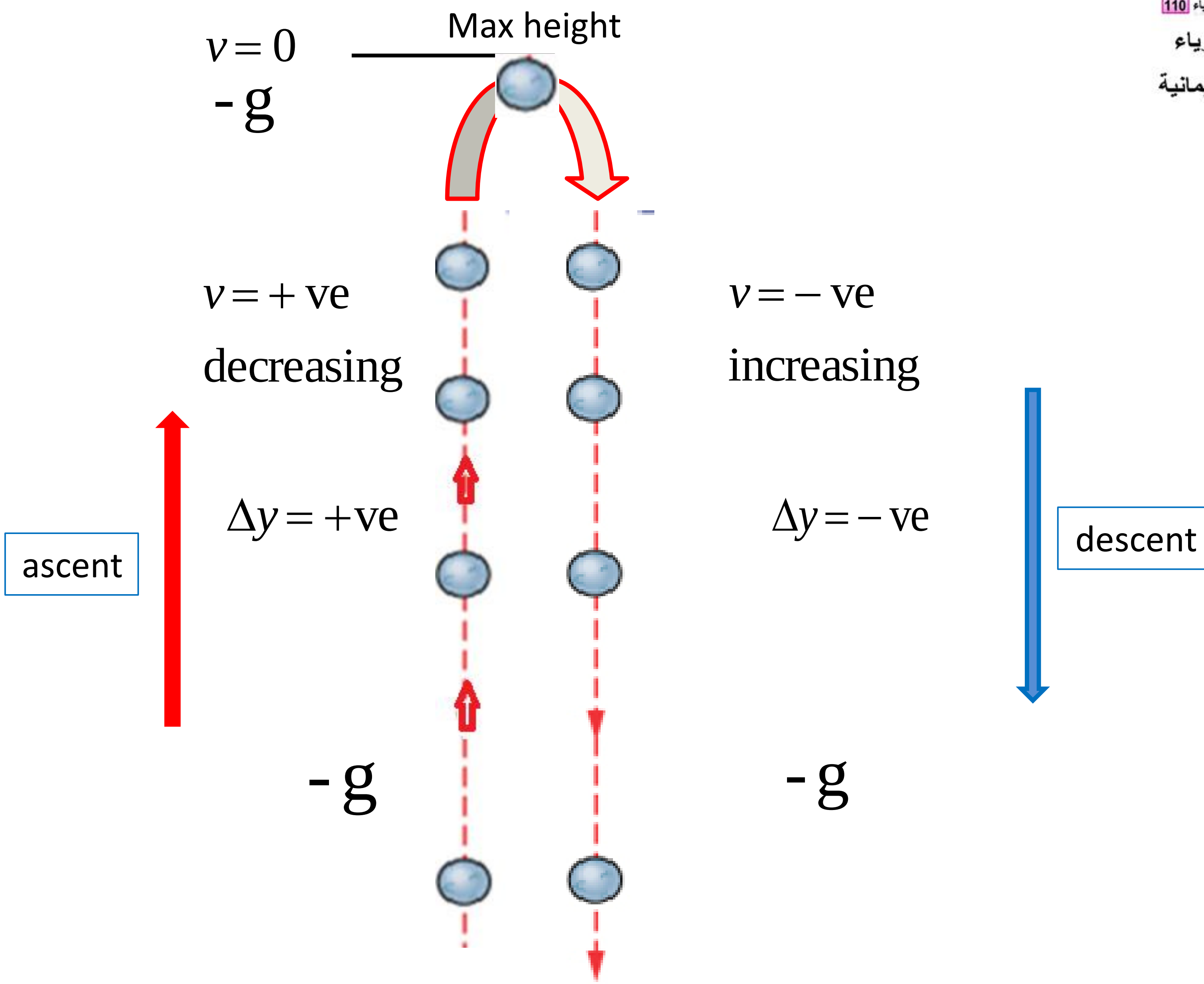

# Equations of motion

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- The motion along y axis $x \rightarrow y$
- $a = -g$

$$v = v_0 + at \quad \rightarrow \quad v = v_0 - gt$$

$$x - x_0 = v_0 + \frac{1}{2}at^2 \quad \rightarrow \quad y - y_0 = v_0 - \frac{1}{2}gt^2$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow \quad v^2 = v_0^2 - 2g(y - y_0)$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t \quad \rightarrow \quad y - y_0 = \frac{1}{2}(v_0 + v)t$$

$$x - x_0 = vt - \frac{1}{2}at^2 \quad \rightarrow \quad y - y_0 = vt + \frac{1}{2}gt^2$$

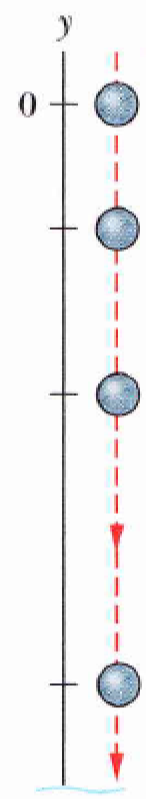
## Rem

- When substituting for g in the equations
 $g = 9.8 m/s.$
- when the object is moving up (ascent).
- When the object is moving down (descent)

## Sample Problem 2-7

On September 26, 1993, Dave Munday went over the Canadian edge of Niagara Falls in a steel ball equipped with an air hole and then fell 48 m to the water (and rocks). Assume his initial velocity was zero, and neglect the effect of the air on the ball during the fall.

(a) How long did Munday fall to reach the water surface?




| Equation |
|---|
| $v = v_0 + at$ |
| $x - x_0 = v_0 t + \frac{1}{2}at^2$ |
| $v^2 = v_0^2 + 2a(x - x_0)$ |
| $x - x_0 = \frac{1}{2}(v_0 + v)t$ |
| $x - x_0 = vt - \frac{1}{2}at^2$ |

(b) Munday could count off the three seconds of free fall but could not see how far he had fallen with each count. Determine his position at each full second.

(c) What was Munday's velocity as he reached the water surface?

| Equation |
|---|
| $v = v_0 + at$ |
| $x - x_0 = v_0 t + \frac{1}{2}at^2$ |
| $v^2 = v_0^2 + 2a(x - x_0)$ |
| $x - x_0 = \frac{1}{2}(v_0 + v)t$ |
| $x - x_0 = vt - \frac{1}{2}at^2$ |

(d) What was Munday's velocity at each count of one full second? Was he aware of his increasing speed?

| Equation |
|---|
| $v = v_0 + at$ |
| $x - x_0 = v_0 t + \frac{1}{2}at^2$ |
| $v^2 = v_0^2 + 2a(x - x_0)$ |
| $x - x_0 = \frac{1}{2}(v_0 + v)t$ |
| $x - x_0 = vt - \frac{1}{2}at^2$ |

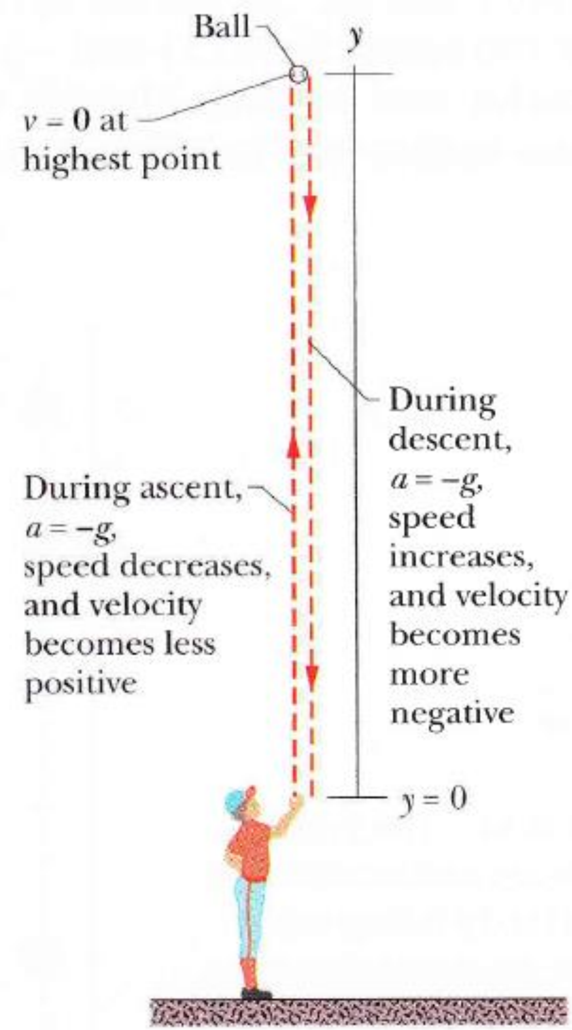
| Equation |
| --- |
| $v = v_0 + at$ |
| $x - x_0 = v_0 t + \frac{1}{2}at^2$ |
| $v^2 = v_0^2 + 2a(x - x_0)$ |
| $x - x_0 = \frac{1}{2}(v_0 + v)t$ |
| $x - x_0 = vt - \frac{1}{2}at^2$ |

## Sample Problem 2-8

In Fig. 2-12, a pitcher tosses a baseball up along a $y$ axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

(b) What is the ball's maximum height above its release point?



Equation

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$
$$x - x_0 = \frac{1}{2}(v_0 + v)t$$
$$x - x_0 = vt - \frac{1}{2}at^2$$

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قسم الفيزياء

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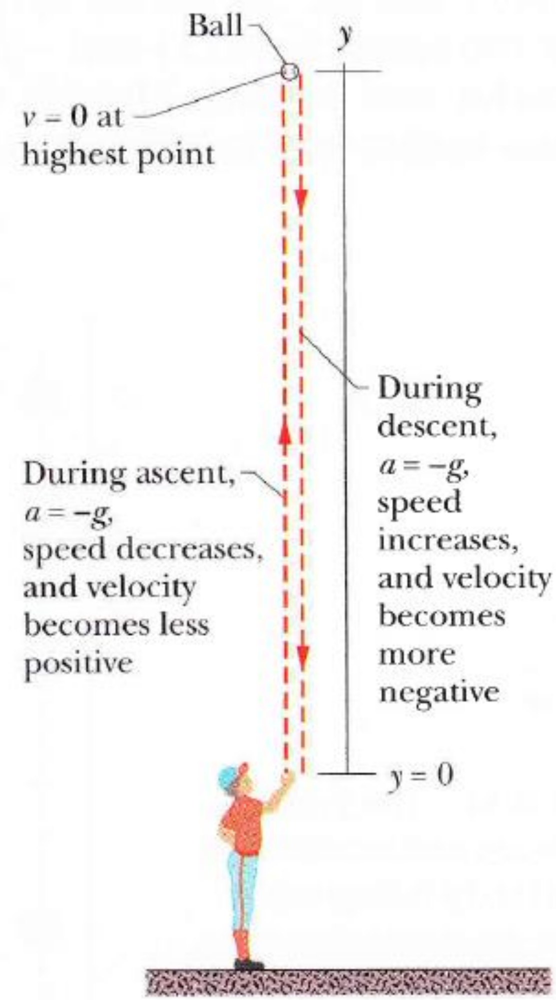
(c) How long does the ball take to reach a point 5.0 m above its release point?



# The End

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CHAPTER

2

# MOTION ALONG A STRAIGHT LINE

## 2-1 WHAT IS PHYSICS?

One purpose of physics is to study the motion of objects—how fast they move, for example, and how far they move in a given amount of time. NASCAR engineers are fanatical about this aspect of physics as they determine the performance of their cars before and during a race. Geologists use this physics to measure tectonic-plate motion as they attempt to predict earthquakes. Medical researchers need this physics to map the blood flow through a patient when diagnosing a partially closed artery, and motorists use it to determine how they might slow sufficiently when their radar detector sounds a warning. There are countless other examples. In this chapter, we study the basic physics of motion where the object (race car, tectonic plate, blood cell, or any other object) moves along a single axis. Such motion is called *one-dimensional motion.*

## 2-2 Motion

The world, and everything in it, moves. Even seemingly stationary things, such as a roadway, move with Earth's rotation, Earth's orbit around the Sun, the Sun's orbit around the center of the Milky Way galaxy, and that galaxy's migration relative to other galaxies. The classification and comparison of motions (called **kinematics**) is often challenging. What exactly do you measure, and how do you compare?

Before we attempt an answer, we shall examine some general properties of motion that is restricted in three ways.

1. The motion is along a straight line only. The line may be vertical, horizontal, or slanted, but it must be straight.
2. Forces (pushes and pulls) cause motion but will not be discussed until Chapter 5. In this chapter we discuss only the motion itself and changes in the motion. Does the moving object speed up, slow down, stop, or reverse direction? If the motion does change, how is time involved in the change?
3. The moving object is either a **particle** (by which we mean a point-like object such as an electron) or an object that moves like a particle (such that every portion moves in the same direction and at the same rate). A stiff pig slipping down a straight playground slide might be considered to be moving like a particle; however, a tumbling tumbleweed would not.

## 2-3 Position and Displacement

To locate an object means to find its position relative to some reference point, often the **origin** (or zero point) of an axis such as the $x$ axis in Fig. 2-1. The **positive direction** of the axis is in the direction of increasing numbers (coordinates), which is to the right in Fig. 2-1. The opposite is the **negative direction.**

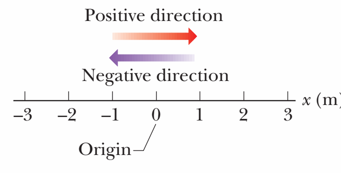


**Fig. 2-1** Position is determined on an axis that is marked in units of length (here meters) and that extends indefinitely in opposite directions. The axis name, here $x$, is always on the positive side of the origin.

For example, a particle might be located at $x = 5$ m, which means it is 5 m in the positive direction from the origin. If it were at $x = -5$ m, it would be just as far from the origin but in the opposite direction. On the axis, a coordinate of $-5$ m is less than a coordinate of $-1$ m, and both coordinates are less than a coordinate of $+5$ m. A plus sign for a coordinate need not be shown, but a minus sign must always be shown.

A change from position $x_1$ to position $x_2$ is called a **displacement** $\Delta x$, where

$$\Delta x = x_2 - x_1. \tag{2-1}$$

(The symbol $\Delta$, the Greek uppercase delta, represents a change in a quantity, and it means the final value of that quantity minus the initial value.) When numbers are inserted for the position values $x_1$ and $x_2$ in Eq. 2-1, a displacement in the positive direction (to the right in Fig. 2-1) always comes out positive, and a displacement in the opposite direction (left in the figure) always comes out negative. For example, if the particle moves from $x_1 = 5$ m to $x_2 = 12$ m, then the displacement is $\Delta x = (12 \text{ m}) - (5 \text{ m}) = +7$ m. The positive result indicates that the motion is in the positive direction. If, instead, the particle moves from $x_1 = 5$ m to $x_2 = 1$ m, then $\Delta x = (1 \text{ m}) - (5 \text{ m}) = -4$ m. The negative result indicates that the motion is in the negative direction.

The actual number of meters covered for a trip is irrelevant; displacement involves only the original and final positions. For example, if the particle moves from $x = 5$ m out to $x = 200$ m and then back to $x = 5$ m, the displacement from start to finish is $\Delta x = (5 \text{ m}) - (5 \text{ m}) = 0$.

A plus sign for a displacement need not be shown, but a minus sign must always be shown. If we ignore the sign (and thus the direction) of a displacement, we are left with the **magnitude** (or absolute value) of the displacement. For example, a displacement of $\Delta x = -4$ m has a magnitude of 4 m.

Displacement is an example of a **vector quantity,** which is a quantity that has both a direction and a magnitude. We explore vectors more fully in Chapter 3 (in fact, some of you may have already read that chapter), but here all we need is the idea that displacement has two features: (1) Its *magnitude* is the distance (such as the number of meters) between the original and final positions. (2) Its *direction,* from an original position to a final position, can be represented by a plus sign or a minus sign if the motion is along a single axis.

*What follows is the first of many checkpoints you will see in this book. Each consists of one or more questions whose answers require some reasoning or a mental calculation, and each gives you a quick check of your understanding of a point just discussed. The answers are listed in the back of the book.*

**CHECKPOINT 1**

Here are three pairs of initial and final positions, respectively, along an $x$ axis. Which pairs give a negative displacement: (a) $-3$ m, $+5$ m; (b) $-3$ m, $-7$ m; (c) 7 m, $-3$ m?

## 2-4 Average Velocity and Average Speed

A compact way to describe position is with a graph of position $x$ plotted as a function of time $t$—a graph of $x(t)$. (The notation $x(t)$ represents a function $x$ of $t$, not the product $x$ times $t$.) As a simple example, Fig. 2-2 shows the position function $x(t)$ for a stationary armadillo (which we treat as a particle) over a 7 s time interval. The animal's position stays at $x = -2$ m.

Figure 2-3 is more interesting, because it involves motion. The armadillo is apparently first noticed at $t = 0$ when it is at the position $x = -5$ m. It moves

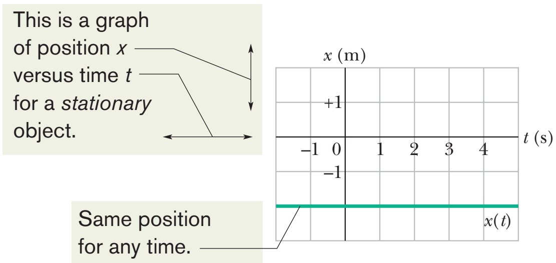


**Fig. 2-2** The graph of $x(t)$ for an armadillo that is stationary at $x = -2$ m. The value of $x$ is $-2$ m for all times $t$.

toward $x = 0$, passes through that point at $t = 3$ s, and then moves on to increasingly larger positive values of $x$. Figure 2-3 also depicts the straight-line motion of the armadillo (at three times) and is something like what you would see. The graph in Fig. 2-3 is more abstract and quite unlike what you would see, but it is richer in information. It also reveals how fast the armadillo moves.

Actually, several quantities are associated with the phrase "how fast." One of them is the **average velocity** $v_{avg}$, which is the ratio of the displacement $\Delta x$ that occurs during a particular time interval $\Delta t$ to that interval:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \tag{2-2}$$

The notation means that the position is $x_1$ at time $t_1$ and then $x_2$ at time $t_2$. A common unit for $v_{avg}$ is the meter per second (m/s). You may see other units in the problems, but they are always in the form of length/time.

On a graph of $x$ versus $t$, $v_{avg}$ is the **slope** of the straight line that connects two particular points on the $x(t)$ curve: one is the point that corresponds to $x_2$ and $t_2$, and the other is the point that corresponds to $x_1$ and $t_1$. Like displacement, $v_{avg}$ has both magnitude and direction (it is another vector quantity). Its magnitude is the magnitude of the line's slope. A positive $v_{avg}$ (and slope) tells us that the line slants upward to the right; a negative $v_{avg}$ (and slope) tells us that the line slants downward to the right. The average velocity $v_{avg}$ always has the same sign as the displacement $\Delta x$ because $\Delta t$ in Eq. 2-2 is always positive.

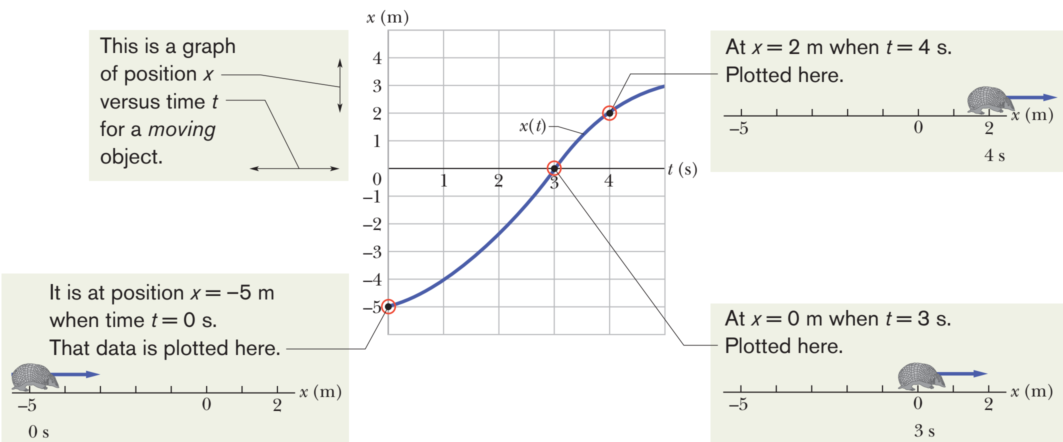


**Fig. 2-3** The graph of $x(t)$ for a moving armadillo. The path associated with the graph is also shown, at three times.

This is a graph of position $x$ versus time $t$.

To find average velocity, first draw a straight line, start to end, and then find the slope of the line.

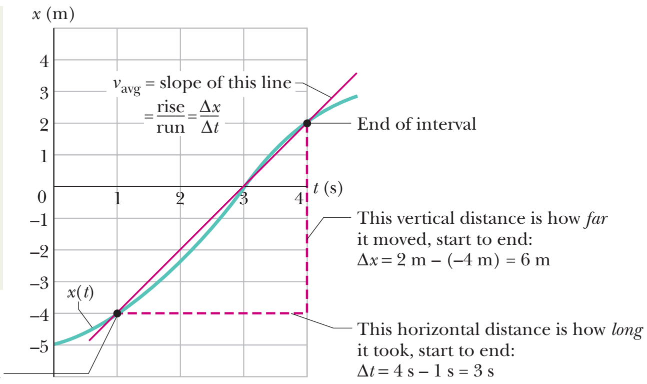


**Fig. 2-4** Calculation of the average velocity between $t = 1$ s and $t = 4$ s as the slope of the line that connects the points on the $x(t)$ curve representing those times.

Figure 2-4 shows how to find $v_{avg}$ in Fig. 2-3 for the time interval $t = 1$ s to $t = 4$ s. We draw the straight line that connects the point on the position curve at the beginning of the interval and the point on the curve at the end of the interval. Then we find the slope $\Delta x/\Delta t$ of the straight line. For the given time interval, the average velocity is

$$v_{avg} = \frac{6\text{ m}}{3\text{ s}} = 2\text{ m/s}.$$

**Average speed** $s_{avg}$ is a different way of describing "how fast" a particle moves. Whereas the average velocity involves the particle's displacement $\Delta x$, the average speed involves the total distance covered (for example, the number of meters moved), independent of direction; that is,

$$s_{avg} = \frac{\text{total distance}}{\Delta t}. \qquad (2\text{-}3)$$

Because average speed does *not* include direction, it lacks any algebraic sign. Sometimes $s_{avg}$ is the same (except for the absence of a sign) as $v_{avg}$. However, the two can be quite different.

## Sample Problem

### Average velocity, beat-up pickup truck

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

**KEY IDEA**

Assume, for convenience, that you move in the positive direction of an $x$ axis, from a first position of $x_1 = 0$ to a second position of $x_2$ at the station. That second position must be at $x_2 = 8.4\text{ km} + 2.0\text{ km} = 10.4$ km. Then your displacement $\Delta x$ along the $x$ axis is the second position minus the first position.

***Calculation:*** From Eq. 2-1, we have

$$\Delta x = x_2 - x_1 = 10.4\text{ km} - 0 = 10.4\text{ km}. \qquad \text{(Answer)}$$

Thus, your overall displacement is 10.4 km in the positive direction of the $x$ axis.

(b) What is the time interval $\Delta t$ from the beginning of your drive to your arrival at the station?

**KEY IDEA**

We already know the walking time interval $\Delta t_{wlk}$ (= 0.50 h), but we lack the driving time interval $\Delta t_{dr}$. However, we know that for the drive the displacement $\Delta x_{dr}$ is 8.4 km and the average velocity $v_{avg,dr}$ is 70 km/h. Thus, this average velocity is the ratio of the displacement for the drive to the time interval for the drive.

***Calculations:*** We first write

$$v_{\text{avg,dr}} = \frac{\Delta x_{\text{dr}}}{\Delta t_{\text{dr}}}.$$

Rearranging and substituting data then give us

$$\Delta t_{\text{dr}} = \frac{\Delta x_{\text{dr}}}{v_{\text{avg,dr}}} = \frac{8.4 \text{ km}}{70 \text{ km/h}} = 0.12 \text{ h}.$$

So, $\Delta t = \Delta t_{\text{dr}} + \Delta t_{\text{wlk}}$

$= 0.12 \text{ h} + 0.50 \text{ h} = 0.62 \text{ h}.$ (Answer)

(c) What is your average velocity $v_{\text{avg}}$ from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

**KEY IDEA**

From Eq. 2-2 we know that $v_{\text{avg}}$ *for the entire trip* is the ratio of the displacement of 10.4 km *for the entire trip* to the time interval of 0.62 h *for the entire trip*.

***Calculation:*** Here we find

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{10.4 \text{ km}}{0.62 \text{ h}}$$

$$= 16.8 \text{ km/h} \approx 17 \text{ km/h}. \qquad \text{(Answer)}$$

To find $v_{\text{avg}}$ graphically, first we graph the function $x(t)$ as shown in Fig. 2-5, where the beginning and arrival points on the graph are the origin and the point labeled as "Station." Your average velocity is the slope of the straight line connecting those points; that is, $v_{\text{avg}}$ is the ratio of the *rise* ($\Delta x$ = 10.4 km) to the *run* ($\Delta t$ = 0.62 h), which gives us $v_{\text{avg}}$ = 16.8 km/h.

(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

**KEY IDEA**

Your average speed is the ratio of the total distance you move to the total time interval you take to make that move.

***Calculation:*** The total distance is 8.4 km + 2.0 km + 2.0 km = 12.4 km. The total time interval is 0.12 h + 0.50 h + 0.75 h = 1.37 h. Thus, Eq. 2-3 gives us

$$s_{\text{avg}} = \frac{12.4 \text{ km}}{1.37 \text{ h}} = 9.1 \text{ km/h}. \qquad \text{(Answer)}$$

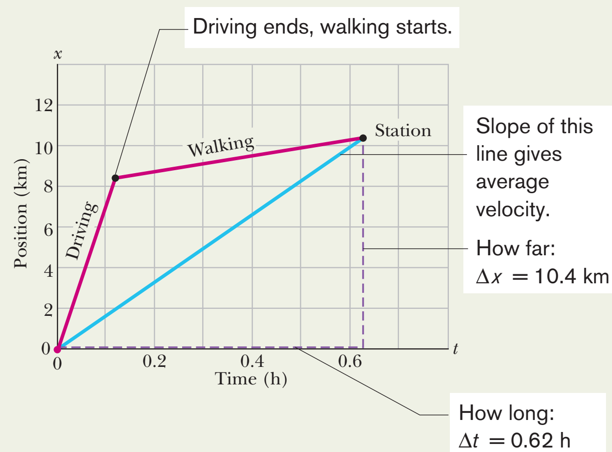


**Fig. 2-5** The lines marked "Driving" and "Walking" are the position–time plots for the driving and walking stages. (The plot for the walking stage assumes a constant rate of walking.) The slope of the straight line joining the origin and the point labeled "Station" is the average velocity for the trip, from the beginning to the station.



## 2-5 Instantaneous Velocity and Speed

You have now seen two ways to describe how fast something moves: average velocity and average speed, both of which are measured over a time interval $\Delta t$. However, the phrase "how fast" more commonly refers to how fast a particle is moving at a given instant—its **instantaneous velocity** (or simply **velocity**) $v$.

The velocity at any instant is obtained from the average velocity by shrinking the time interval $\Delta t$ closer and closer to 0. As $\Delta t$ dwindles, the average velocity approaches a limiting value, which is the velocity at that instant:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}. \qquad (2\text{-}4)$$

Note that $v$ is the rate at which position $x$ is changing with time at a given instant; that is, $v$ is the derivative of $x$ with respect to $t$. Also note that $v$ at any instant is the slope of the position–time curve at the point representing that instant. Velocity is another vector quantity and thus has an associated direction.

**Speed** is the magnitude of velocity; that is, speed is velocity that has been stripped of any indication of direction, either in words or via an algebraic sign. (*Caution:* Speed and average speed can be quite different.) A velocity of $+5$ m/s and one of $-5$ m/s both have an associated speed of 5 m/s. The speedometer in a car measures speed, not velocity (it cannot determine the direction).

**CHECKPOINT 2**

The following equations give the position $x(t)$ of a particle in four situations (in each equation, $x$ is in meters, $t$ is in seconds, and $t > 0$): (1) $x = 3t - 2$; (2) $x = -4t^2 - 2$; (3) $x = 2/t^2$; and (4) $x = -2$. (a) In which situation is the velocity $v$ of the particle constant? (b) In which is $v$ in the negative $x$ direction?

**Sample Problem**

### Velocity and slope of *x* versus *t*, elevator cab

Figure 2-6*a* is an $x(t)$ plot for an elevator cab that is initially stationary, then moves upward (which we take to be the positive direction of $x$), and then stops. Plot $v(t)$.

**KEY IDEA**

We can find the velocity at any time from the slope of the $x(t)$ curve at that time.

***Calculations:*** The slope of $x(t)$, and so also the velocity, is zero in the intervals from 0 to 1 s and from 9 s on, so then the cab is stationary. During the interval $bc$, the slope is constant and nonzero, so then the cab moves with constant velocity. We calculate the slope of $x(t)$ then as

$$\frac{\Delta x}{\Delta t} = v = \frac{24\text{ m} - 4.0\text{ m}}{8.0\text{ s} - 3.0\text{ s}} = +4.0\text{ m/s}. \tag{2-5}$$

The plus sign indicates that the cab is moving in the positive $x$ direction. These intervals (where $v = 0$ and $v = 4$ m/s) are plotted in Fig. 2-6*b*. In addition, as the cab initially begins to move and then later slows to a stop, $v$ varies as indicated in the intervals 1 s to 3 s and 8 s to 9 s. Thus, Fig. 2-6*b* is the required plot. (Figure 2-6*c* is considered in Section 2-6.)

Given a $v(t)$ graph such as Fig. 2-6*b*, we could "work backward" to produce the shape of the associated $x(t)$ graph (Fig. 2-6*a*). However, we would not know the actual values for $x$ at various times, because the $v(t)$ graph indicates only *changes* in $x$. To find such a change in $x$ during any interval, we must, in the language of calculus, calculate the area "under the curve" on the $v(t)$ graph for that interval. For example, during the interval 3 s to 8 s in which the cab has a velocity of 4.0 m/s, the change in $x$ is

$$\Delta x = (4.0\text{ m/s})(8.0\text{ s} - 3.0\text{ s}) = +20\text{ m}. \tag{2-6}$$

(This area is positive because the $v(t)$ curve is above the $t$ axis.) Figure 2-6*a* shows that $x$ does indeed increase by 20 m in that interval. However, Fig. 2-6*b* does not tell us the *values* of $x$ at the beginning and end of the interval. For that, we need additional information, such as the value of $x$ at some instant.

## 2-6 Acceleration

When a particle's velocity changes, the particle is said to undergo **acceleration** (or to accelerate). For motion along an axis, the **average acceleration** $a_{avg}$ over a time interval $\Delta t$ is

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}, \tag{2-7}$$

where the particle has velocity $v_1$ at time $t_1$ and then velocity $v_2$ at time $t_2$. The **instantaneous acceleration** (or simply **acceleration**) is

$$a = \frac{dv}{dt}. \tag{2-8}$$

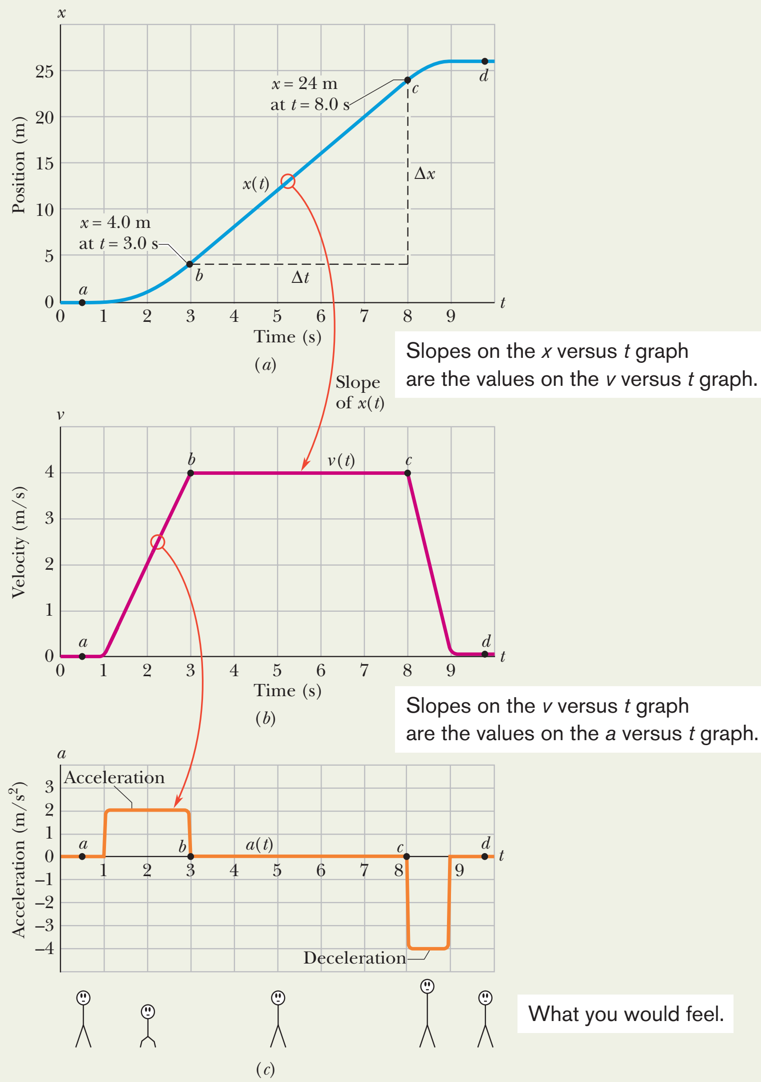


**Fig. 2-6** ($a$) The $x(t)$ curve for an elevator cab that moves upward along an $x$ axis. ($b$) The $v(t)$ curve for the cab. Note that it is the derivative of the $x(t)$ curve ($v = dx/dt$). ($c$) The $a(t)$ curve for the cab. It is the derivative of the $v(t)$ curve ($a = dv/dt$). The stick figures along the bottom suggest how a passenger's body might feel during the accelerations.

Additional examples, video, and practice available at *WileyPLUS*

In words, the acceleration of a particle at any instant is the rate at which its velocity is changing at that instant. Graphically, the acceleration at any point is the slope of the curve of $v(t)$ at that point. We can combine Eq. 2-8 with Eq. 2-4 to write

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}. \tag{2-9}$$

In words, the acceleration of a particle at any instant is the second derivative of its position $x(t)$ with respect to time.

A common unit of acceleration is the meter per second per second: $m/(s \cdot s)$ or $m/s^2$. Other units are in the form of length/(time · time) or length/time$^2$. Acceleration has both magnitude and direction (it is yet another vector quantity). Its algebraic sign represents its direction on an axis just as for displacement and velocity; that is, acceleration with a positive value is in the positive direction of an axis, and acceleration with a negative value is in the negative direction.

**Fig. 2-7** Colonel J. P. Stapp in a rocket sled as it is brought up to high speed (acceleration out of the page) and then very rapidly braked (acceleration into the page). *(Courtesy U.S. Air Force)*

Figure 2-6 gives plots of the position, velocity, and acceleration of an elevator moving up a shaft. Compare the $a(t)$ curve with the $v(t)$ curve—each point on the $a(t)$ curve shows the derivative (slope) of the $v(t)$ curve at the corresponding time. When $v$ is constant (at either 0 or 4 m/s), the derivative is zero and so also is the acceleration. When the cab first begins to move, the $v(t)$ curve has a positive derivative (the slope is positive), which means that $a(t)$ is positive. When the cab slows to a stop, the derivative and slope of the $v(t)$ curve are negative; that is, $a(t)$ is negative.

Next compare the slopes of the $v(t)$ curve during the two acceleration periods. The slope associated with the cab's slowing down (commonly called "deceleration") is steeper because the cab stops in half the time it took to get up to speed. The steeper slope means that the magnitude of the deceleration is larger than that of the acceleration, as indicated in Fig. 2-6*c*.

The sensations you would feel while riding in the cab of Fig. 2-6 are indicated by the sketched figures at the bottom. When the cab first accelerates, you feel as though you are pressed downward; when later the cab is braked to a stop, you seem to be stretched upward. In between, you feel nothing special. In other words, your body reacts to accelerations (it is an accelerometer) but not to velocities (it is not a speedometer). When you are in a car traveling at 90 km/h or an airplane traveling at 900 km/h, you have no bodily awareness of the motion. However, if the car or plane quickly changes velocity, you may become keenly aware of the change, perhaps even frightened by it. Part of the thrill of an amusement park ride is due to the quick changes of velocity that you undergo (you pay for the accelerations, not for the speed). A more extreme example is shown in the photographs of Fig. 2-7, which were taken while a rocket sled was rapidly accelerated along a track and then rapidly braked to a stop.

Large accelerations are sometimes expressed in terms of $g$ units, with

$$1g = 9.8 \text{ m/s}^2 \qquad (g \text{ unit}). \tag{2-10}$$

(As we shall discuss in Section 2-9, $g$ is the magnitude of the acceleration of a falling object near Earth's surface.) On a roller coaster, you may experience brief accelerations up to $3g$, which is $(3)(9.8 \text{ m/s}^2)$, or about 29 m/s$^2$, more than enough to justify the cost of the ride.

In common language, the sign of an acceleration has a nonscientific meaning: positive acceleration means that the speed of an object is increasing, and negative acceleration means that the speed is decreasing (the object is decelerating). In this book, however, the sign of an acceleration indicates a direction, not whether an object's speed is increasing or decreasing. For example, if a car with an initial velocity $v = -25$ m/s is braked to a stop in 5.0 s, then $a_{avg} = +5.0$ m/s$^2$. The acceleration is *positive*, but the car's speed has decreased. The reason is the difference in signs: the direction of the acceleration is opposite that of the velocity.

Here then is the proper way to interpret the signs:

> If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

**CHECKPOINT 3**

A wombat moves along an $x$ axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

**Sample Problem**

### Acceleration and $dv/dt$

A particle's position on the $x$ axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with $x$ in meters and $t$ in seconds.

(a) Because position $x$ depends on time $t$, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

**KEY IDEAS**

(1) To get the velocity function $v(t)$, we differentiate the position function $x(t)$ with respect to time. (2) To get the acceleration function $a(t)$, we differentiate the velocity function $v(t)$ with respect to time.

***Calculations:*** Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad \text{(Answer)}$$

with $v$ in meters per second. Differentiating the velocity function then gives us

$$a = +6t, \qquad \text{(Answer)}$$

with $a$ in meters per second squared.

(b) Is there ever a time when $v = 0$?

***Calculation:*** Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

which has the solution

$$t = \pm 3 \text{ s}. \qquad \text{(Answer)}$$

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

(c) Describe the particle's motion for $t \geq 0$.

***Reasoning:*** We need to examine the expressions for $x(t)$, $v(t)$, and $a(t)$.

At $t = 0$, the particle is at $x(0) = +4$ m and is moving with a velocity of $v(0) = -27$ m/s—that is, in the negative direction of the $x$ axis. Its acceleration is $a(0) = 0$ because just then the particle's velocity is not changing.

For $0 < t < 3$ s, the particle still has a negative velocity, so it continues to move in the negative direction. However, its acceleration is no longer 0 but is increasing and positive. Because the signs of the velocity and the acceleration are opposite, the particle must be slowing.

Indeed, we already know that it stops momentarily at $t = 3$ s. Just then the particle is as far to the left of the origin in Fig. 2-1 as it will ever get. Substituting $t = 3$ s into the expression for $x(t)$, we find that the particle's position just then is $x = -50$ m. Its acceleration is still positive.

For $t > 3$ s, the particle moves to the right on the axis. Its acceleration remains positive and grows progressively larger in magnitude. The velocity is now positive, and it too grows progressively larger in magnitude.

## 2-7 Constant Acceleration: A Special Case

In many types of motion, the acceleration is either constant or approximately so. For example, you might accelerate a car at an approximately constant rate when a traffic light turns from red to green. Then graphs of your position, velocity, and acceleration would resemble those in Fig. 2-8. (Note that $a(t)$ in Fig. 2-8*c* is constant, which requires that $v(t)$ in Fig. 2-8*b* have a constant slope.) Later when you brake the car to a stop, the acceleration (or deceleration in common language) might also be approximately constant.

Such cases are so common that a special set of equations has been derived for dealing with them. One approach to the derivation of these equations is given in this section. A second approach is given in the next section. Throughout both sections and later when you work on the homework problems, keep in mind that *these equations are valid only for constant acceleration (or situations in which you can approximate the acceleration as being constant).*

When the acceleration is constant, the average acceleration and instantaneous acceleration are equal and we can write Eq. 2-7, with some changes in notation, as

$$a = a_{\text{avg}} = \frac{v - v_0}{t - 0}.$$

Here $v_0$ is the velocity at time $t = 0$ and $v$ is the velocity at any later time $t$. We can recast this equation as

$$v = v_0 + at. \tag{2-11}$$

As a check, note that this equation reduces to $v = v_0$ for $t = 0$, as it must. As a further check, take the derivative of Eq. 2-11. Doing so yields $dv/dt = a$, which is the definition of $a$. Figure 2-8*b* shows a plot of Eq. 2-11, the $v(t)$ function; the function is linear and thus the plot is a straight line.

In a similar manner, we can rewrite Eq. 2-2 (with a few changes in notation) as

$$v_{\text{avg}} = \frac{x - x_0}{t - 0}$$

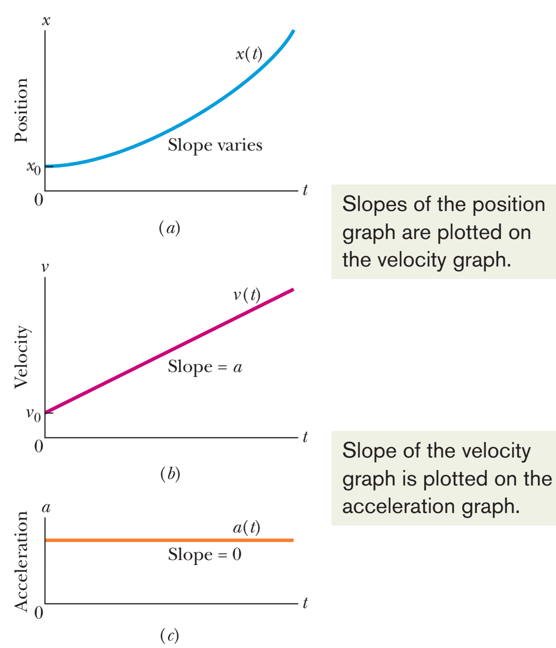


**Fig. 2-8** (*a*) The position $x(t)$ of a particle moving with constant acceleration. (*b*) Its velocity $v(t)$, given at each point by the slope of the curve of $x(t)$. (*c*) Its (constant) acceleration, equal to the (constant) slope of the curve of $v(t)$.

and then as

$$x = x_0 + v_{avg}t, \tag{2-12}$$

in which $x_0$ is the position of the particle at $t = 0$ and $v_{avg}$ is the average velocity between $t = 0$ and a later time $t$.

For the linear velocity function in Eq. 2-11, the *average* velocity over any time interval (say, from $t = 0$ to a later time $t$) is the average of the velocity at the beginning of the interval ($= v_0$) and the velocity at the end of the interval ($= v$). For the interval from $t = 0$ to the later time $t$ then, the average velocity is

$$v_{avg} = \tfrac{1}{2}(v_0 + v). \tag{2-13}$$

Substituting the right side of Eq. 2-11 for $v$ yields, after a little rearrangement,

$$v_{avg} = v_0 + \tfrac{1}{2}at. \tag{2-14}$$

Finally, substituting Eq. 2-14 into Eq. 2-12 yields

$$x - x_0 = v_0t + \tfrac{1}{2}at^2. \tag{2-15}$$

As a check, note that putting $t = 0$ yields $x = x_0$, as it must. As a further check, taking the derivative of Eq. 2-15 yields Eq. 2-11, again as it must. Figure 2-8*a* shows a plot of Eq. 2-15; the function is quadratic and thus the plot is curved.

Equations 2-11 and 2-15 are the *basic equations for constant acceleration;* they can be used to solve any constant acceleration problem in this book. However, we can derive other equations that might prove useful in certain specific situations. First, note that as many as five quantities can possibly be involved in any problem about constant acceleration—namely, $x - x_0$, $v$, $t$, $a$, and $v_0$. Usually, one of these quantities is *not* involved in the problem, *either as a given or as an unknown.* We are then presented with three of the remaining quantities and asked to find the fourth.

Equations 2-11 and 2-15 each contain four of these quantities, but not the same four. In Eq. 2-11, the "missing ingredient" is the displacement $x - x_0$. In Eq. 2-15, it is the velocity $v$. These two equations can also be combined in three ways to yield three additional equations, each of which involves a different "missing variable." First, we can eliminate $t$ to obtain

$$v^2 = v_0^2 + 2a(x - x_0). \tag{2-16}$$

This equation is useful if we do not know $t$ and are not required to find it. Second, we can eliminate the acceleration $a$ between Eqs. 2-11 and 2-15 to produce an equation in which $a$ does not appear:

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t. \tag{2-17}$$

Finally, we can eliminate $v_0$, obtaining

$$x - x_0 = vt - \tfrac{1}{2}at^2. \tag{2-18}$$

Note the subtle difference between this equation and Eq. 2-15. One involves the initial velocity $v_0$; the other involves the velocity $v$ at time $t$.

Table 2-1 lists the basic constant acceleration equations (Eqs. 2-11 and 2-15) as well as the specialized equations that we have derived. To solve a simple constant acceleration problem, you can usually use an equation from this list (*if* you have the list with you). Choose an equation for which the only unknown variable is the variable requested in the problem. A simpler plan is to remember only Eqs. 2-11 and 2-15, and then solve them as simultaneous equations whenever needed.

**Table 2-1**

**Equations for Motion with Constant Acceleration**[a]

| Equation Number | Equation | Missing Quantity |
|---|---|---|
| 2-11 | $v = v_0 + at$ | $x - x_0$ |
| 2-15 | $x - x_0 = v_0t + \frac{1}{2}at^2$ | $v$ |
| 2-16 | $v^2 = v_0^2 + 2a(x - x_0)$ | $t$ |
| 2-17 | $x - x_0 = \frac{1}{2}(v_0 + v)t$ | $a$ |
| 2-18 | $x - x_0 = vt - \frac{1}{2}at^2$ | $v_0$ |

[a]Make sure that the acceleration is indeed constant before using the equations in this table.

**CHECKPOINT 4**

The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

**Sample Problem**

### Constant acceleration, graph of *v* versus *x*

Figure 2-9 gives a particle's velocity $v$ versus its position as it moves along an $x$ axis with constant acceleration. What is its velocity at position $x = 0$?

**KEY IDEA**

We can use the constant-acceleration equations; in particular, we can use Eq. 2-16 ($v^2 = v_0^2 + 2a(x - x_0)$), which relates velocity and position.

***First try:*** Normally we want to use an equation that includes the requested variable. In Eq. 2-16, we can identify $x_0$ as 0 and $v_0$ as being the requested variable. Then we can identify a second pair of values as being $v$ and $x$. From the graph, we have

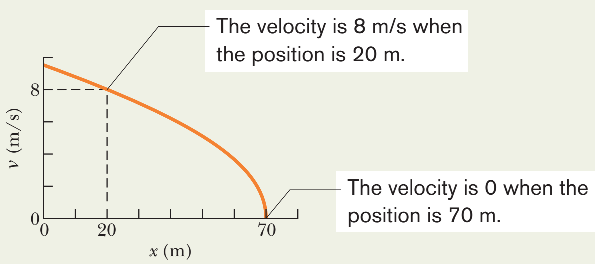


**Fig. 2-9** Velocity versus position.

two such pairs: (1) $v = 8$ m/s and $x = 20$ m, and (2) $v = 0$ and $x = 70$ m. For example, we can write Eq. 2-16 as

$$(8 \text{ m/s})^2 = v_0^2 + 2a(20 \text{ m} - 0). \tag{2-19}$$

However, we know neither $v_0$ nor $a$.

***Second try:*** Instead of directly involving the requested variable, let's use Eq. 2-16 with the two pairs of known data, identifying $v_0 = 8$ m/s and $x_0 = 20$ m as the first pair and $v = 0$ m/s and $x = 70$ m as the second pair. Then we can write

$$(0 \text{ m/s})^2 = (8 \text{ m/s})^2 + 2a(70 \text{ m} - 20 \text{ m}),$$

which gives us $a = -0.64$ m/s$^2$. Substituting this value into Eq. 2-19 and solving for $v_0$ (the velocity associated with the position of $x = 0$), we find

$$v_0 = 9.5 \text{ m/s}. \qquad \text{(Answer)}$$

***Comment:*** Some problems involve an equation that includes the requested variable. A more challenging problem requires you to first use an equation that does *not* include the requested variable but that gives you a value needed to find it. Sometimes that procedure takes *physics courage* because it is so indirect. However, if you build your solving skills by solving lots of problems, the procedure gradually requires less courage and may even become obvious. Solving problems of any kind, whether physics or social, requires practice.



## 2-8 Another Look at Constant Acceleration*

The first two equations in Table 2-1 are the basic equations from which the others are derived. Those two can be obtained by integration of the acceleration with the condition that $a$ is constant. To find Eq. 2-11, we rewrite the definition of acceleration (Eq. 2-8) as

$$dv = a\,dt.$$

We next write the *indefinite integral* (or *antiderivative*) of both sides:

$$\int dv = \int a\,dt.$$

Since acceleration $a$ is a constant, it can be taken outside the integration. We obtain

$$\int dv = a \int dt$$

or

$$v = at + C. \tag{2-20}$$

To evaluate the constant of integration $C$, we let $t = 0$, at which time $v = v_0$. Substituting these values into Eq. 2-20 (which must hold for all values of $t$,

*This section is intended for students who have had integral calculus.

including $t = 0$) yields

$$v_0 = (a)(0) + C = C.$$

Substituting this into Eq. 2-20 gives us Eq. 2-11.

To derive Eq. 2-15, we rewrite the definition of velocity (Eq. 2-4) as

$$dx = v\,dt$$

and then take the indefinite integral of both sides to obtain

$$\int dx = \int v\,dt.$$

Next, we substitute for $v$ with Eq. 2-11:

$$\int dx = \int (v_0 + at)\,dt.$$

Since $v_0$ is a constant, as is the acceleration $a$, this can be rewritten as

$$\int dx = v_0 \int dt + a \int t\,dt.$$

Integration now yields

$$x = v_0 t + \tfrac{1}{2}at^2 + C', \tag{2-21}$$

where $C'$ is another constant of integration. At time $t = 0$, we have $x = x_0$. Substituting these values in Eq. 2-21 yields $x_0 = C'$. Replacing $C'$ with $x_0$ in Eq. 2-21 gives us Eq. 2-15.

## 2-9 Free-Fall Acceleration

If you tossed an object either up or down and could somehow eliminate the effects of air on its flight, you would find that the object accelerates downward at a certain constant rate. That rate is called the **free-fall acceleration,** and its magnitude is represented by $g$. The acceleration is independent of the object's characteristics, such as mass, density, or shape; it is the same for all objects.

Two examples of free-fall acceleration are shown in Fig. 2-10, which is a series of stroboscopic photos of a feather and an apple. As these objects fall, they accelerate downward—both at the same rate $g$. Thus, their speeds increase at the same rate, and they fall together.

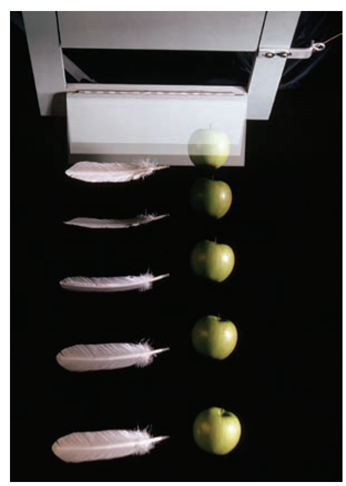

**Fig. 2-10** A feather and an apple free fall in vacuum at the same magnitude of acceleration $g$. The acceleration increases the distance between successive images. In the absence of air, the feather and apple fall together. *(Jim Sugar/Corbis Images)*

The value of $g$ varies slightly with latitude and with elevation. At sea level in Earth's midlatitudes the value is 9.8 m/s$^2$ (or 32 ft/s$^2$), which is what you should use as an exact number for the problems in this book unless otherwise noted.

The equations of motion in Table 2-1 for constant acceleration also apply to free fall near Earth's surface; that is, they apply to an object in vertical flight, either up or down, when the effects of the air can be neglected. However, note that for free fall: (1) The directions of motion are now along a vertical $y$ axis instead of the $x$ axis, with the positive direction of $y$ upward. (This is important for later chapters when combined horizontal and vertical motions are examined.) (2) The free-fall acceleration is negative—that is, downward on the $y$ axis, toward Earth's center—and so it has the value $-g$ in the equations.

The free-fall acceleration near Earth's surface is $a = -g = -9.8$ m/s$^2$, and the *magnitude* of the acceleration is $g = 9.8$ m/s$^2$. Do not substitute $-9.8$ m/s$^2$ for $g$.

Suppose you toss a tomato directly upward with an initial (positive) velocity $v_0$ and then catch it when it returns to the release level. During its *free-fall flight* (from just after its release to just before it is caught), the equations of Table 2-1 apply to its motion. The acceleration is always $a = -g = -9.8 \text{ m/s}^2$, negative and thus downward. The velocity, however, changes, as indicated by Eqs. 2-11 and 2-16: during the ascent, the magnitude of the positive velocity decreases, until it momentarily becomes zero. Because the tomato has then stopped, it is at its maximum height. During the descent, the magnitude of the (now negative) velocity increases.

### CHECKPOINT 5

(a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

## Sample Problem

### Time for full up-down flight, baseball toss

In Fig. 2-11, a pitcher tosses a baseball up along a $y$ axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

**KEY IDEAS**

(1) Once the ball leaves the pitcher and before it returns to his hand, its acceleration is the free-fall acceleration $a = -g$. Because this is constant, Table 2-1 applies to the motion. (2) The velocity $v$ at the maximum height must be 0.

***Calculation:*** Knowing $v$, $a$, and the initial velocity $v_0 = 12$ m/s, and seeking $t$, we solve Eq. 2-11, which contains those four variables. This yields

$$t = \frac{v - v_0}{a} = \frac{0 - 12 \text{ m/s}}{-9.8 \text{ m/s}^2} = 1.2 \text{ s.} \qquad \text{(Answer)}$$

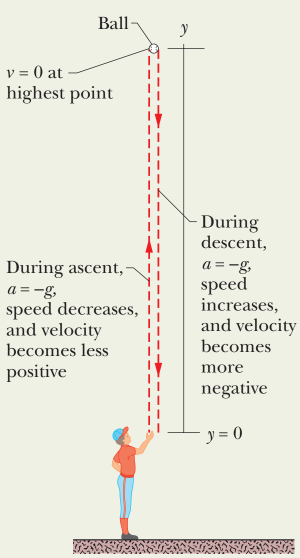


**Fig. 2-11** A pitcher tosses a baseball straight up into the air. The equations of free fall apply for rising as well as for falling objects, provided any effects from the air can be neglected.

(b) What is the ball's maximum height above its release point?

***Calculation:*** We can take the ball's release point to be $y_0 = 0$. We can then write Eq. 2-16 in $y$ notation, set $y - y_0 = y$ and $v = 0$ (at the maximum height), and solve for $y$. We get

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 7.3 \text{ m.} \qquad \text{(Answer)}$$

(c) How long does the ball take to reach a point 5.0 m above its release point?

***Calculations:*** We know $v_0$, $a = -g$, and displacement $y - y_0 = 5.0$ m, and we want $t$, so we choose Eq. 2-15. Rewriting it for $y$ and setting $y_0 = 0$ give us

$$y = v_0 t - \tfrac{1}{2} g t^2,$$

or

$$5.0 \text{ m} = (12 \text{ m/s})t - (\tfrac{1}{2})(9.8 \text{ m/s}^2)t^2.$$

If we temporarily omit the units (having noted that they are consistent), we can rewrite this as

$$4.9t^2 - 12t + 5.0 = 0.$$

Solving this quadratic equation for $t$ yields

$$t = 0.53 \text{ s} \quad \text{and} \quad t = 1.9 \text{ s.} \qquad \text{(Answer)}$$

There are two such times! This is not really surprising because the ball passes twice through $y = 5.0$ m, once on the way up and once on the way down.

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## 2-10 Graphical Integration in Motion Analysis

When we have a graph of an object's acceleration versus time, we can integrate on the graph to find the object's velocity at any given time. Because acceleration $a$ is defined in terms of velocity as $a = dv/dt$, the Fundamental Theorem of Calculus tells us that

$$v_1 - v_0 = \int_{t_0}^{t_1} a\, dt. \tag{2-22}$$

The right side of the equation is a definite integral (it gives a numerical result rather than a function), $v_0$ is the velocity at time $t_0$, and $v_1$ is the velocity at later time $t_1$. The definite integral can be evaluated from an $a(t)$ graph, such as in Fig. 2-12*a*. In particular,

$$\int_{t_0}^{t_1} a\, dt = \begin{pmatrix}\text{area between acceleration curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}. \tag{2-23}$$

If a unit of acceleration is 1 m/s$^2$ and a unit of time is 1 s, then the corresponding unit of area on the graph is

$$(1\ \text{m/s}^2)(1\ \text{s}) = 1\ \text{m/s},$$

which is (properly) a unit of velocity. When the acceleration curve is above the time axis, the area is positive; when the curve is below the time axis, the area is negative.

Similarly, because velocity $v$ is defined in terms of the position $x$ as $v = dx/dt$, then

$$x_1 - x_0 = \int_{t_0}^{t_1} v\, dt, \tag{2-24}$$

where $x_0$ is the position at time $t_0$ and $x_1$ is the position at time $t_1$. The definite integral on the right side of Eq. 2-24 can be evaluated from a $v(t)$ graph, like that shown in Fig. 2-12*b*. In particular,

$$\int_{t_0}^{t_1} v\, dt = \begin{pmatrix}\text{area between velocity curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}. \tag{2-25}$$

If the unit of velocity is 1 m/s and the unit of time is 1 s, then the corresponding unit of area on the graph is

$$(1\ \text{m/s})(1\ \text{s}) = 1\ \text{m},$$

which is (properly) a unit of position and displacement. Whether this area is positive or negative is determined as described for the $a(t)$ curve of Fig. 2-12*a*.

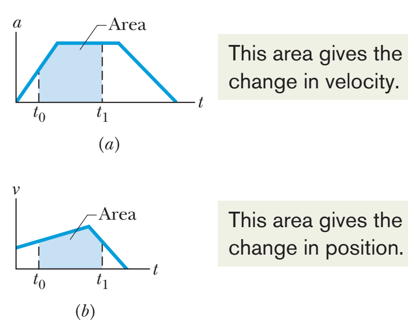


**Fig. 2-12** The area between a plotted curve and the horizontal time axis, from time $t_0$ to time $t_1$, is indicated for (*a*) a graph of acceleration $a$ versus $t$ and (*b*) a graph of velocity $v$ versus $t$.

## Sample Problem

### Graphical integration *a* versus *t*, whiplash injury

"Whiplash injury" commonly occurs in a rear-end collision where a front car is hit from behind by a second car. In the 1970s, researchers concluded that the injury was due to the occupant's head being whipped back over the top of the seat as the car was slammed forward. As a result of this finding, head restraints were built into cars, yet neck injuries in rear-end collisions continued to occur.

In a recent test to study neck injury in rear-end collisions, a volunteer was strapped to a seat that was then moved abruptly to simulate a collision by a rear car moving at 10.5 km/h. Figure 2-13*a* gives the accelerations of the volunteer's torso and head during the collision, which began at time $t = 0$. The torso acceleration was delayed by 40 ms because during that time interval the seat back had to compress against the volunteer. The head acceleration was delayed by an additional 70 ms. What was the torso speed when the head began to accelerate?

**KEY IDEA**

We can calculate the torso speed at any time by finding an area on the torso $a(t)$ graph.

***Calculations:*** We know that the initial torso speed is $v_0 = 0$ at time $t_0 = 0$, at the start of the "collision." We want the torso speed $v_1$ at time $t_1 = 110$ ms, which is when the head begins to accelerate.

Combining Eqs. 2-22 and 2-23, we can write

$$v_1 - v_0 = \begin{pmatrix}\text{area between acceleration curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}. \tag{2-26}$$

For convenience, let us separate the area into three regions (Fig. 2-13*b*). From 0 to 40 ms, region *A* has no area:

$$\text{area}_A = 0.$$

From 40 ms to 100 ms, region *B* has the shape of a triangle, with area

$$\text{area}_B = \tfrac{1}{2}(0.060\ \text{s})(50\ \text{m/s}^2) = 1.5\ \text{m/s}.$$

From 100 ms to 110 ms, region *C* has the shape of a rectangle, with area

$$\text{area}_C = (0.010\ \text{s})(50\ \text{m/s}^2) = 0.50\ \text{m/s}.$$

Substituting these values and $v_0 = 0$ into Eq. 2-26 gives us

$$v_1 - 0 = 0 + 1.5\ \text{m/s} + 0.50\ \text{m/s},$$

or

$$v_1 = 2.0\ \text{m/s} = 7.2\ \text{km/h}. \qquad \text{(Answer)}$$

***Comments:*** When the head is just starting to move forward, the torso already has a speed of 7.2 km/h. Researchers argue that it is this difference in speeds during the early stage of a rear-end collision that injures the neck. The backward whipping of the head happens later and could, especially if there is no head restraint, increase the injury.

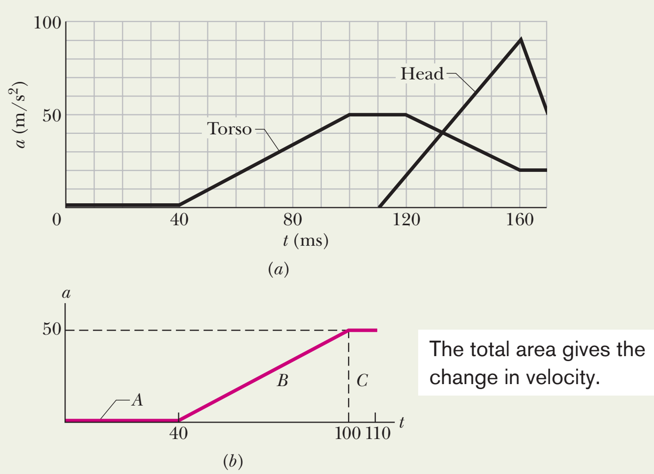


**Fig. 2-13** (*a*) The $a(t)$ curve of the torso and head of a volunteer in a simulation of a rear-end collision. (*b*) Breaking up the region between the plotted curve and the time axis to calculate the area.

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## REVIEW & SUMMARY

**Position** The *position* $x$ of a particle on an $x$ axis locates the particle with respect to the **origin,** or zero point, of the axis. The position is either positive or negative, according to which side of the origin the particle is on, or zero if the particle is at the origin. The **positive direction** on an axis is the direction of increasing positive numbers; the opposite direction is the **negative direction** on the axis.

**Displacement** The *displacement* $\Delta x$ of a particle is the change in its position:

$$\Delta x = x_2 - x_1. \tag{2-1}$$

Displacement is a vector quantity. It is positive if the particle has moved in the positive direction of the $x$ axis and negative if the particle has moved in the negative direction.

**Average Velocity** When a particle has moved from position $x_1$ to position $x_2$ during a time interval $\Delta t = t_2 - t_1$, its *average velocity* during that interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \tag{2-2}$$

The algebraic sign of $v_{\text{avg}}$ indicates the direction of motion ($v_{\text{avg}}$ is a vector quantity). Average velocity does not depend on the actual distance a particle moves, but instead depends on its original and final positions.

On a graph of $x$ versus $t$, the average velocity for a time interval $\Delta t$ is the slope of the straight line connecting the points on the curve that represent the two ends of the interval.

**Average Speed** The *average speed* $s_{\text{avg}}$ of a particle during a time interval $\Delta t$ depends on the total distance the particle moves in that time interval:

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}. \tag{2-3}$$

**Instantaneous Velocity** The *instantaneous velocity* (or simply **velocity**) $v$ of a moving particle is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}, \tag{2-4}$$

where $\Delta x$ and $\Delta t$ are defined by Eq. 2-2. The instantaneous velocity (at a particular time) may be found as the slope (at that particular time) of the graph of $x$ versus $t$. **Speed** is the magnitude of instantaneous velocity.

**Average Acceleration** *Average acceleration* is the ratio of a change in velocity $\Delta v$ to the time interval $\Delta t$ in which the change occurs:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}. \tag{2-7}$$

The algebraic sign indicates the direction of $a_{\text{avg}}$.

**Instantaneous Acceleration** *Instantaneous acceleration* (or simply **acceleration**) $a$ is the first time derivative of velocity $v(t)$ and the second time derivative of position $x(t)$:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}. \tag{2-8, 2-9}$$

On a graph of $v$ versus $t$, the acceleration $a$ at any time $t$ is the slope of the curve at the point that represents $t$.

**Constant Acceleration** The five equations in Table 2-1 describe the motion of a particle with constant acceleration:

$$v = v_0 + at, \tag{2-11}$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2, \tag{2-15}$$

$$v^2 = v_0^2 + 2a(x - x_0), \tag{2-16}$$

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t, \tag{2-17}$$

$$x - x_0 = vt - \tfrac{1}{2}at^2. \tag{2-18}$$

These are *not* valid when the acceleration is not constant.

**Free-Fall Acceleration** An important example of straight-line motion with constant acceleration is that of an object rising or falling freely near Earth's surface. The constant acceleration equations describe this motion, but we make two changes in notation: (1) we refer the motion to the vertical $y$ axis with $+y$ vertically *up*; (2) we replace $a$ with $-g$, where $g$ is the magnitude of the free-fall acceleration. Near Earth's surface, $g = 9.8\ \text{m/s}^2$ ($= 32\ \text{ft/s}^2$).

**1** Figure 2-14 gives the velocity of a particle moving on an $x$ axis. What are (a) the initial and (b) the final directions of travel? (c) Does the particle stop momentarily? (d) Is the acceleration positive or negative? (e) Is it constant or varying?

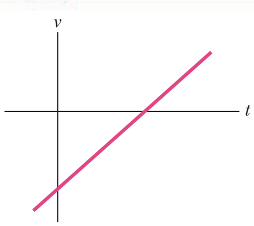


**Fig. 2-14** Question 1.

**2** Figure 2-15 gives the acceleration $a(t)$ of a Chihuahua as it chases

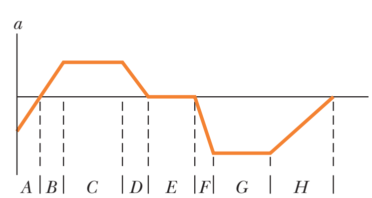


**Fig. 2-15** Question 2.

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a German shepherd along an axis. In which of the time periods indicated does the Chihuahua move at constant speed?

**3** Figure 2-16 shows four paths along which objects move from a starting point to a final point, all in the same time interval. The paths pass over a grid of equally spaced straight lines. Rank the paths according to (a) the average velocity of the objects and (b) the average speed of the objects, greatest first.

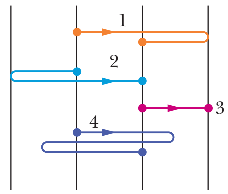


**Fig. 2-16** Question 3.

**4** Figure 2-17 is a graph of a particle's position along an $x$ axis versus time. (a) At time $t = 0$, what is the sign of the particle's position? Is the particle's velocity positive, negative, or 0 at (b) $t = 1$ s, (c) $t = 2$ s, and (d) $t = 3$ s? (e) How many times does the particle go through the point $x = 0$?

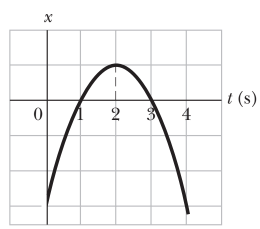


**Fig. 2-17** Question 4.

**5** Figure 2-18 gives the velocity of a particle moving along an axis. Point 1 is at the highest point on the curve; point 4 is at the lowest point; and points 2 and 6 are at the same height. What is the direction of travel at (a) time $t = 0$ and (b) point 4? (c) At which of the six numbered points does the particle reverse its direction of travel? (d) Rank the six points according to the magnitude of the acceleration, greatest first.

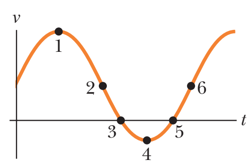


**Fig. 2-18** Question 5.

**6** At $t = 0$, a particle moving along an $x$ axis is at position $x_0 = -20$ m. The signs of the particle's initial velocity $v_0$ (at time $t_0$) and constant acceleration $a$ are, respectively, for four situations: (1) +, +; (2) +, −; (3) −, +; (4) −, −. In which situations will the particle (a) stop momentarily, (b) pass through the origin, and (c) never pass through the origin?

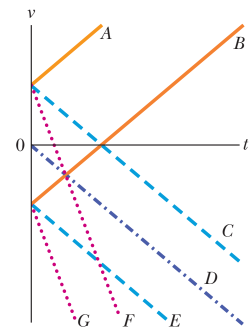


**Fig. 2-19** Question 7.

**7** Hanging over the railing of a bridge, you drop an egg (no initial velocity) as you throw a second egg downward. Which curves in Fig. 2-19 give the velocity $v(t)$ for (a) the dropped egg and (b) the thrown egg? (Curves $A$ and $B$ are parallel; so are $C$, $D$, and $E$; so are $F$ and $G$.)

**8** The following equations give the velocity $v(t)$ of a particle in four situations: (a) $v = 3$; (b) $v = 4t^2 + 2t - 6$; (c) $v = 3t - 4$; (d) $v = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

**9** In Fig. 2-20, a cream tangerine is thrown directly upward past three evenly spaced windows of equal heights. Rank the windows according to (a) the average speed of the cream tangerine while passing them, (b) the time the cream tangerine takes to pass them, (c) the magnitude of the acceleration of the cream tangerine while passing them, and (d) the change $\Delta v$ in the speed of the cream tangerine during the passage, greatest first.

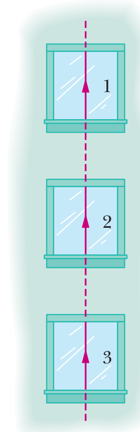


**Fig. 2-20** Question 9.



## PROBLEMS

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS* and WebAssign

SSM Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at

ILW Interactive solution is at

http://www.wiley.com/college/halliday

• – ••• Number of dots indicates level of problem difficulty

Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

### sec. 2-4 Average Velocity and Average Speed

•**1** During a hard sneeze, your eyes might shut for 0.50 s. If you are driving a car at 90 km/h during such a sneeze, how far does the car move during that time?

•**2** Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track. (b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track. (c) Graph $x$ versus $t$ for both cases and indicate how the average velocity is found on the graph.

•**3** SSM WWW An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the same direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during the full 80 km trip? (Assume that it moves in the positive $x$ direction.) (b) What is the average speed? (c) Graph $x$ versus $t$ and indicate how the average velocity is found on the graph.

•**4** A car travels up a hill at a constant speed of 40 km/h and returns down the hill at a constant speed of 60 km/h. Calculate the average speed for the round trip.

•**5** SSM The position of an object moving along an $x$ axis is given by $x = 3t - 4t^2 + t^3$, where $x$ is in meters and $t$ in seconds. Find the position of the object at the following values of $t$: (a) 1 s, (b) 2 s, (c) 3 s, and (d) 4 s. (e) What is the object's displacement between $t = 0$ and $t = 4$ s? (f) What is its average velocity for the time interval from $t = 2$ s to $t = 4$ s? (g) Graph $x$ versus $t$ for $0 \leq t \leq 4$ s and indicate how the answer for (f) can be found on the graph.

•**6** The 1992 world speed record for a bicycle (human-powered vehicle) was set by Chris Huber. His time through the measured

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200 m stretch was a sizzling 6.509 s, at which he commented, "Cogito ergo zoom!" (I think, therefore I go fast!). In 2001, Sam Whittingham beat Huber's record by 19.0 km/h. What was Whittingham's time through the 200 m?

••7 Two trains, each having a speed of 30 km/h, are headed at each other on the same straight track. A bird that can fly 60 km/h flies off the front of one train when they are 60 km apart and heads directly for the other train. On reaching the other train, the bird flies directly back to the first train, and so forth. (We have no idea *why* a bird would behave in this way.) What is the total distance the bird travels before the trains collide?

••8 GO *Panic escape*. Figure 2-21 shows a general situation in which a stream of people attempt to escape through an exit door that turns out to be locked. The people move toward the door at speed $v_s = 3.50$ m/s, are each $d = 0.25$ m in depth, and are separated by $L = 1.75$ m. The arrangement in Fig. 2-21 occurs at time $t = 0$. (a) At what average rate does the layer of people at the door increase? (b) At what time does the layer's depth reach 5.0 m? (The answers reveal how quickly such a situation becomes dangerous.)

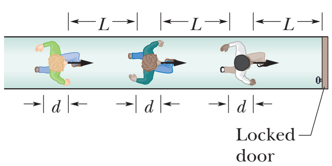


**Fig. 2-21** Problem 8.

••9 ILW In 1 km races, runner 1 on track 1 (with time 2 min, 27.95 s) appears to be faster than runner 2 on track 2 (2 min, 28.15 s). However, length $L_2$ of track 2 might be slightly greater than length $L_1$ of track 1. How large can $L_2 - L_1$ be for us still to conclude that runner 1 is faster?

••10 To set a speed record in a measured (straight-line) distance $d$, a race car must be driven first in one direction (in time $t_1$) and then in the opposite direction (in time $t_2$). (a) To eliminate the effects of the wind and obtain the car's speed $v_c$ in a windless situation, should we find the average of $d/t_1$ and $d/t_2$ (method 1) or should we divide $d$ by the average of $t_1$ and $t_2$? (b) What is the fractional difference in the two methods when a steady wind blows along the car's route and the ratio of the wind speed $v_w$ to the car's speed $v_c$ is 0.0240?

••11 You are to drive to an interview in another town, at a distance of 300 km on an expressway. The interview is at 11:15 A.M. You plan to drive at 100 km/h, so you leave at 8:00 A.M. to allow some extra time. You drive at that speed for the first 100 km, but then construction work forces you to slow to 40 km/h for 40 km. What would be the least speed needed for the rest of the trip to arrive in time for the interview?

•••12 *Traffic shock wave*. An abrupt slowdown in concentrated traffic can travel as a pulse, termed a *shock wave*, along the line of cars, either downstream (in the traffic direction) or upstream, or it can be stationary. Figure 2-22 shows a uniformly spaced line of cars moving at speed $v = 25.0$ m/s toward a uniformly spaced line of slow cars moving at speed $v_s = 5.00$ m/s. Assume that each faster car adds length $L = 12.0$ m (car length plus buffer zone) to the line of slow cars when it joins the line, and assume it slows abruptly at the last instant. (a) For what separation distance $d$ between the faster cars does the shock wave remain stationary? If the separation is twice that amount, what are the (b) speed and (c) direction (upstream or downstream) of the shock wave?

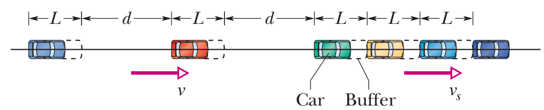


**Fig. 2-22** Problem 12.

•••13 ILW You drive on Interstate 10 from San Antonio to Houston, half the *time* at 55 km/h and the other half at 90 km/h. On the way back you travel half the *distance* at 55 km/h and the other half at 90 km/h. What is your average speed (a) from San Antonio to Houston, (b) from Houston back to San Antonio, and (c) for the entire trip? (d) What is your average velocity for the entire trip? (e) Sketch $x$ versus $t$ for (a), assuming the motion is all in the positive $x$ direction. Indicate how the average velocity can be found on the sketch.

### sec. 2-5 Instantaneous Velocity and Speed

•14 GO An electron moving along the $x$ axis has a position given by $x = 16te^{-t}$ m, where $t$ is in seconds. How far is the electron from the origin when it momentarily stops?

•15 GO (a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where $t$ is in seconds and $x$ is in meters), what is its velocity at $t = 1$ s? (b) Is it moving in the positive or negative direction of $x$ just then? (c) What is its speed just then? (d) Is the speed increasing or decreasing just then? (Try answering the next two questions without further calculation.) (e) Is there ever an instant when the velocity is zero? If so, give the time $t$; if not, answer no. (f) Is there a time after $t = 3$ s when the particle is moving in the negative direction of $x$? If so, give the time $t$; if not, answer no.

•16 The position function $x(t)$ of a particle moving along an $x$ axis is $x = 4.0 - 6.0t^2$, with $x$ in meters and $t$ in seconds. (a) At what time and (b) where does the particle (momentarily) stop? At what (c) negative time and (d) positive time does the particle pass through the origin? (e) Graph $x$ versus $t$ for the range $-5$ s to $+5$ s. (f) To shift the curve rightward on the graph, should we include the term $+20t$ or the term $-20t$ in $x(t)$? (g) Does that inclusion increase or decrease the value of $x$ at which the particle momentarily stops?

••17 The position of a particle moving along the $x$ axis is given in centimeters by $x = 9.75 + 1.50t^3$, where $t$ is in seconds. Calculate (a) the average velocity during the time interval $t = 2.00$ s to $t = 3.00$ s; (b) the instantaneous velocity at $t = 2.00$ s; (c) the instantaneous velocity at $t = 3.00$ s; (d) the instantaneous velocity at $t = 2.50$ s; and (e) the instantaneous velocity when the particle is midway between its positions at $t = 2.00$ s and $t = 3.00$ s. (f) Graph $x$ versus $t$ and indicate your answers graphically.

### sec. 2-6 Acceleration

•18 The position of a particle moving along an $x$ axis is given by $x = 12t^2 - 2t^3$, where $x$ is in meters and $t$ is in seconds. Determine (a) the position, (b) the velocity, and (c) the acceleration of the particle at $t = 3.0$ s. (d) What is the maximum positive coordinate reached by the particle and (e) at what time is it reached? (f) What is the maximum positive velocity reached by the particle and (g) at

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what time is it reached? (h) What is the acceleration of the particle at the instant the particle is not moving (other than at $t = 0$)? (i) Determine the average velocity of the particle between $t = 0$ and $t = 3$ s.

**•19** SSM At a certain time a particle had a speed of 18 m/s in the positive $x$ direction, and 2.4 s later its speed was 30 m/s in the opposite direction. What is the average acceleration of the particle during this 2.4 s interval?

**•20** (a) If the position of a particle is given by $x = 20t - 5t^3$, where $x$ is in meters and $t$ is in seconds, when, if ever, is the particle's velocity zero? (b) When is its acceleration $a$ zero? (c) For what time range (positive or negative) is $a$ negative? (d) Positive? (e) Graph $x(t)$, $v(t)$, and $a(t)$.

**••21** From $t = 0$ to $t = 5.00$ min, a man stands still, and from $t = 5.00$ min to $t = 10.0$ min, he walks briskly in a straight line at a constant speed of 2.20 m/s. What are (a) his average velocity $v_{avg}$ and (b) his average acceleration $a_{avg}$ in the time interval 2.00 min to 8.00 min? What are (c) $v_{avg}$ and (d) $a_{avg}$ in the time interval 3.00 min to 9.00 min? (e) Sketch $x$ versus $t$ and $v$ versus $t$, and indicate how the answers to (a) through (d) can be obtained from the graphs.

**••22** The position of a particle moving along the $x$ axis depends on the time according to the equation $x = ct^2 - bt^3$, where $x$ is in meters and $t$ in seconds. What are the units of (a) constant $c$ and (b) constant $b$? Let their numerical values be 3.0 and 2.0, respectively. (c) At what time does the particle reach its maximum positive $x$ position? From $t = 0.0$ s to $t = 4.0$ s, (d) what distance does the particle move and (e) what is its displacement? Find its velocity at times (f) 1.0 s, (g) 2.0 s, (h) 3.0 s, and (i) 4.0 s. Find its acceleration at times (j) 1.0 s, (k) 2.0 s, (l) 3.0 s, and (m) 4.0 s.

### sec. 2-7 Constant Acceleration: A Special Case

**•23** SSM An electron with an initial velocity $v_0 = 1.50 \times 10^5$ m/s enters a region of length $L = 1.00$ cm where it is electrically accelerated (Fig. 2-23). It emerges with $v = 5.70 \times 10^6$ m/s. What is its acceleration, assumed constant?

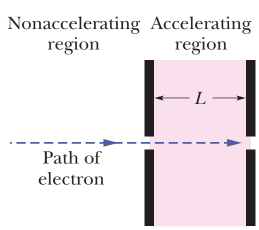



**Fig. 2-23** Problem 23.

**•24** *Catapulting mushrooms*. Certain mushrooms launch their spores by a catapult mechanism. As water condenses from the air onto a spore that is attached to the mushroom, a drop grows on one side of the spore and a film grows on the other side. The spore is bent over by the drop's weight, but when the film reaches the drop, the drop's water suddenly spreads into the film and the spore springs upward so rapidly that it is slung off into the air. Typically, the spore reaches a speed of 1.6 m/s in a 5.0 $\mu$m launch; its speed is then reduced to zero in 1.0 mm by the air. Using that data and assuming constant accelerations, find the acceleration in terms of $g$ during (a) the launch and (b) the speed reduction.

**•25** An electric vehicle starts from rest and accelerates at a rate of 2.0 m/s$^2$ in a straight line until it reaches a speed of 20 m/s. The vehicle then slows at a constant rate of 1.0 m/s$^2$ until it stops. (a) How much time elapses from start to stop? (b) How far does the vehicle travel from start to stop?

**•26** A muon (an elementary particle) enters a region with a speed of $5.00 \times 10^6$ m/s and then is slowed at the rate of $1.25 \times 10^{14}$ m/s$^2$. (a) How far does the muon take to stop? (b) Graph $x$ versus $t$ and $v$ versus $t$ for the muon.

**•27** An electron has a constant acceleration of +3.2 m/s$^2$. At a certain instant its velocity is +9.6 m/s. What is its velocity (a) 2.5 s earlier and (b) 2.5 s later?

**•28** On a dry road, a car with good tires may be able to brake with a constant deceleration of 4.92 m/s$^2$. (a) How long does such a car, initially traveling at 24.6 m/s, take to stop? (b) How far does it travel in this time? (c) Graph $x$ versus $t$ and $v$ versus $t$ for the deceleration.

**•29** ILW A certain elevator cab has a total run of 190 m and a maximum speed of 305 m/min, and it accelerates from rest and then back to rest at 1.22 m/s$^2$. (a) How far does the cab move while accelerating to full speed from rest? (b) How long does it take to make the nonstop 190 m run, starting and ending at rest?

**•30** The brakes on your car can slow you at a rate of 5.2 m/s$^2$. (a) If you are going 137 km/h and suddenly see a state trooper, what is the minimum time in which you can get your car under the 90 km/h speed limit? (The answer reveals the futility of braking to keep your high speed from being detected with a radar or laser gun.) (b) Graph $x$ versus $t$ and $v$ versus $t$ for such a slowing.

**•31** SSM Suppose a rocket ship in deep space moves with constant acceleration equal to 9.8 m/s$^2$, which gives the illusion of normal gravity during the flight. (a) If it starts from rest, how long will it take to acquire a speed one-tenth that of light, which travels at $3.0 \times 10^8$ m/s? (b) How far will it travel in so doing?

**•32** A world's land speed record was set by Colonel John P. Stapp when in March 1954 he rode a rocket-propelled sled that moved along a track at 1020 km/h. He and the sled were brought to a stop in 1.4 s. (See Fig. 2-7.) In terms of $g$, what acceleration did he experience while stopping?

**•33** SSM ILW A car traveling 56.0 km/h is 24.0 m from a barrier when the driver slams on the brakes. The car hits the barrier 2.00 s later. (a) What is the magnitude of the car's constant acceleration before impact? (b) How fast is the car traveling at impact?

**••34** GO In Fig. 2-24, a red car and a green car, identical except for the color, move toward each other in adjacent lanes and parallel to an $x$ axis. At time $t = 0$, the red car is at $x_r = 0$ and the green car is at $x_g = 220$ m. If the red car has a constant velocity of 20 km/h, the cars pass each other at $x = 44.5$ m, and if it has a constant velocity of 40 km/h, they pass each other at $x = 76.6$ m. What are (a) the initial velocity and (b) the constant acceleration of the green car?


**Fig. 2-24** Problems 34 and 35.

**••35** Figure 2-24 shows a red car and a green car that move toward each other. Figure 2-25 is a graph of their motion, showing the positions $x_{g0} = 270$ m and $x_{r0} = -35.0$ m at time $t = 0$. The green car has a constant speed of 20.0 m/s and the red car begins from rest. What is the acceleration magnitude of the red car?

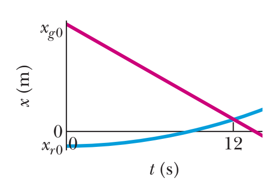



**Fig. 2-25** Problem 35.

••36 A car moves along an $x$ axis through a distance of 900 m, starting at rest (at $x = 0$) and ending at rest (at $x = 900$ m). Through the first $\frac{1}{4}$ of that distance, its acceleration is $+2.25$ m/s$^2$. Through the rest of that distance, its acceleration is $-0.750$ m/s$^2$. What are (a) its travel time through the 900 m and (b) its maximum speed? (c) Graph position $x$, velocity $v$, and acceleration $a$ versus time $t$ for the trip.

••37 Figure 2-26 depicts the motion of a particle moving along an $x$ axis with a constant acceleration. The figure's vertical scaling is set by $x_s = 6.0$ m. What are the (a) magnitude and (b) direction of the particle's acceleration?

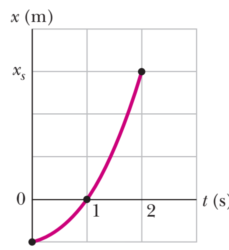



Fig. 2-26 Problem 37.

••38 (a) If the maximum acceleration that is tolerable for passengers in a subway train is 1.34 m/s$^2$ and subway stations are located 806 m apart, what is the maximum speed a subway train can attain between stations? (b) What is the travel time between stations? (c) If a subway train stops for 20 s at each station, what is the maximum average speed of the train, from one start-up to the next? (d) Graph $x$, $v$, and $a$ versus $t$ for the interval from one start-up to the next.

••39 Cars $A$ and $B$ move in the same direction in adjacent lanes. The position $x$ of car $A$ is given in Fig. 2-27, from time $t = 0$ to $t = 7.0$ s. The figure's vertical scaling is set by $x_s = 32.0$ m. At $t = 0$, car $B$ is at $x = 0$, with a velocity of 12 m/s and a negative constant acceleration $a_B$. (a) What must $a_B$ be such that the cars are (momentarily) side by side (momentarily at the same value of $x$) at $t = 4.0$ s? (b) For that value of $a_B$, how many times are the cars side by side? (c) Sketch the position $x$ of car $B$ versus time $t$ on Fig. 2-27. How many times will the cars be side by side if the magnitude of acceleration $a_B$ is (d) more than and (e) less than the answer to part (a)?

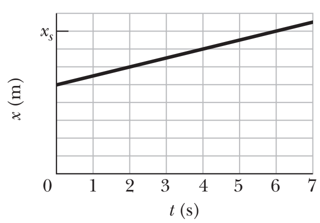



Fig. 2-27 Problem 39.

••40 You are driving toward a traffic signal when it turns yellow. Your speed is the legal speed limit of $v_0 = 55$ km/h; your best deceleration rate has the magnitude $a = 5.18$ m/s$^2$. Your best reaction time to begin braking is $T = 0.75$ s. To avoid having the front of your car enter the intersection after the light turns red, should you brake to a stop or continue to move at 55 km/h if the distance to the intersection and the duration of the yellow light are (a) 40 m and 2.8 s, and (b) 32 m and 1.8 s? Give an answer of brake, continue, either (if either strategy works), or neither (if neither strategy works and the yellow duration is inappropriate).

••41 As two trains move along a track, their conductors suddenly notice that they are headed toward each other. Figure 2-28 gives their velocities $v$ as functions of time $t$ as the conductors slow the trains. The figure's vertical scaling is set by $v_s = 40.0$ m/s. The slowing processes begin when the trains are 200 m apart. What is their separation when both trains have stopped?

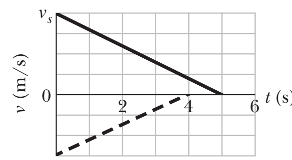



Fig. 2-28 Problem 41.

•••42 You are arguing over a cell phone while trailing an unmarked police car by 25 m; both your car and the police car are traveling at 110 km/h. Your argument diverts your attention from the police car for 2.0 s (long enough for you to look at the phone and yell, "I won't do that!"). At the beginning of that 2.0 s, the police officer begins braking suddenly at 5.0 m/s$^2$. (a) What is the separation between the two cars when your attention finally returns? Suppose that you take another 0.40 s to realize your danger and begin braking. (b) If you too brake at 5.0 m/s$^2$, what is your speed when you hit the police car?

•••43 GO When a high-speed passenger train traveling at 161 km/h rounds a bend, the engineer is shocked to see that a locomotive has improperly entered onto the track from a siding and is a distance $D = 676$ m ahead (Fig. 2-29). The locomotive is moving at 29.0 km/h. The engineer of the high-speed train immediately applies the brakes. (a) What must be the magnitude of the resulting constant deceleration if a collision is to be just avoided? (b) Assume that the engineer is at $x = 0$ when, at $t = 0$, he first spots the locomotive. Sketch $x(t)$ curves for the locomotive and high-speed train for the cases in which a collision is just avoided and is not quite avoided.

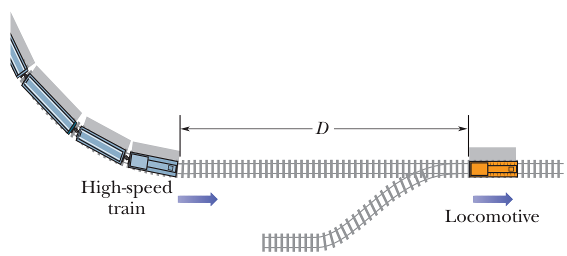



Fig. 2-29 Problem 43.

### sec. 2-9 Free-Fall Acceleration

•44 When startled, an armadillo will leap upward. Suppose it rises 0.544 m in the first 0.200 s. (a) What is its initial speed as it leaves the ground? (b) What is its speed at the height of 0.544 m? (c) How much higher does it go?

•45 SSM WWW (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50 m? (b) How long will it be in the air? (c) Sketch graphs of $y$, $v$, and $a$ versus $t$ for the ball. On the first two graphs, indicate the time at which 50 m is reached.

•46 Raindrops fall 1700 m from a cloud to the ground. (a) If they were not slowed by air resistance, how fast would the drops be moving when they struck the ground? (b) Would it be safe to walk outside during a rainstorm?

•47 SSM At a construction site a pipe wrench struck the ground with a speed of 24 m/s. (a) From what height was it inadvertently dropped? (b) How long was it falling? (c) Sketch graphs of $y$, $v$, and $a$ versus $t$ for the wrench.

•48 A hoodlum throws a stone vertically downward with an initial speed of 12.0 m/s from the roof of a building, 30.0 m above the ground. (a) How long does it take the stone to reach the ground? (b) What is the speed of the stone at impact?

•49 SSM A hot-air balloon is ascending at the rate of 12 m/s and is 80 m above the ground when a package is dropped over the side. (a) How long does the package take to reach the ground? (b) With what speed does it hit the ground?

••50 At time $t = 0$, apple 1 is dropped from a bridge onto a roadway beneath the bridge; somewhat later, apple 2 is thrown down from the same height. Figure 2-30 gives the vertical positions $y$ of the apples versus $t$ during the falling, until both apples have hit the roadway. The scaling is set by $t_s = 2.0$ s. With approximately what speed is apple 2 thrown down?

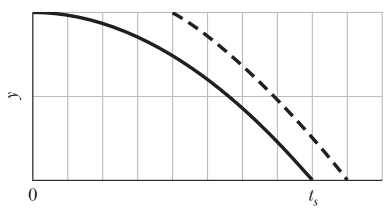



**Fig. 2-30** Problem 50.

••51 As a runaway scientific balloon ascends at 19.6 m/s, one of its instrument packages breaks free of a harness and free-falls. Figure 2-31 gives the vertical velocity of the package versus time, from before it breaks free to when it reaches the ground. (a) What maximum height above the break-free point does it rise? (b) How high is the break-free point above the ground?

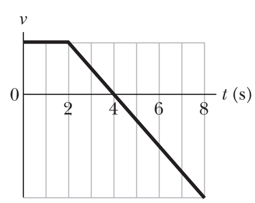



**Fig. 2-31** Problem 51.

••52 GO A bolt is dropped from a bridge under construction, falling 90 m to the valley below the bridge. (a) In how much time does it pass through the last 20% of its fall? What is its speed (b) when it begins that last 20% of its fall and (c) when it reaches the valley beneath the bridge?

••53 SSM ILW A key falls from a bridge that is 45 m above the water. It falls directly into a model boat, moving with constant velocity, that is 12 m from the point of impact when the key is released. What is the speed of the boat?

••54 A stone is dropped into a river from a bridge 43.9 m above the water. Another stone is thrown vertically down 1.00 s after the first is dropped. The stones strike the water at the same time. (a) What is the initial speed of the second stone? (b) Plot velocity versus time on a graph for each stone, taking zero time as the instant the first stone is released.

••55 SSM A ball of moist clay falls 15.0 m to the ground. It is in contact with the ground for 20.0 ms before stopping. (a) What is the magnitude of the average acceleration of the ball during the time it is in contact with the ground? (Treat the ball as a particle.) (b) Is the average acceleration up or down?

••56 GO Figure 2-32 shows the speed $v$ versus height $y$ of a ball tossed directly upward, along a $y$ axis. Distance $d$ is 0.40 m. The speed at height $y_A$ is $v_A$. The speed at height $y_B$ is $\frac{1}{3}v_A$. What is speed $v_A$?

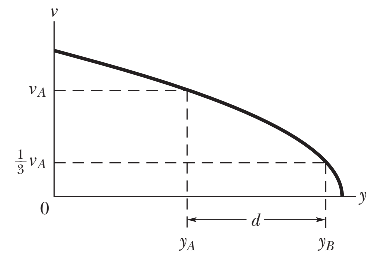



**Fig. 2-32** Problem 56.

••57 To test the quality of a tennis ball, you drop it onto the floor from a height of 4.00 m. It rebounds to a height of 2.00 m. If the ball is in contact with the floor for 12.0 ms, (a) what is the magnitude of its average acceleration during that contact and (b) is the average acceleration up or down?

••58 An object falls a distance $h$ from rest. If it travels $0.50h$ in the last 1.00 s, find (a) the time and (b) the height of its fall. (c) Explain the physically unacceptable solution of the quadratic equation in $t$ that you obtain.

••59 Water drips from the nozzle of a shower onto the floor 200 cm below. The drops fall at regular (equal) intervals of time, the first drop striking the floor at the instant the fourth drop begins to fall. When the first drop strikes the floor, how far below the nozzle are the (a) second and (b) third drops?

••60 A rock is thrown vertically upward from ground level at time $t = 0$. At $t = 1.5$ s it passes the top of a tall tower, and 1.0 s later it reaches its maximum height. What is the height of the tower?

•••61 GO A steel ball is dropped from a building's roof and passes a window, taking 0.125 s to fall from the top to the bottom of the window, a distance of 1.20 m. It then falls to a sidewalk and bounces back past the window, moving from bottom to top in 0.125 s. Assume that the upward flight is an exact reverse of the fall. The time the ball spends below the bottom of the window is 2.00 s. How tall is the building?

•••62 A basketball player grabbing a rebound jumps 76.0 cm vertically. How much total time (ascent and descent) does the player spend (a) in the top 15.0 cm of this jump and (b) in the bottom 15.0 cm? Do your results explain why such players seem to hang in the air at the top of a jump?

•••63 GO A drowsy cat spots a flowerpot that sails first up and then down past an open window. The pot is in view for a total of 0.50 s, and the top-to-bottom height of the window is 2.00 m. How high above the window top does the flowerpot go?

•••64 A ball is shot vertically upward from the surface of another planet. A plot of $y$ versus $t$ for the ball is shown in Fig. 2-33, where $y$ is the

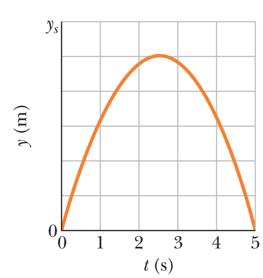



**Fig. 2-33** Problem 64.

height of the ball above its starting point and $t = 0$ at the instant the ball is shot. The figure's vertical scaling is set by $y_s = 30.0$ m. What are the magnitudes of (a) the free-fall acceleration on the planet and (b) the initial velocity of the ball?

### sec. 2-10 Graphical Integration in Motion Analysis

**•65** Figure 2-13*a* gives the acceleration of a volunteer's head and torso during a rear-end collision. At maximum head acceleration, what is the speed of (a) the head and (b) the torso?

**••66** In a forward punch in karate, the fist begins at rest at the waist and is brought rapidly forward until the arm is fully extended. The speed $v(t)$ of the fist is given in Fig. 2-34 for someone skilled in karate. The vertical scaling is set by $v_s = 8.0$ m/s. How far has the fist moved at (a) time $t = 50$ ms and (b) when the speed of the fist is maximum?

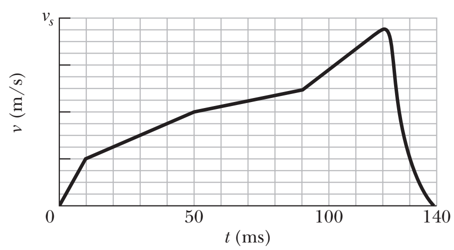



**Fig. 2-34** Problem 66.

**••67** When a soccer ball is kicked toward a player and the player deflects the ball by "heading" it, the acceleration of the head during the collision can be significant. Figure 2-35 gives the measured acceleration $a(t)$ of a soccer player's head for a bare head and a helmeted head, starting from rest. The scaling on the vertical axis is set by $a_s = 200$ m/s$^2$. At time $t = 7.0$ ms, what is the difference in the speed acquired by the bare head and the speed acquired by the helmeted head?

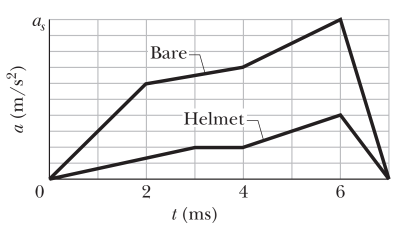



**Fig. 2-35** Problem 67.

**••68** A salamander of the genus *Hydromantes* captures prey by launching its tongue as a projectile: The skeletal part of the tongue is shot forward, unfolding the rest of the tongue, until the outer portion lands on the prey, sticking to it. Figure 2-36 shows the acceleration magnitude $a$ versus time $t$ for the acceleration phase of the launch in a typical situation. The indicated accelerations are $a_2 = 400$ m/s$^2$ and $a_1 = 100$ m/s$^2$. What is the outward speed of the tongue at the end of the acceleration phase?

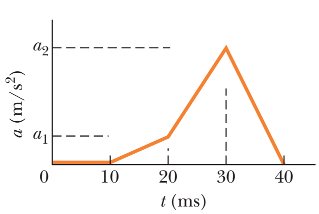



**Fig. 2-36** Problem 68.

**••69** ILW How far does the runner whose velocity–time graph is shown in Fig. 2-37 travel in 16 s? The figure's vertical scaling is set by $v_s = 8.0$ m/s.

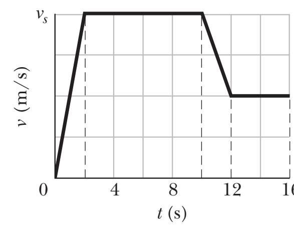



**Fig. 2-37** Problem 69.

**•••70** Two particles move along an $x$ axis. The position of particle 1 is given by $x = 6.00t^2 + 3.00t + 2.00$ (in meters and seconds); the acceleration of particle 2 is given by $a = -8.00t$ (in meters per second squared and seconds) and, at $t = 0$, its velocity is 20 m/s. When the velocities of the particles match, what is their velocity?

### Additional Problems

**71** In an arcade video game, a spot is programmed to move across the screen according to $x = 9.00t - 0.750t^3$, where $x$ is distance in centimeters measured from the left edge of the screen and $t$ is time in seconds. When the spot reaches a screen edge, at either $x = 0$ or $x = 15.0$ cm, $t$ is reset to 0 and the spot starts moving again according to $x(t)$. (a) At what time after starting is the spot instantaneously at rest? (b) At what value of $x$ does this occur? (c) What is the spot's acceleration (including sign) when this occurs? (d) Is it moving right or left just prior to coming to rest? (e) Just after? (f) At what time $t > 0$ does it first reach an edge of the screen?

**72** A rock is shot vertically upward from the edge of the top of a tall building. The rock reaches its maximum height above the top of the building 1.60 s after being shot. Then, after barely missing the edge of the building as it falls downward, the rock strikes the ground 6.00 s after it is launched. In SI units: (a) with what upward velocity is the rock shot, (b) what maximum height above the top of the building is reached by the rock, and (c) how tall is the building?

**73** GO At the instant the traffic light turns green, an automobile starts with a constant acceleration $a$ of 2.2 m/s$^2$. At the same instant a truck, traveling with a constant speed of 9.5 m/s, overtakes and passes the automobile. (a) How far beyond the traffic signal will the automobile overtake the truck? (b) How fast will the automobile be traveling at that instant?

**74** A pilot flies horizontally at 1300 km/h, at height $h = 35$ m above initially level ground. However, at time $t = 0$, the pilot begins to fly over ground sloping upward at angle $\theta = 4.3°$ (Fig. 2-38). If the pilot does not change the airplane's heading, at what time $t$ does the plane strike the ground?

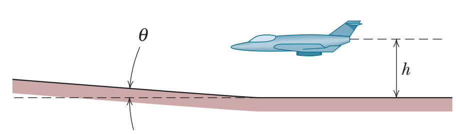



**Fig. 2-38** Problem 74.

**75** To stop a car, first you require a certain reaction time to begin braking; then the car slows at a constant rate. Suppose that the total distance moved by your car during these two phases is 56.7 m when its initial speed is 80.5 km/h, and 24.4 m when its initial speed is 48.3 km/h. What are (a) your reaction time and (b) the magnitude of the acceleration?

**76** Figure 2-39 shows part of a street where traffic flow is to be controlled to allow a *platoon* of cars to move smoothly along the street. Suppose that the platoon leaders have just reached intersection 2, where the green appeared when they were distance $d$ from the intersection. They continue to travel at a certain speed $v_p$ (the speed limit) to reach intersection 3, where the green appears when they are distance $d$ from it. The intersections are separated by distances $D_{23}$ and $D_{12}$. (a) What should be the time delay of the onset of green at intersection 3 relative to that at intersection 2 to keep the platoon moving smoothly?

Suppose, instead, that the platoon had been stopped by a red light at intersection 1. When the green comes on there, the leaders require a certain time $t_r$ to respond to the change and an additional time to accelerate at some rate $a$ to the cruising speed $v_p$. (b) If the green at intersection 2 is to appear when the leaders are distance $d$ from that intersection, how long after the light at intersection 1 turns green should the light at intersection 2 turn green?

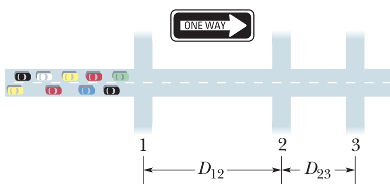



**Fig. 2-39** Problem 76.

**77** SSM A hot rod can accelerate from 0 to 60 km/h in 5.4 s. (a) What is its average acceleration, in m/s$^2$, during this time? (b) How far will it travel during the 5.4 s, assuming its acceleration is constant? (c) From rest, how much time would it require to go a distance of 0.25 km if its acceleration could be maintained at the value in (a)?

**78** A red train traveling at 72 km/h and a green train traveling at 144 km/h are headed toward each other along a straight, level track. When they are 950 m apart, each engineer sees the other's train and applies the brakes. The brakes slow each train at the rate of 1.0 m/s$^2$. Is there a collision? If so, answer yes and give the speed of the red train and the speed of the green train at impact, respectively. If not, answer no and give the separation between the trains when they stop.

**79** At time $t = 0$, a rock climber accidentally allows a piton to fall freely from a high point on the rock wall to the valley below him. Then, after a short delay, his climbing partner, who is 10 m higher on the wall, throws a piton downward. The positions $y$ of the pitons versus $t$ during the falling are given in Fig. 2-40. With what speed is the second piton thrown?

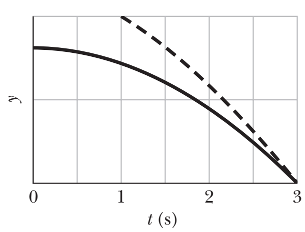



**Fig. 2-40** Problem 79.

**80** A train started from rest and moved with constant acceleration. At one time it was traveling 30 m/s, and 160 m farther on it was traveling 50 m/s. Calculate (a) the acceleration, (b) the time required to travel the 160 m mentioned, (c) the time required to attain the speed of 30 m/s, and (d) the distance moved from rest to the time the train had a speed of 30 m/s. (e) Graph $x$ versus $t$ and $v$ versus $t$ for the train, from rest.

**81** SSM A particle's acceleration along an $x$ axis is $a = 5.0t$, with $t$ in seconds and $a$ in meters per second squared. At $t = 2.0$ s, its velocity is $+17$ m/s. What is its velocity at $t = 4.0$ s?

**82** Figure 2-41 gives the acceleration $a$ versus time $t$ for a particle moving along an $x$ axis. The $a$-axis scale is set by $a_s = 12.0$ m/s$^2$. At $t = -2.0$ s, the particle's velocity is 7.0 m/s. What is its velocity at $t = 6.0$ s?

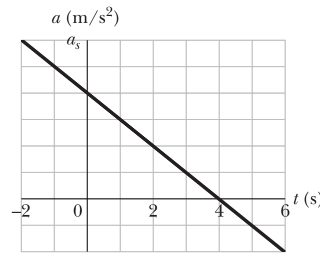



**Fig. 2-41** Problem 82.

**83** Figure 2-42 shows a simple device for measuring your reaction time. It consists of a cardboard strip marked with a scale and two large dots. A friend holds the strip *vertically,* with thumb and forefinger at the dot on the right in Fig. 2-42. You then position your thumb and forefinger at the other dot (on the left in Fig. 2-42), being careful not to touch the strip. Your friend releases the strip, and you try to pinch it as soon as possible after you see it begin to fall. The mark at the place where you pinch the strip gives your reaction time. (a) How far from the lower dot should you place the 50.0 ms mark? How much higher should you place the marks for (b) 100, (c) 150, (d) 200, and (e) 250 ms? (For example, should the 100 ms marker be 2 times as far from the dot as the 50 ms marker? If so, give an answer of 2 times. Can you find any pattern in the answers?)

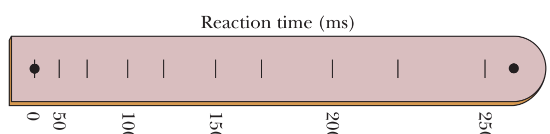



**Fig. 2-42** Problem 83.

**84** A rocket-driven sled running on a straight, level track is used to investigate the effects of large accelerations on humans. One such sled can attain a speed of 1600 km/h in 1.8 s, starting from rest. Find (a) the acceleration (assumed constant) in terms of $g$ and (b) the distance traveled.

**85** A mining cart is pulled up a hill at 20 km/h and then pulled back down the hill at 35 km/h through its original level. (The time required for the cart's reversal at the top of its climb is negligible.) What is the average speed of the cart for its round trip, from its original level back to its original level?

**86** A motorcyclist who is moving along an $x$ axis directed toward the east has an acceleration given by $a = (6.1 - 1.2t)$ m/s$^2$

for $0 \le t \le 6.0$ s. At $t = 0$, the velocity and position of the cyclist are 2.7 m/s and 7.3 m. (a) What is the maximum speed achieved by the cyclist? (b) What total distance does the cyclist travel between $t = 0$ and 6.0 s?

**87** SSM When the legal speed limit for the New York Thruway was increased from 55 mi/h to 65 mi/h, how much time was saved by a motorist who drove the 700 km between the Buffalo entrance and the New York City exit at the legal speed limit?

**88** A car moving with constant acceleration covered the distance between two points 60.0 m apart in 6.00 s. Its speed as it passed the second point was 15.0 m/s. (a) What was the speed at the first point? (b) What was the magnitude of the acceleration? (c) At what prior distance from the first point was the car at rest? (d) Graph $x$ versus $t$ and $v$ versus $t$ for the car, from rest ($t = 0$).

**89** SSM A certain juggler usually tosses balls vertically to a height $H$. To what height must they be tossed if they are to spend twice as much time in the air?

**90** A particle starts from the origin at $t = 0$ and moves along the positive $x$ axis. A graph of the velocity of the particle as a function of the time is shown in Fig. 2-43; the $v$-axis scale is set by $v_s = 4.0$ m/s. (a) What is the coordinate of the particle at $t = 5.0$ s? (b) What is the velocity of the particle at $t = 5.0$ s? (c) What is the acceleration of the particle at $t = 5.0$ s? (d) What is the average velocity of the particle between $t = 1.0$ s and $t = 5.0$ s? (e) What is the average acceleration of the particle between $t = 1.0$ s and $t = 5.0$ s?

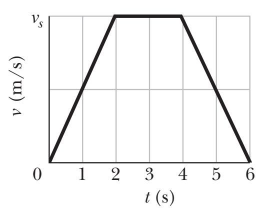



**Fig. 2-43** Problem 90.

**91** A rock is dropped from a 100-m-high cliff. How long does it take to fall (a) the first 50 m and (b) the second 50 m?

**92** Two subway stops are separated by 1100 m. If a subway train accelerates at $+1.2$ m/s$^2$ from rest through the first half of the distance and decelerates at $-1.2$ m/s$^2$ through the second half, what are (a) its travel time and (b) its maximum speed? (c) Graph $x$, $v$, and $a$ versus $t$ for the trip.

**93** A stone is thrown vertically upward. On its way up it passes point $A$ with speed $v$, and point $B$, 3.00 m higher than $A$, with speed $\frac{1}{2}v$. Calculate (a) the speed $v$ and (b) the maximum height reached by the stone above point $B$.

**94** A rock is dropped (from rest) from the top of a 60-m-tall building. How far above the ground is the rock 1.2 s before it reaches the ground?

**95** SSM An iceboat has a constant velocity toward the east when a sudden gust of wind causes the iceboat to have a constant acceleration toward the east for a period of 3.0 s. A plot of $x$ versus $t$ is shown in Fig. 2-44, where $t = 0$ is taken to be the instant the wind starts to blow and the positive $x$ axis is toward the east. (a) What is the acceleration of the iceboat during the 3.0 s interval? (b) What is the velocity of the iceboat at the end of the 3.0 s interval? (c) If the acceleration remains constant for an additional 3.0 s, how far does the iceboat travel during this second 3.0 s interval?

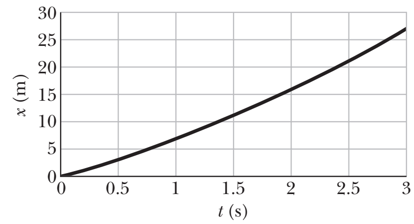



**Fig. 2-44** Problem 95.

**96** A lead ball is dropped in a lake from a diving board 5.20 m above the water. It hits the water with a certain velocity and then sinks to the bottom with this same constant velocity. It reaches the bottom 4.80 s after it is dropped. (a) How deep is the lake? What are the (b) magnitude and (c) direction (up or down) of the average velocity of the ball for the entire fall? Suppose that all the water is drained from the lake. The ball is now thrown from the diving board so that it again reaches the bottom in 4.80 s. What are the (d) magnitude and (e) direction of the initial velocity of the ball?

**97** The single cable supporting an unoccupied construction elevator breaks when the elevator is at rest at the top of a 120-m-high building. (a) With what speed does the elevator strike the ground? (b) How long is it falling? (c) What is its speed when it passes the halfway point on the way down? (d) How long has it been falling when it passes the halfway point?

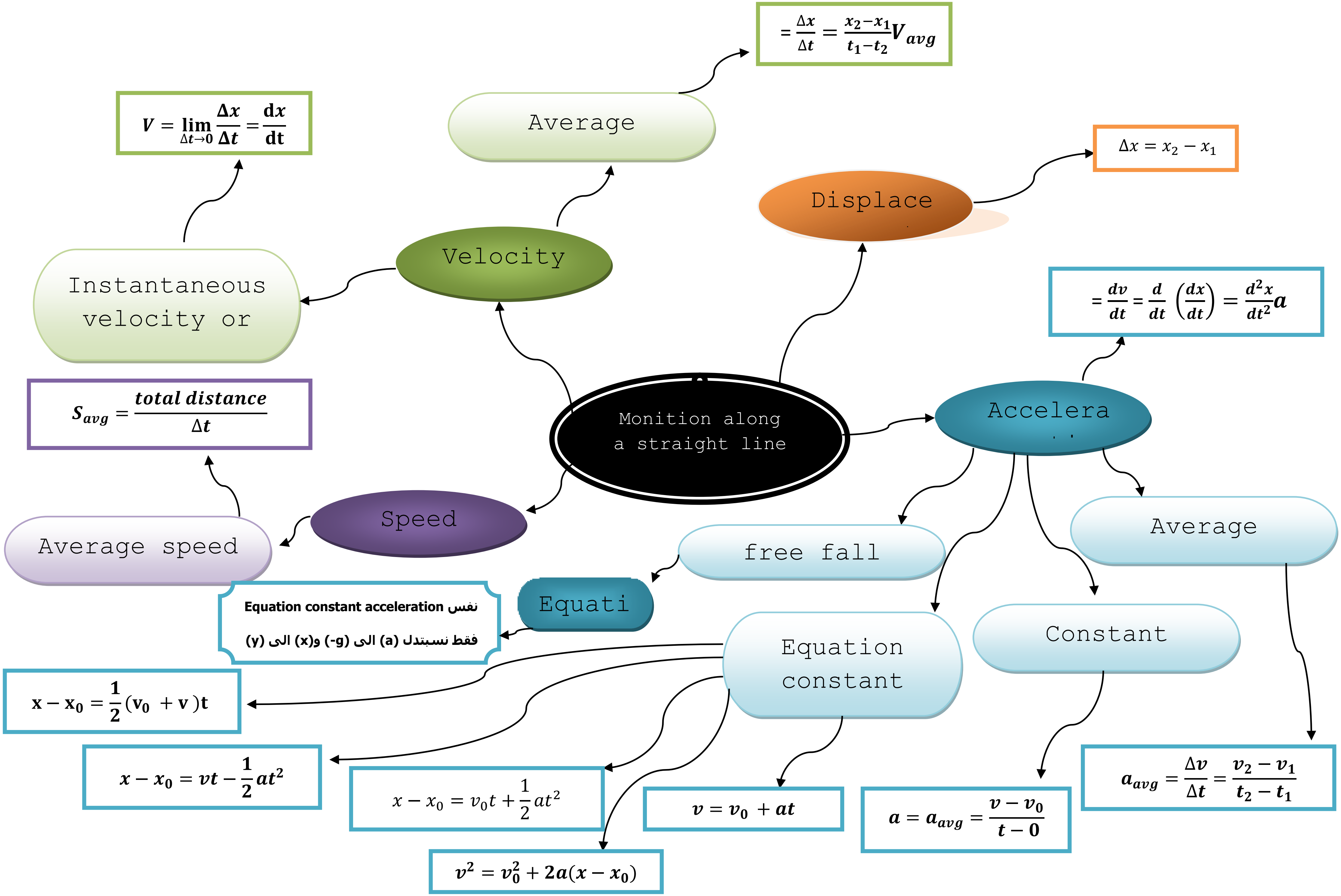
**98** Two diamonds begin a free fall from rest from the same height, 1.0 s apart. How long after the first diamond begins to fall will the two diamonds be 10 m apart?

**99** A ball is thrown vertically downward from the top of a 36.6-m-tall building. The ball passes the top of a window that is 12.2 m above the ground 2.00 s after being thrown. What is the speed of the ball as it passes the top of the window?

**100** A parachutist bails out and freely falls 50 m. Then the parachute opens, and thereafter she decelerates at 2.0 m/s$^2$. She reaches the ground with a speed of 3.0 m/s. (a) How long is the parachutist in the air? (b) At what height does the fall begin?

**101** A ball is thrown *down* vertically with an initial *speed* of $v_0$ from a height of $h$. (a) What is its speed just before it strikes the ground? (b) How long does the ball take to reach the ground? What would be the answers to (c) part a and (d) part b if the ball were thrown *upward* from the same height and with the same initial speed? Before solving any equations, decide whether the answers to (c) and (d) should be greater than, less than, or the same as in (a) and (b).

**102** The sport with the fastest moving ball is jai alai, where measured speeds have reached 303 km/h. If a professional jai alai player faces a ball at that speed and involuntarily blinks, he blacks out the scene for 100 ms. How far does the ball move during the blackout?


Monition along a straight line
Velocity
Average
$= \frac{\Delta x}{\Delta t} = \frac{x_2-x_1}{t_1-t_2} V_{avg}$
Instantaneous velocity or
$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$
Displace
$\Delta x = x_2 - x_1$
Speed
Average speed
$S_{avg} = \frac{total\ distance}{\Delta t}$
Accelera
$= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} a$
free fall
Equati
Equation constant acceleration نفس
فقط نستبدل (a) الى (-g) و(x) الى (y)
Average
$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$
Constant
$a = a_{avg} = \frac{v - v_0}{t - 0}$
Equation constant
$x - x_0 = \frac{1}{2}(v_0 + v)t$
$x - x_0 = vt - \frac{1}{2}at^2$
$x - x_0 = v_0 t + \frac{1}{2}at^2$
$v = v_0 + at$
$v^2 = v_0^2 + 2a(x - x_0)$

# Chapter 2: MOTION ALONG A STRAIGHT LINE

Choose the correct answer:

**1.** Suppose the motion of a particle is described by the equation**:** $\mathbf{X = 20 + 4\ t^{2}}$**.** Find the **instantaneous velocity at t =5 s** ?

(a) 16 m/s (b) 60 m/s (c) 40 m/s (d) 36 m/s

---

**2.** A ball thrown vertically upward with an initial velocity of **12 m/s**, **what is the ball's maximum height**?

(a) 7.35 m (b) 14.7 m (c) 0.61 m (d) 1.22 m

---

**3.** A body moves along the x-axis with constant acceleration $\mathbf{a = 4\ m/s^2}$ . At **t=0** the body is at $\mathbf{x_0 = 5\ m}$ and has velocity $\mathbf{v_0 = 3\ m/s}$. Find its **position at t = 2 s** ?

(a) 14 m (b) 19 m (c) 15 m (d) 18 m

---

**4.** Suppose the velocity of the particle is given by the: $\mathbf{v = 10 + 2\ t^{2}}$ where **v** is in m/s and **t** is in s . Find **the change in velocity** of the particle in the time interval between $\mathbf{t_1 = 2\ s}$ **and** $\mathbf{t_2 = 5\ s}$ ?

(a) 41 m/s (b) 14 m/s (c) 24 m/s (d) 42 m/s

---

**5.** **In question 4**, Find **the instantaneous acceleration when t = 2 s** ?

(a) 4 $m/s^2$ (b) 14 $m/s^2$ (c) 8 $m/s^2$ (d) 18 $m/s^2$

---

**6.** Which pair of the following **initial and final positions** along the x-axis give a **positive** displacement?

(a) - 3m, +5m (b) - 3m, - 4m (c) 5m, - 3m (d) 4m, 3m

---

**7.** You walk a distance **1.22 m in 1 s** and then run a distance **3.05 m in 1 s**, what is your **average speed**?

(a) 0.92 m/s (b) 4.27 m/s (c) 2.14 m/s (d) 1.83 m/s

---

**8.** The following are equations of the velocity v(t) of a particle, in which situation the **acceleration is constant**?

(a) $v = 3t + 6$ (b) $v = 4\ t^{2}$ (c) $v = 3\ t^{2} - 4\ t$ (d) $v = 5\ t^{3} - 3$

---

**9.** A particle's position on the x-axis is given by $X = 8 - 5\ t + 25\ t^{2}$, with X in meters and t in seconds. Find the particles **velocity function**?

(a) $v = -5 + 25\ t$ (b) $v = -5 + 50\ t$ (c) $v = 8 - 5 + 25\ t$ (d) $v = 8 + 5 + 50\ t$

---

**10.** A rocket ship moves with **constant acceleration** equal to 9.8 $m/s^2$, if it starts from rest **how long** will it take to reach a velocity $\frac{1}{10}$ the velocity of light? ( $V_{light} = 3 \times 10^8$ m/s)

(a) $3.1 \times 10^5$ s (b) $3.1 \times 10^7$ s (c) $3.1 \times 10^6$ s (d) $3.1 \times 10^4$ s

**11.** In **question 10**, **how far** will the rocket ship travel?

(a) $4.6 \times 10^{13}$ m (b) $4.6 \times 10^{10}$ m (c) $4.6 \times 10^{12}$ m (d) $4.6 \times 10^{11}$ m

**12.** A ball thrown vertically upward with an initial velocity of **12 m/s**, **how long** does the ball take to reach its **maximum height**?

(a) 0.74 s (b) 1.35 s (c) 0.82 s (d) 1.22 s

**13.** A car moving with a constant acceleration covered a **distance** between two points **60 m** apart in **6 s**, what was its **initial speed** if the **final speed was 15 m/s**?

(a) -10 m/s (b) -5 m/s (c) 5 m/s (d) 17.5 m/s

**14.** The **instantaneous acceleration** a equals:

(a) $\frac{dx}{dt}$ (b) $\frac{d}{dt}\left(\frac{d^2x}{dt^2}\right)$ (c) $\frac{d^2}{dt^2}\left(\frac{dx}{dt}\right)$ (d) $\frac{d}{dt}\left(\frac{dx}{dt}\right)$

**15.** Suppose the motion of a particle is described by the equation**: $X = 20 + 4t^2$**. Find the **average velocity** of the particle in the time interval **$t_1 = 2$ s to $t_2 = 5$ s** ?

(a) 29 m/s (b) 28 m/s (c) 84 m/s (d) 10 m/s

**16.** **In question 15**, Find the **instantaneous velocity at t =5 s** ?

(a) 16 m/s (b) 60 m/s (c) 40 m/s (d) 36 m/s

**17.** A rock is dropped from rest from the top of a **100 m** tall building**, how long does it take to fall the first 50 m** ?

(a) 3.2 s (b) 10.2 s (c) 20.4 s (d) 4.5 s

**18.** The following are equations of the position of a particle, in which situation the **velocity of the particle is constant** ?

(a) $x = 4t^2 - 2$ (b) $x = -2t^3$ (c) $x = -3t - 2$ (d) $x = 4t^{-2}$

**19.** A ball thrown vertically upward with an initial velocity of **12 m/s**, **what is the ball's maximum height**?

(a) 7.35 m (b) 14.7 m (c) 0.61 m (d) 1.22 m

**20.** A body moves along the x-axis with constant acceleration **a =4 m/s²** . At **t=0** the body is at **x=5 m** and has velocity **v = 3 m/s**. Find its **position at t = 2 s** ?

(a) 14 m (b) 19 m (c) 15 m (d) 18 m

**21. In question 20,** where is the body when its velocity is **5 m/s** ?

(a) 7 m (b) 9 m (c) 11 m (d) 2 m

**22.** A man runs a distance of **1 mile** in exactly **4 minutes**, What is his **average velocity** in **mi/hr** ?

(a) 900 mi/hr (b) 15 mi/hr (c) 6.71 mi/hr (d)15000 mi/hr

**23.** You walk a distance of **73.2 m** at a speed of **1.22 m/s** and then run **73.2 m** in **24 s**. What is your overall **displacement**?

(a) 97.2 m (b) 73.2 m (c) 146.4 m (d) zero

**24.** In question **23,** what is the **time interval** from the start to the end?

(a) 24 s (b) 84 s (c) 36 s (d) 4.27 s

**25.** If $t_1$**=2 s** and $t_2$**=4 s** find the **average acceleration** when the velocity changes from **8 m/s** to **12 m/s**?

(a) 1 $m/s^2$ (b) 3.33 $m/s^2$ (c) 5 $m/s^2$ (d) 2 $m/s^2$

**26.** What is the **initial speed** of a car moving a **distance** of **60 m** in **6 s** if the **final speed was 15 m/s**?

(a) -10 m/s (b) -5 m/s (c) 5 m/s (d) 17.5 m/s

**27.** If the total distance moved by a bus before stopping was **56.7 m** with **initial speed** of **22.36 m/s**. What is the magnitude of the **acceleration**?

(a) 8.82 $m/s^2$ (b) 4.41 $m/s^2$ (c) 17.63 $m/s^2$ (d) 2.21 $m/s^2$

**28.** A pipe dropped from a building struck the ground with **a speed of 24 m/s**. what **height** was it dropped from?

(a) 58.8 m (b) 2.44 m (c) 1.22 m (d) 29.4 m

**29.** What is the **initial speed** of a ball thrown upward vertically reaching a **height of 0.544 m** in **0.2 s** ?

(a) 4.68 m/s (b) 3.7 m/s (c) 2.1 m/s (d) 0.74 m/s

**30.** The **initial and the final positions** of a particle moving along the x-axis are **-2 m, 10 m**, then its **displacement Δx** equals:

(a) +12 m (b) +8 m (c) -12 m (d) -8 m

**31.** In which situation of the following the displacement is **positive**?

| Situation | $X_1$(m) | $X_2$(m) |
|---|---|---|
| **A** | -3 | 5 |
| **B** | -3 | -7 |
| **C** | -3 | -3 |
| **D** | 2 | 5 |

(a) **A** and **B** (b) **A** and **C** (c) **A** and **D** (d) **B** and **C**

**32.** The position of a body moving along the x axis is given by $\mathbf{x = 3\,t - 4\,t^2 + t^3}$ . Its position at **t = 2 s** is:

(a) 6 m (b) 2 m (c) -6 m (d) –2 m

**33.** **In question 32,** the **displacement** of the object in the time interval **t = 0** to **t = 4 s** is:

(a) $\Delta x = 3m$ (b) $\Delta x = 12m$ (c) $\Delta x = -3m$ (d) $\Delta x = -12m$

**34.** A car travelled **40 km in 0.5 h**, then travelled **40 km in 1 h**. Its **average speed** is:

(a) 26.7 km/h (b) 160 km/h (c) 80 km/h (d) 53.3 km/h

**35.** A car starts from point **A** moved a distance **50 km** to point **B** then returns to point **A** in a time interval of **2 hours**. Its **average velocity** is:

(a) zero (b) 50 km/h (c) 100 km/h (d) 25 km/h

**36.** The position of a particle moving along the x-axis is given by: $\mathbf{x = 2\,t^3}$ **. Its acceleration is:**

(a) $6t^2$ m/$s^2$ (b) 12t m/$s^2$ (c) constant (d) zero

**37.** A ball dropped from a building ,its **velocity and position** after **1 s** are:

(a) V= -9.8 m/s y= -9.8 m (b) V= -4.9 m/s y= -9.8 m (c) V= -9.8 m/s y= -4.9 m (d) V= -4.9 m/s y= -4.9 m

**38.** An electron has an initial velocity $V_0$**= 1x10$^5$ m/s** travels a distance **0.01 m**, if the final velocity was **V=2x10$^6$ m/s**, then its **acceleration is:**

(a) 1995 x $10^{14}$ m/$s^2$  (b) 195 x $10^6$ m/$s^2$  (c) 95 x $10^6$ m/$s^2$  (d) 1.995 x $10^{14}$ m/$s^2$

**39.** A particle moving in the **+ x direction** with **increasing speed** :

(a) Its velocity is positive and acceleration is negative
(b) Its velocity is negative and acceleration positive
(c) Its velocity and acceleration are both positive
(d) Its velocity is positive and acceleration is zero

**40.** In which situation of the following the **velocity** is in the **negative** x direction?

| Situation | Position of the particle |
|---|---|
| **A** | $X = -2t^2 - 2$ |
| **B** | $X = 3t^3 - 5$ |
| **C** | $X = -2t^{-2} + 1$ |
| **D** | $X = -5 + 5t$ |

(a) **A**  (b) **B**  (c) **C**  (d) **D**

**41.** A ball is thrown vertically upward. Its **displacement** is:

(a) positive during rising and negative during falling
(b) negative during rising and positive during falling
(c) positive during rising and falling
(d) negative during rising and falling

**42.** A man walks **4 m** from point A **due east**, then **3 m due north**. What is his **displacement** from the point A?

(a) 7 m  (b) 6 m  (c) 5 m  (d) 10 m

**43.** The following are equations of the velocity v(t) of a particle, in which situation the **acceleration is constant**?

(a) $v = 3t + 6$  (b) $v = 4t^2$  (c) $v = 3t^2 - 4t$  (d) $v = 5t^3 - 3$

**44.** You are throwing a ball straight up in the air. At the highest point, the ball's velocity and acceleration are**:**

(a) $v = 0$, $a = -g$  (b) $v = v_0$, $a = 0$  (c) $v > v_0$, $a = -g$  (d) $v < v_0$, $a < -g$

**45.** If the **sign** of the **velocity and acceleration** of a particle are **opposite**, then the **speed of the particle**

(a) is zero  (b) decreases  (c) increases  (d) does not change

**46.** A particle moves **from $x_1$= 5 m to $x_2$= 12 m, then:**

(a) $\Delta$x is positive (b) $\Delta$x is negative (c) $\Delta$x is zero (d) $\Delta$x = 12m

**47.** You **walked a distance of 2 km** along a road **in 0.5 h**, then walked back to **the initial position in 0.75 h.** Your overall **displacement** is:

(a) 6 km (b) 0 (c) 4 km (d) 2 km

**48.** **In question 62**, your **average speed** is :

(a) 5.3 km/h (b) 1.6 km/h (c) 3.2 km/h (d) 0

**49.** The position of a car changes from **$x_1$= 20 m to $x_2$= 100 m** in the **time interval** from **2s** to **4s**, the **average velocity** of the car is:

(a) 40 m/s (b) 30 m/s (c) 45 m/s (d) 25 m/s

**50.** The position of a particle is given by: **x(t)= 10 + $t^2$** ,the **instantaneous acceleration at t= 1 s** is:

(a) 8 m/s$^2$ (b) 6 m/s$^2$ (c) 4 m/s$^2$ (d) 2 m/s$^2$

**51.** The **free fall acceleration** is:

(a) zero (b) – 9.8 m/s$^2$ (c) +9.8 m/s$^2$ (d) – 32 m/s$^2$

**52.** In which situation of the following the **velocity is constant** ?

| Situation | Position of the particle |
|---|---|
| **A** | **X= 3 t – 2** |
| **B** | **X= 2 $t^2$ – 2** |
| **C** | **X= - 2 $t^3$** |
| **D** | **X= 2 – 5 $t^2$** |

(a) **A** (b) **B** (c) **C** (d) **D**

**53.** **A car starts from rest,** travels with constant accelertion a distance **500 m**, the final velocity is **50 m/s.** Its **acceleration** is:

(a) 1.6 m/s$^2$ (b) 2.5 m/s$^2$ (c) 3.6 m/s$^2$ (d) 4.9 m/s$^2$

**54.** The equation that represents the **motion with constant acceleration** is:

(a) $v^2 = v_0^2 + 2at$ (b) $v = v_0 + 2a(x - x_0)$ (c) $x - x_0 = v_0 t + \frac{1}{2}at^2$ (d) $v = v_0 + \frac{1}{2}at^2$

**55.** When an object is thrown **vertically upward ↑,** while it is **rising:**

**(a)** its velocity and acceleration are both upward ↑

**(b)** its velocity is upward ↑ and its acceleration is downward ↓

**(c)** its velocity and acceleration are both downward ↓

**(d)** its velocity is downward ↓ and its acceleration is upward ↑

**Are the following statements** (**True ✓**) or (**False ×** ) ?

**56.** Speed is the magnitude of instantaneous velocity.

(a) True (b) False

---

**57.** Average acceleration is the ratio of (النسـبة بـين) the change of velocity $\Delta v$ to the time interval $\Delta t$.

(a) True (b) False

---

**58.** **The free fall motion** is an example of motion along a straight line with constant acceleration.

(a) True (b) False

# CH 2

1- $x = 20 + 4t^2$

$v = 8t$

$= 8(5) = 40$ m/s

ملاحظة / إذا أعطانا x في معادلة : نفاضل مرة وحدة عشان نجيب السرعة v

نفاضل مرتين عشان نجيب التسارع a .

2- $v_0 = 12$ , $v = 0$ , $g = 9.8$ , $\boxed{\Delta y = ??}$

$v^2 = v_0^2 - 2g\Delta y$

$0 = 144 - 2 \times 9.8 \times \Delta y$

$\frac{-144}{-19.6} = \frac{-19.6\,\Delta y}{-19.6}$

$\boxed{\Delta y = 7.35 \text{ m}}$

* إذا شفنا كلمة upward يعني نستخدم معادلات السقوط الحر .
* كلمة Maximum height يعني السرعة النهائية $v = 0$ .

3- $t = 0$ , $x_0 = 5$ , $v_0 = 3$ , $\boxed{x = ??\ \text{عندما}\ t = 2}$

$x - x_0 = v_0 t + \frac{1}{2} a t^2$

$x - 5 = 3(2) + \frac{1}{2}(4)(2^2)$

$x - 5 = 14$

$x = 14 + 5 = 19$ m

4- $v = 10 + 2t^2$

$v_1 = 10 + 2(2)^2 = 18$ m/s

$v_2 = 10 + 2(5)^2 = 60$ m/s

$\Delta v = 60 - 18 = 42$ m/s

* هنا يعني التغير في السرعة يعني $\Delta v$ وقانونها

$\Delta v = v_2 - v_1$

يعني لازم نعوض ونجيب $v_1$ و $v_2$

5- $v = 10 + 2t^2$ نفاضل عشان

$a = 4t$ نجيب التسارع

$= 4(2) = 8$ m/s²

6- a) $5 + 3 = \boxed{8 \text{ m}}$

b) $-4 + 3 = -1$ m

c) $-3 - 5 = -8$ m

d) $3 - 4 = -1$ m

* أي واحد من الخيارات الإزاحة فيه موجبة ؟

$\Delta x = x_f - x_i$

تابع CH2

7- $x_1 = 0,\ t_1 = 0 \xrightarrow[1\,s]{1.22\,m} \xrightarrow[1\,s]{3.05\,m} x_2 = 1.22 + 3.05 = 4.27\ m,\ t_2 = 2s$

$$S_{avg} = \frac{4.27}{2} = 2.135\ m/s$$

8- a) $v = 3t + 6 \rightarrow a = 3$

* أي المعادلات التسارع فيها ثابت؟
يعني رقم بس بدون t.

b) $v = 4t^2 \rightarrow a = 8t$

c) $v = 3t^2 - 4t \rightarrow a = 6t - 4$

d) $v = 5t^3 - 3 \rightarrow a = 15t^2$

9- $x = 8 - 5t + 25t^2$ نفاضل مرة وحدة

$v = -5 + 50t$ ونجيب السرعة

10- $v_0 = 0$, $v = \frac{1}{10}(3\times10^8)$, $a = 9.8$, $\boxed{t = ??}$

* لما نشوف كلمة starts the rest
يعني السرعة الابتدائية $v_0 = 0$

$v = v_0 + at$

$(\frac{1}{10}\times3\times10^8) = 0 + 9.8t$

$\frac{(3\times10^7)}{9.8} = \frac{\cancel{9.8}\,t}{\cancel{9.8}}$

$t = 3.1\times10^6\ s$

* هذا السؤال يبغى الزمن المستغرق إلى أن توصل
الحركة إلى عُشر سرعة الضوء يعني السرعة النهائية
تكون $v = \frac{1}{10}(3\times10^8)$

11- $v^2 = v_0^2 + 2a\Delta x$

$(\frac{1}{10}\times3\times10^8)^2 = 0 + 2\times9.8\,\Delta x$

$\Delta x = 4.59\times10^{13}\ m$

* حنستخدم نفس المعطيات في سؤال 10
بس نغير القانون المناسب.

12- $v_0 = 12$, $v = 0$, $g = 9.8$, $\boxed{t = ?}$

$v = v_0 - gt$

$0 = 12 - 9.8t$

$\frac{-12}{-9.8} = \frac{\cancel{-9.8}\,t}{\cancel{-9.8}}$

$$\boxed{t = 1.22\ s}$$

CH 2 تابع

13– $\Delta x = 60$ , $t = 6$ , $v = 15$ , $\boxed{v_0 = ??}$

$x - x_0 = \frac{1}{2}(v_0 + v)t$

$2 \times (60) = (\frac{1}{\cancel{2}}(v_0 + 15)6) \times \cancel{2}$

$120 = (v_0 + 15)6$

$120 = 6v_0 + 90$

$\frac{\cancel{6}v_0}{\cancel{6}} = \frac{30}{6}$

$\boxed{v_0 = 5 \text{ m/s}}$

14– $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$ ← ☺ حفظ

15– $x_1 = 20 + 4(2)^2 = 36$ m

$x_2 = 20 + 4(5)^2 = 120$ m

$v_{avg} = \frac{120 - 36}{5 - 2} = 28$ m/s

16– $v = 8t$

$= 8(5) = 40$ m/s

17– $v_0 = 0$ , $g = 9.8$ , $y = 50$ , $\boxed{t = ??}$

$\Delta y = \cancel{v_0 t} - \frac{1}{2}gt^2$

$-50 = -\frac{1}{2} \times 9.8\, t^2$

$\frac{-50}{4.9} = \frac{\cancel{4.9}}{\cancel{4.9}}t^2$

$\sqrt{t^2} = \sqrt{10.2}$

$\boxed{t = 3.19 \text{ s}}$

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18 – a) $x = 4t^2 - 2 \rightarrow v = 8t$

b) $x = -2t^3 \rightarrow v = -6t^2$

c) $x = -3t - 2 \rightarrow v = -3$

d) $x = 4t^{-2} \rightarrow v = -8t^{-1}$

* أي المعادلات فيها السرعة ثابتة ؟
لازم نفاضل وإذا طلع عدد بس يعني ثابتة .

19 – $v_0 = 12$ , $v = 0$ , $g = 9.8$ , $\boxed{y = ??}$

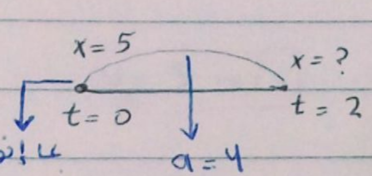
$v^2 = v_0^2 - 2g\,\Delta y$

$0 = 144 - 2 \times 9.8\,\Delta y$

$\frac{-144}{-19.6} = \frac{-19.6\,\Delta y}{-19.6}$

$\boxed{\Delta y = 7.346}$

20 –




عادةً الزمن
صفر وأكيد السرعة الي
أعطانا ابتدائية . $v_0 = 3$

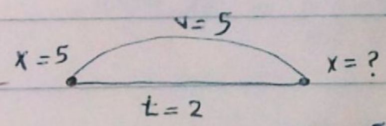
$x - x_0 = v_0 t + \frac{1}{2} a t^2$

$x - 5 = 3(2) + \frac{1}{2}(4)(2)^2$

$x - 5 = 14$

$\boxed{x = 19}$

21 –




$x - 5 = 5(2) - \frac{1}{2}(4)(2)^2$

$x - 5 = 2$

$\boxed{x = 7\ m}$

22 – $v_{avg} = \frac{1}{\left(\frac{4}{60}\right)} = 15\ mi/hr$

للتحويل من دقيقة إلى ساعة .

23 – displacement = $73.2 + 73.2 = 146.4\ m$

* لما يجيب كلمة overall displacment يعني نجمع على طول .

CH2 تابع

24- $t_1 = \frac{d}{v} = \frac{73.2}{1.22} = 60$

$t_2 = 24$ ← أعطانا ياها في السؤال

$60 + 24 = 84\ s$

25- $a_{avg} = \frac{12-8}{4-2} = 2\ m/s^2$

26- $x = 60$ , $t = 6$ , $v = 15$ , $\boxed{v_0 = ??}$

$\Delta x = \frac{1}{2}(v_0 + v)t$

$2 \times (60) = (\frac{1}{\cancel{2}}(v_0 + 15)6) \times \cancel{2}$

$120 = 6v_0 + 90$

$\frac{\cancel{6}v_0}{\cancel{6}} = \frac{30}{6}$

$\boxed{v_0 = 5\ m/s}$

27- $v^2 = v_0^2 + 2a(x - x_0)$

$0 = (22.36)^2 + 2a(56.7)$

$\frac{-499.96}{113.4} = \frac{\cancel{113.4}}{\cancel{113.4}}a$

$a = -4.41\ m/s^2$

* إذا شفنا كلمة stop يعني السرعة النهائية $v = 0$

هنا قال يعني القيمة المطلقة يعني الإجابة ← ناخذها موجبة .

28- $v^2 = v_0^2 - 2g\Delta y$

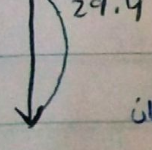
$(-24)^2 = 0 - 2 \times 9.8\ \Delta y$

$\frac{576}{-19.6} = \frac{-\cancel{19.6}}{-\cancel{19.6}}\Delta y$

$\Delta y = -29.38\ m$

ملاحظة ! إذا كان الجسم نازل لأسفل يعني المسافة سالبة .

وإذا شفنا كلمة dropped يعني السرعة الابتدائية $v_0 = 0$




* هنا بس يعني كم كان الارتفاع مالو صلاح في الإشارة .

29- $\Delta y = v_0 t - \frac{1}{2} g t^2$

$0.544 = v_0 (0.2) - \frac{1}{2} \times 9.8 \times (0.2)^2$

$0.544 = 0.2 v_0 - 0.196$

$\frac{0.74}{0.2} = \frac{\cancel{0.2}}{\cancel{0.2}} v_0$

$\boxed{v_0 = 3.7 \text{ m/s}}$

30- $\Delta x = 10 - (-2) = 10 + 2 = 12 \text{ m}$

31- A) $5 + 3 = 8$ ✓      * أي الحالات فيها الإزاحة موجبة ؟

B) $-7 + 3 = -4$

C) $-3 + 3 = 0$

D) $5 - 2 = 3$ ✓

32- $x = 3t - 4t^2 + t^3$

$x = 3(2) - 4(2)^2 + (2)^3 = -2 \text{ m}$

33- $x_1 = 3(0) - 4(0)^2 + (0)^3 = 0$

$x_2 = 3(4) - 4(4)^2 + (4)^3 = 12$

$\Delta x = 12 - 0 = 12 \text{ m}$

34- $S_{avg} = \frac{40 + 40}{0.5 + 1} = 53.3 \text{ km/h}$

35-

$\longleftarrow$

$A \xrightarrow[2 \text{ hours}]{50 \text{ km}} B$      * لما يقول إنو مشي مسافة كذا ... وبعدين رجع يعني نطرح.

$V_{avg} = \frac{50 - 50}{2} = 0$

CH2 تابع

36- $x = 2t^3$

$v = 6t^2$ * تفاضل مرة وحدة ← السرعة .

$a = 12t$ * تفاضل مرتين ← التسارع .

37- $v_0 = 0$ , $g = 9.8$ , $t = 1$ , $\boxed{v = ??}$ , $\boxed{y = ??}$

* $v = v_0 - gt$

$v = 0 - 9.8(1) = -9.8$ m/s

* $\Delta y = v_0 t - \frac{1}{2} g t^2$

$\Delta y = 0(1) - \frac{1}{2}(9.8)(1)^2$

$= -4.9$ m

38- $v^2 = v_0^2 + 2a(x - x_0)$

$(2\times10^6)^2 = (1\times10^5)^2 + 2a(0.01)$

$4\times10^{12} = 1\times10^{10} + 0.02a$

$\frac{3.99\times10^{12}}{0.02} = \frac{0.02a}{0.02}$

$\boxed{a = 1.995\times10^{14} \text{ m/s}}$

39- speed increase ← إذا السرعة والتسارع كلها موجبة

أو كلها سالبة .

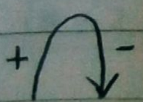
40- A) $v = -4t$ ✓ أي وحدة منها لو فاضلنا وجبنا السرعة

B) $v = 9t^2$ حتكون سالبة .

C) $v = 4t^{-1}$

D) $v = 5$

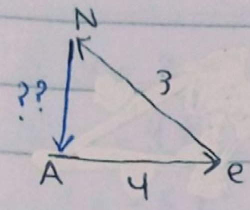
41-

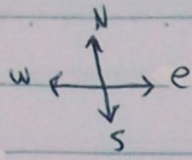



الإزاحة موجبة في الصعود

وسالبة في الهبوط .

CH2 تابع

42- 


$3^2 = 4^2 + x^2$ * نظرية فيثاغورس *

$\sqrt{x^2} = \sqrt{25}$

$\boxed{x = 5}$

43- a) $v = 3t + 6 \rightarrow a = 3$ ✓

b) $v = 4t^2 \rightarrow a = 8t$ * أي المعادلات لو فاضلنا حيطلع التسارع

c) $v = 3t^2 - 4t \rightarrow a = 6t$ ثابت [عدد بس] ؟

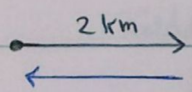
d) $v = 5t^3 - 3 \rightarrow a = 15t^2$

44- $v = 0$ * في أقصى ارتفاع السرعة دايمًا تكون صفر

$a = -g$ والتسارع في كل الاتجاهات دايمًا يكون g-

45- speed decrease ← إذا السرعة والتسارع بإشارات مختلفة

46- $\Delta x = 12 - 5 = +8$ [موجبة]

47- 


إذا رجعنا للخلف يعني نطرح

∴ displacement $= 2 - 2 = 0$

48- $S_{avg} = \frac{2+2}{0.5+0.75} = 3.2$ m/s المسافة الكلية / الزمن الكلي .

49- $V_{avg} = \frac{100-20}{4-2} = 40$ m/s

50- $x(t) = 10 + t^2$

$v = 2t$

$\boxed{a = 2}$

51- free fall acceleration $= -9.8$ m/s$^2$ تسارع السقوط الحر ← 9.8-

تابع CH2

52- A) $v = 3$

B) $v = 4t$ — أي المعادلات لو فاضلنا السرعة بتكون

C) $v = -6t^2$ — ثابتة [ عدد بس ].

D) $v = 10t$

53- $v_0 = 0$ , $\Delta x = 500$ , $v = 50$ , $\boxed{a = ??}$

$v^2 = v_0^2 + 2a\Delta x$

$(50)^2 = 0 + 2a(500)$

$\frac{2500}{1000} = \frac{\cancel{1000}a}{\cancel{1000}}$

$\boxed{a = 2.5}$

* $v_0=0$ يعني start the rest

54- كلها خطأ ما عدا فقرة C صح.

* مهم حفظ معادلات الحركة كلها ☺

55- السرعة في اتجاه الأعلى في حالة الصعود ↑

والتسارع دايمًا اتجاهه نفسو في اتجاه الأسفل ↓

56- لا السرعة هي مقدار أو القيمة المطلقة للسرعة اللحظية " صح

57- " التسارع المتوسط هو النسبة بين التغير في السرعة ~~على~~ في فترة زمنية "

$a_{avg} = \frac{\Delta v}{\Delta t}$ صح

58- " السقوط الحر هو مثال على الحركة في خط مستقيم بتسارع ثابت "

صح، لأن التسارع فيه دايمًا $-9.8$

Chapter 2

## Motion Along a Straight Line

**In this chapter we will study the motion of objects and the basic physics of motion .**

### Examples of motion :

**1-Earth's orbit around the Sun**

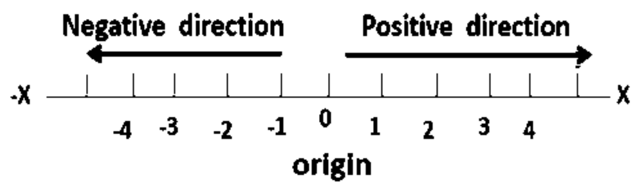
**2-Earth's rotation on its axis**

**Every thing in the world moves, even the stationary objects move with Earth's rotation.**

**The motion on a straight line may be vertical, horizontal, or slanted, but it must be straight .**

## 2-3 Position and Displacement

**To locate an object means to find its position relative to a reference point origin (or Zero point) of an axis, such as the X axis. FIG. 2-1**




## The displacement

**A change from initial position to final position**

$$\Delta X = \underbrace{X_2}_{\text{final position}} - \underbrace{X_1}_{\text{initial position}}$$

- **unit of $\Delta X$ is meter**

- **$\Delta X$ is a vector quantity**

## Example page 15

(a) A particle moves from $\vec{x_1} = (5\,\text{m})$ to $\vec{x_2} = (12\,\text{m})$

So $\Delta x$ =

(b) A particle moves from $\vec{x_1} = (12\,\text{m})$ to $\vec{x_2} = (5\,\text{m})$

So $\Delta x$ =

**(c) Find the distance in (a) and (b)**

**(d) a particle moves from $y_1$= 2m to $y_2$ = 8m, so $\Delta y$ =**

---

# Features of a Displacement

**1- Its magnitude is the distance, such as the number of meters, between the original and final positions.**

**2- Its direction from an original position to a final position can be represented by a plus sign (+) or a minus sign (-) if a motion is along a single axis.**

**- Distance is a scalar quantity** **(Absolute value)**

**( number of meters )**

**CHECKPOINT 1** Here are three pairs of initial and final positions, respectively, along an $x$ axis. Which pairs give a negative displacement: (a) $-3$ m, $+5$ m; (b) $-3$ m, $-7$ m; (c) $7$ m, $-3$ m?

## 2-4 Average Velocity and Average Speed

***Average Velocity is: The ratio of the displacement to time interval ($\Delta t$)**

$$* \; V_{avg} = \frac{displacement}{\Delta t} = \frac{\Delta X}{\Delta t} = \frac{X_2 - X_1}{t_2 - t_1}$$

*** Unit of the $V_{avg}$ is m/s , Km/s**

*** $V_{avg}$ Is vector quantity**

*** $V_{avg}$ Is the slope of the Straight line**

**Example motion of armadillo**

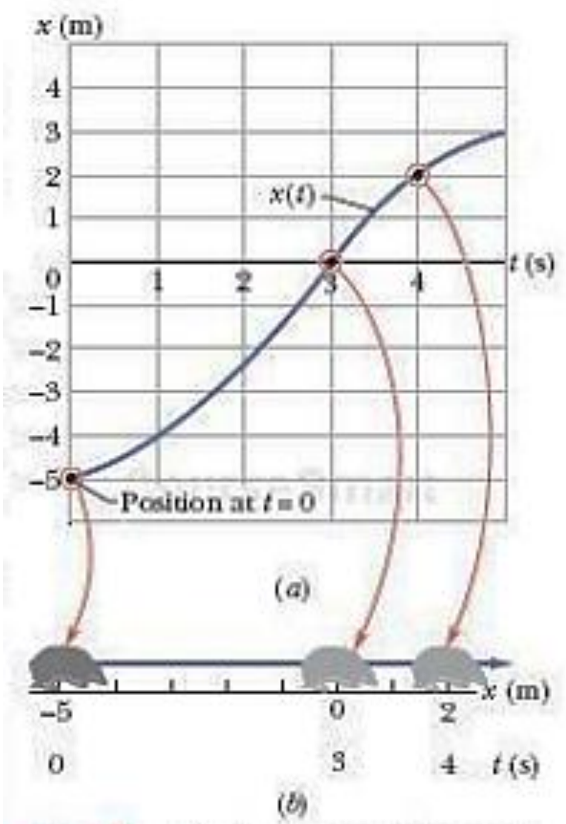



Fig 2-2

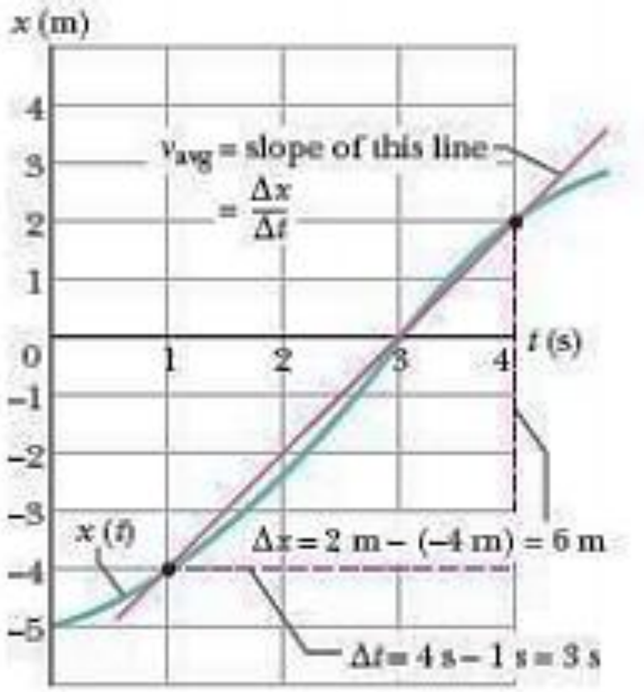



Fig 2-3

**Fig 2-3 shows how to find $V_{avg}$ for the time interval $t_1$=1s to $t_2$=4 s**

**Position is $x_1$= -4m and $x_2$= 2m**

**The average velocity is 6m/3s = 2 m/second**

# Average Speed

Average Speed: **is a ratio of the total distance that occurs during a particular time interval Δ t to that interval.**

$$S_{avg} = \frac{total\ distance}{\Delta t} = \frac{numbers\ of\ meters}{\Delta t}$$

**Unit of $S_{avg}$ is m/s , Km/s**

$S_{avg}$ **Is Scalar quantity**

**Sample Problem 2-1**

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

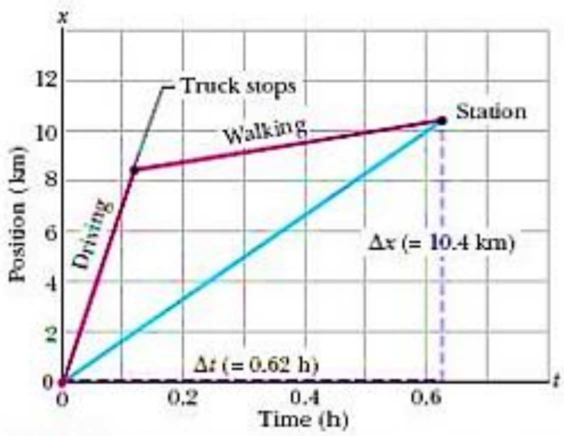
(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

$$\Delta x = x_2 - x_1 = 10.4\ \text{km} - 0 = 10.4\ \text{km}.$$

**In the positive direction of the x axis**

---

(b) What is the time interval $\Delta t$ from the beginning of your drive to your arrival at the station?




**Calculations:** We first write

$$v_{avg,dr} = \frac{\Delta x_{dr}}{\Delta t_{dr}}.$$

Rearranging and substituting data then give us

$$\Delta t_{dr} = \frac{\Delta x_{dr}}{v_{avg,dr}} = \frac{8.4\ \text{km}}{70\ \text{km/h}} = 0.12\ \text{h}.$$

So, $\Delta t = \Delta t_{dr} + \Delta t_{wlk}$
$= 0.12\ \text{h} + 0.50\ \text{h} = 0.62\ \text{h}.$ (Answer)

(c) What is your average velocity $v_{avg}$ from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

**Calculation:** Here we find

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{10.4 \text{ km}}{0.62 \text{ h}}$$
$$= 16.8 \text{ km/h} \approx 17 \text{ km/h}. \quad \text{(Answer)}$$

(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

**Calculation:** The total distance is 8.4 km + 2.0 km + 2.0 km = 12.4 km. The total time interval is 0.12 h + 0.50 h + 0.75 h = 1.37 h. Thus, Eq. 2-3 gives us

$$s_{avg} = \frac{12.4 \text{ km}}{1.37 \text{ h}} = 9.1 \text{ km/h}. \quad \text{(Answer)}$$

## 2-5 Instantaneous Velocity & Speed

**Velocity at any instant**

$$V_{ins} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

**Unit of the $V_{ins}$ is m/s**

$V_{ins}$ **Is vector quantity**

**Speed is the magnitude of Velocity**

**The Speedometer in a car measures speed, not velocity**

**A velocity of +5 m/s or -5m/s**

**Then the speed is 5m/s**

**CHECKPOINT 2** The following equations give the position $x(t)$ of a particle in four situations (in each equation, $x$ is in meters, $t$ is in seconds, and $t > 0$): (1) $x = 3t - 2$; (2) $x = -4t^2 - 2$; (3) $x = 2/t^2$; and (4) $x = -2$. (a) In which situation is the velocity $v$ of the particle constant? (b) In which is $v$ in the negative $x$ direction?

**Check point 2**

**Page 18**

**SP 2-3 Page 19**

**Sample Problem 2-3**

The position of a particle moving on an $x$ axis is given by

$$x = 7.8 + 9.2t - 2.1t^3, \qquad (2\text{-}5)$$

with $x$ in meters and $t$ in seconds. What is its velocity at $t = 3.5$ s? Is the velocity constant, or is it continuously changing?

$$v = \frac{dx}{dt} = \frac{d}{dt}(7.8 + 9.2t - 2.1t^3),$$

which becomes

$$v = 0 + 9.2 - (3)(2.1)t^2 = 9.2 - 6.3t^2. \qquad (2\text{-}6)$$

At $t = 3.5$ s,

$$v = 9.2 - (6.3)(3.5)^2 = -68 \text{ m/s.} \qquad \text{(Answer)}$$

At $t = 3.5$ s, the particle is moving in the negative direction of $x$ (note the minus sign) with a speed of 68 m/s. Since the quantity $t$ appears in Eq. 2-6, the velocity $v$ depends on $t$ and so is continuously changing.

## 2-6 Acceleration

**Acceleration is the ratio of the velocity Δv to time interval (Δt)**

$$a_{avg} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t}$$

**Where $V_1$ is Velocity at $t_1$ ,$V_2$ is Velocity at $t_2$**

**Unit of $a_{avg}$ is $m/S^2$ ( length /Time$^2$)**

**$a_{avg}$ is Vector quantity .**

### Instantaneous acceleration

$$a_{ins} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt}$$

$$a_{ins} = \frac{dV}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

**$a_{ins}$ is the second derivative of position (x) with respect to time .**

**The unit of $a_{ins}$ is $m/s^2$ OR Length/time$^2$ .**

**$a_{ins}$ is vector quantity (magnitude and direction ).**

## An acceleration's sign

If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

**CHECKPOINT 3** A wombat moves along an $x$ axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

Checkpoint 3

Sample problem 2-4

Page 20-21(a,b)

**Answer: (a) Plus**

**(b) minus**

**(c) minus**

**(d) plus**

**Sample Problem 2-4 Build your skill**

A particle's position on the $x$ axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with $x$ in meters and $t$ in seconds.

(a) Because position $x$ depends on time $t$, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

***Calculations:*** Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad \text{(Answer)}$$

with $v$ in meters per second. Differentiating the velocity function then gives us

$$a = +6t, \qquad \text{(Answer)}$$

with $a$ in meters per second squared.

(b) Is there ever a time when $v = 0$?

***Calculation:*** Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

which has the solution

$$t = \pm 3\text{ s}. \qquad \text{(Answer)}$$

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

---

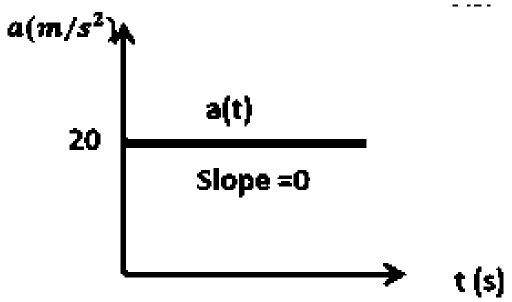
## 2-7Constant Acceleration: a special case

**What is constant acceleration ?**

**Constant acceleration is the movement of particles with constant velocity in equal time.**

**Example:**

| T(s) | V(m/s) | a=V/t ($m/s^2$) |
|---|---|---|
| 1 | 20 | 20 |
| 2 | 40 | 20 |
| 3 | 60 | 20 |
| 4 | 80 | 20 |
| 5 | 100 | 20 |




$$a_{avg} = \frac{V_2 - V_1}{t_2 - t_1}$$

**When the acceleration is constant, the average acceleration and instantaneous acceleration are equal**

$$V_2 \to V \qquad t_2 \to t \qquad X_2 \to X$$
$$V_1 \to V_0 \qquad t_1 \to 0 \qquad X_1 \to X_0$$

$$a = a_{avg} = \frac{V - V_0}{t - 0}$$

**TABLE 2-1**

**Equations for Motion with Constant Acceleration[a]** Page 23

| Equation Number | Equation | Missing Quantity |
|---|---|---|
| 2-11 | $v = v_0 + at$ | $x - x_0$ |
| 2-15 | $x - x_0 = v_0 t + \frac{1}{2}at^2$ | $v$ |
| 2-16 | $v^2 = v_0^2 + 2a(x - x_0)$ | $t$ |
| 2-17 | $x - x_0 = \frac{1}{2}(v_0 + v)t$ | $a$ |
| 2-18 | $x - x_0 = vt - \frac{1}{2}at^2$ | $v_0$ |

**Check point 4**

**Page 23**

**Sample problem 2-5**

**Page 23**

**CHECKPOINT 4** The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

## Sample Problem 2-5

The head of a woodpecker is moving forward at a speed of 7.49 m/s when the beak makes first contact with a tree limb. The beak stops after penetrating the limb by 1.87 mm. Assuming the acceleration to be constant, find the acceleration magnitude in terms of $g$.

# 2-9 Free-Fall Acceleration

**If you toss an object either up or down without an air effect, you would find that the object accelerates downward at a certain constant rate .**

**That rate is called the free-fall acceleration and its magnitude is g = 9.8 . The acceleration is independent of the object's characteristics, such as mass, density, or shape.**

**The free –fall acceleration near Earth's surface is**
**$a = -g = -9.8 m/s^2$**

**and the magnitude of the acceleration is $g = 9.8\ m/s^2$**

**-As these objects fall, they acceleration downward at the same rate $g = 9.8\ m/s^2$**

**-Their speed increases at the same rate**

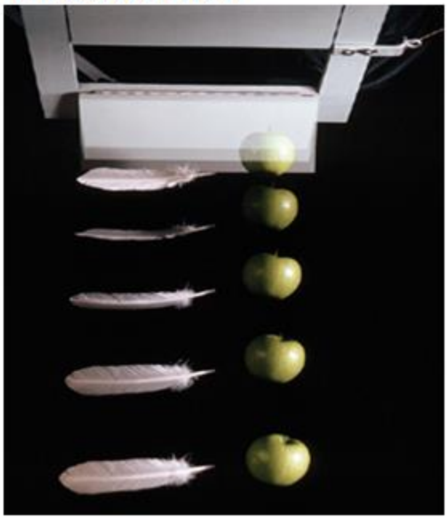
**-They fall together**

**-The value of g varies slightly with the latitude and with the elevation .**

**Examples of free –fall acceleration:**

**- Feather**

**- An apple**



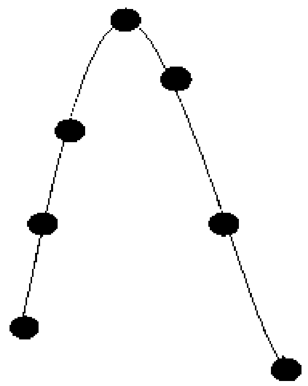
**The direction of motion are now along a vertical y axis instead of the x axis**

**Direction:**

Upward → positive (y)

Downward → negative (y)

**Upward**
**V (+)**
**$V_0$ (+)**
**Y (+)**
**a (-)**



**downward**
**V (-)**
**$V_0$ (-)**
**Y (-)**
**a (-)**

**free-fall acceleration is always negative and thus downward**

**The equation of motion in table 2-1 for constant acceleration also apply to free-fall near Earth's surface**

**$a = -g$ , $x_0 = y_0$ , $x = y$**

$$V = V_0 - gt \qquad\qquad y - y_0 = V_0 - \frac{1}{2} g t^2$$

$$V^2 = V_0^2 - 2g(y - y_0) \qquad\qquad y - y_0 = \frac{1}{2}(V_0 + V)t$$

$$y - y_0 = V t + \frac{1}{2} g t^2$$

Checkpoint 5
page 25

**CHECKPOINT 5** (a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

## Sample Problem 2-7

On September 26, 1993, Dave Munday went over the Canadian edge of Niagara Falls in a steel ball equipped with an air hole and then fell 48 m to the water (and rocks). Assume his initial velocity was zero, and neglect the effect of the air on the ball during the fall.


| $t$ (s) | $y$ (m) | $v$ (m/s) | $a$ (m/s$^2$) |
|---|---|---|---|
| 0 | 0 | 0 | −9.8 |
| 1 | −4.9 | −9.8 | −9.8 |
| 2 | −19.6 | −19.6 | −9.8 |
| 3 | −44.1 | −29.4 | −9.8 |
| | −48.0 | | −9.8 |

(a) How long did Munday fall to reach the water surface?

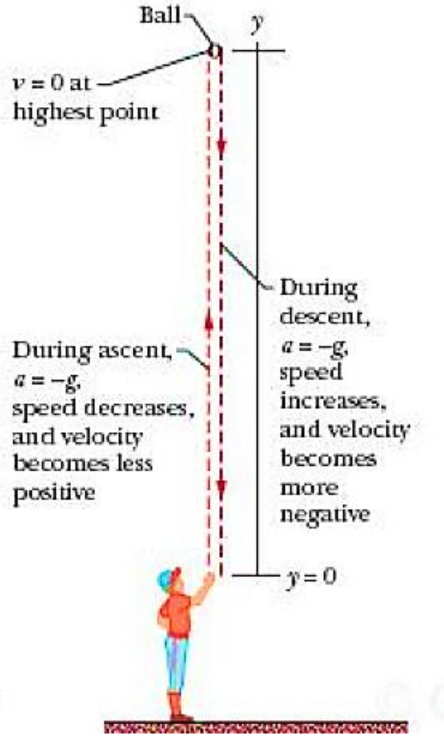
(b) Munday could count off the three seconds of free fall but could not see how far he had fallen with each count. Determine his position at each full second.

(c) What was Munday's velocity as he reached the water surface?

(d) What was Munday's velocity at each count of one full second? Was he aware of his increasing speed?

## Sample Problem 2-8

In Fig. 2-12, a pitcher tosses a baseball up along a $y$ axis, with an initial speed of 12 m/s.




(a) How long does the ball take to reach its maximum height?

(b) What is the ball's maximum height above its release point?

(c) How long does the ball take to reach a point 5.0 m above its release point?