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# 2.7 Implicit Differentiation

**Exercise :**

Find $\frac{dy}{dx}$ if $y^3 + 7y = x^3$

**Solution**

$$3y^2\frac{dy}{dx} + 7\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx}(3y^2 + 7) = 3x^2$$

$$\frac{dy}{dx} = \frac{\mathbf{3x^2}}{\mathbf{3y^2 + 7}}$$

Example 1 : Find $\frac{dy}{dx}$ if $4x^2y - 3y = x^3 - 1$

**Solution**

$$4\left(2xy + x^2\frac{dy}{dx}\right) - 3\frac{dy}{dx} = 3x^2$$

$$8xy + 4x^2\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2$$

$$4x^2\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2 - 8xy$$

$$\frac{dy}{dx}(4x^2 - 3) = 3x^2 - 8xy$$

$$\frac{dy}{dx} = \frac{\mathbf{3x^2 - 8xy}}{\mathbf{4x^2 - 3}}$$

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Example 2 : Find $\frac{dy}{dx}$ if $x^2 + 5y^3 = x + 9$

**Solution**

$$2x + 15y^2 \frac{dy}{dx} = 1$$

$$15y^2 \frac{dy}{dx} = 1 - 2x$$

$$\frac{dy}{dx} = \frac{\mathbf{1 - 2x}}{\mathbf{15y^2}}$$

Example 3 : Find the equation of the tangent line to the curve of $y^3 - xy^2 + \cos xy = 2$ , at the point (0,1)

**Solution**

$$3y^2 \frac{dy}{dx} - \left(y^2 + x \cdot 2y \frac{dy}{dx}\right) - \left(y + x \frac{dy}{dx}\right) \sin xy = 0$$

$$3y^2 \frac{dy}{dx} - y^2 - 2xy \frac{dy}{dx} - y \sin xy - x \frac{dy}{dx} \sin xy = 0$$

$$3y^2 \frac{dy}{dx} - 2xy \frac{dy}{dx} - x \frac{dy}{dx} \sin xy = y^2 + y \sin xy$$

$$\frac{dy}{dx} (3y^2 - 2xy - x \sin xy) = y^2 + y \sin xy$$

$$\frac{dy}{dx} = \frac{\mathbf{y^2 + y \sin xy}}{\mathbf{3y^2 - 2xy - x \sin xy}}$$

$$At\ (0,1) \rightarrow y' = \frac{1}{3}$$

$\therefore$ Equation of tangent line: $y - 1 = \frac{1}{3}(x - 0)$

$$\mathbf{y = \frac{1}{3} x + 1}$$

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Example 4 : If $y = 2x^{5/3} + \sqrt{x^2+1}$, find $D_x y$

**Solution**

$$y = 2x^{5/3} + (x^2+1)^{\frac{1}{2}}$$

$$D_x y = 2 \cdot \frac{5}{3} x^{\frac{5}{3}-1} + \frac{1}{2}(x^2+1)^{\frac{1}{2}-1}(2x)$$

$$= \frac{10}{3} x^{\frac{2}{3}} + \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)$$

$$= \frac{10}{3} x^{\frac{2}{3}} + x(x^2+1)^{-\frac{1}{2}}$$

$$= \frac{10}{3} x^{\frac{2}{3}} + \frac{x}{(x^2+1)^{\frac{1}{2}}}$$

$$= \mathbf{\frac{10}{3} x^{\frac{2}{3}} + \frac{x}{\sqrt{x^2+1}}}$$