

**Question 3 (10 + 10 + 10 + 10 = 40 Points):**

A uniform sheet of charge equal to  $27.9 \text{ nC/m}^2$  lies in  $z = 0$  plane, and a uniform line charge density of  $133.5 \text{ nC/m}$  lies along  $x$ -axis. Find a)  $\vec{D}$  at  $A(0, 0, 4)$  and b)  $\vec{D}$  at  $B(1, 2, 4)$ .

8.3  
40

$\rho_s = 27.9 \text{ nC/m}^2$  in  $z=0$  (And  $\rho_L = 133.5 \text{ nC/m}$  along  $x$ -axis)

a)  $A(0, 0, 4) \Rightarrow \vec{a}_r = \frac{4\vec{a}_z}{4}$  (10)  
 $\Rightarrow \vec{D}_s = \frac{\rho_s}{2} \cdot \vec{a}_r = \frac{27.9 \times 10^{-9}}{2} \cdot \frac{4\vec{a}_z}{4} = 13.95\vec{a}_z \text{ nC/m}^2$

$\vec{a}_r = 4\vec{a}_z$  And  $|\vec{a}_r| = \sqrt{4^2} = 4$

$\Rightarrow \vec{D}_L = \frac{\rho_L}{2\pi r} \cdot \vec{a}_r = \frac{133.5 \times 10^{-9}}{2\pi \times 4} \cdot (4\vec{a}_z) = 5.314\vec{a}_z \text{ nC/m}$  (10)

$\vec{D} = \vec{D}_L + \vec{D}_s = 19.264 \text{ nC/m}$

b)  $B(1, 2, 4) \Rightarrow \vec{a}_r = \vec{a}_z$

$\vec{D}_s = \frac{\rho_s}{2} \cdot \vec{a}_r = \frac{27.9 \times 10^{-9}}{2} \cdot \vec{a}_z = 13.95\vec{a}_z \text{ nC/m}^2$  (10)

$\vec{a}_r = 2\vec{a}_y + 4\vec{a}_z$  so  $|\vec{a}_r| = \sqrt{2^2 + 4^2} = \sqrt{20}$   $\vec{a}_r = \frac{2\vec{a}_y + 4\vec{a}_z}{\sqrt{20}}$

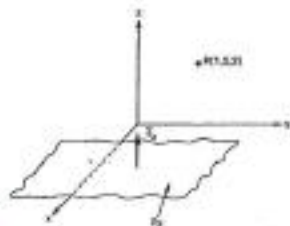
$\vec{D}_L = \frac{\rho_L}{2\pi r} \cdot \vec{a}_r = \frac{133.5 \times 10^{-9}}{2\pi \times \sqrt{20}} \cdot (2\vec{a}_y + 4\vec{a}_z) = 2.125 \times 10^{-9} \vec{a}_y + 4.25 \times 10^{-9} \vec{a}_z$   
 $= (2.125\vec{a}_y + 4.25\vec{a}_z) \text{ nC/m}$  (10)

$\vec{D} = \vec{D}_L + \vec{D}_s = (2.125\vec{a}_y + 18.2\vec{a}_z) \text{ nC/m}$   
 (v/m)

**Question 2 (30 Points):**

Given a sheet of charge  $\rho_s = 25 \text{ nC/m}^2$  over the plane  $z = -1$  and the uniform line charge density  $\rho_L = 180 \text{ nC/m}$  along x-axis. Find  $\vec{E}$  at P (1, 5, 2) in free space. In the end, find total  $\vec{E}$  at point P.

**Constant:**  $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9} = 8.854 \times 10^{-12} \text{ F/m}$



2.2  
30  
$$E_s = \frac{\rho_s}{2\epsilon_0} \cdot a_r$$

$$a_r = a_z$$

So 
$$\Rightarrow E_s = \frac{\rho_s}{2\epsilon_0} \cdot a_r = \frac{25 \times 10^{-9}}{2 \times 8.854 \times 10^{-12}} \cdot (a_z) = 1411.791 \text{ a}_z \text{ V/m (C/m}^2)$$

And 
$$E_L = \frac{\rho_L}{2\pi\epsilon_0 r} \cdot a_r$$

$$r = 5a_y + 2a_z \quad \text{so } |\vec{r}| = \sqrt{5^2 + 2^2} = \sqrt{29}$$

So 
$$a_r = \frac{5a_y + 2a_z}{\sqrt{29}}$$

$$\Rightarrow E_L = \frac{180 \times 10^{-9}}{2 \times 31.4 \times 8.854 \times 10^{-12} \times \sqrt{29}} \cdot (5a_y + 2a_z) = 558.14 a_y + 223.257 a_z \text{ V/m (C/m)}$$

So 
$$\Rightarrow E_{\text{Total}} = E_L + E_s = 558.14 a_y + 223.257 a_z + 1411.791 a_z$$
  

$$= 558.14 a_y + 1635.048 a_z \text{ V/m}$$

Q.1 Question 1 (30 Points):

Consider Spherical co-ordinate system and integrate to find the area of the region  $0 \leq \varphi \leq \beta$  on the spherical shell of radius  $a$ . What will be the area for  $\beta = 2\pi$ ?

Area of region =  $\int_0^\beta \int_0^\pi a^2 \sin \theta \, d\theta \, d\phi$

$$= \int_0^\beta a^2 \left[ -\cos \theta \right]_0^\pi \, d\phi = \int_0^\beta a^2 (-\cos \pi + \cos 0) \, d\phi = \int_0^\beta 2a^2 \, d\phi$$

$$= 2a^2 \phi \Big|_0^\beta = 2a^2 \beta$$

when  $\beta = 2\pi$

$$A = 4a^2 \pi$$

$$\text{Area of region} = 4\pi a^2$$

Good



EE 282 – ELECTROMAGNETIC FIELD THEORY

Fall Semester 2017 - 2018

Mid Term # 01

Date: November 02<sup>nd</sup>, 2017; Duration: 70 minutes

Student's Full Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

Section #:1052

Signature: \_\_\_\_\_

Instructions:

- Write your student ID number on the top of each page.
- Write the solution in the space provided under each question.
- Show all the steps of your calculations.
- Bring your own Calculators, use of mobile phone as calculators and sharing of calculators are strictly NOT allowed.

Question No.	Points Assigned	Points Awarded
1. [CO_1, PI_1_62, SO_1]	30	30
2. [CO_2, PI_1_46, SO_1]	30	30
3. [CO_3, PI_5_23, SO_5]	40	40
Total	100	100

100  
100

Instructor's Full Name	Dr. Khawaja Bilal Mahmood
Signature	