

الأستاذ : ياسر مدرس رياضيات وإحصاء جامعي جوال: 0551807200

23 a) $P(X=k) = \frac{1}{k}$; $k=1,2,3,4,5$

$k: r.v$	1	2	3	4	5
$P(X=k)$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

1) $P(X=k) \geq 0$

2) $\sum_1^5 P(X=k) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$
 $= 2.28 \neq 1$

$P(X=k)$ not mass function

b) $P(X=q) = \frac{1}{q}$; $k=-2, 1, 2, 3, 4, 5$

$k: r.v$	-2	1	2	3	4	5
$P(X=q)$	$\frac{1}{-2}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

$P(X=-2) = -\frac{1}{2} < 0 \rightarrow$ not mass function

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Q3 c)

$$P(X=j) = \frac{1}{9} \quad j = 1, 2, 3, 4, 5, 6, 7, 8, 9$$

k \ nV	1	2	3	4	5	6	7	8	9
$P(X=j)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

$$1) \quad P(X=j) = \frac{1}{9} > 0$$

$$2) \quad \sum_{j=1}^9 P(X=j) = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{9}{9} = 1$$

$$P(X=j) = \frac{1}{9} \text{ mass function}$$

$$d) \quad P(X=x) = -x \quad ; \quad x \in [0, 1]$$

not mass function

x not discrete random variable

$$4) a) f(x) = \begin{cases} \frac{8}{3}x & \text{for } x \in [-0.5, 1] \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^{-0.5} 0 dx + \int_{-0.5}^1 f(x) dx + \int_1^{\infty} 0 dx$$

$$= \int_{-0.5}^1 \frac{8}{3}x dx = \frac{8x^2}{6} \Big|_{-0.5}^1$$

$$= \frac{8}{6} - \frac{8(-0.5)^2}{6} = 1$$

$f(x) \rightarrow$ density function (DF)

$$b) f(x) = \begin{cases} 2x & \text{for } x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^0 0 dx + \int_0^1 f(x) dx + \int_1^{\infty} 0 dx$$

$$= \int_0^1 2x dx = x^2 \Big|_0^1 = 1^2 - 0^2 = 1$$

$f(x)$ density function (DF)

Q 4) c)

$$f(x) = \begin{cases} x & x \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^0 0 dx + \int_0^1 f(x) dx + \int_1^{\infty} 0 dx$$

$$= \int_0^1 x dx = \left. \frac{x^2}{2} \right|_0^1 = \frac{1}{2} - 0$$

$$= \frac{1}{2} \neq 1$$

$f(x)$ not density function

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Q5) ai | $8! = 40320$

ii | $(8-1)! = 7! = 5040$

b) i | ${}_{12}P_5 = \frac{12!}{(12-5)!} = 95040$

ii | ${}_{12}C_5 = \frac{12!}{5!(12-5)!} = 792$

c) ${}_{52}C_7 = 133784560$

d) $28^2 \times 10^4 + 26^3 \times 10^5 = 111460$

e) ${}_{16}C_6 \times {}_6C_3 = 160160$

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Q: 7

S_1 , S_2 , S_3 , S_4
6% 5% 2% 7%

Let E selected student is excellent

$$P(S_1) = P(S_2) = P(S_3) = P(S_4) = 0.25$$

$$P(E) = \sum_{k=1}^4 P(S_k) \cdot P(E|S_k)$$

$$P(E) = P(S_1) \cdot P(E|S_1) + P(S_2) \cdot P(E|S_2) \\ + P(S_3) \cdot P(E|S_3) + P(S_4) \cdot P(E|S_4)$$

$$P(E) = 0.25 \times 0.06 + 0.25 \times 0.05 \\ + 0.25 \times 0.02 + 0.25 \times 0.07$$

$$P(E) = 0.05$$

$$P(\bar{E}) = 1 - 0.05 = 0.95$$

سَمَة السَّوَال] Q7

$$P(S_3 | \bar{E}) = \frac{P(S_3) \cdot P(\bar{E} | S_3)}{P(\bar{E})}$$

$$= \frac{0.25 \times (1 - 0.02)}{0.95}$$

$$= \frac{49}{190} = 0.2579$$

8) a)

$$P(A) = \frac{2}{13} \times \frac{2}{13} \times \frac{0}{13} \times \frac{0}{13} = 0$$

$$b) P(B) = \frac{7}{13} \times \frac{6}{12} \times \frac{5}{11} \times \frac{4}{10} + \frac{4}{13} \times \frac{3}{12} + \frac{2}{11} + \frac{1}{10} =$$

$$P(B) = \frac{7}{143} + \frac{2401}{2860} = 0.888$$

$$c) P(C) = \frac{{}^7C_4 \times {}^6C_0}{{}^{13}C_4} + \frac{{}^4C_4 \times {}^9C_0}{{}^{13}C_4} = 0.050$$

$$c) P(D) = \frac{7}{13} \times \frac{4}{12} \times \frac{2}{11} \times \frac{0}{10}$$
$$P(D) = 0$$

Q 9 | Let A the car wants foretold

$$P(A) = 0.95, P(\bar{A}) = 0.05$$

$$a) P(\bar{A}) = 1 - 0.95 = 0.05$$

$$b) P(A \cap B) = P(A) \cdot P(B)$$

$$= 0.95 \times 0.95 = 0.9025$$

$$c) P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$$

$$= 0.05 \times 0.05 \times 0.05$$

$$= 0.000125$$

$$d) P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(X=k) = \binom{10}{k} (0.95)^k (0.05)^{10-k}$$

سنة الدور و

Q9

د

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) = \\ &= 1 - P(X = 0) \\ &= 1 - \binom{10}{0} (0.95)^0 (0.05)^{10} \\ &= 1 \end{aligned}$$

$$\begin{aligned} &P(X=1) + P(X=2) + P(X=3) + P(X=4) \\ &+ P(X=5) + P(X=6) + P(X=7) = \\ &1 - [P(X=8) + P(X=9) + P(X=10)] \\ &1 - \left[\binom{10}{8} (0.95)^8 (0.05)^2 + \binom{10}{9} (0.95)^9 (0.05)^1 \right. \\ &\quad \left. + \binom{10}{10} (0.95)^{10} (0.05)^0 \right] \\ &= 0.0115 \end{aligned}$$

$$e) \quad \lambda = 5 \rightarrow \mu = \frac{1}{5}$$

$$f_x(x) = \frac{1}{5} e^{-\frac{x}{5}} \quad x \geq 0$$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - \left(\int_0^2 \frac{1}{5} e^{-\frac{x}{5}} dx \right)$$

$$= 1 - \frac{1}{5} \int_0^2 e^{-\frac{x}{5}} dx$$

$$= 1 - \frac{1}{5} (-5) \left[e^{-\frac{x}{5}} \right]_0^2$$

$$= 1 + e^{-\frac{2}{5}} - e^0$$

$$= 1 + e^{-\frac{2}{5}} - 1 = e^{-\frac{2}{5}}$$

$$= 0.67$$

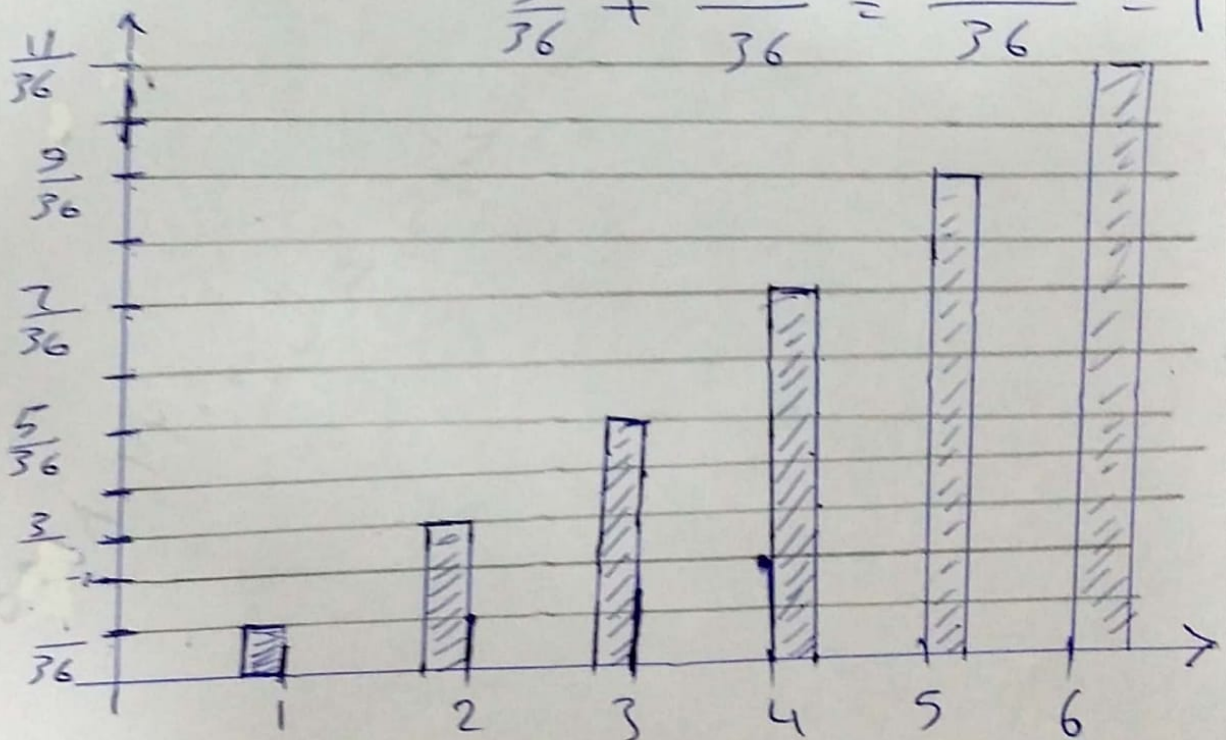
Q.10 a) $X = 1, 2, 3, 4, 5, 6$
 (b)

k	n_{v_x}	1	2	3	4	5	6
$P(X=k)$		$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

v) $P(X=k) > 0$

2) $\sum_{k=1}^6 P(X=k) = \frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{7}{36} +$

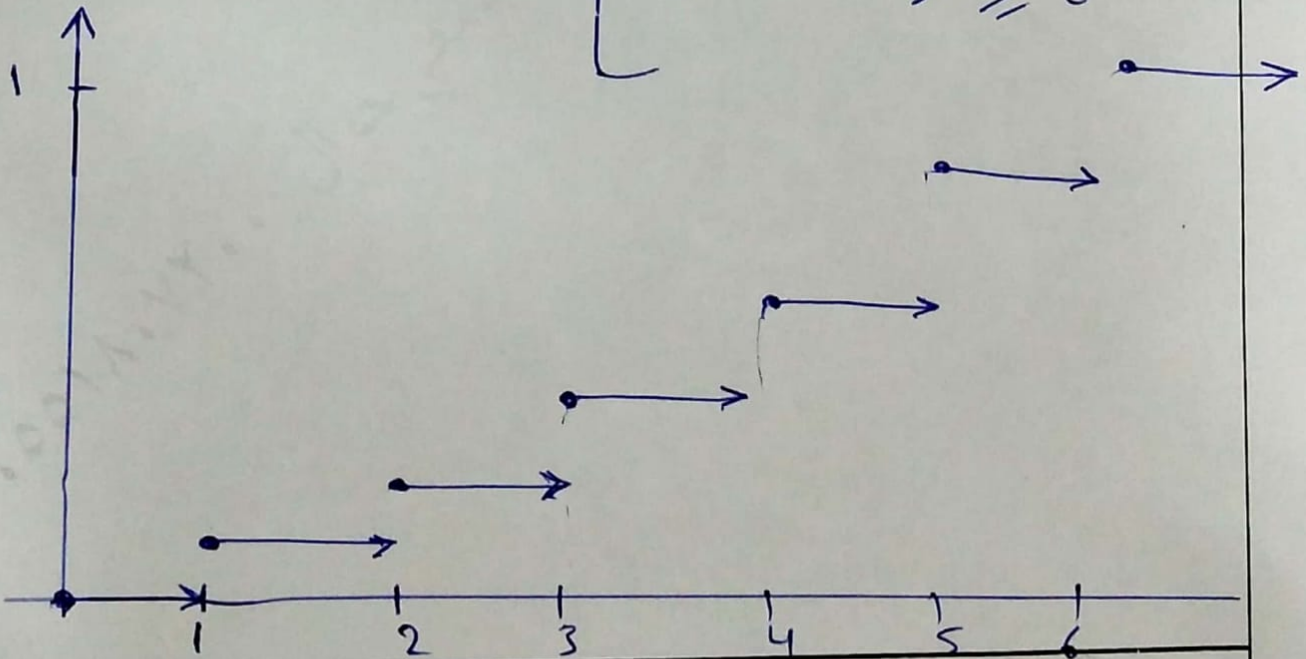
c) $\frac{9}{36} + \frac{11}{36} = \frac{36}{36} = 1$



d)

$$F(x) = \sum_1^x P(X=x)$$

0	$x < 1$
$\frac{1}{36}$	$1 \leq x < 2$
$\frac{4}{36}$	$2 \leq x < 3$
$\frac{9}{36}$	$3 \leq x < 4$
$\frac{16}{36}$	$4 \leq x < 5$
$\frac{25}{36}$	$5 \leq x < 6$
1	$x \geq 6$



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Q2 10 | f

X	$P(X=.)$	$X \cdot P(X)$	X^2	$X^2 \cdot P(X)$
1	$\frac{1}{36}$	$\frac{1}{36}$	1	$\frac{1}{36}$
2	$\frac{3}{36}$	$\frac{6}{36}$	4	$\frac{12}{36}$
3	$\frac{5}{36}$	$\frac{15}{36}$	9	$\frac{45}{36}$
4	$\frac{7}{36}$	$\frac{28}{36}$	16	$\frac{112}{36}$
5	$\frac{9}{36}$	$\frac{45}{36}$	25	$\frac{225}{36}$
6	$\frac{11}{36}$	$\frac{66}{36}$	36	$\frac{396}{36}$
Total	1	$\frac{161}{36}$		$\frac{791}{36}$

$$\begin{aligned}\mu = E(X) &= \sum_{i=1}^6 x_i \cdot P(X=x_i) \\ &= \frac{161}{36} = 4.472\end{aligned}$$

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g) 10)

$$\begin{aligned} \text{g)} \quad \text{Var}(X) &= E(X^2) - (E(X))^2 \\ &= 21.97 - (4.472)^2 \\ &= 21.97 - 19.99 \end{aligned}$$

$$\text{Var}(X) = 1.97$$

$$\sigma = \sqrt{1.97} = 1.40$$

$$\text{h)} \quad \text{Var}(Y) = \text{Var}(2X + 5)$$

$$= 2^2 \text{Var}(X)$$

$$= 4(1.97) = 7.88$$

$$\sigma_Y = 2.81$$

Q 11 a)

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_{-\infty}^{-1} 0 dx + \int_{-1}^1 (ax+b) dx + \int_1^{\infty} 0 dx = 1$$

$$= \int_{-1}^1 (ax+b) dx = 1$$

$$\int_{-1}^1 (ax+b) dx = \left[\frac{ax^2}{2} + bx \right]_{-1}^1 =$$

$$\left(\frac{a}{2} + b \right) - \left(\frac{a}{2} - b \right) =$$

$$\frac{a}{2} + b - \frac{a}{2} + b = 2b$$

$$2b = 1 \rightarrow \boxed{b = \frac{1}{2} = 0.5}$$

$$P(X > 0) = 0.25$$

$$P(X > 0) = 1 - F_x(x) = 0.25$$

Q11) a

سمة السؤال 11

$$P(X \leq 0) = \int_{-\infty}^0 f(x) \cdot dx = \int_{-\infty}^0 (ax+b) dx$$

$$= \int_{-\infty}^{-1} 0 dx + \int_{-1}^0 (ax+b) dx =$$

$$= \left. \frac{ax^2}{2} + bx \right|_{-1}^0 = 0 - \left(\frac{a}{2} - b \right)$$

$$= -\frac{a}{2} + b = b - \frac{a}{2}$$

$$1 - P(X \leq 0) = 0.25$$

$$1 - \left(b - \frac{a}{2} \right) = 0.25$$

$$1 - \frac{1}{2} + \frac{a}{2} = 0.25$$

$$\frac{1}{2} + \frac{a}{2} = 0.25 \rightarrow \frac{a}{2} = 0.25 - 0.5$$

$$\frac{a}{2} = -0.25 \rightarrow \boxed{a = -0.5}$$

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32 11) a)

$$a = -0.5, \quad b = 0.5$$

$$f(x) = \begin{cases} -0.5x + 0.5 & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$b) P(-1.2 < X < 0.5) = \int_{-1.2}^{0.5} f(x) \cdot dx =$$

$$= \int_{-1.2}^{-1} 0 dx + \int_{-1}^{0.5} (-0.5x + 0.5) dx =$$

$$= \left[\frac{-0.5x^2}{2} + 0.5x \right]_{-1}^{0.5} =$$

$$= \left[\frac{-0.5(-1)^2}{2} + 0.5(-1) \right] - \left[\frac{-0.5(0.5)^2}{2} + 0.5(0.5) \right]$$

$$= \frac{16}{15} = 0.9375$$

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ع) 11

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(t) \cdot dt$$

$$= \int_{-\infty}^{-1} 0 dx + \int_{-1}^x f(x) \cdot dt$$

$$= \int_{-1}^x (-0.5t + 0.5) dt$$

$$= \left[-\frac{0.5t^2}{2} + 0.5t \right]_{-1}^x$$

$$= -0.25t^2 + 0.5t \Big|_{-1}^x$$

$$= -0.25X^2 + 0.5X - \left(-0.5(-1)^2 + 0.5(-1) \right)$$

$$= -0.25X^2 + 0.5X + 0.75$$

$$F(x) = \begin{cases} 0 & x < -1 \\ -0.25X^2 + 0.5X + 0.75 & -1 < x < 1 \\ 1 & x > 1 \end{cases}$$

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$$Q_{11}) \quad F(x) = \begin{cases} 0 & x < -1 \\ -0.25x^2 + 0.5x + 0.75 & -1 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

$$\begin{aligned} \mu = E(x) &= \int_{-\infty}^{\infty} x \cdot f(x) dx = \int_{-\infty}^{-1} 0 dx + \int_{-1}^1 x f(x) dx + \int_1^{\infty} 0 dx = \int_{-1}^1 x(-0.5x + 0.5) dx \\ &= \int_{-1}^1 (-0.5x^2 + 0.5x) dx = \left[\frac{-0.5x^3}{3} + \frac{0.5x^2}{2} \right]_{-1}^1 \\ &= \left[\frac{-0.5(1)^3}{3} + \frac{0.5(1)^2}{2} \right] - \left[\frac{-0.5(-1)^3}{3} + \frac{0.5(-1)^2}{2} \right] \\ &= \frac{-1}{3} \end{aligned}$$

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سنة الأول 11 Q111

$$\downarrow E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x) dx$$

$$= \int_{-1}^1 x^2 (-0.5x + 0.5) dx$$

$$= \int_{-1}^1 (-0.5x^3 + 0.5x^2) dx$$

$$= \left. \frac{-0.5x^4}{4} + \frac{0.5x^3}{3} \right|_{-1}^1$$

$$= \left(\frac{-0.5(1)^4}{4} + \frac{0.5(1)^3}{3} \right) - \left(\frac{-0.5(-1)^4}{4} + \frac{0.5(-1)^3}{3} \right)$$

$$= \frac{1}{3}$$

$$\text{Var}(X) = E(x^2) - (E(x))^2$$

$$= \frac{1}{3} - \left(-\frac{1}{3}\right)^2 = \frac{2}{9}$$

$$\sigma = \sqrt{\frac{2}{9}} = 0.471$$

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Q: 12 | $\mu = 175, \sigma = 15$

$$\begin{aligned} \text{a) } P(X < 155) &= P\left(\frac{X - \mu}{\sigma} < \frac{155 - 175}{15}\right) \\ &= P(Z < -1.33) = \Phi(-1.33) \\ &= 0.0918 \end{aligned}$$

b) $P(177 < X < 188)$

$$\begin{aligned} P\left(\frac{177 - 175}{15} < \frac{X - \mu}{\sigma} < \frac{188 - 175}{15}\right) \\ P(0.13 < Z < 0.87) &= \\ P(Z < 0.87) - P(Z < 0.13) &= \\ \Phi(0.87) - \Phi(0.13) &= \\ 0.8078 - 0.5517 &= \\ 0.2561 \end{aligned}$$

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Q 12

$$\Rightarrow P(X > 195) = 1 - P(X < 195)$$

$$= 1 - P\left(\frac{X - \mu}{\sigma} < \frac{195 - 175}{15}\right)$$

$$= 1 - P(Z < 1.33)$$

$$= 1 - \Phi(1.33) =$$

$$= 1 - 0.9082$$

$$= 0.0918$$

$$13) \quad P(X=k) = \frac{c}{5k} ; k=1,2,3,4,5$$

$$a) \quad \sum_{k=1}^5 P(X=k) = 1$$

$$\sum_{k=1}^5 \frac{c}{5k} = \sum_{k=1}^5 \frac{1}{5} \cdot \frac{c}{k} = \frac{1}{5} \sum_{k=1}^5 \frac{c}{k} = 1$$

$$\frac{1}{5} \left(\frac{c}{1} + \frac{c}{2} + \frac{c}{3} + \frac{c}{4} + \frac{c}{5} \right) = 1$$

60 30 20 15 12

$$\frac{1}{5} \left(\frac{60c + 30c + 20c + 15c + 12c}{60} \right) = 1$$

$$\frac{137c}{5 \times 60} = 1 \rightarrow 137c = 300$$

$$c = \frac{300}{137}$$

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13) ب

$$P(X=k) = \frac{C}{5k} = \frac{\frac{300}{137}}{\frac{5k}{1}} = \frac{300}{137} \times \frac{1}{5k}$$
$$= \frac{60}{137k} \quad k = 1, 2, 3, 4, 5$$

X	1	2	3	4	5
P(X=k)	$\frac{60}{137}$	$\frac{30}{137}$	$\frac{20}{137}$	$\frac{15}{137}$	$\frac{12}{137}$

14)

$$k = 1 - (0.01 + 0.29 + 0.20 + 0.35 + 0.02)$$

$$k = 1 - 0.87 = 0.13$$

$$k = 0.13$$

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Q 6 a)

$$P(A) = P(A|B) + P(A \cap B) \\ = 0.25 + 0.25 = 0.5$$

$$P(B) = P(B|A) + P(A \cap B) = \\ = 0.25 + 0.25 = 0.5$$

$$P(C) = P(C|A) + P(A \cap C) \\ = 0.25 + 0.25 = 0.5$$

$$P(A|C) = P(A) - P(A \cap C) \\ = 0.5 - 0.25 = 0.25$$

$$P(C|B) = P(C) - P(C \cap B) \\ = 0.5 - 0.25 = 0.25$$

سنة الأول ادسي ١٩

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١٩ ١٩

$$P(B|C) = P(B) - P(B \cap C) \\ = 0.5 - 0.25 = 0.25$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.5} = 0.5$$

$$P(C|A \cap B) = \frac{P(C \cap A \cap B)}{P(A \cap B)} = \frac{0.125}{0.25} \\ = 0.5$$

$$b) P(\bar{A} \cup \bar{B} \cup \bar{C}) = P(\overline{A \cap B \cap C}) \\ = 1 - P(A \cap B \cap C) = 1 - 0.125 \\ = 0.875$$

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سؤال الـ ٦ ب

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ق26 : ب

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = 0.5 + 0.5 + 0.5 - 0.25 - 0.25 - 0.25 + 0.125 = 0.875$$

c) $P(A) \cdot P(B) = 0.5 \times 0.5 = 0.25$

$$P(A \cap B) = 0.25$$

$$P(A \cap B) = P(A) \cdot P(B)$$

A and B independent

$$P(A) \cdot P(C) = 0.5 \times 0.5 = 0.25$$

$$P(A \cap C) = 0.25 = P(A) \cdot P(C)$$

A and C independent

$$P(B) \cdot P(C) = 0.5 \times 0.5 = 0.25$$

$$P(B \cap C) = 0.25 = P(B) \cdot P(C)$$

B and C independent

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Q 1

- a) Number of heads in 10 tosses of coin
- b) The random variable that recording the number of heads by tossing a coin infinite times
- c) The variable that measures the life. TV

Q 2 a) discrete

b) discrete

c) discrete

d) discrete

e) continuous