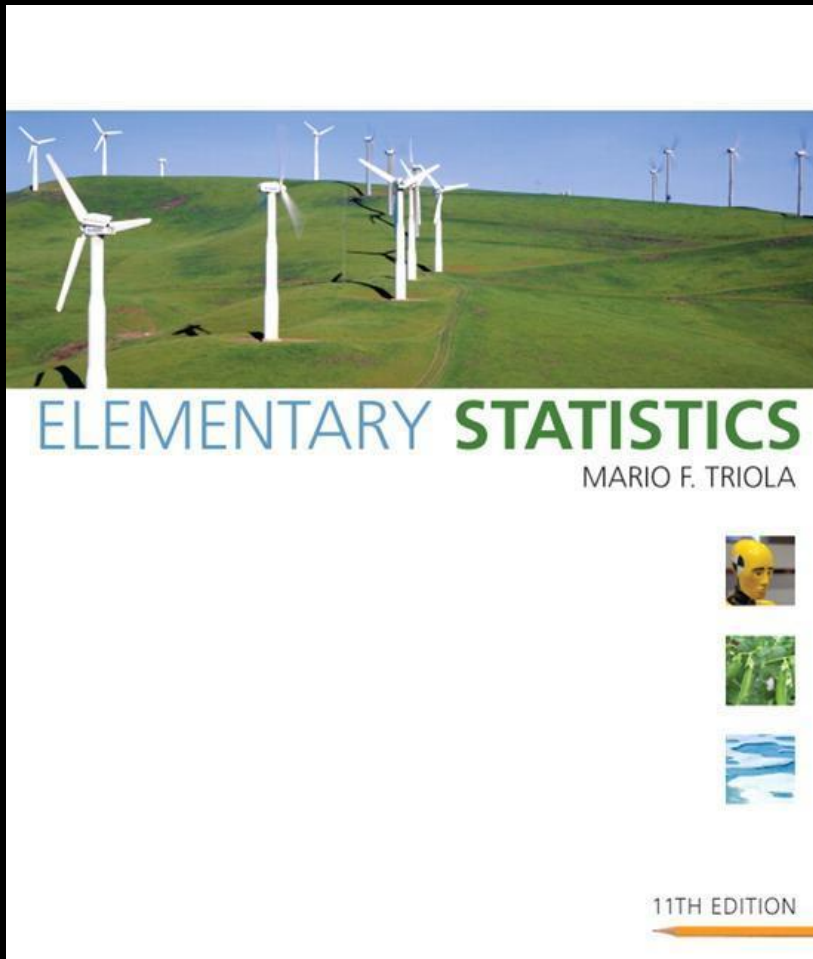


# Lecture Slides

## *Elementary Statistics* Eleventh Edition

and the Triola Statistics Series

by Mario F. Triola



# Chapter 4

# Probability

## 4-1 Review and Preview

## 4-2 Basic Concepts of Probability

## 4-3 Addition Rule

## 4-4 Multiplication Rule: Basics

## 4-5 Multiplication Rule: Complements and Conditional Probability

## 4-6 Probabilities Through Simulations

## 4-7 Counting



**Section 4-1**  
**Review and Preview**

# Review

**Necessity of sound sampling methods.**

**Common measures of characteristics of data**

**Mean**

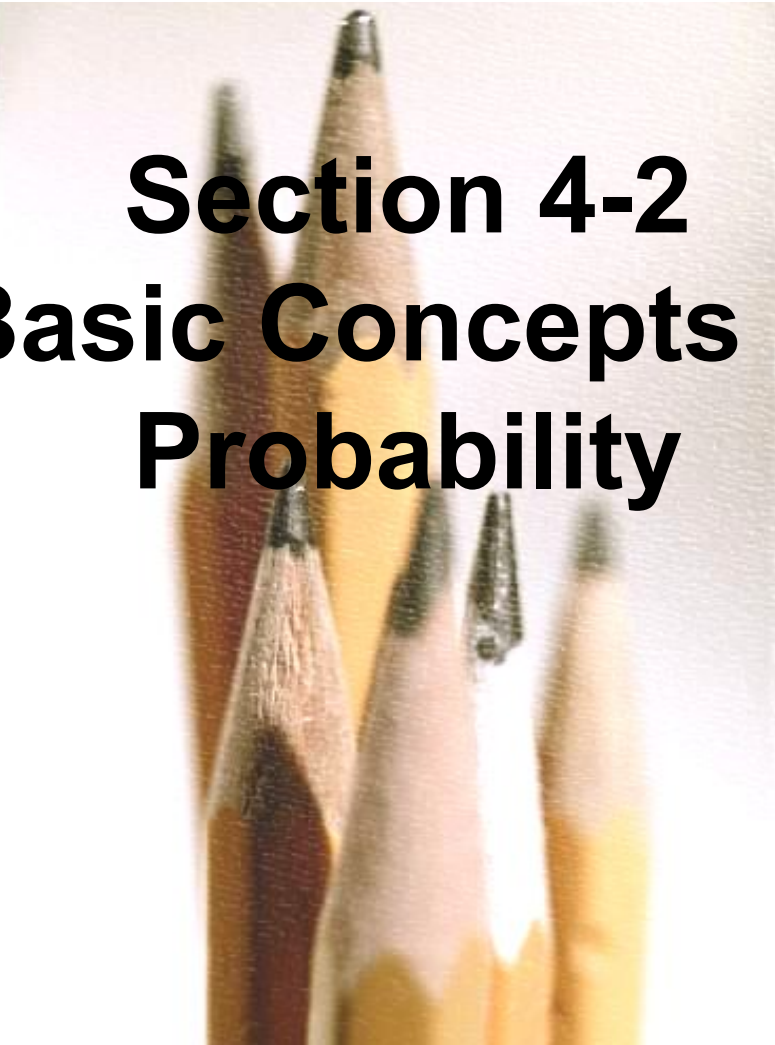
**Standard deviation**

# Preview

## Rare Event Rule for Inferential Statistics:

**If, under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.**

**Statisticians use the rare event rule for inferential statistics.**



**Section 4-2**  
**Basic Concepts of**  
**Probability**

# Key Concept

This section presents three approaches to finding the **probability** of an event.

The most important objective of this section is to learn how to **interpret** probability values.

# Part 1

## Basics of Probability



# Events and Sample Space



## Event

any collection of results or outcomes of a procedure



## Simple Event

an outcome or an event that cannot be further broken down into simpler components



## Sample Space

for a procedure consists of all possible **simple** events; that is, the sample space consists of all outcomes that cannot be broken down any further

# Notation for Probabilities

**$P$**  - denotes a probability.

**$A$ ,  $B$ , and  $C$**  - denote specific events.

**$P(A)$**  - denotes the probability of event  $A$  occurring.

# Basic Rules for Computing Probability

## Rule 1: Relative Frequency Approximation of Probability

Conduct (or observe) a procedure, and count the number of times event  $A$  actually occurs. Based on these actual results,  $P(A)$  is **approximated** as follows:

$$P(A) = \frac{\text{\# of times } A \text{ occurred}}{\text{\# of times procedure was repeated}}$$

# Basic Rules for Computing Probability - continued

## Rule 2: Classical Approach to Probability (Requires Equally Likely Outcomes)

Assume that a given procedure has  $n$  different simple events and that **each of those simple events has an equal chance of occurring**. If event  $A$  can occur in  $s$  of these  $n$  ways, then

$$P(A) = \frac{s}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}}$$

# Basic Rules for Computing Probability - continued

## Rule 3: Subjective Probabilities

$P(A)$ , the probability of event  $A$ , is **estimated** by using knowledge of the relevant circumstances.

# Law of Large Numbers

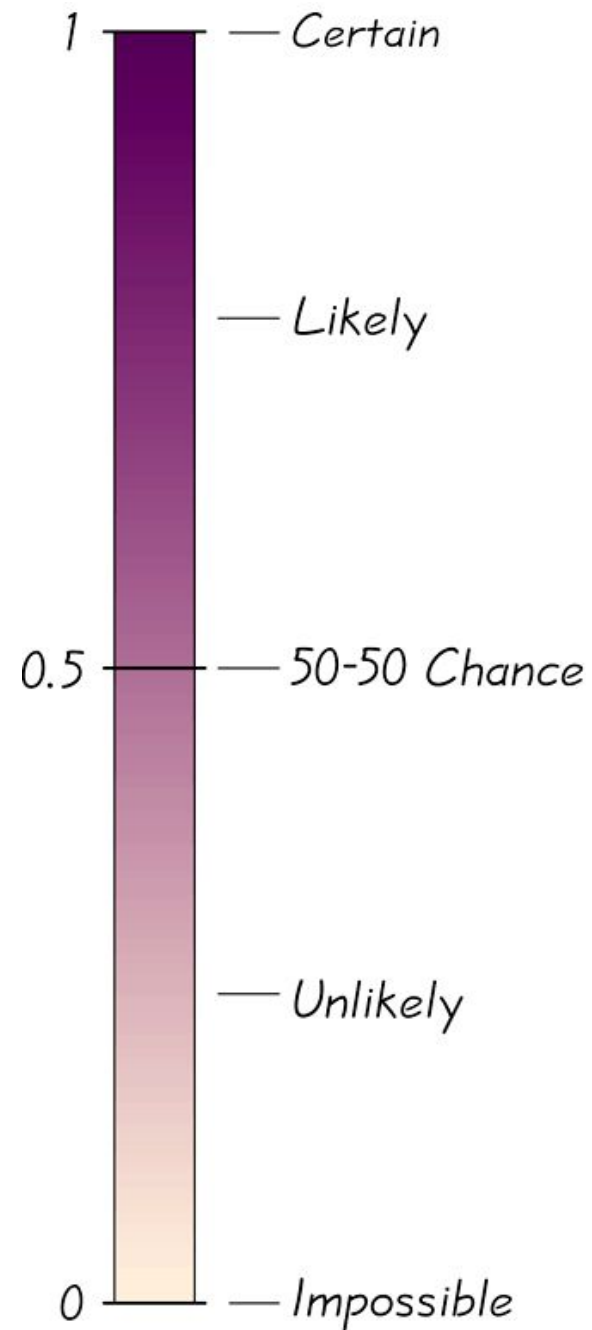
**As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.**

# Probability Limits

**Always express a probability as a fraction or decimal number between 0 and 1.**

- ❖ **The probability of an impossible event is 0.**
- ❖ **The probability of an event that is certain to occur is 1.**
- ❖ **For any event  $A$ , the probability of  $A$  is between 0 and 1 inclusive.  
That is,  $0 \leq P(A) \leq 1$ .**

# Possible Values for Probabilities





# Complementary Events

The complement of event  $A$ , denoted by  $\bar{A}$ , consists of all outcomes in which the event  $A$  does **not** occur.

# Rounding Off Probabilities

When expressing the value of a probability, either give the **exact** fraction or decimal or round off final decimal results to three significant digits. (*Suggestion: When a probability is not a simple fraction such as  $2/3$  or  $5/9$ , express it as a decimal so that the number can be better understood.*)

# Part 2

## Beyond the Basics of Probability: Odds

# Odds

The **actual odds against** event  $A$  occurring are the ratio  $P(A)/P(\bar{A})$ , usually expressed in the form of  $a:b$  (or “ $a$  to  $b$ ”), where  $a$  and  $b$  are integers having no common factors.

The **actual odds in favor** of event  $A$  occurring are the ratio  $P(\bar{A})/P(A)$ , which is the reciprocal of the actual odds against the event. If the odds against  $A$  are  $a:b$ , then the odds in favor of  $A$  are  $b:a$ .

The **payoff odds** against event  $A$  occurring are the ratio of the net profit (if you win) to the amount bet.

**payoff odds against event  $A = (\text{net profit}) : (\text{amount bet})$**

# Recap

**In this section we have discussed:**

- ❖ **Rare event rule for inferential statistics.**
- ❖ **Probability rules.**
- ❖ **Law of large numbers.**
- ❖ **Complementary events.**
- ❖ **Rounding off probabilities.**
- ❖ **Odds.**



**Section 4-3**  
**Addition Rule**

# Key Concept

This section presents the **addition rule** as a device for finding probabilities that can be expressed as  $P(A \text{ or } B)$ , the probability that either event  $A$  occurs or event  $B$  occurs (or they both occur) as the single outcome of the procedure.

The key word in this section is “or.” It is the *inclusive or*, which means either one or the other or both.

# Compound Event

## Compound Event

any event combining 2 or more simple events

### Notation

$P(A \text{ or } B) = P$  (in a single trial, event  $A$  occurs or event  $B$  occurs or they both occur)



# General Rule for a Compound Event

When finding the probability that event  $A$  occurs or event  $B$  occurs, find the total number of ways  $A$  can occur and the number of ways  $B$  can occur, but **find that total in such a way that no outcome is counted more than once.**

# Compound Event

## Formal Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

where  $P(A \text{ and } B)$  denotes the probability that  $A$  and  $B$  both occur at the same time as an outcome in a trial of a procedure.

# Compound Event

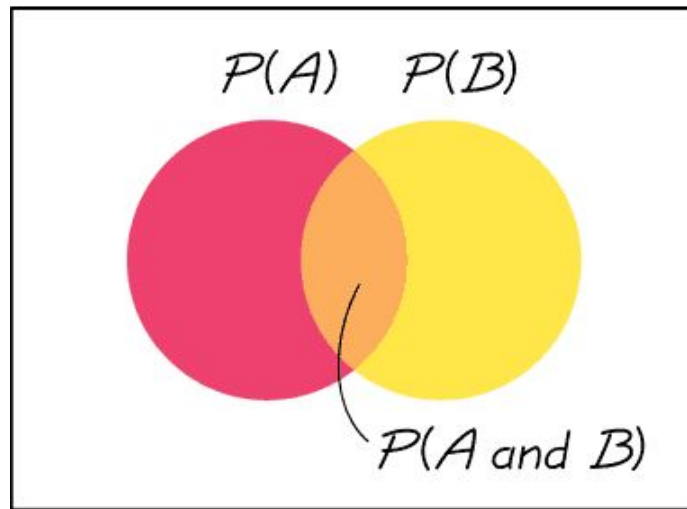
## Intuitive Addition Rule

To find  $P(A \text{ or } B)$ , find the sum of the number of ways event  $A$  can occur and the number of ways event  $B$  can occur, **adding in such a way that every outcome is counted only once**.  $P(A \text{ or } B)$  is equal to that sum, divided by the total number of outcomes in the sample space.

# Disjoint or Mutually Exclusive

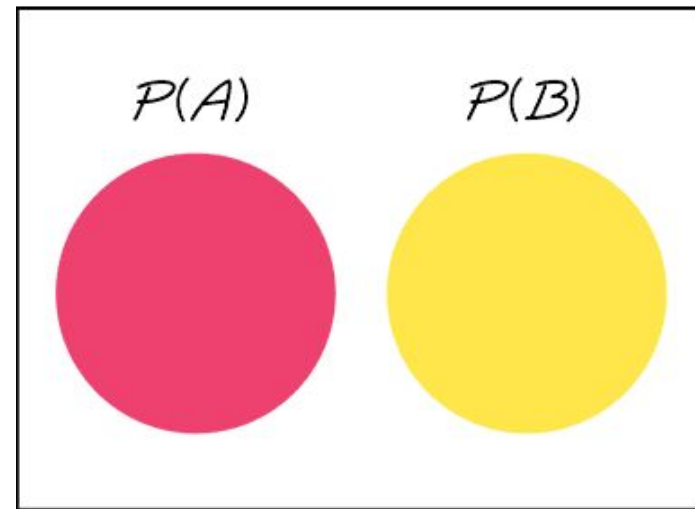
Events  $A$  and  $B$  are **disjoint** (or **mutually exclusive**) if they cannot occur at the same time. (That is, disjoint events do not overlap.)

Total Area = 1



Venn Diagram for Events That Are Not Disjoint

Total Area = 1



Venn Diagram for Disjoint Events

# Complementary Events

$P(A)$  and  $P(\bar{A})$   
are disjoint

**It is impossible for an event and its complement to occur at the same time.**

# Rule of Complementary Events

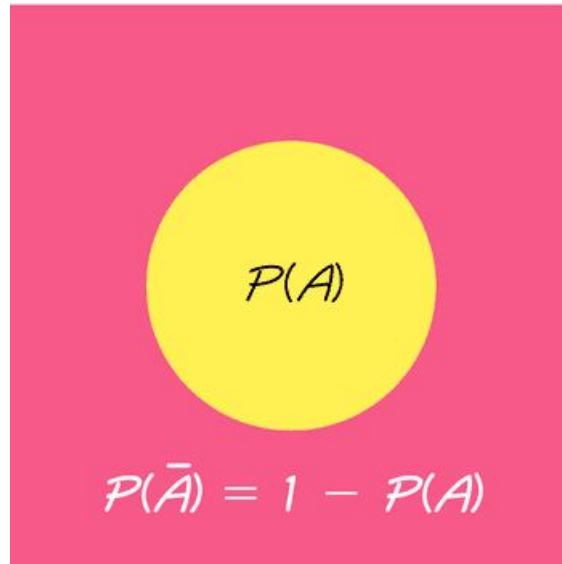
$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\bar{A})$$

# Venn Diagram for the Complement of Event A

Total Area = 1

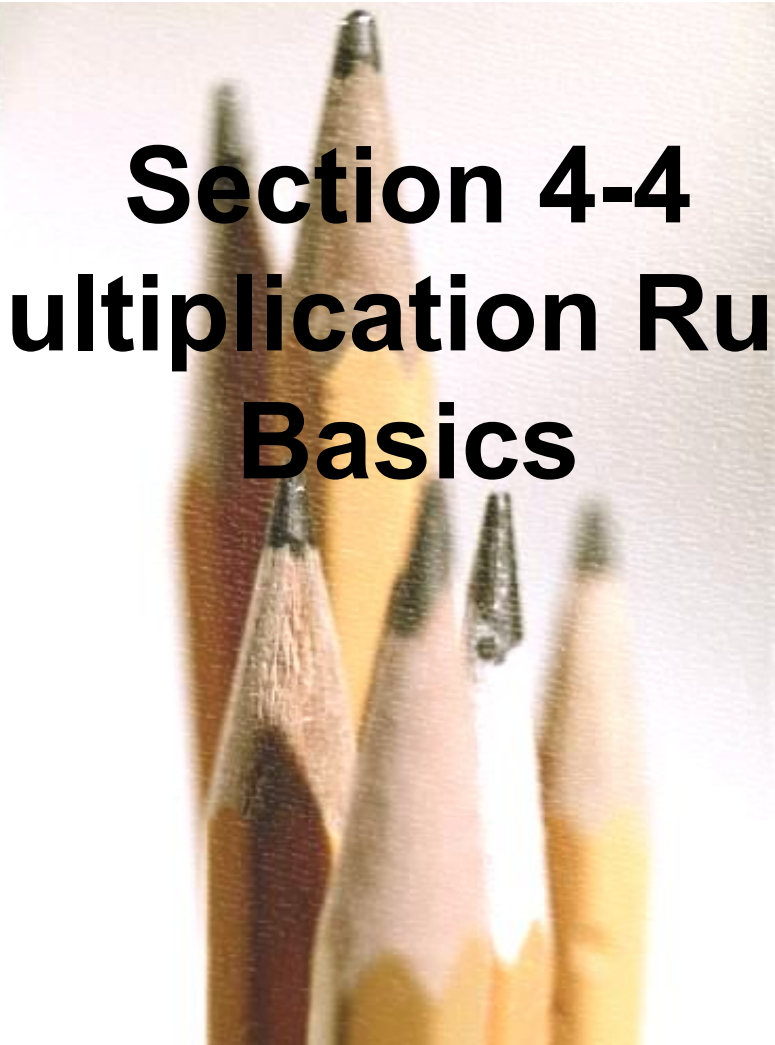


# Recap

In this section we have discussed:

- ❖ **Compound events.**
- ❖ **Formal addition rule.**
- ❖ **Intuitive addition rule.**
- ❖ **Disjoint events.**
- ❖ **Complementary events.**





**Section 4-4**  
**Multiplication Rule:**  
**Basics**

# Key Concept

**The basic multiplication rule is used for finding  $P(A \text{ and } B)$ , the probability that event  $A$  occurs in a first trial and event  $B$  occurs in a second trial.**

**If the outcome of the first event  $A$  somehow affects the probability of the second event  $B$ , it is important to adjust the probability of  $B$  to reflect the occurrence of event  $A$ .**

# Notation

**$P(A \text{ and } B) =$**

**$P(\text{event } A \text{ occurs in a first trial and}$   
 **$\text{event } B \text{ occurs in a second trial})$****

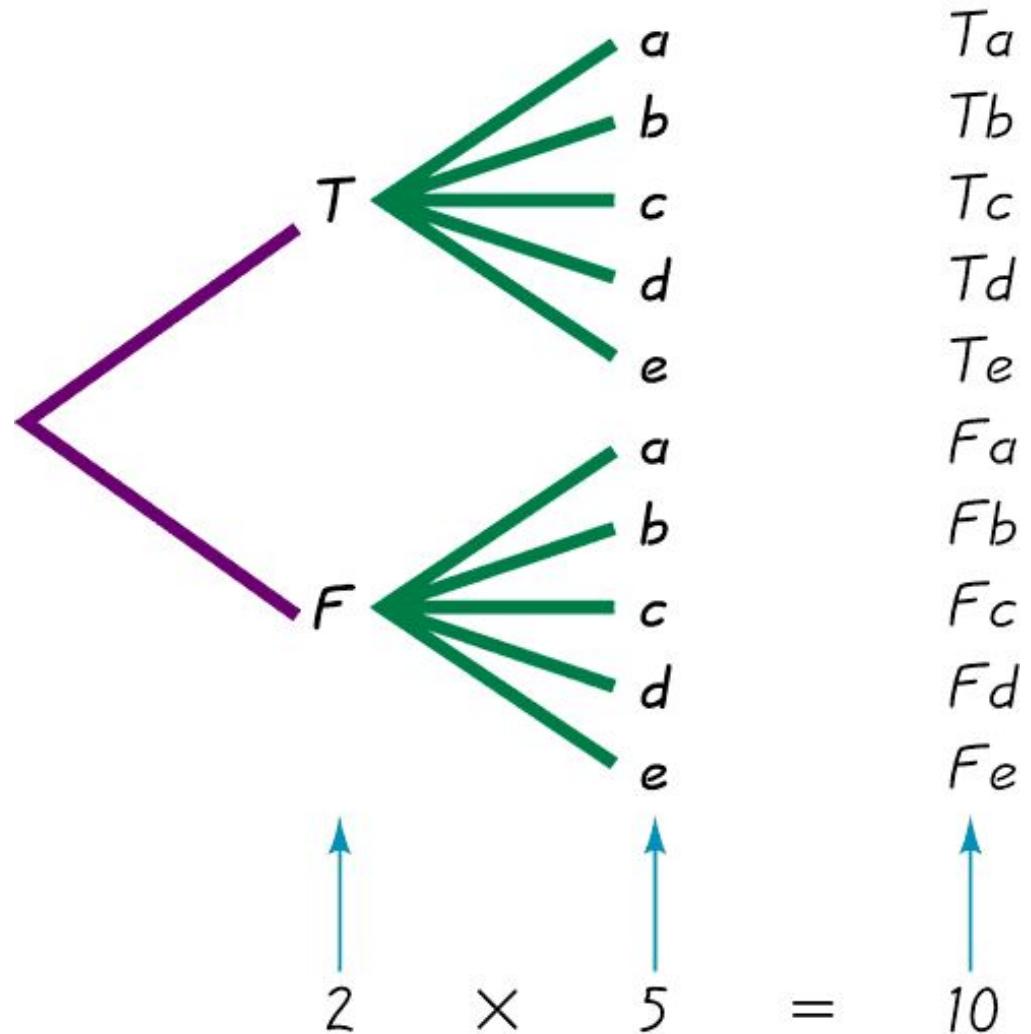
# Tree Diagrams

A **tree diagram** is a picture of the possible outcomes of a procedure, shown as line segments emanating from one starting point. These diagrams are sometimes helpful in determining the number of possible outcomes in a sample space, if the number of possibilities is not too large.

# Tree Diagrams

This figure summarizes the possible outcomes for a true/false question followed by a multiple choice question.

Note that there are 10 possible combinations.



# Conditional Probability

## Key Point

**We must adjust the probability of the second event to reflect the outcome of the first event.**

# Conditional Probability

## Important Principle

**The probability for the second event  $B$  should take into account the fact that the first event  $A$  has already occurred.**

# Notation for Conditional Probability

**$P(B|A)$  represents the probability of event  $B$  occurring after it is assumed that event  $A$  has already occurred (read  $B|A$  as “ $B$  given  $A$ .”)**



# Dependent and Independent

Two events  $A$  and  $B$  are **independent** if the occurrence of one does not affect the *probability* of the occurrence of the other. (Several events are similarly independent if the occurrence of any does not affect the probabilities of the occurrence of the others.) If  $A$  and  $B$  are not independent, they are said to be **dependent**.

# Dependent Events

**Two events are dependent if the occurrence of one of them affects the *probability* of the occurrence of the other, but this does not necessarily mean that one of the events is a *cause* of the other.**

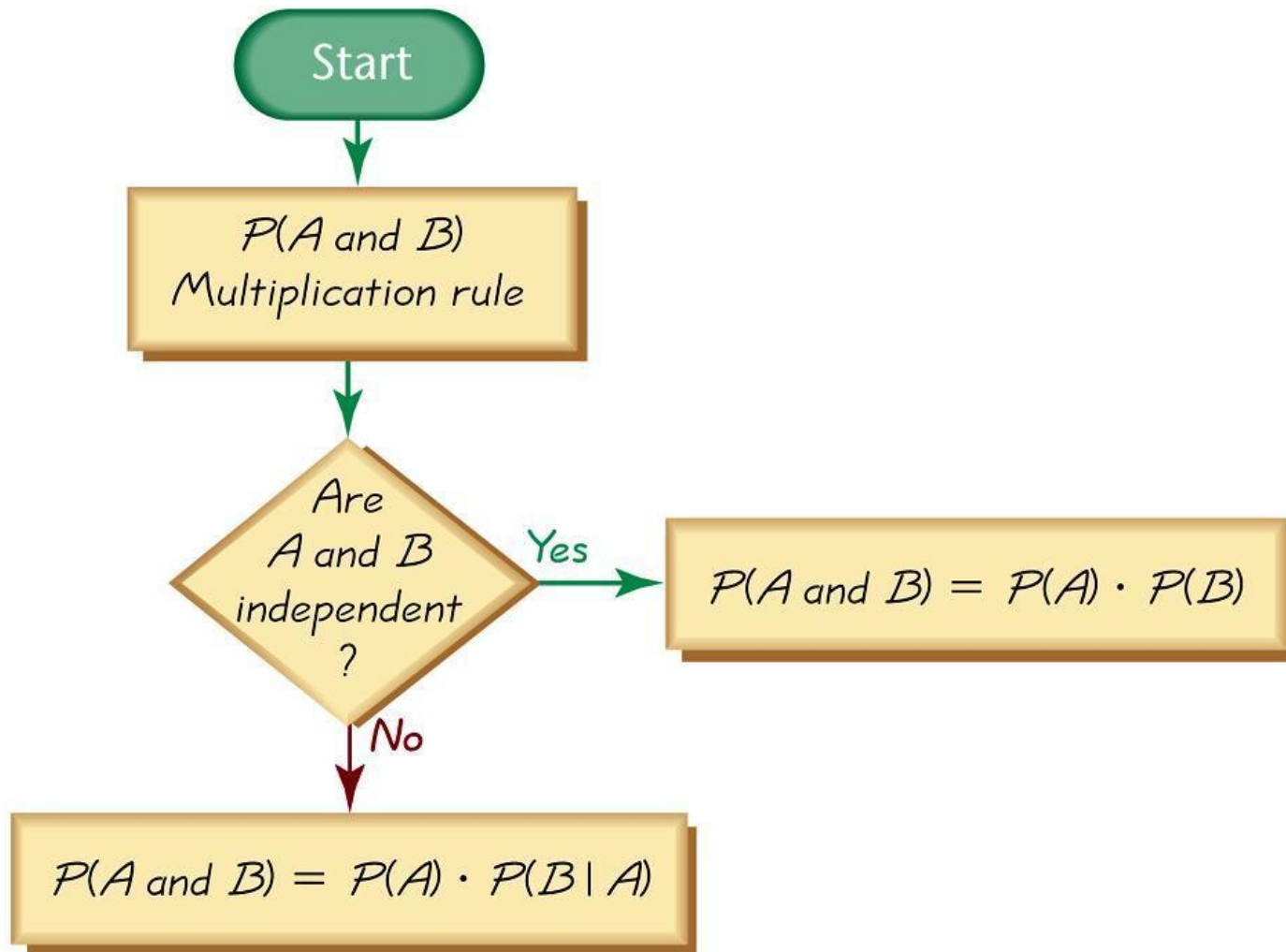
# Formal Multiplication Rule

- ❖  $P(A \text{ and } B) = P(A) \cdot P(B|A)$
- ❖ Note that if A and B are independent events,  $P(B|A)$  is really the same as  $P(B)$ .

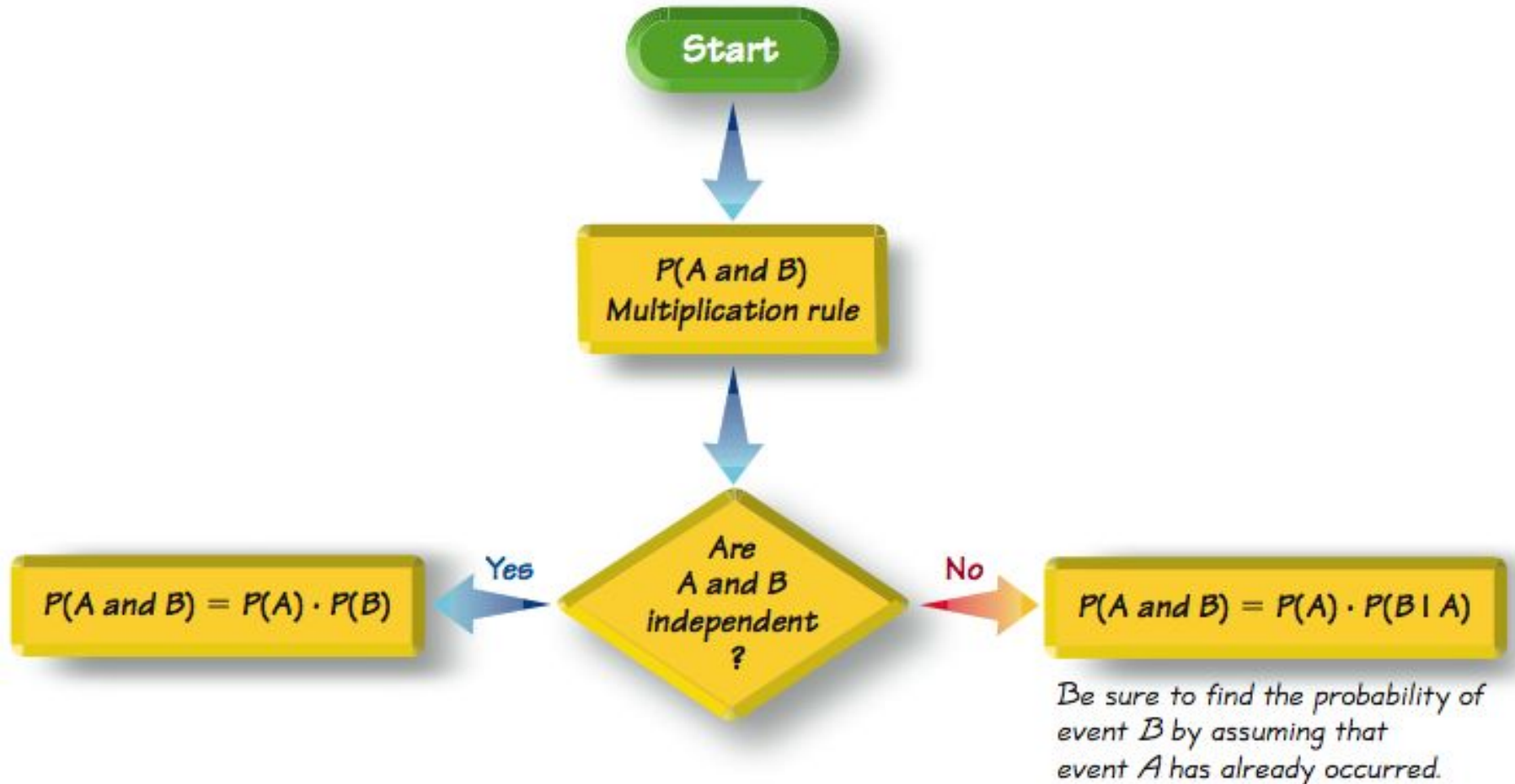
# Intuitive Multiplication Rule

**When finding the probability that event  $A$  occurs in one trial and event  $B$  occurs in the next trial, multiply the probability of event  $A$  by the probability of event  $B$ , but be sure that the probability of event  $B$  takes into account the previous occurrence of event  $A$ .**

# Applying the Multiplication Rule



# Applying the Multiplication Rule



# Caution

**When applying the multiplication rule, always consider whether the events are independent or dependent, and adjust the calculations accordingly.**

# Multiplication Rule for Several Events

**In general, the probability of any sequence of independent events is simply the product of their corresponding probabilities.**



# Treating Dependent Events as Independent

**Some calculations are cumbersome, but they can be made manageable by using the common practice of treating events as independent when small samples are drawn from large populations. In such cases, it is rare to select the same item twice.**

# The 5% Guideline for Cumbersome Calculations

If a sample size is no more than 5% of the size of the population, treat the selections as being **independent** (even if the selections are made without replacement, so they are technically dependent).

# Principle of Redundancy

**One design feature contributing to reliability is the use of redundancy, whereby critical components are duplicated so that if one fails, the other will work. For example, single-engine aircraft now have two independent electrical systems so that if one electrical system fails, the other can continue to work so that the engine does not fail.**

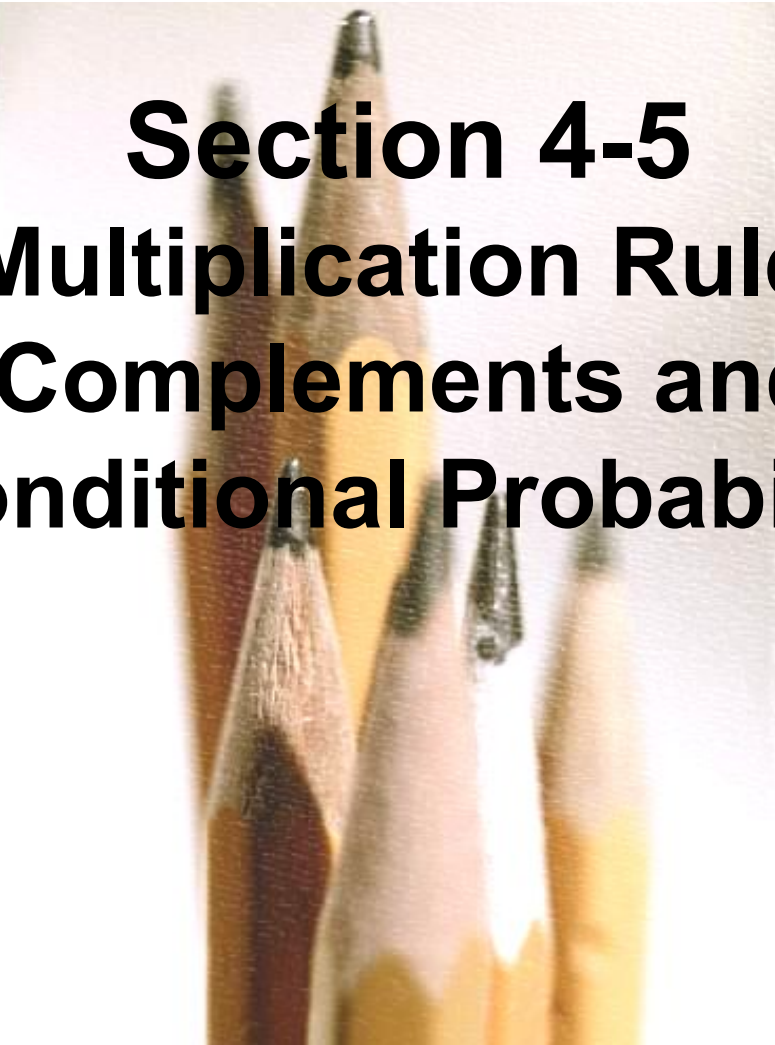
# Summary of Fundamentals

- ❖ In the addition rule, the word “or” in  $P(A \text{ or } B)$  suggests addition. Add  $P(A)$  and  $P(B)$ , being careful to add in such a way that every outcome is counted only once.
- ❖ In the multiplication rule, the word “and” in  $P(A \text{ and } B)$  suggests multiplication. Multiply  $P(A)$  and  $P(B)$ , but be sure that the probability of event  $B$  takes into account the previous occurrence of event  $A$ .

# Recap

**In this section we have discussed:**

- ❖ **Notation for  $P(A \text{ and } B)$ .**
- ❖ **Tree diagrams.**
- ❖ **Notation for conditional probability.**
- ❖ **Independent events.**
- ❖ **Formal and intuitive multiplication rules.**



**Section 4-5**  
**Multiplication Rule:**  
**Complements and**  
**Conditional Probability**

# Key Concepts

**Probability of “at least one”:**

Find the probability that among several trials, we get **at least one** of some specified event.

**Conditional probability:**

Find the probability of an event when we have additional information that some other event has already occurred.

# Complements: The Probability of “At Least One”

- ❖ “At least one” is equivalent to “one or more.”
- ❖ The complement of getting at least one item of a particular type is that you get **no** items of that type.



# Finding the Probability of “At Least One”

To find the probability of **at least one** of something, calculate the probability of **none**, then subtract that result from 1. That is,

$$P(\text{at least one}) = 1 - P(\text{none}).$$

# Conditional Probability

A **conditional probability** of an event is a probability obtained with the additional information that some other event has already occurred.  $P(B|A)$  denotes the conditional probability of event  $B$  occurring, given that event  $A$  has already occurred, and it can be found by dividing the probability of events  $A$  and  $B$  both occurring by the probability of event  $A$ :

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)}$$

# Intuitive Approach to Conditional Probability

**The conditional probability of  $B$  given  $A$  can be found by assuming that event  $A$  has occurred, and then calculating the probability that event  $B$  will occur.**

# Confusion of the Inverse

**To incorrectly believe that  $P(A|B)$  and  $P(B|A)$  are the same, or to incorrectly use one value for the other, is often called confusion of the inverse.**

# Recap

**In this section we have discussed:**

- ❖ **Concept of “at least one.”**
- ❖ **Conditional probability.**
- ❖ **Intuitive approach to conditional probability.**



**Section 4-6**  
**Probabilities Through**  
**Simulations**

# Key Concept

**In this section we use simulations as an alternative approach to finding probabilities. The advantage to using simulations is that we can overcome much of the difficulty encountered when using the formal rules discussed in the preceding sections.**

# Simulation

A **simulation** of a procedure is a process that behaves the same way as the procedure, so that similar results are produced.



# Simulation Example

**Gender Selection** In a test of the MicroSort method of gender selection developed by the Genetics & IVF Institute, 127 boys were born among 152 babies born to parents who used the YSORT method for trying to have a baby boy. In order to properly evaluate these results, we need to know the probability of getting at least 127 boys among 152 births, assuming that boys and girls are equally likely. Assuming that male and female births are equally likely, describe a simulation that results in the genders of 152 newborn babies.

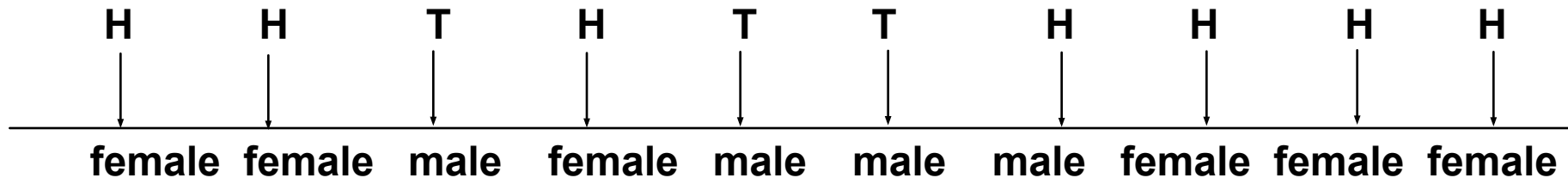
# Solution

**One approach is simply to flip a fair coin 152 times, with heads representing females and tails representing males. Another approach is to use a calculator or computer to randomly generate 152 numbers that are 0s and 1s, with 0 representing a male and 1 representing a female. The numbers must be generated in such a way that they are equally likely. Here are typical results:**

# Simulation Examples

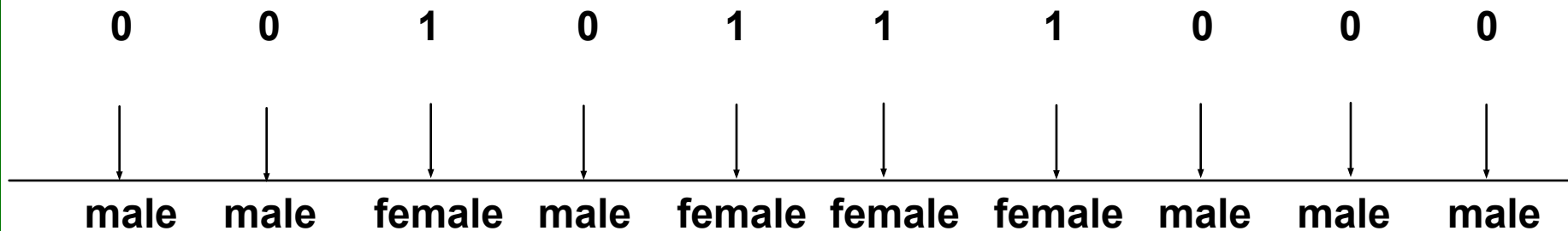
## Solution 1:

- ❖ Flipping a fair coin 100 times where  
heads = female  
tails = male



## Solution 2:

- ❖ Generating 0's and 1's with a computer or calculator where  
0 = male  
1 = female



# Random Numbers

In many experiments, **random numbers** are used in the simulation of naturally occurring events. Below are some ways to generate random numbers.

- ❖ A table of random of digits
- ❖ STATDISK
- ❖ Minitab
- ❖ Excel
- ❖ TI-83/84 Plus calculator

# Random Numbers

## STATDISK

Row	1 Ran...
1	7
2	8
3	16
4	38
5	42
6	46
7	68
8	68
9	104
10	117
11	140
12	195
13	204
14	244
15	271
16	274

## Minitab

↓	C1	C2
1	38	
2	48	
3	59	
4	71	
5	101	
6	107	
7	122	
8	129	
9	153	
10	153	
11	163	

# Random Numbers

Excel

	A
1	15
2	3
3	15
4	362
5	164
6	184
7	158
8	59
9	143
10	85
11	134

TI-83/84 Plus calculator

```
randInt(1,365,25  
→L1  
{79 206 340 133...  
SortA(L1)  
Done  
L1  
{17 34 46 70 79...
```

# Recap

**In this section we have discussed:**

- ❖ **The definition of a simulation.**
- ❖ **How to create a simulation.**
- ❖ **Ways to generate random numbers.**



# **Section 4-7 Counting**



# Key Concept

**In many probability problems, the big obstacle is finding the total number of outcomes, and this section presents several methods for finding such numbers without directly listing and counting the possibilities.**

# Fundamental Counting Rule

For a sequence of two events in which the first event can occur  $m$  ways and the second event can occur  $n$  ways, the events together can occur a total of  $m \cdot n$  ways.

# Notation

The **factorial symbol !** denotes the product of decreasing positive whole numbers.

For example,

$$4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24.$$

By special definition,  $0! = 1$ .

# Factorial Rule

A collection of  $n$  different items can be arranged in order  $n!$  different ways.  
(This **factorial rule** reflects the fact that the first item may be selected in  $n$  different ways, the second item may be selected in  $n - 1$  ways, and so on.)

# Permutations Rule

## (when items are all different)

### Requirements:

1. There are  $n$  different items available. (This rule does not apply if some of the items are identical to others.)
2. We select  $r$  of the  $n$  items (without replacement).
3. We consider rearrangements of the same items to be different sequences. (The permutation of **ABC** is different from **CBA** and is counted separately.)

If the preceding requirements are satisfied, the number of **permutations** (or sequences) of  $r$  items selected from  $n$  available items (without replacement) is

$${}_n P_r = \frac{n!}{(n - r)!}$$

# Permutations Rule

(when some items are **identical to others**)

Requirements:

1. There are  $n$  items available, and some items are identical to others.
2. We select all of the  $n$  items (without replacement).
3. We consider rearrangements of distinct items to be different sequences.

If the preceding requirements are satisfied, and if there are  $n_1$  alike,  $n_2$  alike, . . .  $n_k$  alike, the number of **permutations** (or sequences) of all items selected without replacement is

$$\frac{n!}{n_1! \cdot n_2! \cdot \dots \cdot n_k!}$$

# Combinations Rule

## Requirements:

1. There are  $n$  different items available.
2. We select  $r$  of the  $n$  items (without replacement).
3. We consider rearrangements of the same items to be the same. (The combination of **ABC** is the same as **CBA**.)

If the preceding requirements are satisfied, the number of combinations of  $r$  items selected from  $n$  different items is

$${}_n C_r = \frac{n!}{(n-r)! r!}$$

# Permutations versus Combinations

**When different orderings of the same items are to be counted separately, we have a permutation problem, but when different orderings are not to be counted separately, we have a combination problem.**



# Recap

**In this section we have discussed:**

- ❖ The fundamental counting rule.**
- ❖ The factorial rule.**
- ❖ The permutations rule (when items are all different).**
- ❖ The permutations rule (when some items are identical to others).**
- ❖ The combinations rule.**