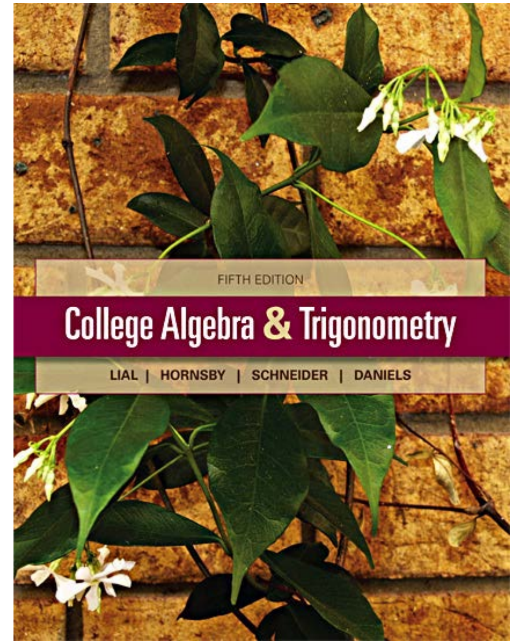
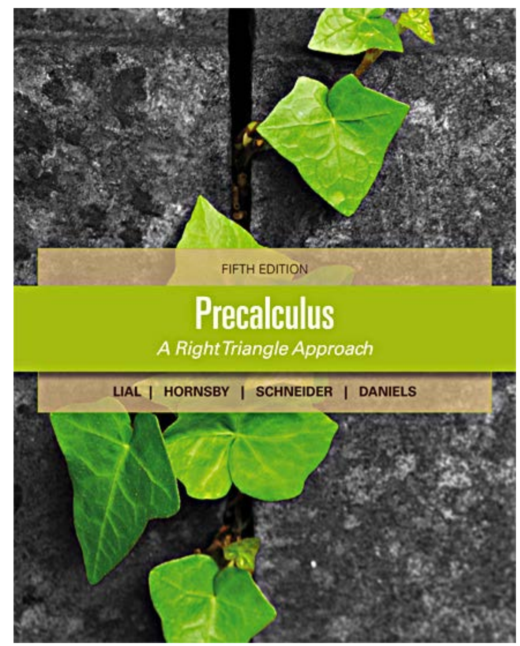
3

Graphs and Functions





3.1 Functions

- Relations and Functions
- Domain and Range
- Determining Whether Relations Are Functions
- Function Notation
- Increasing, Decreasing, and Constant Functions

Ordered Pairs الازواج المرتبة

An **ordered pair** consists of two components, written inside parentheses.

(x,y), x is the first component and y is the second component

Ex. (2,3), (-5,4),…

$(2,3) \neq (3,2)$…..

Relation العلاقة

A **relation** is a set of ordered pairs.

العلاقة هي مجموعة من الازواج المرتبة

Function

A **function** is a relation in which, for each distinct value of the first component of the ordered pairs, there is *exactly one* value of the second component.

الدالة هي علاقة فيها كل مركبة اولى ترتبط بمركبة ثانية واحدة فقط أو المركبة الاولى لا تتكرر حتى لو تكررت المركبة الثانية.

Example 1 DECIDING WHETHER RELATIONS DEFINE FUNCTIONS

Decide whether the relation defines a function.

$$F = \{(1,2),(-2,4)(3,4)\}$$

Solution Relation *F* is a function, because for each different *x*-value there is exactly one *y*-value. We can show this correspondence as follows.

$$\{1, -2, 3\} \quad x\text{-values of } F$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\{2, 4, 4\} \quad y\text{-values of } F$$

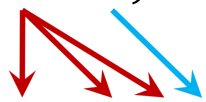
Example 1 DECIDING WHETHER RELATIONS DEFINE FUNCTIONS

Decide whether the relation defines a function.

$$G = \{(1,1),(1,2)(1,3)(2,3)\}$$

Solution As the correspondence below shows, relation *G* is not a function because one first component corresponds to *more than one* second component.

$\{1, 2\}$ *x*-values of *G*



$\{1, 2, 3\}$ *y*-values of *G*

Example 1 DECIDING WHETHER RELATIONS DEFINE FUNCTIONS

Decide whether the relation defines a function.

$$H = \{(-4,1),(-2,1)(-2,0)\}$$

Solution In relation *H* the last two ordered pairs have the same *x*-value paired with two different *y*-values, so *H* is a relation but not a function.

Different *y*-values

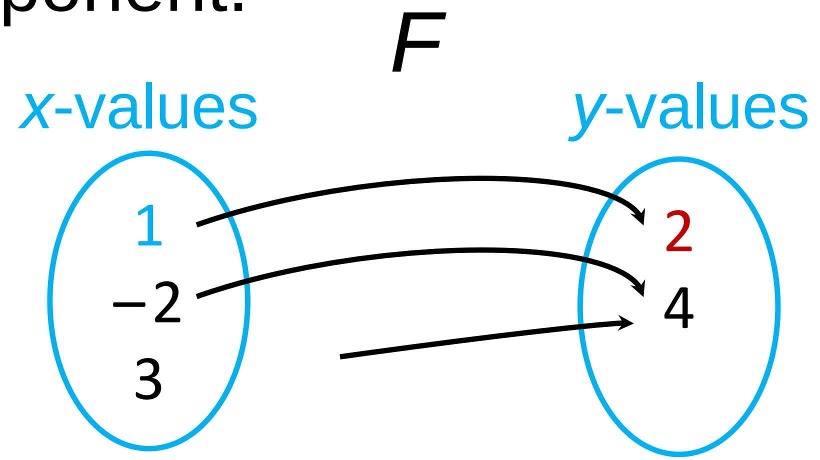
$$H = \{(-4,1),(-2,1)(-2,0)\}$$

Not a function

Same *x*-value

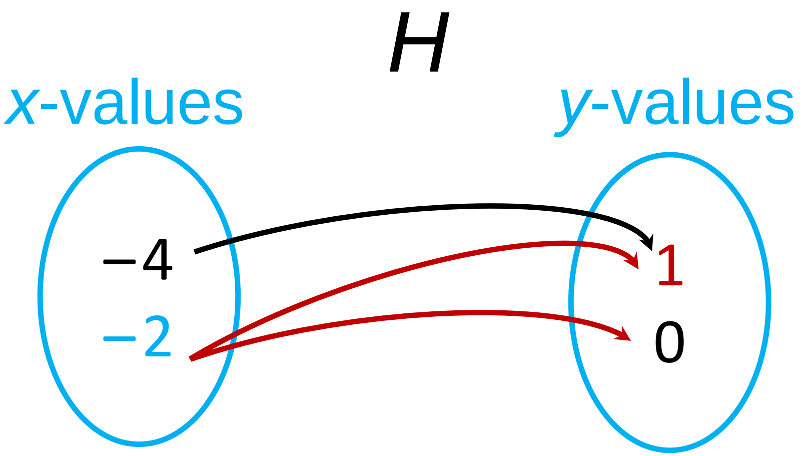
Mapping

Relations and functions can also be expressed as a correspondence or *mapping* from one set to another. In the example below the arrow from 1 to 2 indicates that the ordered pair (1, 2) belongs to *F*. Each first component is paired with exactly one second component.



Mapping

In the mapping for relation *H*, which is not a function, the first component –2 is paired with two different second components, 1 and 0.



Relations

Note Another way to think of a function relationship is to think of the independent variable as an input and the dependent variable as an output.

Domain and Range

In a relation consisting of ordered pairs (x, y), the set of all values of the independent variable (x) is the **domain.** The set of all values of the dependent variable (y) is the **range.**

Homework 1 FINDING DOMAINS AND RANGES OF RELATIONS

Give the domain and range of each relation. Tell whether the relation defines a function.

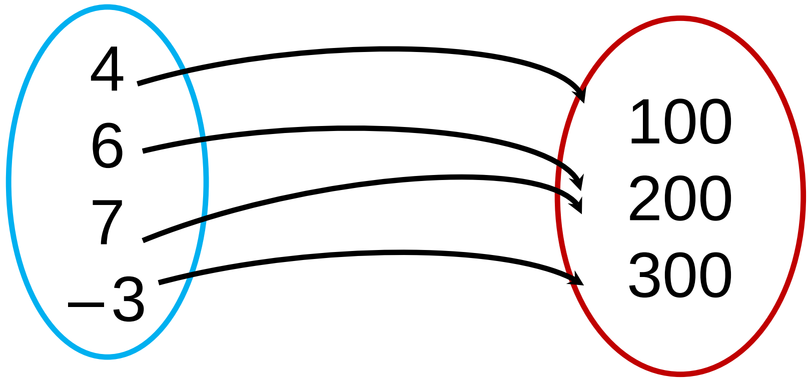
(a) $\{(3,-1),(4,2),(4,5),(6,8)\}$

The domain is the set of *x*-values, {3, 4, 6}.
The range is the set of *y*-values, {–1, 2, 5, 8}.
This relation is not a function because the same *x*-value, 4, is paired with two different *y*-values, 2 and 5.

FINDING DOMAINS AND RANGES OF RELATIONS

Give the domain and range of each relation. Tell whether the relation defines a function.

(b)



The domain is {4, 6, 7, –3} and the range is {100, 200, 300}. This mapping defines a function. Each *x*-value corresponds to exactly one *y*-value.

FINDING DOMAINS AND RANGES OF RELATIONS

Give the domain and range of each relation. Tell whether the relation defines a function.

(c)

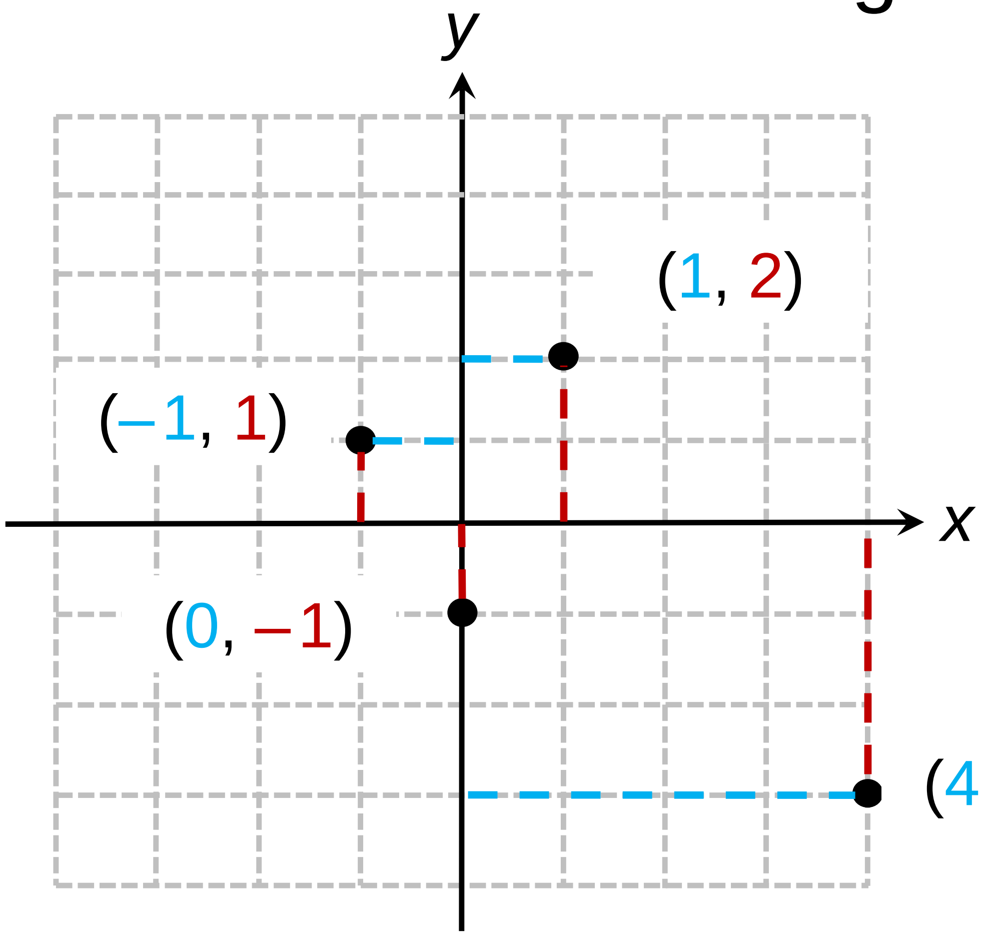
x	***y***
–5	2
0	2
5	2

This relation is a set of ordered pairs, so the domain is the set of *x*-values {– 5, 0, 5} and the range is the set of *y*-values {2}. The table defines a function because each different *x*-value corresponds to exactly one *y*-value.

Example 2 FINDING DOMAINS AND RANGES FROM GRAPHS

Give the domain and range of each relation.

(a)

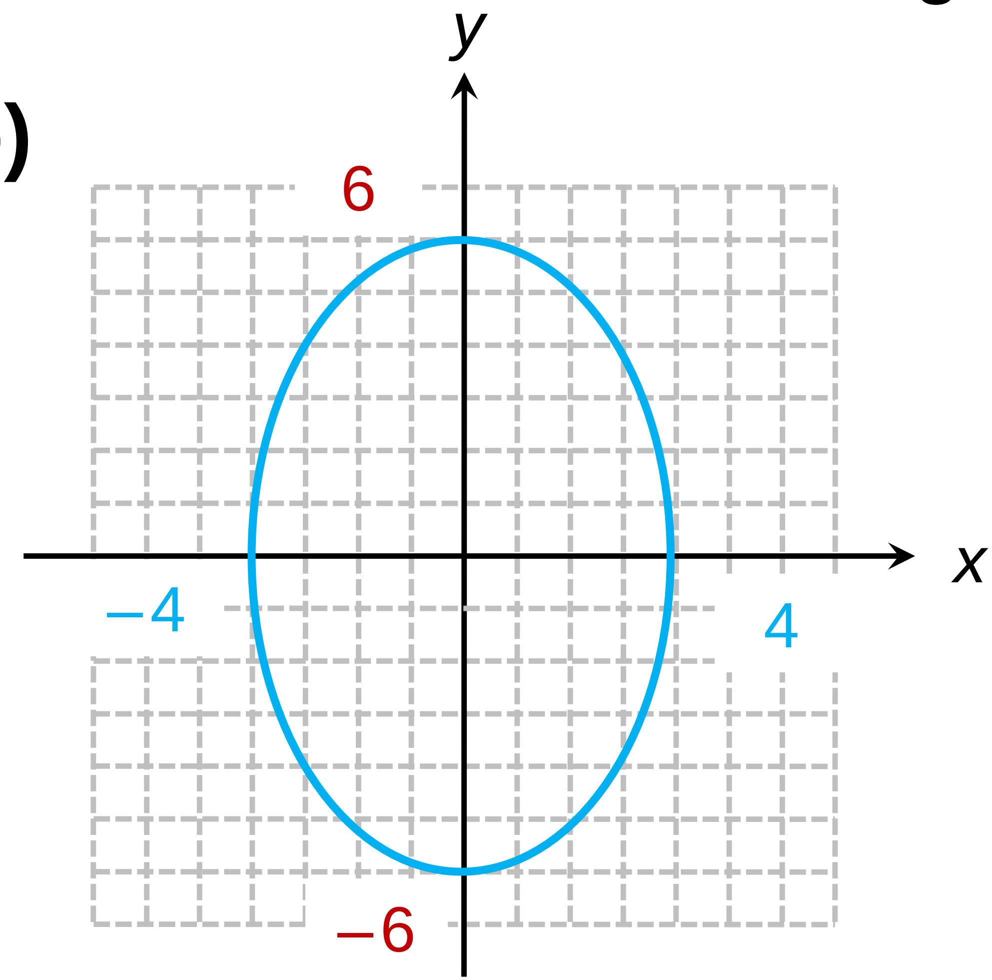


The domain is the set of *x*-values which are {–1, 0, 1, 4}.
The range is the set of *y*-values, {–3, –1, 1, 2}.

Example 2 FINDING DOMAINS AND RANGES FROM GRAPHS

Give the domain and range of each relation.

(b)



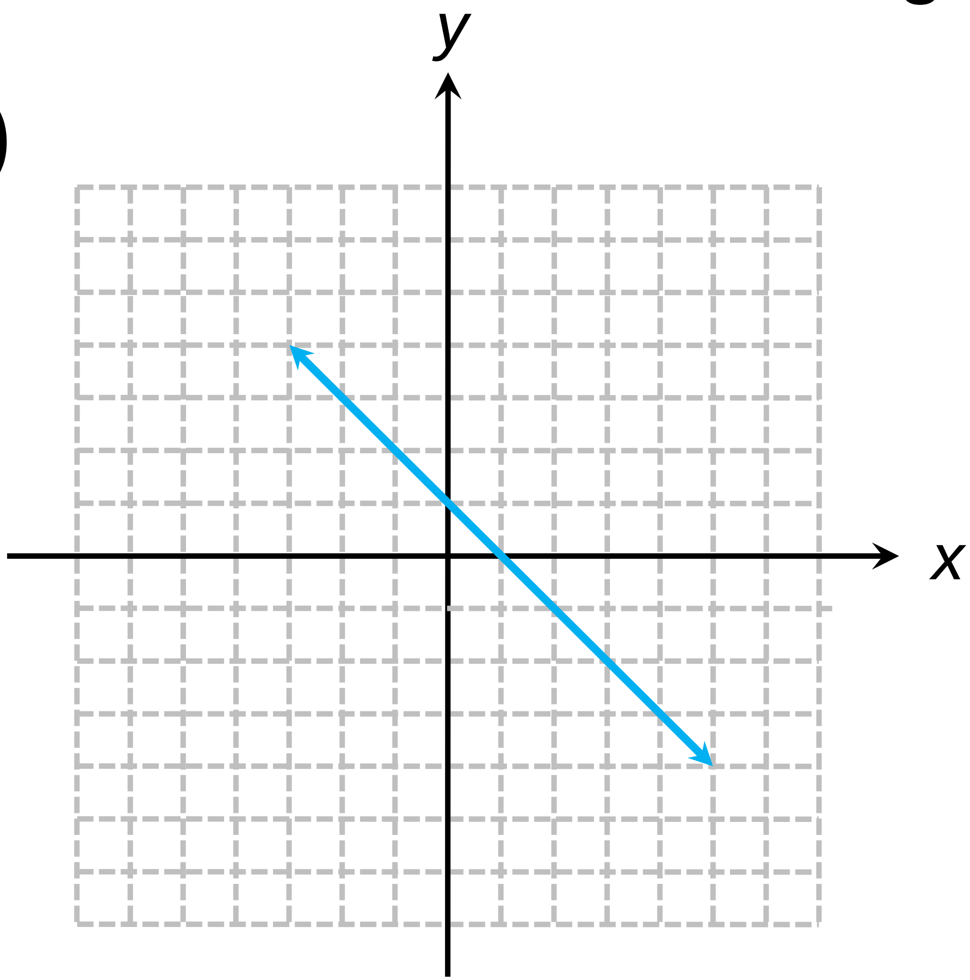
The *x*-values of the points on the graph include all numbers between –4 and 4, inclusive. The *y*-values include all numbers between –6 and 6, inclusive.

The domain is [–4, 4].
The range is [–6, 6].

Example 2 FINDING DOMAINS AND RANGES FROM GRAPHS

Give the domain and range of each relation.

(c)

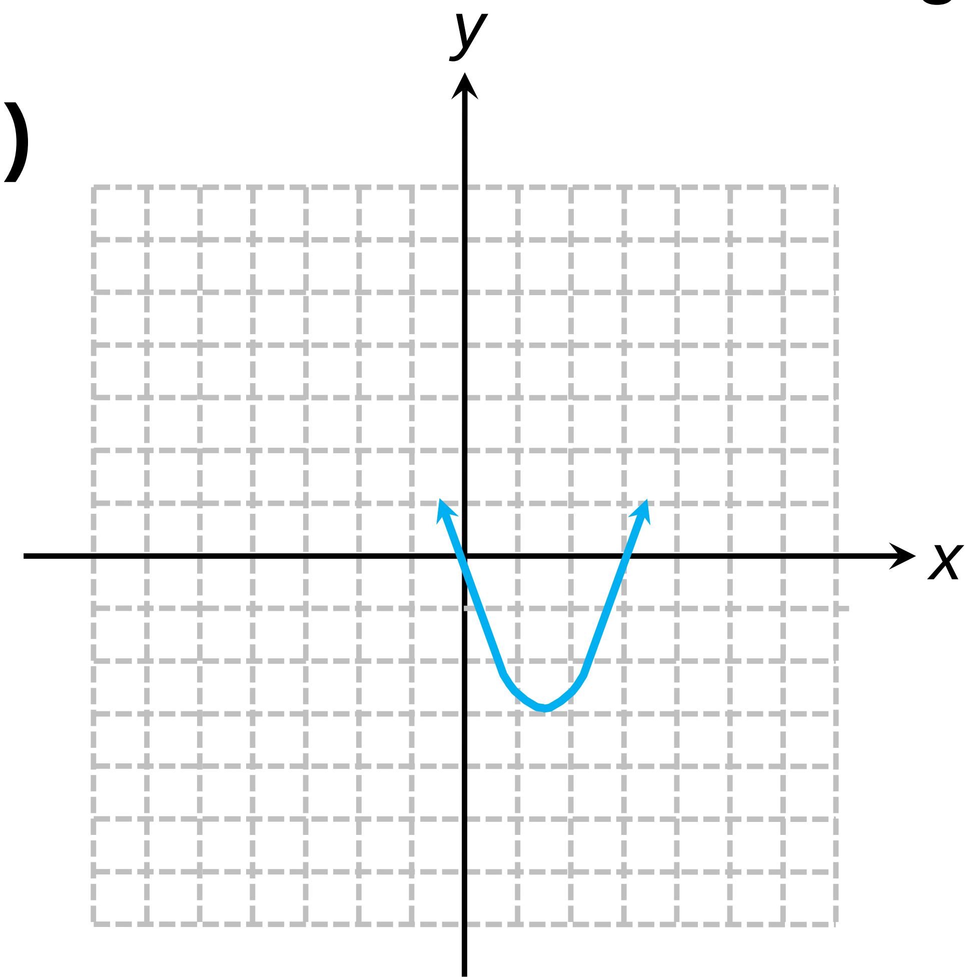


The arrowheads indicate that the line extends indefinitely left and right, as well as up and down. Therefore, both the domain and the range include all real numbers, which is written $(-\infty, \infty)$.

Example 2 FINDING DOMAINS AND RANGES FROM GRAPHS

Give the domain and range of each relation.

(d)



The arrowheads indicate that the line extends indefinitely left and right, as well as upward. The domain is $(-\infty, \infty)$. Because there is a least *y*-value, –3, the range includes all numbers greater than or equal to –3, written $[-3, \infty)$.

Agreement on Domain

Unless specified otherwise, the domain of a relation is assumed to be all real numbers that produce real numbers when substituted for the independent variable.

Agreement on Domain

For example:

a) y=2x+3 Domain =all real numbers

=(-∞,∞).

b) y=$\frac{1}{x}, Domain = R\backslash\{0\}$

c) $y = \frac{2}{x-3}, Domain = R\backslash\{3\}$

d)$y = \sqrt{x-4}$, Domain : $x - 4 \geq 0$

$x \geq 4 = [4, \infty)$

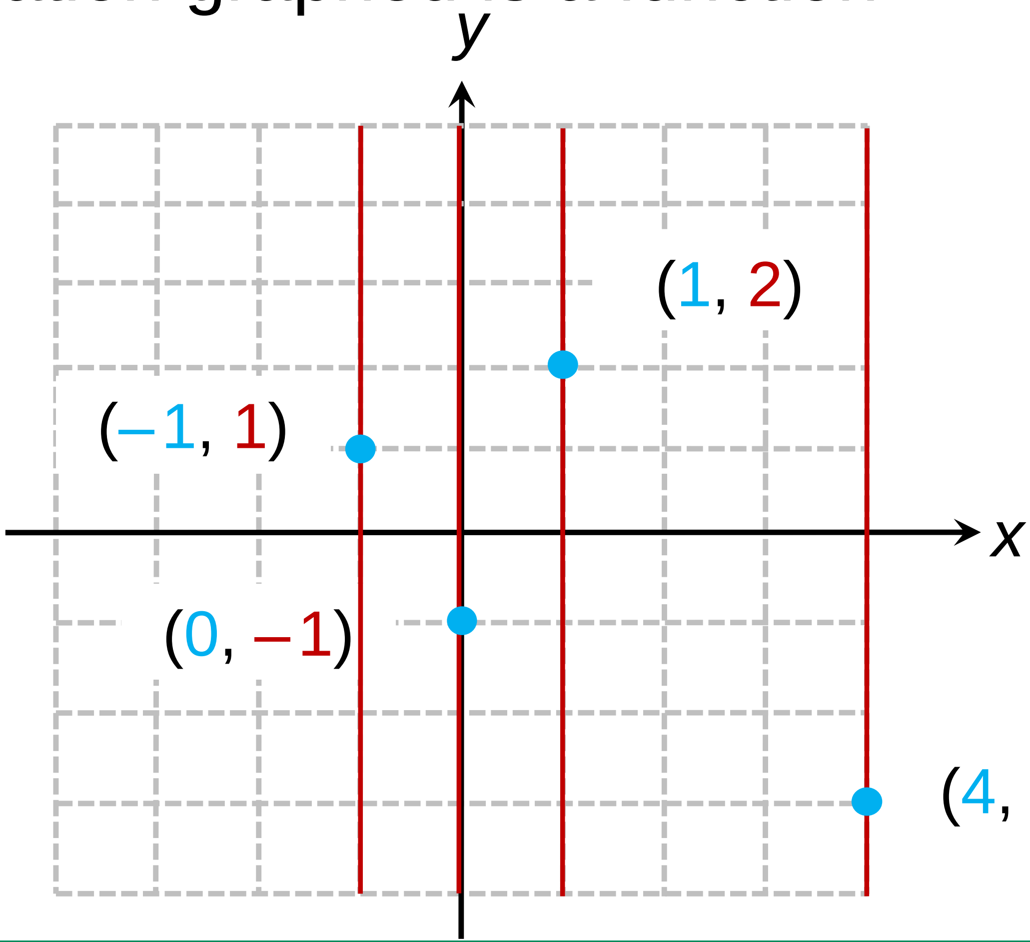
Vertical Line Test

If every vertical line intersects the graph of a relation in no more than one point, then the relation is a function.

Example 4 USING THE VERTICAL LINE TEST

Use the vertical line test to determine whether each relation graphed is a function.

(a)

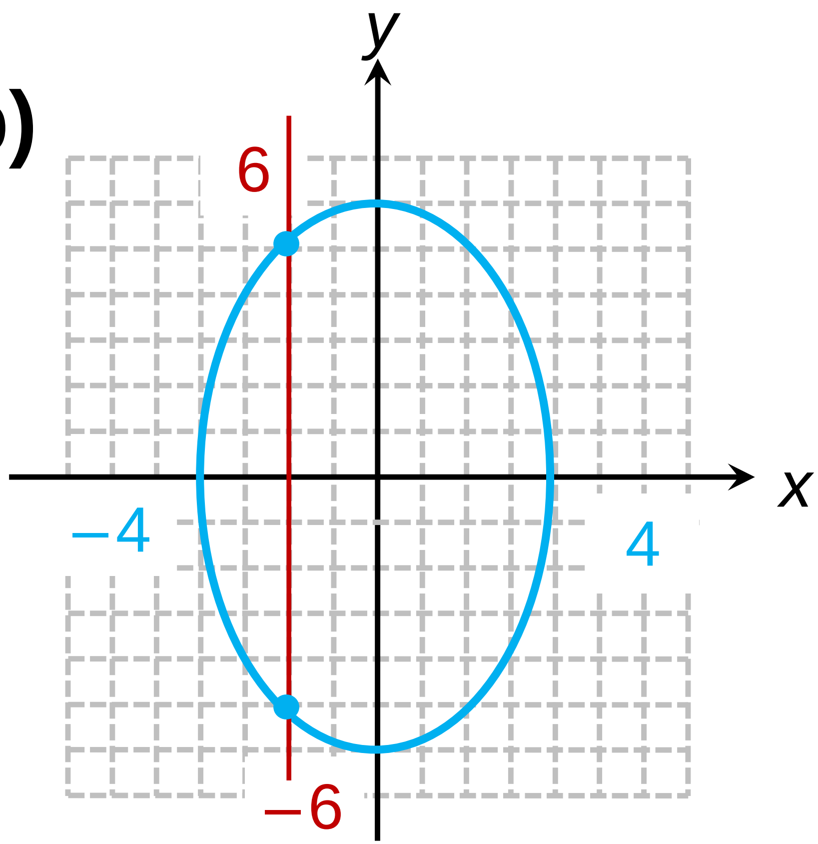


The graph of this relation passes the vertical line test, since every vertical line intersects the graph no more than once. Thus, this graph represents a function.

Example 4 USING THE VERTICAL LINE TEST

Use the vertical line test to determine whether each relation graphed is a function.

(b)

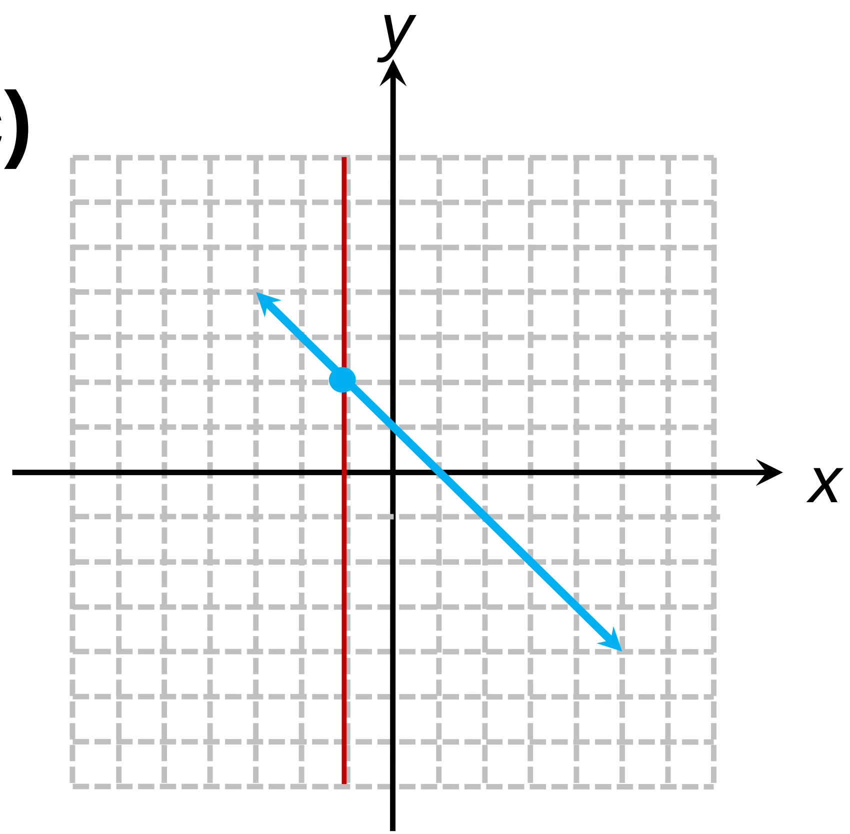


The graph of this relation fails the vertical line test, since the same *x*-value corresponds to two different *y*-values. Therefore, it is not the graph of a function.

Example 4 USING THE VERTICAL LINE TEST

Use the vertical line test to determine whether each relation graphed is a function.

(c)

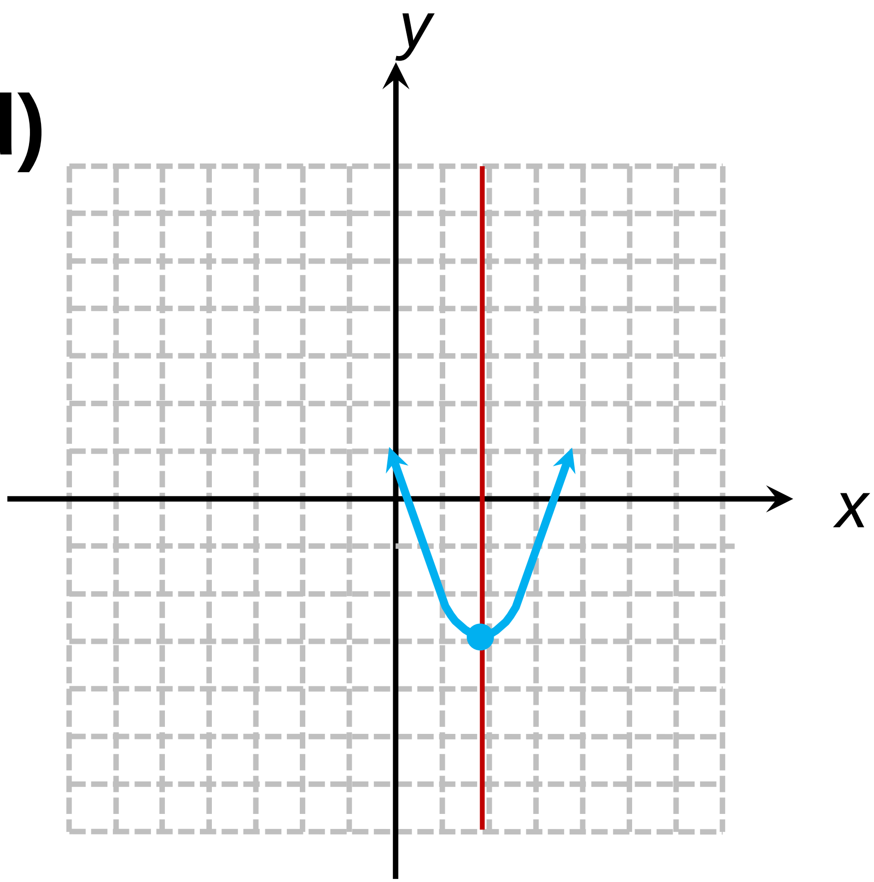


The graph of this relation passes the vertical line test, since every vertical line intersects the graph no more than once. Thus, this graph represents a function.

Example 4 USING THE VERTICAL LINE TEST

Use the vertical line test to determine whether each relation graphed is a function.

(d)



The graph of this relation passes the vertical line test, since every vertical line intersects the graph no more than once. Thus, this graph represents a function.

Example 3 IDENTIFYING FUNCTIONS, DOMAINS, AND RANGES

Decide whether each relation defines a function and give the domain and range.

(a) $y = x + 4$

Solution In the defining equation (or rule), *y* is always found by adding 4 to *x*. Thus, each value of *x* corresponds to just one value of *y*, and the relation defines a function. The variable *x* can represent any real number, so the domain is

$$\{x \mid x \text{ is a real number}\} \quad \text{or} \quad (-\infty, \infty).$$

Since *y* is always 4 more than *x*, *y* also may be any real number, and so the range is $(-\infty, \infty)$.

Example 3 IDENTIFYING FUNCTIONS, DOMAINS, AND RANGES

Decide whether each relation defines a function and give the domain and range.

(b) $y = \sqrt{2x - 1}$

Solution For any choice of *x* in the domain, there is exactly one corresponding value for y (the radical is a nonnegative number), so this equation defines a function. Since the equation involves a square root, the quantity under the radical cannot be negative.

$$2x - 1 \geq 0 \quad \text{Solve the inequality.}$$

$$2x \geq 1 \quad \text{Add 1.}$$

$$x \geq \frac{1}{2} \quad \text{Divide by 2.}$$

Example 3 IDENTIFYING FUNCTIONS, DOMAINS, AND RANGES

Decide whether each relation defines a function and give the domain and range.

(b) $y = \sqrt{2x - 1}$

Solution

$$x \geq \frac{1}{2}$$

The domain is $\left[\frac{1}{2}, \infty\right)$.

Because the radical must represent a non-negative number, as *x* takes values greater than or equal to 1/2, the range is $\{y \mid y \geq 0\}$, or $[0, \infty)$.

Example 3 IDENTIFYING FUNCTIONS, DOMAINS, AND RANGES

Decide whether each relation defines a function and give the domain and range.

(c) $y^2 = x$

Solution The ordered pairs (16, 4) and (16, –4) both satisfy the equation. Since one value of *x*, 16, corresponds to two values of *y*, 4 and –4, this equation does not define a function.

The domain is $[0, \infty)$.

Any real number can be squared, so the range of the relation is $(-\infty, \infty)$.

Example 3 IDENTIFYING FUNCTIONS, DOMAINS, AND RANGES

Decide whether each relation defines a function and give the domain and range.

(d) $y \leq x - 1$

Solution By definition, *y* is a function of *x* if every value of *x* leads to exactly one value of *y*. Substituting a particular value of *x* into the inequality corresponds to many values of *y*.

The ordered pairs (1, 0), (1, –1), (1, –2), and (1, –3) all satisfy the inequality. Any number can be used for *x* or for *y*, so the domain and range are both the set of real numbers, or $(-\infty, \infty)$.

Example 3 IDENTIFYING FUNCTIONS, DOMAINS, AND RANGES

Decide whether each relation defines a function and give the domain and range.

(e) $y = \dfrac{5}{x-1}$

Solution Given any value of *x* in the domain of the relation, we find *y* by subtracting 1 from *x* and then dividing the result into 5. This process produces exactly one value of *y* for each value in the domain, so this equation defines a function.

Example 3 IDENTIFYING FUNCTIONS, DOMAINS, AND RANGES

Decide whether each relation defines a function and give the domain and range.

(e) $y = \dfrac{5}{x-1}$

Solution The domain includes all real numbers except those making the denominator 0.

$$x - 1 = 0$$

$$x = 1 \qquad \text{Add 1.}$$

Thus, the domain includes all real numbers except 1 and is written $(-\infty, 1) \cup (1, \infty)$.

The range is the interval $(-\infty, 0) \cup (0, \infty)$.

Variations of the Definition of Function

1. A **function** is a relation in which, for each distinct value of the first component of the ordered pairs, there is exactly one value of the second component.
2. A **function** is a set of ordered pairs in which no first component is repeated.
3. A **function** is a rule or correspondence that assigns exactly one range value to each distinct domain value.

Function Notation

When a function f is defined with a rule or an equation using *x* and *y* for the independent and dependent variables, we say "*y* is a function of *x*" to emphasize that *y depends on x*. We use the notation

$$y = f(x)$$

called **function notation,** to express this and read $f(x)$ as "f of *x*." The letter f is the name given to this function. For example, if $y = 3x - 5$, we can name the function f and write

$$f(x) = 3x - 5.$$

Function Notation

Note that $f(x)$ is just another name for the dependent variable y. For example, if

$y = f(x) = 3x - 5$ and $x = 2$, then we find y, or $f(2)$, by replacing x with 2.

$$f(2) = 3 \times 2 - 5 = 1$$

The statement "if $x = 2$, then $y = 1$" represents the ordered pair (2, 1) and is abbreviated with function notation as

$$f(2) = 1.$$

The symbol $f(2)$ is read "f of 2" or "f at 2."

Function Notation

These ideas can be illustrated as follows.

Name of the function

Defining expression

$$y = f(x) = 3x - 5$$

Value of the function

Name of the independent variable

Caution The symbol $f(x)$ does not indicate "f times x," but represents the y-value for the indicated x-value. As just shown, $f(2)$ is the y-value that corresponds to the x-value 2.

USING FUNCTION NOTATION

Let $f(x) = -x^2 + 5x - 3$ and $g(x) = 2x + 3$. Find and simplify each of the following.

(a) $f(2)$

Solution

$$f(x) = -x^2 + 5x - 3$$

$$f(2) = -2^2 + 5 \times 2 - 3$$ Replace *x* with 2.

$$= -4 + 10 - 3$$ Apply the exponent; multiply.

$$= 3$$ Add and subtract.

Thus, $f(2) = 3$; the ordered pair (2, 3) belongs to f.

Let $f(x) = -x^2 + 5x - 3$ and $g(x) = 2x + 3$. Find and simplify each of the following.

(b) $f(q)$

Solution

$$f(x) = -x^2 + 5x - 3$$

$$f(q) = -q^2 + 5q - 3$$ Replace *x* with *q*.

USING FUNCTION NOTATION

Let $f(x) = x^2 + 5x - 3$ and $g(x) = 2x + 3$. Find and simplify each of the following.

(c) $g(a+1)$

Solution

$$g(x) = 2x + 3$$

$$g(a+1) = 2(a+1) + 3$$

Replace x with $a + 1$.

$$= 2a + 2 + 3$$

$$= 2a + 5$$

Example 4 **USING FUNCTION NOTATION**

For each function, find $f(3)$.

(a) $f(x) = 3x - 7$

Solution

$$f(x) = 3x - 7$$

$$f(3) = 3(3) - 7$$ Replace x with 3.

$$f(3) = 2$$

Example 4 **USING FUNCTION NOTATION**

For each function, find $f(3)$.

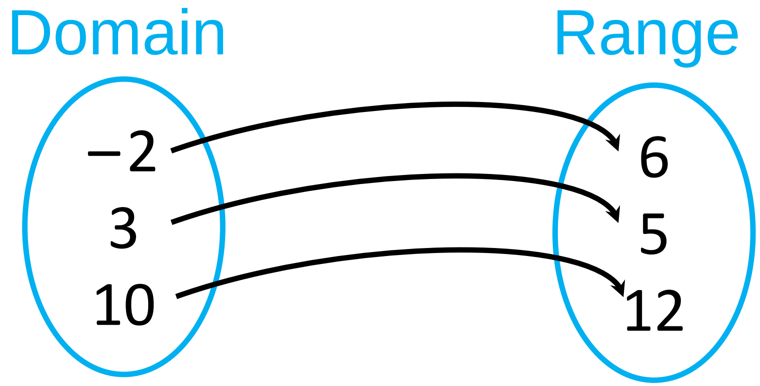
(b) $f = \{(-3,5),(0,3),(3,1),(6,-1)\}$

Solution For $f = \{(-3,5), (0,3), (3,1), (6,-1)\}$, we want $f(3)$, the *y*-value of the ordered pair where $x = 3$. As indicated by the ordered pair (3, 1), when $x = 3$, $y = 1$, so $f(3) = 1$.

Example 4 USING FUNCTION NOTATION

For each function, find $f(3)$.

(c)



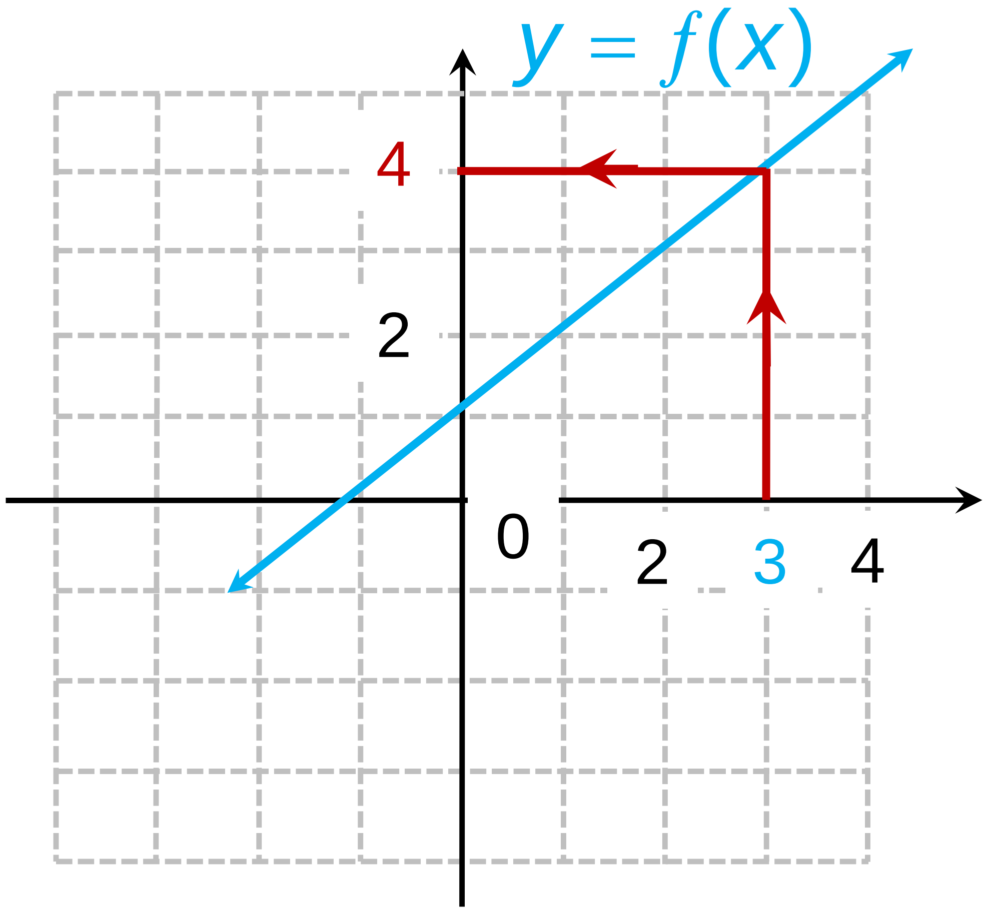
Solution

In the mapping, the domain element 3 is paired with 5 in the range, so $f(3) = 5$.

Example 4 USING FUNCION NOTATION

For each function, find $f(3)$.

(d)



Solution

Find 3 on the *x*-axis. Then move up until the graph of f is reached. Moving horizontally to the *y*-axis gives 4 for the corresponding *y*-value. Thus $f(3) = 4$.

Finding an Expression for $f(x)$

Consider an equation involving *x* and *y*. Assume that *y* can be expressed as a function f of *x*.

To find an expression for $f(x)$ use the following steps.

Step 1 Solve the equation for *y*.

Step 2 Replace *y* with $f(x)$.

WRITING EQUATIONS USING FUNCTION NOTATION

Assume that y is a function of x. Rewrite each equation using function notation. Then find $f(-2)$ and $f(a)$.

(a) $y = x^2 + 1$

Solution

$$y = x^2 + 1$$

$$f(x) = x^2 + 1 \qquad \text{Let } y = f(x).$$

Now find $f(-2)$ and $f(a)$.

$$f(-2) = (-2)^2 + 1 \qquad \text{Let } x = -2.$$

$$f(-2) = 4 + 1$$

$$f(-2) = 5$$

$$f(a) = a^2 + 1 \qquad \text{Let } x = a.$$

WRITING EQUATIONS USING FUNCTION NOTATION

Assume that *y* is a function of *x*. Rewrite each equation using function notation. Then find $f(-2)$ and $f(a)$.

(b) $x - 4y = 5$

Solution

$$x - 4y = 5$$

Solve for *y*.

$$-4y = -x + 5$$

$$y = \frac{x-5}{4}$$

Multiply by –1; divide by 4.

$$f(x) = \frac{1}{4}x - \frac{5}{4}$$

$$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

WRITING EQUATIONS USING FUNCTION NOTATION

Assume that y is a function of x. Rewrite each equation using function notation. Then find $f(-2)$ and $f(a)$.

(b) $x - 4y = 5$

Solution Now find $f(-2)$ and $f(a)$.

$$f(-2) = \frac{1}{4}(-2) - \frac{5}{4} = -\frac{7}{4}$$ Let $x = -2$.

$$f(a) = \frac{1}{4}a - \frac{5}{4}$$ Let $x = a$.

Increasing, Decreasing, and Constant Functions

Suppose that a function f is defined over an interval I and x_1 and x_2 are in I.

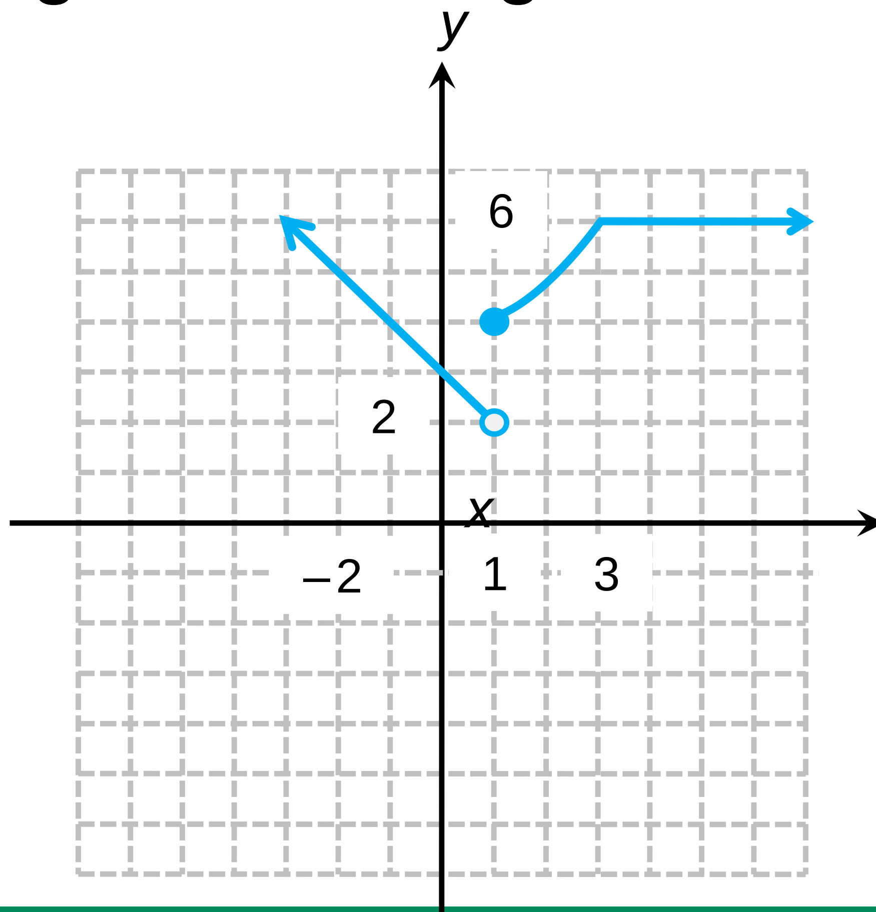
(a) f **increases** on I if, whenever $x_1 < x_2$, $f(x_1) < f(x_2)$.

(b) f **decreases** on I if, whenever $x_1 < x_2$, $f(x_1) > f(x_2)$.

(c) f is **constant** on I if, for every x_1 and x_2, $f(x_1) = f(x_2)$.

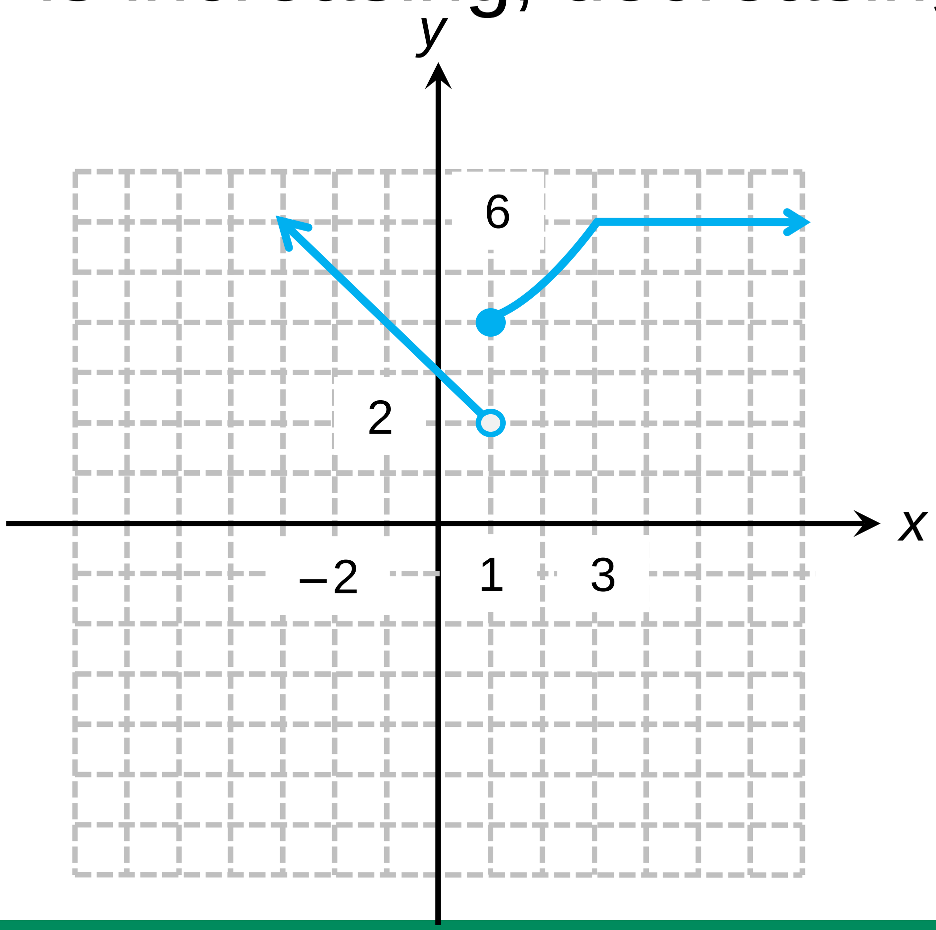
Example 5 **DETERMINING INTERVALS OVER WHICH A FUNCTION IS INCREASING, DECREASING, OR CONSTANT**

Determine the intervals over which the function is increasing, decreasing, or constant.



Example 5 DETERMINING INTERVALS OVER WHICH A FUNCTION IS INCREASING, DECREASING, OR CONSTANT

Determine the intervals over which the function is increasing, decreasing, or constant.

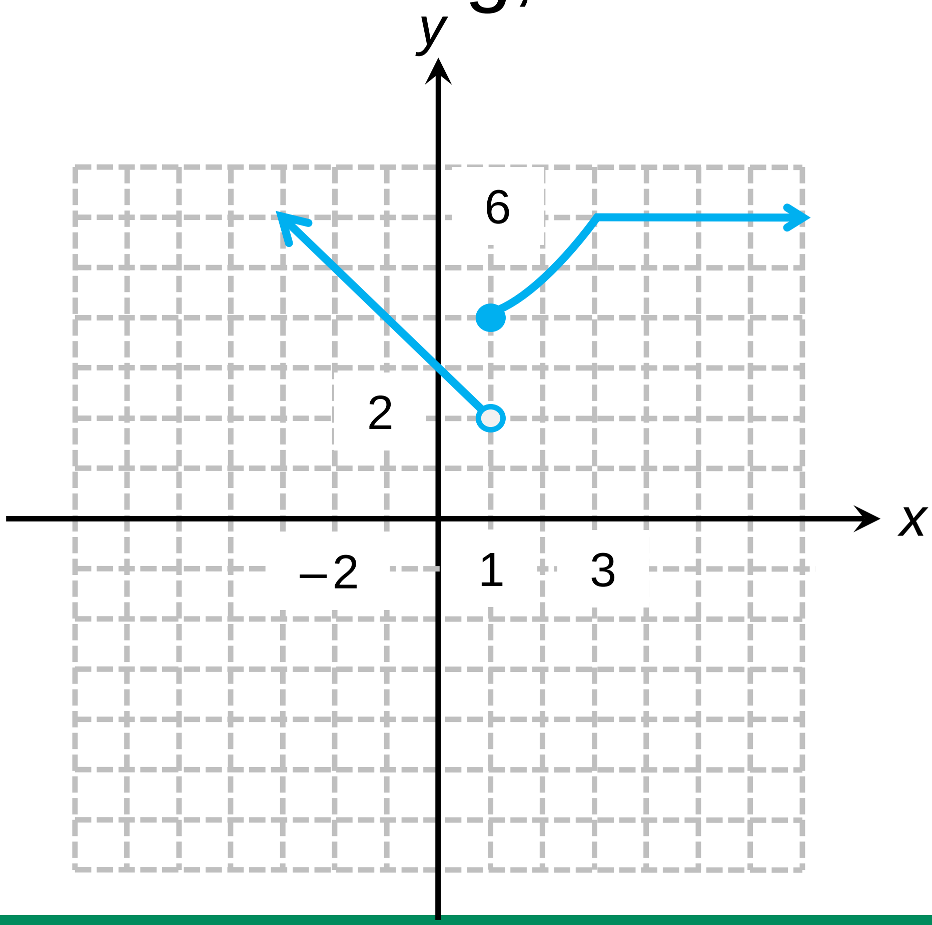


Solution

On the interval $(-\infty, 1)$, the *y*-values are *decreasing*; on the interval $[1,3]$, the *y*-values are *increasing*; on the interval $[3, \infty)$, the *y*-values are *constant* (and equal to 6).

Example 5 DETERMINING INTERVALS OVER WHICH A FUNCTION IS INCREASING, DECREASING, OR CONSTANT

Determine the intervals over which the function is increasing, decreasing, or constant.

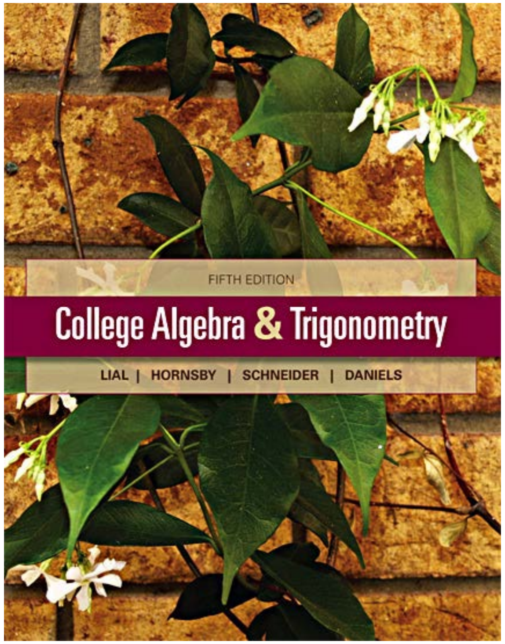


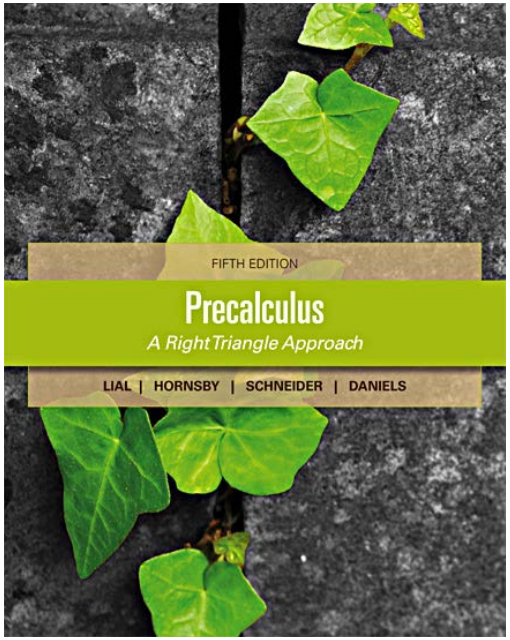
Solution

Therefore, the function is decreasing on $(-\infty, 1)$, increasing on $[1,3]$, and constant on $[3, \infty)$.

3

Graphs and Functions





3.2 Equations of Lines; Linear Modeling

Objectives:

- Point-Slope Form
- Slope-Intercept Form
- Vertical and Horizontal Lines
- Parallel and Perpendicular Lines
- Modeling Data
- Solving Linear Equations in One Variable by Graphing

3.2 Equations of Lines; Curve Fitting

Bridge-in:

- What are the different forms of Lines?
- When the two lines are parallel and perpendicular?

Remember that:

- The slope $$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1},$$
- The standard form is Ax+By=C
- The slope of the vertical line is undefined.
- The slope of the horizontal line is 0.

Point-Slope Form

The **point–slope form** of the equation of the line with slope *m* passing through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

$$or:$$

$$y = y_1 + m(x - x_1)$$

Example 1 USING THE POINT-SLOPE FORM (GIVEN A POINT AND THE SLOPE)

Write an equation of the line through (–4, 1) having slope –3.

Solution Here $x_1 = -4$, $y_1 = 1$, and $m = -3$.

$$y - y_1 = m(x - x_1)$$ Point-slope form

$$y - 1 = -3[x - (-4)]$$ $x_1 = -4$, $y_1 = 1$, $m = -3$

Be careful with signs.

$$y - 1 = -3(x + 4)$$

$$y - 1 = -3x - 12$$ Distributive property

$$y = -3x - 11$$ Add 1.

Exercise 3 USING THE POINT-SLOPE FORM (GIVEN A POINT AND THE SLOPE)

Write an equation of the line through (1, 3) having slope –2.

Solution Here $x_1 = 1, y_1 = 3,$ and $m = -2.$

$$y - y_1 = m(x - x_1)$$ Point-slope form

$$y - 3 = -2[x - (1)]$$ $x_1 = 1, y_1 = 3, m = -2$

$$y - 3 = -2(x - 1)$$

$$y - 3 = -2x + 2$$ Distributive property

$$y = -2x + 5$$ Add 3.

Homework 1 USING THE POINT-SLOPE FORM (GIVEN TWO POINTS)

Write an equation of the line through (–3, 2) and (2, –4). Write the result in standard form $Ax + By = C$.

Solution Find the slope first.

$$m = \frac{-4 - 2}{2 - (-3)} = -\frac{6}{5}$$ Definition of slope

The slope m is $-\frac{6}{5}$. Either $(-3, 2)$ or $(2, -4)$ can be used for (x_1, y_1). We choose $(-3, 2)$.

Homework 1 USING THE POINT-SLOPE FORM (GIVEN TWO POINTS)

Write an equation of the line through (–3, 2) and (2, –4). Write the result in standard form $Ax + By = C$.

Solution

$$y - y_1 = m(x - x_1)$$ Point-slope form

$$y - 2 = -\frac{6}{5}[x - (-3)]$$ $x_1 = -3$, $y_1 = 2$, $m = -6/5$

$$5(y - 2) = -6(x + 3)$$ Multiply by 5.

$$5y - 10 = -6x - 18$$ Distributive property.

$$6x + 5y = -8$$ Standard form

Note

In standard form $Ax + By = C$.

We have the slope

$$m = -\frac{A}{B} = \frac{-A}{B} = \frac{A}{-B}$$

Exercise 2 USING THE POINT-SLOPE FORM (GIVEN A POINT AND THE SLOPE)

Give the slope of the following lines:

a) y=3x-1, b) 4x-y=7, c) x+2y=-4

Solution

a) $\because$ y=3x-1

standard form: 3x-y=1

then

A=3, B=-1, C=1

$$m=-\frac{A}{B}=-\frac{3}{-1}=3$$

Exercise 2 **USING THE POINT-SLOPE FORM (GIVEN A POINT AND THE SLOPE)**

Give the slope of the following lines:

a) y=3x-1, b) 4x-y=7, c) x+2y=-4,

Solution

b) $\because 4x - y = 7$

standard form: $4x - y = 7$

then

A=4, B=-1, C=7

$$m = -\frac{A}{B} = -\frac{4}{-1} = 4$$

Exercise 2 **USING THE POINT-SLOPE FORM (GIVEN A POINT AND THE SLOPE)**

Give the slope of the following lines:

a) y=3x-1, b) 4x-y=7, c) x+2y=-4

Solution

c) $\because x + 2y = -4$

standard form: $x + 2y = -4$

then

A=1, B=2, C=-4

$$m = -\frac{A}{B} = -\frac{1}{2}$$

Pre-Assessment.

Who can tell me the point- slope form? And when we use it?

Slope-Intercept Form

As a special case, suppose that a line passes through the point (0, *b*), so the line has *y*-intercept *b*. If the line has slope *m*, then using the point-slope form with $x_1 = 0$ and $y_1 = b$ gives the following.

$$y - y_1 = m(x - x_1)$$

$$y - b = m(x - 0)$$

$$y = mx + b$$

Slope (m) and *y*-intercept (b)

Slope-Intercept Form

The **slope-intercept form** of the equation of the line with slope m and y-intercept b is

$$y = mx + b.$$

Example 2 FINDING THE SLOPE AND *y*-INTERCEPT FROM AN EQUATION OF A LINE

Find the slope and *y*-intercept of the line with equation $4x + 5y = -10$.

Solution Write the equation in slope-intercept form.

$$4x + 5y = -10$$

$$5y = -4x - 10 \qquad \text{Subtract } 4x.$$

$$y = -\frac{4}{5}x - 2 \qquad \text{Divide by 5.}$$

$m = -\frac{4}{5}$, $b = -2$

The slope is $-\frac{4}{5}$ and the *y*-intercept is –2.

NOTE:

In standard form $Ax + By = C$.

We have the y-intercept $b = \frac{C}{B}$

X-intercept a $= \frac{C}{A}$

Homework 2 USING THE SLOPE-INTERCEPT FORM (GIVEN TWO POINTS)

Write an equation of a line through (1,1) and (2,4). Then graph the line using the slope-intercept form.

Solution Use the slope intercept form. First, find the slope.

$$m = \frac{4-1}{2-1} = \frac{3}{1} = 3$$ Definition of slope.

Substitute 3 for *m* in *y* = *mx* + *b* and choose one of the given points, say (1,1), to find *b*.

USING THE SLOPE-INTERCEPT FORM (GIVEN TWO POINTS)

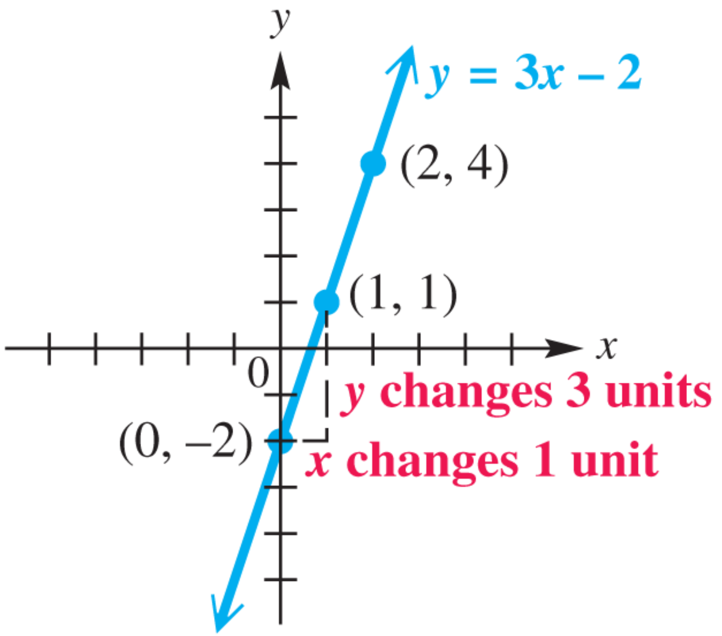
Solution $y = mx + b$ Slope-intercept form

$1 = 3(1) + b$ $m = 3, x = 1, y = 1$

y-intercept ⟶ $b = -2$ Solve for *b*.

The slope intercept form is $y = 3x - 2$.

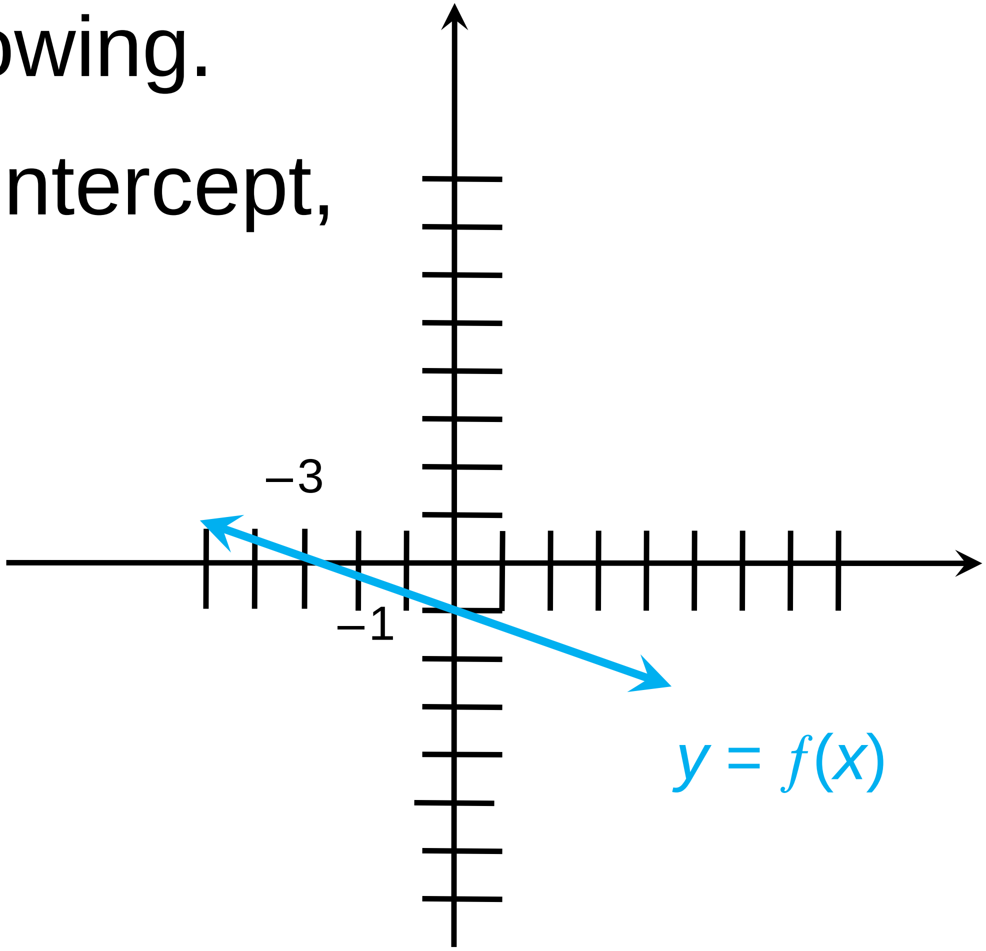
Plot (0,–2) and then use the definition of slope to arrive at (1,1).



Example 3 FINDING AN EQUATION FROM A GRAPH

Use the graph of the linear function f shown to complete the following.

(a) Find the slope, *y*-intercept, and *x*-intercept.



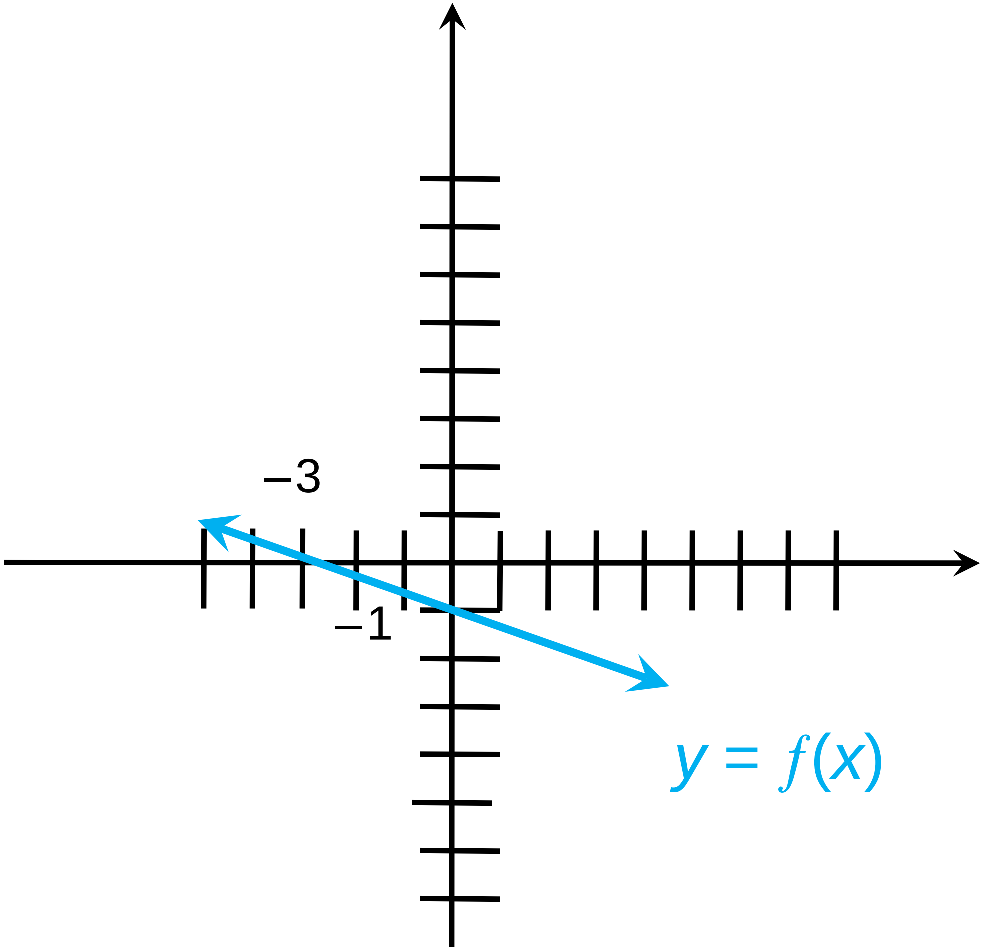
Solution The line falls 1 unit each time the *x*-value increases by 3 units.

$$\text{Slope} = \frac{-1}{3} = -\frac{1}{3}.$$

Example 3 **FINDING AN EQUATION FROM A GRAPH**

Solution

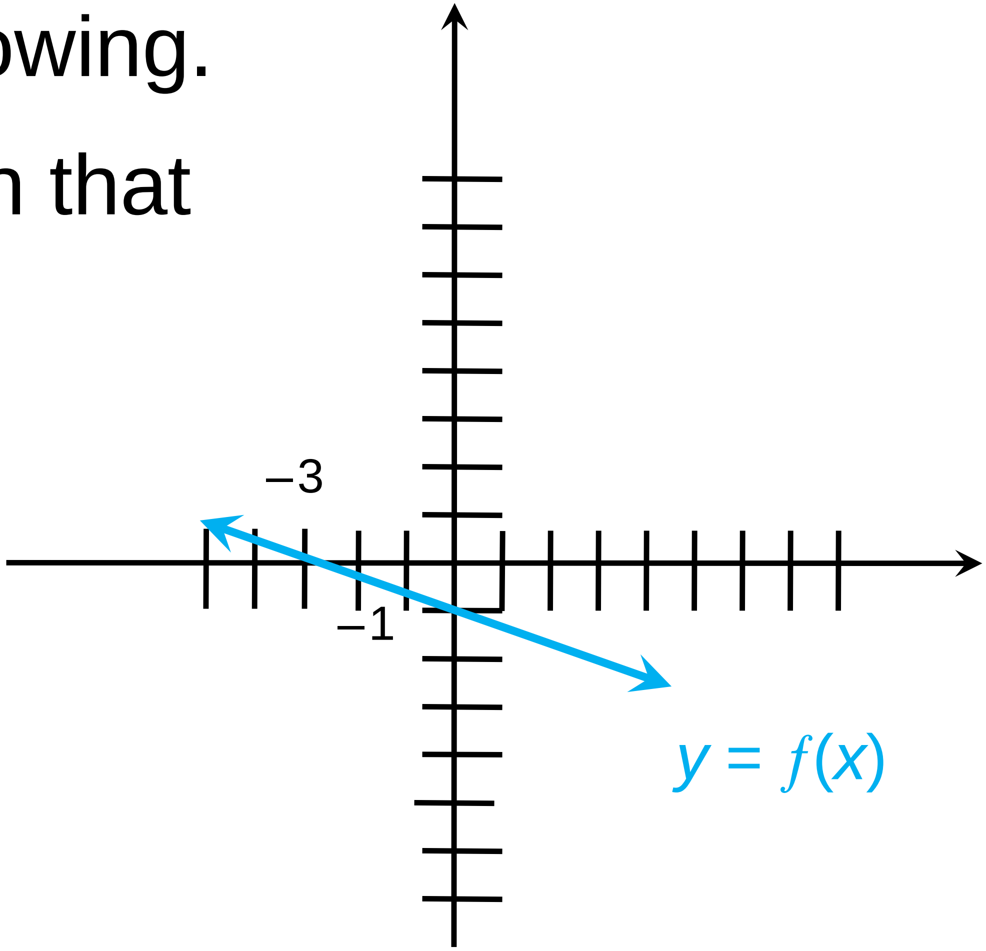
The graph intersects the *y*-axis at the point (0,–1) and intersects the *x*-axis at the point (–3,0). The *y*-intercept is –1 and the *x*-intercept is –3.



Example 3 FINDING AN EQUATION FROM A GRAPH

Use the graph of the linear function f shown to complete the following.

(b) Write the equation that defines f.



Solution

The slope is $m = -\frac{1}{3}$ and the y-intercept is $b = -1$.

$$f(x) = -\frac{1}{3}x - 1$$

Pre-Assessment.

Who can tell me the slope-intercept form and when we use it?

Equations of Vertical and Horizontal lines

An equation of the **vertical line** through the point (a, b) is $\boldsymbol{x = a}$**.**

An equation of the **horizontal line** through the point (a, b) is $\boldsymbol{y = b}$**.**

Parallel Lines

Two distinct nonvertical lines are parallel if and only if they have the same slope. $m_1 = m_2$

Perpendicular Lines

Two lines, neither of which is vertical, are perpendicular if and only if their slopes have a product of –1. Thus, the slopes of perpendicular lines, neither of which are vertical, are *negative reciprocals*. $m_1 \times m_2 = -1$ *or* $m_1 = \frac{-1}{m_2}$

Quiz 1

Give the slope of the line that parallel to the following lines:

a) $x+3y=5$

b) $3x+5y=1$

c) $y=-5$

Quiz 2

Give the slope of the line that perpendicular to the following lines:

a) $x+3y=5$

b) $3x+5y=1$

c) $y=-5$

Write the equation in both slope-intercept and standard form of the line that passes through the point (3, 5) and satisfies the given condition.

(a) parallel to the line $2x + 5y = 4$

Solution The point (3, 5) is on the line, so we need only to find the slope to use the point-slope form. We find the slope by writing the equation of the given line in slope-intercept form. (That is, we solve for *y*.)

Write the equation in both slope-intercept and standard form of the line that passes through the point (3, 5) and satisfies the given condition.

(a) parallel to the line $2x + 5y = 4$

Solution

$$2x + 5y = 4$$

$$5y = -2x + 4 \quad \text{Subtract } 2x.$$

$$y = -\frac{2}{5}x + \frac{4}{5} \quad \text{Divide by 5.}$$

Write the equation in both slope-intercept and standard form of the line that passes through the point (3, 5) and satisfies the given condition.

(a) parallel to the line $2x + 5y = 4$

Solution

$$y = -\frac{2}{5}x + \frac{4}{5}$$

The slope is –2/5. Since the lines are parallel, –2/5 is also the slope of the line whose equation is to be found.

Write the equation in both slope-intercept and standard form of the line that passes through the point (3, 5) and satisfies the given condition.

(a) parallel to the line $2x + 5y = 4$

Solution

$$y - y_1 = m(x - x_1)$$ Point-slope form

$$y - 5 = -\frac{2}{5}(x - 3)$$ $m = -2/5$, $x_1 = 3$, $y_1 = 5$

$$y - 5 = -\frac{2}{5}x + \frac{6}{5}$$ Distributive property

Write the equation in both slope-intercept and standard form of the line that passes through the point (3, 5) and satisfies the given condition.

(a) parallel to the line $2x + 5y = 4$

Solution

slope-intercept form → $y = -\frac{2}{5}x + \frac{31}{5}$ Add 5 = 25/5.

$5y = -2x + 31$ Multiply by 5.

standard form → $2x + 5y = 31$ Add $2x$.

Write the equation in both slope-intercept and standard form of the line that passes through the point (3, 5) and satisfies the given condition.

(b) perpendicular to the line 2x + 5y = 4

Solution In part (a) we found that the slope of the line 2x + 5y = 4 is –2/5. The slope of any line perpendicular to it is 5/2.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{5}{2}(x - 3) \qquad m = \frac{5}{2}, x_1 = 3, y_1 = 5$$

Homework 3 FINDING EQUATIONS OF PARALLEL AND PERPENDICULAR LINES

Write the equation in both slope-intercept and standard form of the line that passes through the point (3, 5) and satisfies the given condition.

(b) perpendicular to the line $2x + 5y = 4$

Solution

$$y - 5 = \frac{5}{2}x - \frac{15}{2}$$ Distributive property

slope-intercept form →

$$y = \frac{5}{2}x - \frac{5}{2}$$ Add 5 = 10/2.

$$2y = 5x - 5$$ Multiply by 2.

standard form →

$$5x - 2y = 5$$ Subtract 2y, add 5, and rewrite.

Closure and Summary

Equation	*Description*	*When to Use*
$y = mx + b$	**Slope-Intercept Form** Slope is m. y-intercept is b.	Slope and y-intercept easily identified and used to quickly graph the equation. Also used to find the equation of a line given a point and the slope.
$y - y_1 = m(x - x_1)$	**Point-Slope Form** Slope is m. Line passes through (x_1, y_1)	Ideal for finding the equation of a line if the slope and a point on the line or two points on the line are known.

Equation	Description	When to Use
$Ax + By = C$	**Standard Form** (If the coefficients and constant are rational, then *A*, *B*, and *C* are expressed as relatively prime integers, with $A \geq 0$). Slope is $-\frac{A}{B}$ $(B \neq 0)$. *x*-intercept is $\frac{C}{A}$ $(A \neq 0)$. *y*-intercept is $\frac{C}{B}$ $(B \neq 0)$.	The *x*- and *y*-intercepts can be found quickly and used to graph the equation. The slope must be calculated.

Equation	*Description*	*When to Use*
$y = b$	**Horizontal Line** Slope is 0. y-intercept is b.	If the graph intersects only the y-axis, then y is the only variable in the equation.
$x = a$	**Vertical Line** Slope is undefined. x-intercept is a.	If the graph intersects only the x-axis, then x is the only variable in the equation.

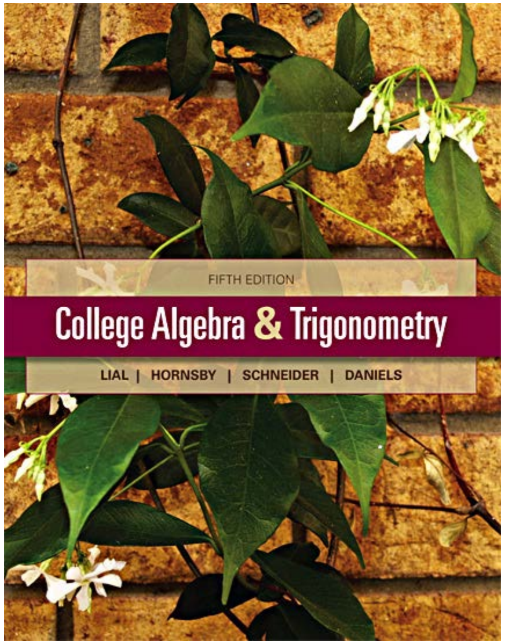
Guidelines for Modeling

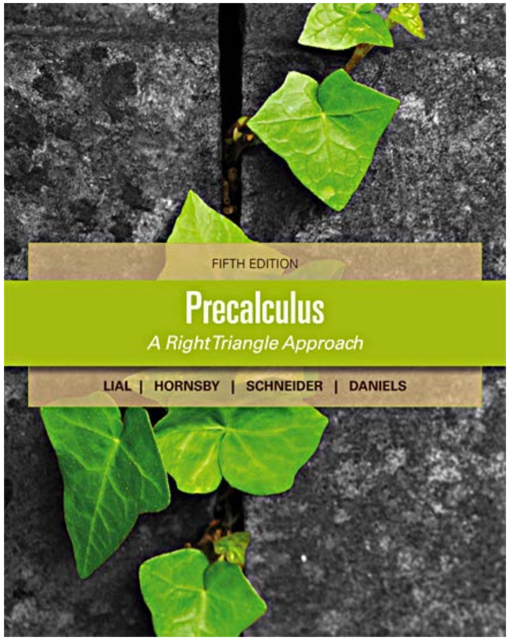
Step 1 Make a scatter diagram of the data.

Step 2 Find an equation that models the data. For a line, this involves selecting two data points and finding the equation of the line through them.

3

Graphs and Functions





3.3 Function Operations and Composition

- Arithmetic Operations on Functions
- The Difference Quotient
- Composition of Functions and Domain

Operations on Functions and Domains

Given two functions f and g, then for all values of x for which both $f(x)$ and $g(x)$ are defined, the functions $f + g$, $f - g$, fg, and $\frac{f}{g}$ are defined as follows.

$$(f+g)(x) = f(x) + g(x)$$ Sum

$$(f-g)(x) = f(x) - g(x)$$ Difference

$$(fg)(x) = f(x) \cdot g(x)$$ Product

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$ Quotient

Domains

For functions f and g, the **domains of** $\boldsymbol{f + g}$**,** $\boldsymbol{f - g}$, and $\boldsymbol{fg}$ include all real numbers in the intersection of the domains of f and g, while the **domain of** $\boldsymbol{\frac{f}{g}}$ includes those real numbers in the intersection of the domains of f and g for which $g(x) \neq 0$.

Note The condition $g(x) \neq 0$ in the definition of the quotient means that the domain of $\left(\frac{f}{g}\right)(x)$ is restricted to all values of x for which $g(x)$ is not 0. The condition does not mean that $g(x)$ is a function that is never 0.

Example 1 USING OPERATIONS ON FUNCTIONS

Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find each of the following.

(a) $(f+g)(1)$

Solution First determine $f(1) = 2$ and $g(1) = 8$. Then use the definition.

$$(f+g)(1) = f(1) + g(1) \qquad (f+g)(x) = f(x) + g(x)$$

$$= 2 + 8$$

$$= 10$$

Example 1 USING OPERATIONS ON FUNCTIONS

Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find each of the following.

(b) $(f - g)(-3)$

Solution First determine that $f(-3) = 10$ and $g(-3) = -4$. Then use the definition.

$$(f - g)(-3) = f(-3) - g(-3) \qquad (f - g)(x) = f(x) - g(x)$$

$$= 10 - (-4)$$

$$= 14$$

Example 1 USING OPERATIONS ON FUNCTIONS

Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find each of the following.

(c) $(fg)(5)$

Solution

$$(fg)(5) = f(5) \cdot g(5)$$

$$= (5^2 + 1)(3 \times 5 + 5)$$

$$= 26 \cdot 20$$

$$= 520$$

Example 1 USING OPERATIONS ON FUNCTIONS

Let $f(x) = x^2 + 1$ and $g(x) = 3x + 5$. Find each of the following.

(d) $\left(\dfrac{f}{g}\right)(0)$

Solution

$$\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{0^2+1}{3(0)+5} = \frac{1}{5}$$

Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x-1}$.
Find each function in (a)-(d).

(a) $(f+g)(x)$

Solution

$$(f+g)(x) = f(x) + g(x) = 8x - 9 + \sqrt{2x-1}$$

USING OPERATIONS OF FUNCTIONS AND DETERMINING DOMAINS

Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x - 1}$.

Find each function in (a)-(d).

(b) $(f - g)(x)$

Solution

$$(f - g)(x) = f(x) - g(x) = 8x - 9 - \sqrt{2x - 1}$$

Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x-1}$.
Find each function.

(c) $(fg)(x)$

Solution

$$(fg)(x) = f(x) \cdot g(x) = (8x-9)\sqrt{2x-1}$$

Homework 1 USING OPERATIONS OF FUNCTIONS AND DETERMINING DOMAINS

Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x-1}$. Find each function.

(d) $\left(\dfrac{f}{g}\right)(x)$

Solution

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{8x-9}{\sqrt{2x-1}}$$

Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x - 1}$.

Find each function.

(e) Give the domains of the functions in parts (a)-(d).

Solution To find the domains of the functions, we first find the domains of f and g.

The domain of f is the set of all real numbers $(-\infty, \infty)$.

Because *g* is defined by a square root radical, the radicand must be non-negative (that is, greater than or equal to 0).

Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x-1}$.
Find each function.

(e) Give the domains of the functions in parts (a)-(d).

Solution $g(x) = \sqrt{2x-1}$

$$2x - 1 \geq 0$$

$$2x \geq 1$$

$$x \geq \frac{1}{2}$$

Thus, the domain of *g* is $\left[\frac{1}{2}, \infty\right)$.

Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x-1}$.
Find each function.

(e) Give the domains of the functions in parts (a)-(d).

Solution The domains of $f + g$, $f - g$, fg are the intersection of the domains of f and g, which is

$$(-\infty, \infty) \cap \left[\frac{1}{2}, \infty\right) = \left[\frac{1}{2}, \infty\right).$$

Let $f(x) = 8x - 9$ and $g(x) = \sqrt{2x - 1}$.
Find each function.

(e) Give the domains of the functions in parts (a)-(d).

Solution The domain of $\frac{f}{g}$ includes those real numbers in the intersection of the domains for which $g(x) = \sqrt{2x - 1} \neq 0$.
That is, the domain of $\frac{f}{g}$ is $\left(\frac{1}{2}, \infty\right)$.

Example 2 EVALUATING COMBINATIONS OF FUNCTIONS

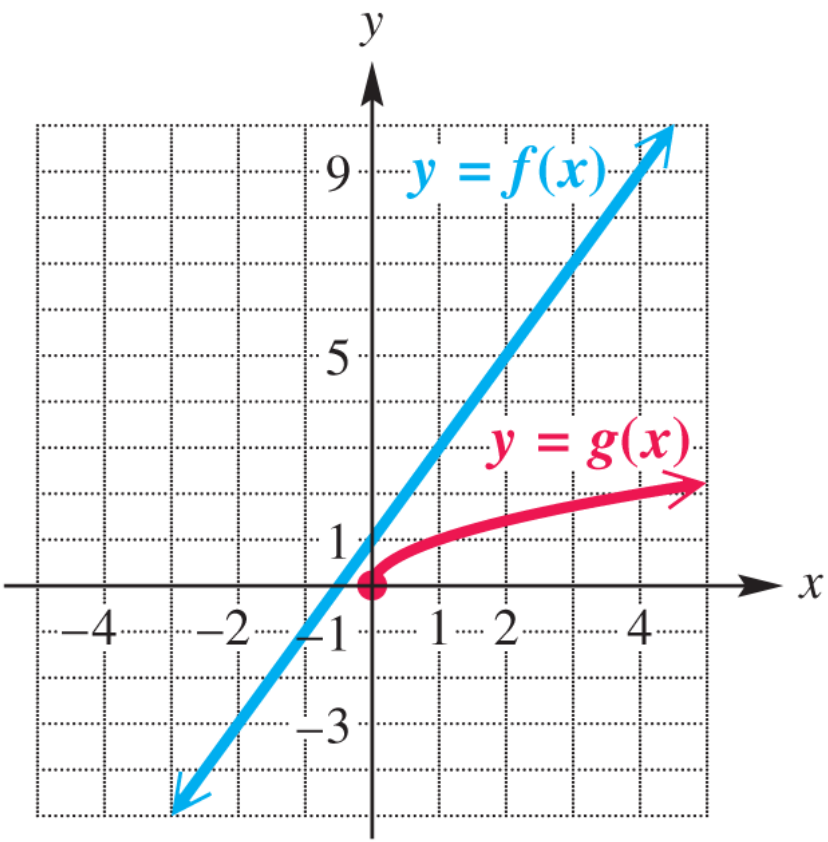
If possible, use the given representations of functions f and g to evaluate

$$(f+g)(4),\quad (f-g)(-2),\quad (fg)(1),\quad \text{and}\quad \left(\frac{f}{g}\right)(0).$$

Example 2 **EVALUATING COMBINATIONS OF FUNCTIONS**

$$(f+g)(4),\quad (f-g)(-2),\quad (fg)(1),\quad \text{and}\quad \left(\frac{f}{g}\right)(0).$$

(a)



$f(4) = 9 \qquad g(4) = 2$

$$(f+g)(4) = f(4) + g(4)$$

$$= 9 + 2 = 11$$

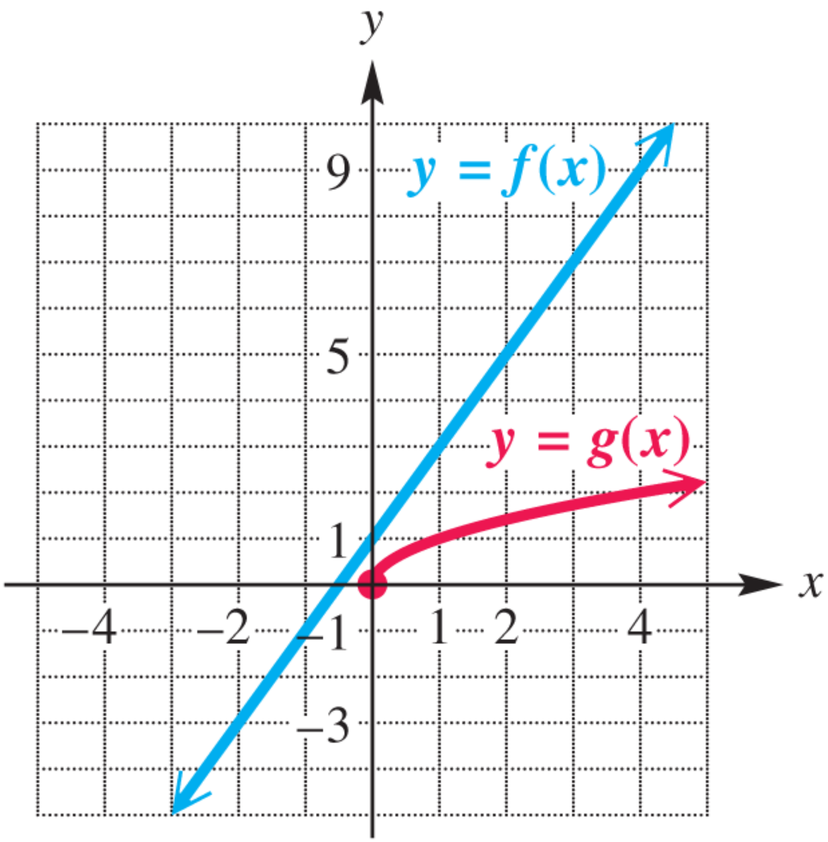
For $(f - g)(-2)$, although $f(-2) = -3$, $g(-2)$ is undefined because –2 is not in the domain of g.

Example 2

EVALUATING COMBINATIONS OF FUNCTIONS

$(f+g)(4)$, $(f-g)(-2)$, $(fg)(1)$, and $\left(\frac{f}{g}\right)(0)$.

(a)



The domains of f and g include 1, so

$$(fg)(1) = f(1)\cdot g(1) = 3\cdot 1 = 3$$

The graph of g includes the origin, so $g(0) = 0.$

Thus, $\left(\frac{f}{g}\right)(0)$ is undefined.

Example 2 EVALUATING COMBINATIONS OF FUNCTIONS

If possible, use the given representations of functions f and g to evaluate

$$(f+g)(4), \quad (f-g)(-2), \quad (fg)(1), \quad \text{and} \quad \left(\frac{f}{g}\right)(0).$$

(b)

x	$f(x)$	$g(x)$
–2	–3	undefined
0	1	0
1	3	1
4	9	2

$f(4) = 9 \quad g(4) = 2$

$$(f+g)(4) = f(4) + g(4)$$
$$= 9 + 2 = 11$$

In the table, $g(-2)$ is undefined. Thus, $(f-g)(-2)$ is undefined.

Example 2

EVALUATING COMBINATIONS OF FUNCTIONS

If possible, use the given representations of functions f and g to evaluate

$$(f+g)(4),\quad (f-g)(-2),\quad (fg)(1),\quad \text{and}\quad \left(\frac{f}{g}\right)(0).$$

(b)

x	$f(x)$	$g(x)$
–2	–3	undefined
0	1	0
1	3	1
4	9	2

$$(fg)(1) = f(1)\cdot(1) = 3(1) = 3$$

$$\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)}$$

is undefined since $g(0) = 0$

Example 2 EVALUATING COMBINATIONS OF FUNCTIONS

If possible, use the given representations of functions f and g to evaluate

$$(f+g)(4), \quad (f-g)(-2), \quad (fg)(1), \quad \text{and} \quad \left(\frac{f}{g}\right)(0).$$

(c) Using $f(x)=2x+1$ and $g(x)=\sqrt{x}$, we can find $(f+g)(4)$ and $(fg)(1)$. Since –2 is not in the domain of g, $(f-g)(-2)$ is not defined.

$$(f+g)(4)=f(4)+g(4)=(2\cdot 4+1)+\sqrt{4}=9+2=11$$

$$(fg)(1)=f(1)\cdot g(1)=(2\cdot 1+1)\sqrt{1}=3(1)=3$$

$\left(\frac{f}{g}\right)$ is undefined since $g(0)=0.$

Homework 2 FINDING THE DIFFERENCE QUOTIENT

Let $f(x) = 2x^2 - 3x$. Find and simplify the expression for the difference quotient,

$$\frac{f(x+h) - f(x)}{h}.$$

Solution We use a three-step process.

Step 1 Find the first term in the numerator, $f(x + h)$. Replace x in $f(x)$ with $x + h$.

$$f(x+h) = 2(x+h)^2 - 3(x+h)$$

Homework 2 FINDING THE DIFFERENCE QUOTIENT

Let $f(x) = 2x^2 - 3x$. Find and simplify the expression for the difference quotient,

$$\frac{f(x+h) - f(x)}{h}.$$

Solution

Step 2 Find the entire numerator $f(x+h) - f(x)$.

$$f(x+h) - f(x) = \left[2(x+h)^2 - 3(x+h)\right] - (2x^2 - 3x)$$

Substitute

$$= 2(x^2 + 2xh + h^2) - 3(x+h) - (2x^2 - 3x)$$

Remember this term when squaring *x* + *h*

Square $x + h$

Homework 2 FINDING THE DIFFERENCE QUOTIENT

Let $f(x) = 2x^2 – 3x$. Find and simplify the expression for the difference quotient,

Solution

$$\frac{f(x+h) - f(x)}{h}.$$

Step 2

$$= 2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x$$

Distributive property

$$= 4xh + 2h^2 - 3h$$

Combine like terms.

Homework 2 FINDING THE DIFFERENCE QUOTIENT

Let $f(x) = 2x^2 - 3x$. Find and simplify the expression for the difference quotient,

Solution $$\frac{f(x+h)-f(x)}{h}.$$

Step 3 Find the difference quotient by dividing by *h*.

$$\frac{f(x+h)-f(x)}{h} = \frac{4xh+2h^2-3h}{h}$$ Substitute.

$$= \frac{h(4x+2h-3)}{h}$$ Factor out *h*.

$$= 4x+2h-3$$ Divide.

Caution In **Example 4,** notice that the expression $f(x+h)$ is not equivalent to $f(x)+f(h)$.

$$f(x+h) = 2(x+h)^2 - 3(x+h)$$
$$= 2x^2 + 4xh + 2h^2 - 3x - 3h$$
$$f(x) + f(h) = (2x^2 - 3x) + (2h^2 - 3h)$$
$$= 2x^2 - 3x + 2h^2 - 3h$$

These expressions differ by $4xh$. In general, $f(x+h)$ is not equivalent to $f(x)+f(h)$.

Composition of Functions and Domain

If f and g are functions, then the **composite function,** or **composition,** of g and f is defined by

$$(g \circ f)(x) = g(f(x)).$$

The **domain of** $g \circ f$ is the set of all numbers x in the domain of f such that $f(x)$ is in the domain of g.

Example 3 EVALUATING COMPOSITE FUNCTIONS

Let $f(x) = 2x - 1$ and $g(x)\ \dfrac{4}{x-1}$

(a) Find $(f \circ g)(2)$.

Solution First find $g(2)$:

$$g(2) = \frac{4}{2-1} = \frac{4}{1} = 4$$

Now find $(f \circ g)(2)$:

$$(f \circ g)(2) = f(g(2)) = f(4) = 2(4) - 1 = 7$$

Example 3 EVALUATING COMPOSITE FUNCTIONS

Let $f(x) = 2x - 1$ and $g(x) = \dfrac{4}{x-1}$

(b) Find $(g \circ f)(-3)$.

Solution $(g \circ f)(-3) = g\left(f(-3)\right)$

$$= g[2(-3) - 1]$$

$$= g(-7)$$

$$= \frac{4}{-7-1} = \frac{4}{-8}$$

$$= -\frac{1}{2}.$$

Given that $f(x) = \sqrt{x}$ and $g(x) = 4x + 2,$
Find each of the following.

(a) $(f \circ g)(x)$ and its domain.

Solution

$$(f \circ g)(x) = f(g(x)) = f(4x + 2) = \sqrt{4x + 2}$$

The domain and range of g are both the set of real numbers. The domain of f is the set of all nonnegative real numbers. Thus, $g(x)$, which is defined as $4x + 2$, must be greater than or equal to zero.

Homework 3 DETERMINING COMPOSITE FUNCTIONS AND THEIR DOMAINS

Given that $f(x) = \sqrt{x}$ and $g(x) = 4x + 2$,
Find each of the following.

(a) $(f \circ g)(x)$ and its domain.

Solution $4x + 2 \geq 0$

$$4x \geq -2$$

$$x \geq -\frac{1}{2}$$

Therefore, the domain of $f \circ g$ is $\left[-\frac{1}{2}, \infty\right)$.

Given that $f(x) = \sqrt{x}$ and $g(x) = 4x + 2$,
Find each of the following.

(b) $(g \circ f)(x)$ and its domain.

Solution $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = 4\sqrt{x} + 2$

The domain and range of *f* are both the set of all nonnegative real numbers. The domain of g is the set of all real numbers. Therefore, the domain of $g \circ f$ is $[0, \infty)$.

Example 4 DETERMINING COMPOSITE FUNCTIONS AND THEIR DOMAINS

Given that $f(x) = \dfrac{6}{x-3}$ and $g(x) = \dfrac{1}{x}$, find each of the following.

(a) $(f \circ g)(x)$ and its domain

Solution

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right)$$

$$(f \circ g)(x) = \frac{6}{\frac{1}{x} - 3}$$

$$(f \circ g)(x) = \frac{6x}{1 - 3x}$$

Multiply the numerator and denominator by *x*.

Example 4 DETERMINING COMPOSITE FUNCTIONS AND THEIR DOMAINS

Given that $f(x) = \dfrac{6}{x-3}$ and $g(x) = \dfrac{1}{x}$, find each of the following.

(a) $(f \circ g)(x)$ and its domain

Solution The domain of g is all real numbers *except* 0, which makes $g(x)$ undefined. The domain of f is all real numbers *except* 3. The expression for $g(x)$, therefore, cannot equal 3. We determine the value that makes $g(x) = 3$ and *exclude* it from the domain of $f \circ g$.

Example 4 DETERMINING COMPOSITE FUNCTIONS AND THEIR DOMAINS

Given that $f(x) = \frac{6}{x-3}$ and $g(x) = \frac{1}{x}$, find each of the following.

(a) $(f \circ g)(x)$ and its domain

Solution

$$\frac{1}{x} = 3$$ The solution must be excluded.

$$1 = 3x$$ Multiply by *x*.

$$x = \frac{1}{3}$$ Divide by 3.

Example 4 DETERMINING COMPOSITE FUNCTIONS AND THEIR DOMAINS

Given that $f(x) = \dfrac{6}{x-3}$ and $g(x) = \dfrac{1}{x}$, find each of the following.

(a) $(f \circ g)(x)$ and its domain

Solution

Therefore the domain of $f \circ g$ is the set of all real numbers *except* 0 and 1/3, written in interval notation as

$$(-\infty, 0) \cup \left(0, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right).$$

Example 4 DETERMINING COMPOSITE FUNCTIONS AND THEIR DOMAINS

Given that $f(x)=\dfrac{6}{x-3}$ and $g(x)=\dfrac{1}{x}$, find each of the following.

(b) $(g\circ f)(x)$ and its domain

Solution $(g\circ f)(x)=g(f(x))=g\left(\dfrac{6}{x-3}\right)$

$$=\frac{1}{\dfrac{6}{x-3}}$$

Note that this is meaningless if $x = 3$

$$=\frac{x-3}{6}$$

Example 4 DETERMINING COMPOSITE FUNCTIONS AND THEIR DOMAINS

Given that $f(x) = \frac{6}{x-3}$ and $g(x) = \frac{1}{x}$, find each of the following.

(b) $(g \circ f)(x)$ and its domain

Solution The domain of f is all real numbers *except* 3, and the domain of g is all real numbers *except* 0. The expression for $f(x)$, which is $\frac{6}{x-3}$, is never zero, since the numerator is the nonzero number 6. Therefore, the domain of $g \circ f$ is the set of all real numbers *except* 3, written $(-\infty, 3) \cup (3, \infty)$.

Homework 4 SHOWING THAT $(g \circ f)(x)$ IS NOT EQUIVALENT TO $(f \circ g)(x)$

Let $f(x) = 4x + 1$ and $g(x) = 2x^2 + 5x$.

Show that $(f \circ g)(x) \neq (g \circ f)(x)$.

Solution First, find $(g \circ f)(x)$.

$$(g \circ f)(x) = g(f(x)) = g(4x+1) \quad f(x) = 4x+1$$

$$= 2(4x+1)^2 + 5(4x+1) \quad g(x) = 2x^2 + 5x$$

Square $4x + 1$; distributive property.

$$= 2(16x^2 + 8x + 1) + 20x + 5$$

Homework 4 SHOWING THAT $(g \circ f)(x)$ IS NOT EQUIVALENT TO $(f \circ g)(x)$

Let $f(x) = 4x + 1$ and $g(x) = 2x^2 + 5x$.

Show that $(f \circ g)(x) \neq (g \circ f)(x)$.

Solution

$$= 32x^2 + 16x + 2 + 20x + 5$$ Distributive property.

$$(g \circ f)(x) = 32x^2 + 36x + 7$$ Combine like terms.

Let $f(x) = 4x + 1$ and $g(x) = 2x^2 + 5x$.

Show that $(f \circ g)(x) \neq (g \circ f)(x)$.

Solution Now find $(f \circ g)(x)$.

$$(f \circ g)(x) = f(g(x))$$

$$= f(2x^2 + 5x) \qquad g(x) = 2x^2 + 5x$$

$$= 4(2x^2 + 5x) + 1 \qquad f(x) = 4x + 1$$

$$(f \circ g)(x) = 8x^2 + 20x + 1$$ Distributive property

Thus, $(g \circ f)(x) \neq (f \circ g)(x)$.

Example 5 FINDING FUNCTIONS THAT FORM A GIVEN COMPOSITE

Find functions f and g such that

$$(f \circ g)(x) = (x^2 - 5)^3 - 4(x^2 - 5) + 3.$$

Solution Note the repeated quantity $x^2 - 5$. If we choose $g(x) = x^2 - 5$ and $f(x) = x^3 - 4x + 3$, then

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x^2 - 5) \\ &= (x^2 - 5)^3 - 4(x^2 - 5) + 3\end{aligned}$$

There are other pairs of functions f and g that also work.

Example 5 FINDING FUNCTIONS THAT FORM A GIVEN COMPOSITE

Find functions *f* and *g* such that

$$(f \circ g)(x) = (x^2 - 5)^3 - 4(x^2 - 5) + 3.$$

Solution Note the repeated quantity x^2. If we choose $g(x) = x^2$ and $f(x) = (x-5)^3 - 4(x-5) + 3$, then

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x^2 - 5) \\ &= (x^2 - 5)^3 - 4(x^2 - 5) + 3\end{aligned}$$

There are other pairs of functions *f* and *g* that also work.