Random Variables:

- $0 \leq P(X = x) \leq 1$
- $\sum P(X = x) = 1$
- $E(X) = \sum x\, P(X = x)$

Question 1:

Given the following discrete distribution:

x	-1	0	1	2	3	4
P(X=x)	0.15	0.30	M	0.15	0.10	0.10

1. *The value of M is equal to*

$$M = 1 - (0.15 + 0.30 + 0.15 + 0.10 + 0.10) = 1 - 0.80 = 0.20$$

(A) 0.20	(B) 0.0	(C) 0.10	(D) 0.25

2. *P(X ≤0.5) =*

$$0.15 + 0.30 = 0.45$$

(A) 0.0	(B) 0.50	(C) 0.45	(D) 1.0

3. *P(X=0) =*

(A) 0	(B)0.30	(C) 0.80	(D) 1.0

4. *The expected (mean) value E[X] is equal to*

$$E(X) = (-1 \times 0.15) + (0 \times 0.30) + (1 \times 0.20) + (2 \times 0.15) + (3 \times 0.10) + (4 \times 0.10) = 1.05$$

(A) 0.0	(B) 1.35	(C) 1.05	(D) 1.20

Question 2:

The average length of stay in a hospital is useful for planning purposes. Suppose that the following is the probability distribution of the length of stay (X) in a hospital after a minor operation:

Length of stay (days)	3	4	5	6
Probability	0.4	0.2	0.1	k

(1) The value of k is

$$k = 1 - (0.4 + 0.2 + 0.1) = 1 - 0.7 = 0.3$$

A	0.0	B	1	C	0.3	D	6

(2) $P(X < 0) =$

A	0.0	B	0.5	C	1	D	0.75

(3) $P(0 < X \leq 5) =$

$$0.4 + 0.2 + 0.1 = 0.7$$

A	0.32	B	0.5	C	0.7	D	0.1

(4) $P(X \leq 5.5) =$

$$0.4 + 0.2 + 0.1 = 0.7$$

A	0.7	B	0.6	C	0	D	0.1

(5) The probability that the patient will stay at most 4 days in a hospital after a minor operation is equal to

$$0.4 + 0.2 = 0.6$$

A	0.4	B	0.1	C	0.2	D	0.6

(6) The average length of stay in a hospital is

$$E(X) = (3 \times 0.4) + (4 \times 0.2) + (5 \times 0.1) + (6 \times 0.3) = 4.3$$

A	2.3	B	0.7	C	1	D	4.3

<u>Binomial Distribution:</u>

$$P(X = x) = \binom{n}{x} p^x q^{n-x} \quad ; \quad x = 0,1 \dots, n$$

$$* \, E(X) = np \qquad * \, Var(X) = npq$$

$$q = 1 - p$$

<u>Question 1:</u>

Suppose that 25% of the people in a certain large population have high blood pressure. A Sample of 7 people is selected at random from this population. Let X be the number of people in the sample who have high blood pressure, follows a binomial distribution then

1) *The values of the parameters of the distribution are:*

$$\boxed{p = 0.25 \quad q = 0.75 \quad n = 7}$$

(A) 7, 0.75	*(B) <u>7, 0.25</u>*	*(C) 0.25, 0.75*	*(D) 25, 7*

2) *The probability that we find exactly one person with high blood pressure, is:*

$$\boxed{P(X = 1) = \binom{7}{1} (0.25)^1 (0.75)^6 = 0.31146}$$

(A) <u>0.31146</u>	*(B) 0.143*	*(C) 0.125*	*(D) 0.25*

3) *The probability that there will be at most one person with high blood pressure, is:*

$$\boxed{P(X \leq 1) = \binom{7}{0} (0.25)^0 (0.75)^7 + \binom{7}{1} (0.25)^1 (0.75)^6 = 0.4449}$$

(A) 0.311	*(B) 0.25*	*(C) <u>0.4449</u>*	*(D) 0.5551*

4) *The probability that we find more than one person with high blood pressure, is:*

$$\boxed{P(X > 1) = 1 - P(X \leq 1) = 1 - 0.4449 = 0.5551}$$

(A) 0.689	*(B) 0.857*	*(C) 0.4449*	*(D) <u>0.5551</u>*

Question 2:

In some population it was found that the percentage of adults who have hypertension is 24 percent. Suppose we select a simple random sample of five adults from this population. Then the probability that the number of people who have hypertension in this sample, will be:

$$p = 0.24 \quad q = 0.76 \quad n = 5$$

1. *Zero:*

$$P(X = 0) = \binom{5}{0}(0.24)^0(0.76)^5 = 0.2536$$

2. *Exactly one*

$$P(X = 1) = \binom{5}{1}(0.24)^1(0.76)^4 = 0.4003$$

3. *Between one and three, inclusive*

$$P(1 \le X \le 3) = \binom{5}{1}(0.24)^1(0.76)^4 + \binom{5}{2}(0.24)^2(0.76)^3 + \binom{5}{3}(0.24)^3(0.76)^2 = 0.7330$$

4. *Two or fewer (at most two):*

$$P(X \le 2) = \binom{5}{0}(0.24)^0(0.76)^5 + \binom{5}{1}(0.24)^1(0.76)^4 + \binom{5}{2}(0.24)^2(0.76)^3 = 0.9067$$

5. *Five:*

$$P(X = 5) = \binom{5}{5}(0.24)^5(0.76)^0 = 0.0008$$

6. *The mean of the number of people who have hypertension is equal to:*

$$E(X) = np = 5 \times 0.24 = 1.2$$

7. *The variance of the number of people who have hypertension is equal to:*

$$Var(X) = npq = 5 \times 0.24 \times 0.76 = 0.912$$

Poisson distribution:

$$P(X = x) = \frac{e^{-\lambda}\ \lambda^{x}}{x!} \quad ; \quad x = 0,1,2,\ldots$$

$$E(X) = Var(X) = \lambda$$

Question 1:

The number of serious cases coming to a hospital during a night follows a Poisson distribution with an average of 10 persons per night, then:

1) *The probability that 12 serious cases coming in the next night, is:*

$$\lambda_{one\ night} = 10$$

$$P(X = 12) = \frac{e^{-10}\ 10^{12}}{12!} = 0.09478$$

(A) <u>0.09478</u>	*(B) 0.3456*	*(C) 12*	*(D) 0.5*

2) *The average number of serious cases in a two nights period is:*

$$\lambda_{two\ nights} = 20$$

(A) 10.5	*(B) <u>20</u>*	*(C) 0.2065*	*(D) 0.0867*

3) *The probability that 20 serious cases coming in next two nights is:*

$$\lambda_{two\ nights} = 20$$

$$P(X = 20) = \frac{e^{-20}\ 20^{20}}{20!} = 0.0888$$

(A) 10.5	*(B) 0.7694*	*(C) 0.20*	*(D) <u>0.0888</u>*

Question 2:

Given the mean number of serious accidents per year in a large factory is five. If the number of accidents follows a Poisson distribution, then the probability that in the next year there will be:

1. *Exactly seven accidents:*

$$\lambda_{one\ year} = 5$$

$$P(X = 7) = \frac{e^{-5}\ 5^7}{7!} = 0.1044$$

2. *No accidents*

$$P(X = 0) = \frac{e^{-5}\ 5^0}{0!} = 0.0067$$

3. *one or more accidents*

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - P(X = 0) \\ &= 1 - 0.0067 = 0.9933 \end{aligned}$$

4. *The expected number (mean) of serious accidents in the next two years is equal to*

$$\lambda_{two\ years} = 10$$

5. *The probability that in the next two years there will be three accidents*

$$\lambda_{two\ years} = 10$$

$$P(X = 3) = \frac{e^{-10}\ 10^3}{3!} = 0.0076$$

The Normal Distribution:

Question 1:

Given the standard normal distribution, Z~N(0,1), find:

1. *The area under the curve between and z=0 and z=1.43*

(A) 0.4236	(B) 0.2330	(C) 0.5396	(D) 0.7864

2. *P(Z ≥2.71)=*

(A) 0.7088	(B)0.0034	(C) 0.3645	(D) 0.1875

3. *P(-1.96 < Z < 1.96) =*

(A) 0.0746	(B) 0.9950	(C) 0.9500	(D) 0.9750

4. *If P(Z < a) = 0.9929, then the value of a =*

(A) -2.54	(B) 0	(C) 1.64	(D) 2.45

5. *If P(-k < Z < k)= 0.8132, then the value of k =*

(A) 2.54	(B) 2.31	(C) 1.32	(D) 0.5

6. *P(Z=1.33)=*

(A) 0.1220	(B) 0.1660	(C) 0.1550	(D) 0.0

<u>***Question 2:***</u>

Given the standard normal distribution, then:

1) $P(-1.1 \leq z \leq 1.1)$ *is:*

(A) 0.3254	*(B) 0.8691*	*(C) <u>0.7286</u>*	*(D) 0.1475*

2) $P(z > -0.15)$ *is:*

(A) <u>0.5596</u>	*(B) 0.9394*	*(C) 0.0606*	*(D) 0.4404*

3) *The z value that has an area of 0.883 to its right, is:*

(A) -0.811	*(B) 1.19*	*(C) 0.811*	*(D) -1.19*

Question 3:

A nurse supervisor has found that staff nurses, on the average, complete a certain task in 10 minutes. If the times required to complete the task are approximately normally distributed with a standard deviation of 3 minutes, then:

1) *The probability that a nurse will complete the task in less than 8 minutes is:*

(A) 0.3221	*(B) <u>0.2514</u>*	*(C) 0.5288*	*(D) 0.1565*

2) *The probability that a nurse will complete the task in more than 4 minutes is:*

(A) 0.5461	*(B) 0.7558*	*(C) <u>0.9772</u>*	*(D) 0.8712*

3) *If eight nurses were assigned the task, the expected number of them who will complete it within 8 minutes is approximately equal to:*

(A) 4	*(B) 1*	*(C) 5*	*(D) <u>2</u>*

4) *If a certain nurse completes the task within k minutes with probability 0.6293; then k equals approximately:*

(A) 15	*(B) 11*	*(C) 7*	*(D) 21*

Question 4:

Given the normally distributed random variable X with mean 491 and standard deviation 119,

1) *If $P(X < k) = 0.9082$, the value of k is equal to*

(A) 649.27	(B) 390.58	(C) 128.90	(D) 132.65

2) *If $P(292 < X < M) = o.8607$, the value of M is equal to*

(A) 766	(B) 649	(C) 108	(D) 136

Question 5:

The IQ (Intelligent Quotient) of individuals admitted to a state school for the mentally retarded are approximately normally distributed with a mean of 60 and a standard deviation of 10, then:

1) *The probability that an individual picked at random will have an IQ greater than 75 is:*

(A) 0.9332	(B) 0.8691	(C) 0.7286	(D) 0.0668

2) *The probability that an individual picked at random will have an IQ between 55 and 75 is:*

(A) 0.3085	(B) 0.6915	(C) 0.6247	(D) 0.9332

3) *If the probability that an individual picked at random will have an IQ less than k is 0.1587. Then the value of k*

(A) 50	(B) 45	(C) 51	(D) 40

Question 6:

In a simple random sample of size 36 drawn from a population with a mean of 100 and a standard deviation of 36, then

1) *the probability that the sample mean will be less than 91is:*

(A) 0.1549	*(B) 0.0753*	*(C) 0.0668*	*(D) 0.0875*

2) *the probability that the sample mean will be more than 98 is:*

(A) 0.5468	*(B) 0.6293*	*(C) 0.8527*	*(D) 0.7169*

3) *the probability that the sample mean will be between 95 and 105 is:*

(A) 0.5934	*(B) 0.6174*	*(C) 0.8432*	*(D) 0.7647*