# Chapter(6)

## The Normal Distribution

# Introduction

- 6-1 Properties of the Normal Distribution and the Standard Normal Distribution.
- 6-2 Applications of the Normal Distribution.
- 6-3 The Central Limit Theorem

Note: This PowerPoint is only a summary and your main source should be the book.

# The Normal Distribution

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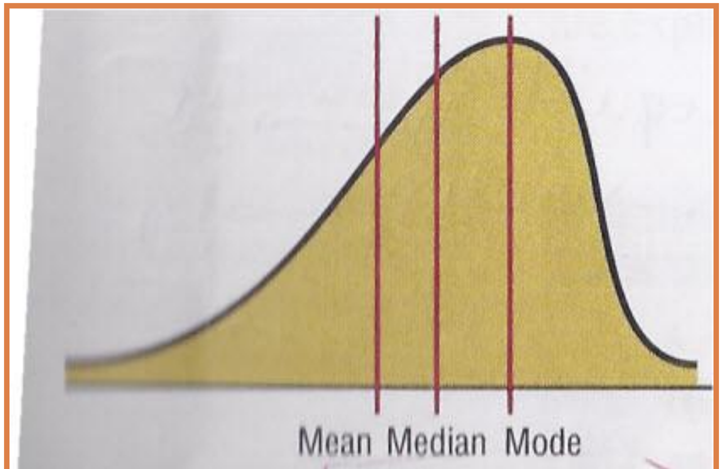
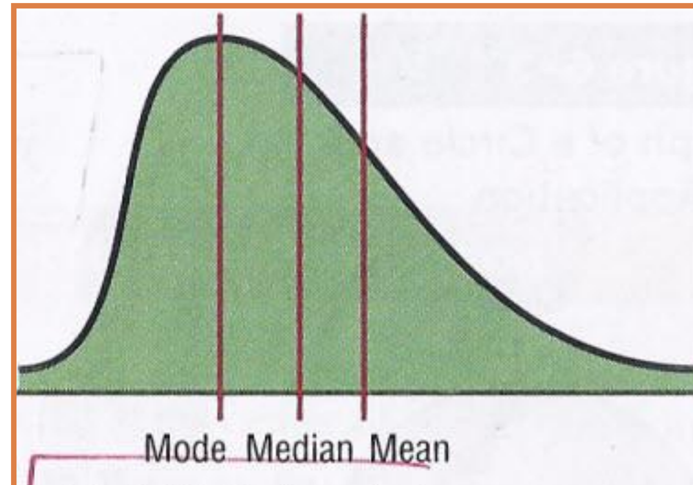
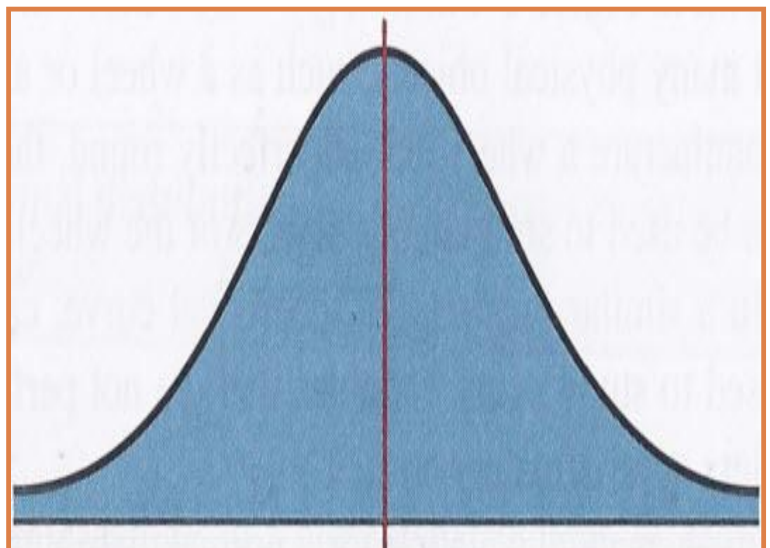


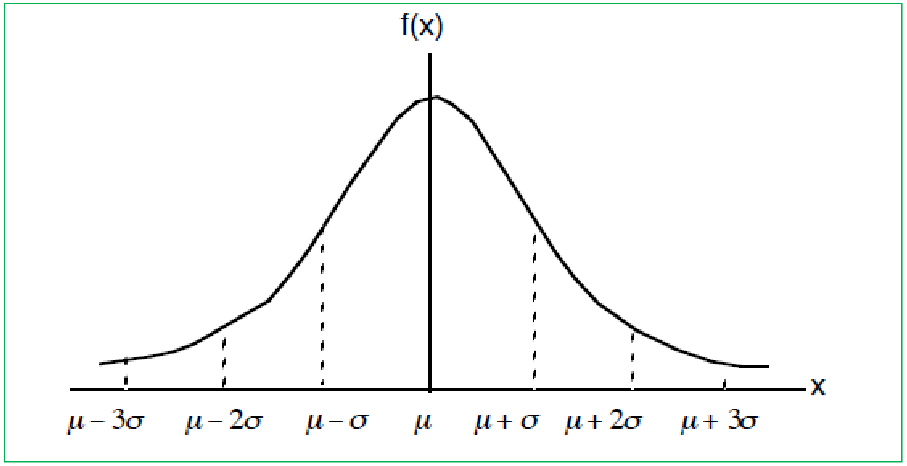
(b) Negatively skewed



(c) Positively skewed


(a) Normal

**A normal distribution** is a continuous , symmetric , bell shaped distribution of a variable.




- A normal distribution curve depend on two parameters .

  $\mu$ ⟶ Position parameter

  $\sigma$ ⟶ shape parameter

*The mathematical equation for the normal distribution:*

$$y = \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$$

*where*

$e \approx 2\ 718$

$\pi \approx 3\ 14$

$\mu \approx$ *population mean*

$\sigma \approx$ *population standard deviation*

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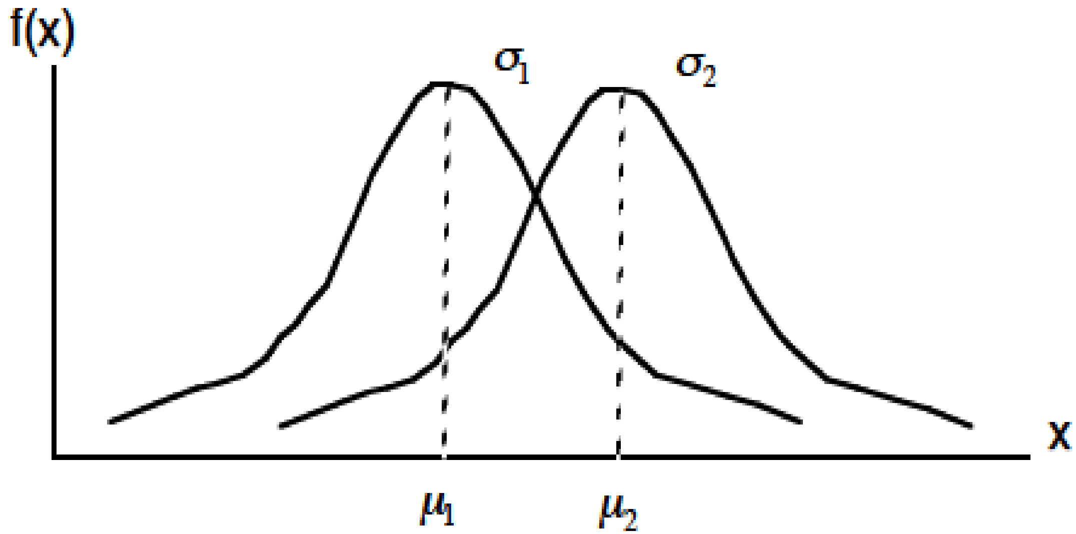
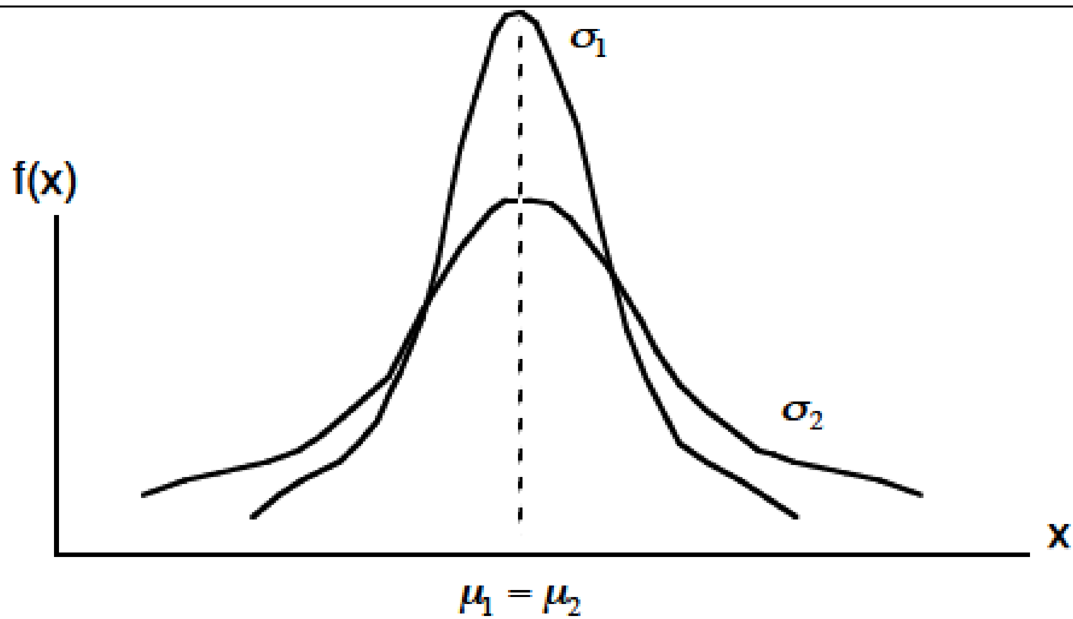
1




Normal curves with $\mu_1 \neq \mu_2$ and $\sigma_1 = \sigma_2$

**(1)** Different means but same standard deviations.

2

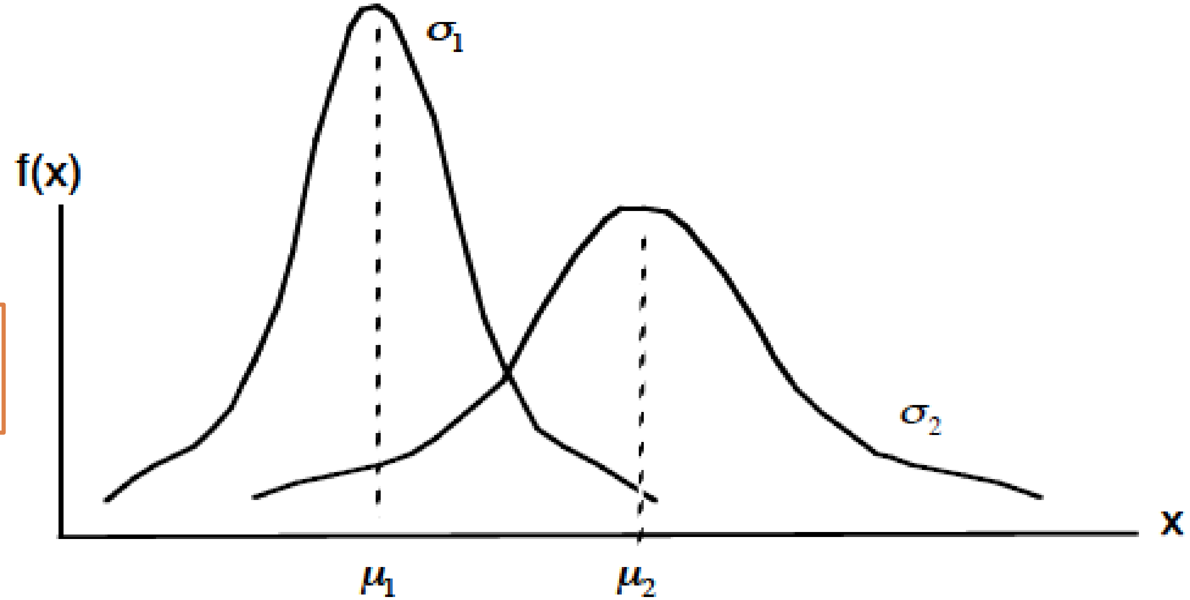



**Normal curves with $\mu 1 = \mu 2$ and $\sigma_1 < \sigma_2$**

**(2)** Same means but different standard deviations .

Note: This PowerPoint is only a summary and your main source should be the book.

3




Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$

**(3)** Different means and different standard deviations .

Note: This PowerPoint is only a summary and your main source should be the book.

# Properties of the Normal Distribution

- The normal distribution curve is **bell-shaped**.
- The mean, median, and mode are **equal** and located at the center of the distribution.
- The normal distribution curve is **unimodal** (single mode).
- The curve is **symmetrical** about the mean.
- The curve is **continuous**.
- The curve **never touches the *x*-axis**.
- The total area under the normal distribution curve is **equal to 1 or 100%.**

Note: This PowerPoint is only a summary and your main source should be the book.

- The area under the normal curve that lies within

  - one standard deviation of the mean is approximately 0.68 (68%).

  - two standard deviations of the mean is approximately 0.95 (95%).

  - three standard deviations of the mean is approximately 0.997 (99.7%).

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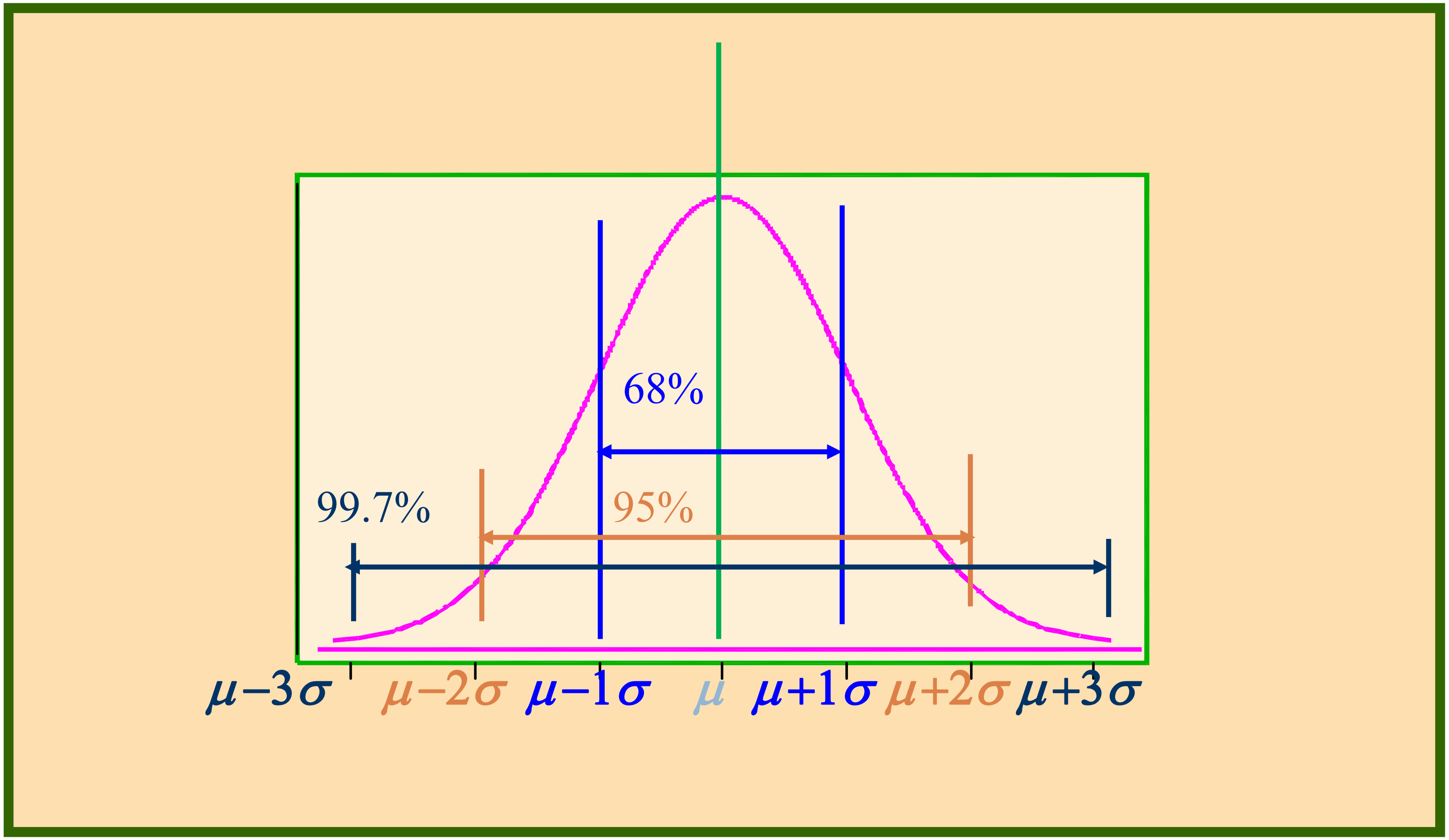
# Empirical Rule: Normal Distribution

## (Areas Under the Normal Curve)



# The Standard Normal Distribution

Note: This PowerPoint is only a summary and your main source should be the book.

- The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1.

- The formula for the standard normal distribution is

$$y = \frac{e^{\frac{-z^2}{2}}}{\sqrt{2\pi}}$$

$\mu = 0$

$\sigma = 1$

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- All Normal Distribution can be transformed into standard Distribution.

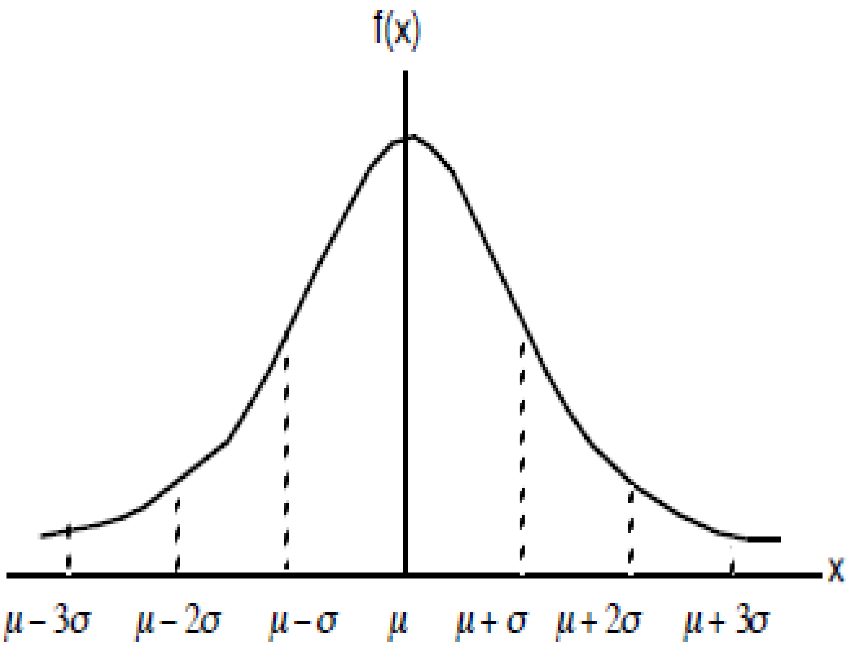
$$z = \frac{value - mean}{standard\,deviation}$$

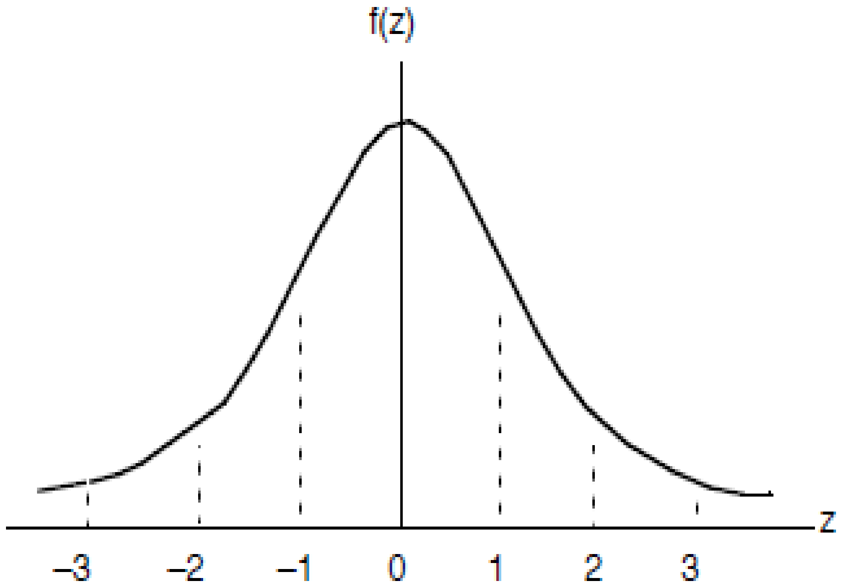
or

$$z = \frac{x - \mu}{\sigma}$$

Note: This PowerPoint is only a summary and your main source should be the book.

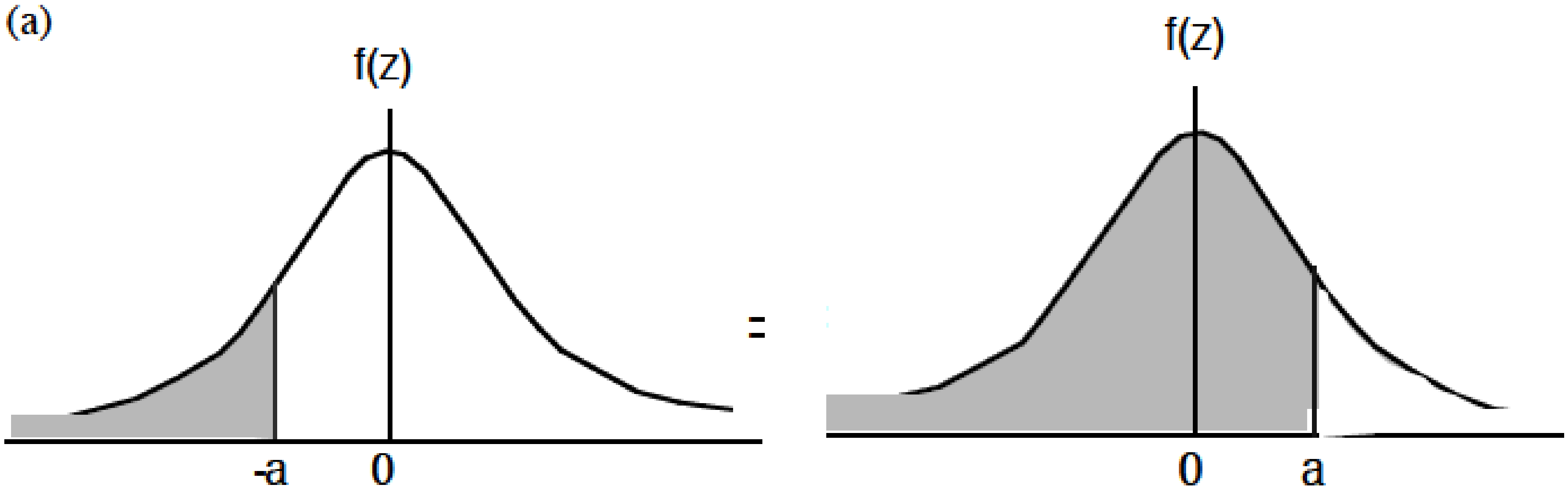
# Empirical Rule: Standard Normal Distribution



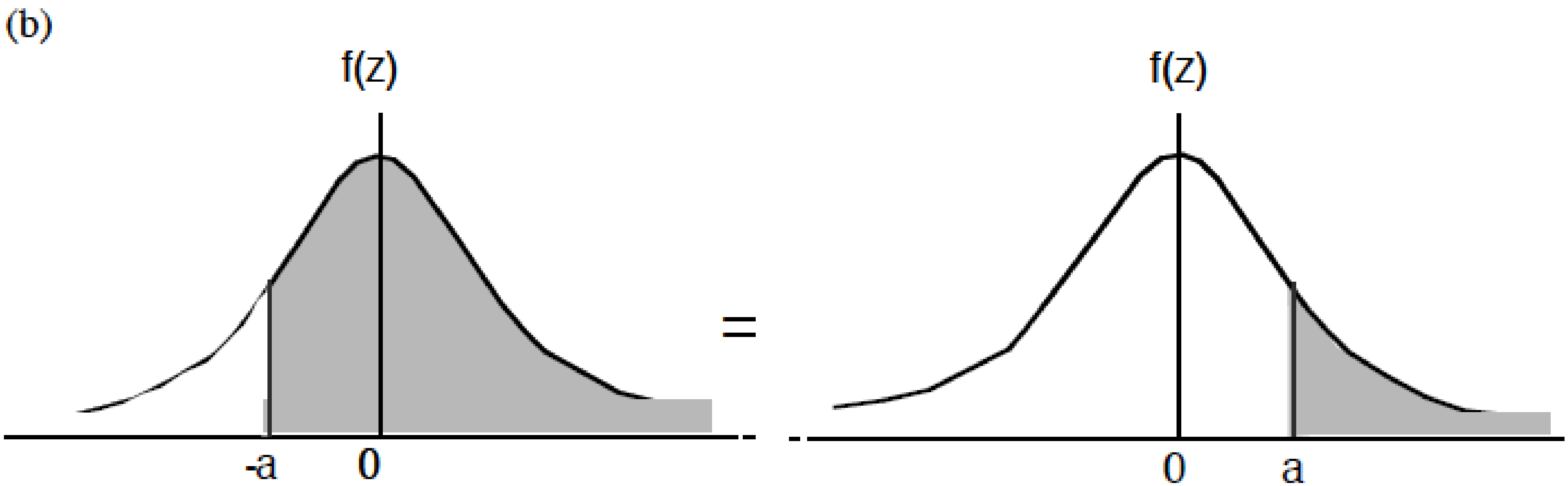
# Procedure To Finding the area under Standard Normal Distribution:

1. To the left of any Z value




$$P(z<-a) = P(z<a) = Q(a)$$

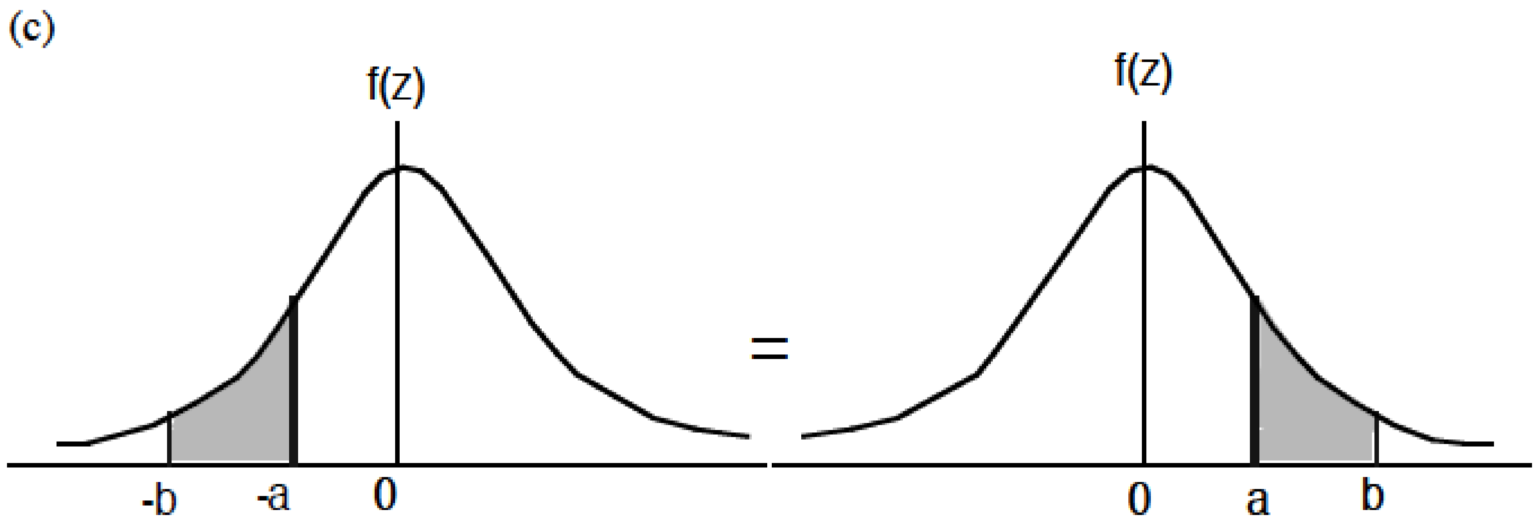
# 2.To the right of any Z value




$$P(z>-a) = P(z>a) = 1 - Q(a)$$

Note: This PowerPoint is only a summary and your main source should be the book.

# 3.Between any two Z values



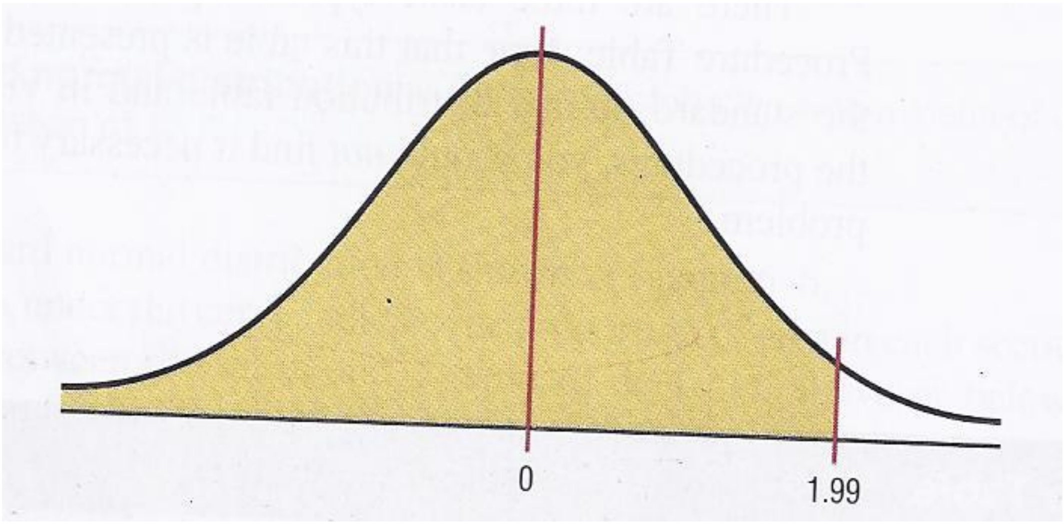

$$P(-b<z<-a) = P(a<z<b) = P(z>a) - P(z>b)$$

$$= Q(a) - Q(b)$$

Note: This PowerPoint is only a summary and your main source should be the book.

# Example 6-1:

Find the area to the left of $z=1.99$




$P(Z<1.99)=0.9767$

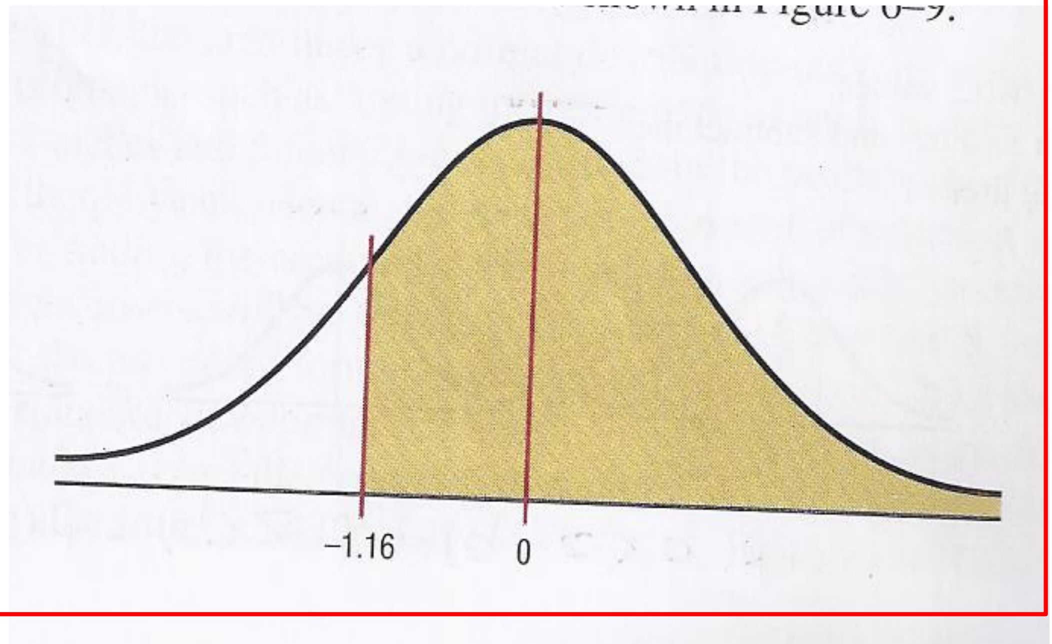
Table E (continued)

**Cumulative Standard Normal Distribution**

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |

## Example 6-2:

Find the area to the right of $z=-1.16$




$$P(Z>-1.16)=1-P(Z<-1.16)$$
$$=1-0.1230$$
$$=0.8770$$

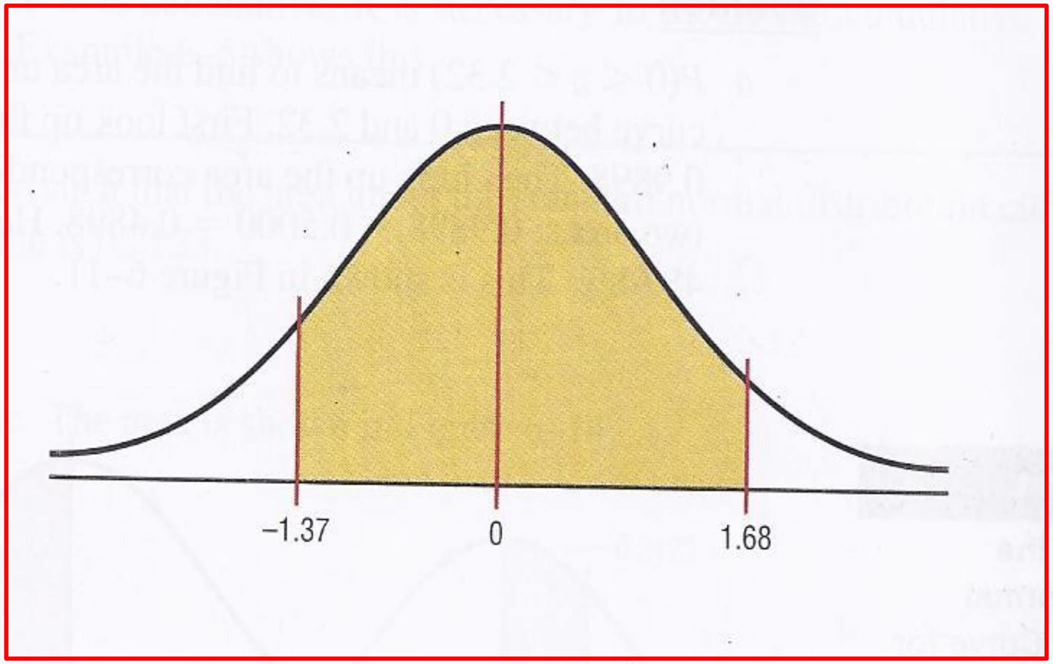
**Table E** The Standard Normal Distribution

**Cumulative Standard Normal Distribution**

| *z* | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| −3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| −3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| −3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |

## Example 6-3:

Find the area between z=1.68 and z=-1.37



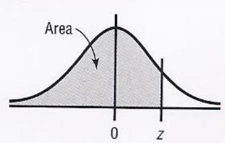

$$P(-1.37<Z<1.68)=P(Z<1.68)-P(Z<-1.37)$$
$$= 0.9535-0.0853$$
$$= 0.8682$$

## Table E (continued)

**Cumulative Standard Normal Distribution**

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

For $z$ values greater than 3.49, use 0.9999.



## Table E The Standard Normal Distribution

**Cumulative Standard Normal Distribution**

| *z* | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| −3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| −3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| −3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |

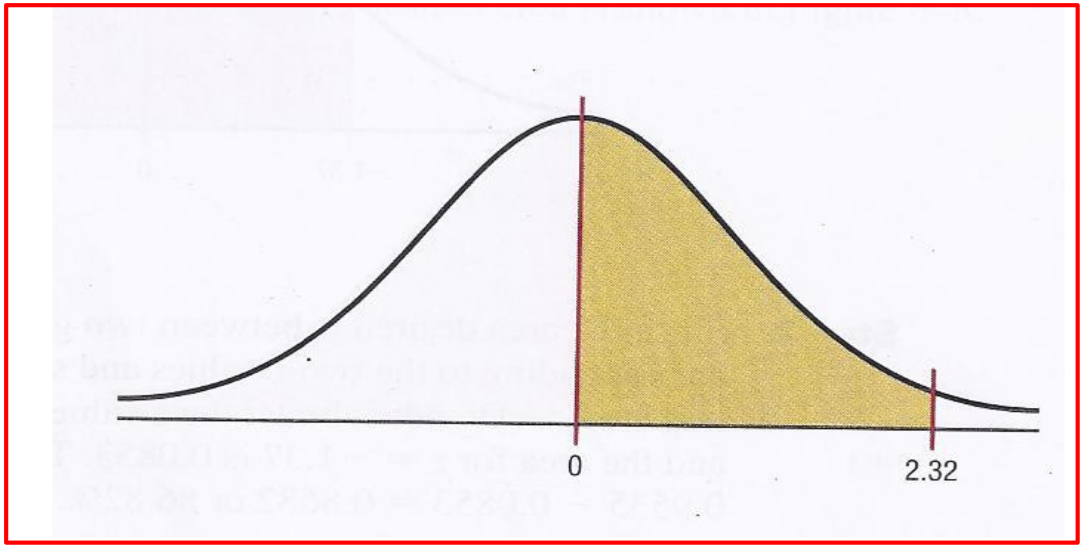
# Example 6-4:

Find probability for each

a) $P(0<z<2.23)$

b) $P(z<1.65)$

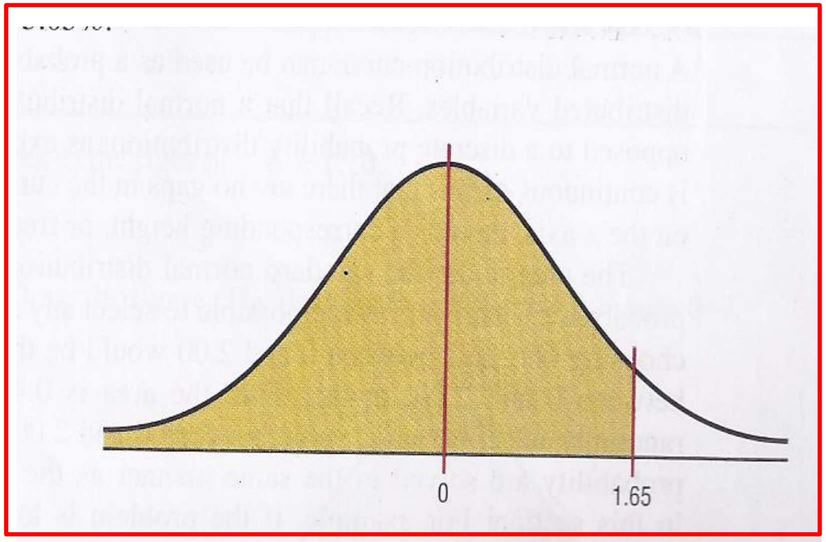
c) $P(z>1.91)$



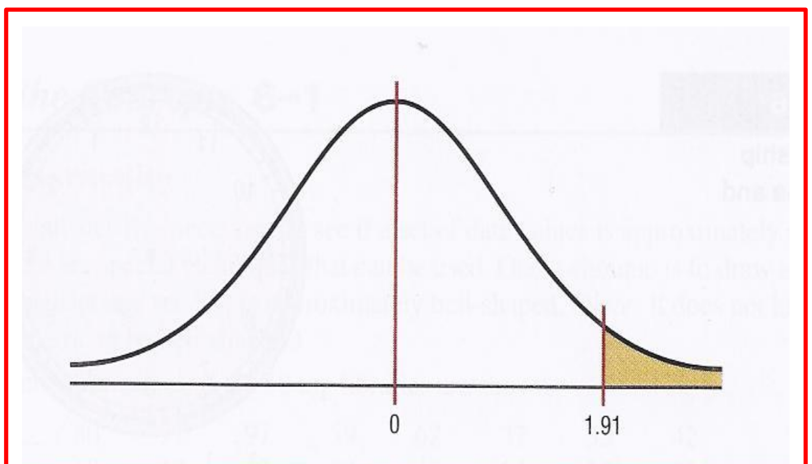

a) $P(0<Z<2.23)=P(Z<2.23)-P(Z<0)$

$=0.9898-0.5000$

$=0.4898$




b) P(Z<1.65)=0.9505



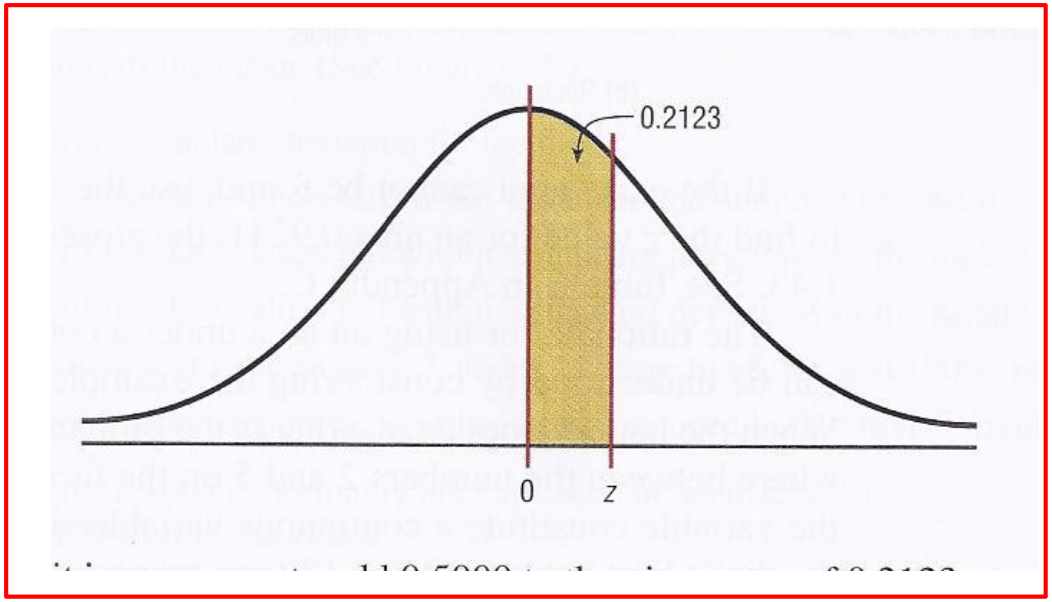

c) P(Z>1.91)=1-P(Z<1.91)

=1-0.9719

=0.0281

## Example 6-5:

Find the z value such that the area under the standard normal distribution curve between 0 and the z value is 0.2123




0.2123+0.5000=0.7123

Z=0.56

**Table E** *(continued)*

**Cumulative Standard Normal Distribution**

| *z* | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 |
|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 |

# Applications of the Normal Distribution

- All **normally distributed** variables can be transformed into the **standard normally distribution** by using the formula for the standard score:

$$z = \frac{value - mean}{standard\ deviation}$$

or

$$z = \frac{x - \mu}{\sigma}$$

Note: This PowerPoint is only a summary and your main source should be the book.

## Example 6-6:

A survey by the National Retail Federation found that women spend on average \$146.21 for the Christmas holidays. Assume the standard deviation is \$29.44. Find the percentage of women who spend less than \$160. Assume the variable is normally distributed.

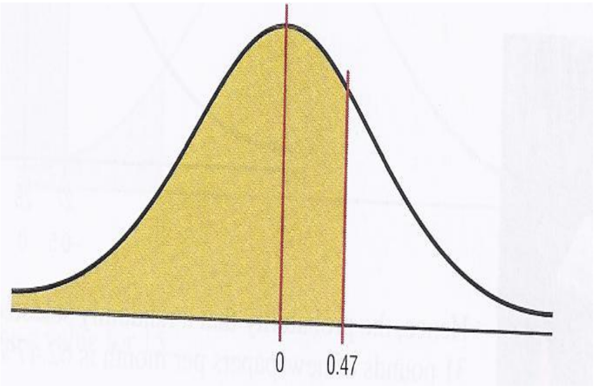
Solution:

Solution:

**Step 1** : Find the z value .

$$Z = \frac{X-\mu}{\sigma} = \frac{160-146.21}{29.44} = 0.47$$

**Step 2** :Draw the figure




**Step 3** :Find the area ,using table E.

P(Z<0.47)=0.6808

68.08% of the women spend less than 160$ at Christmas time.

**Example 6-7:**

Each month, an American household generates an average of 28 pounds of newspaper for garbage or recycling. Assume the standard deviation is 2 pounds. If a household is selected at random, Find the probability of its generating.

a) Between 27 and 31 pounds per month

b) More than 30.2 pounds per month

Assume the variable is approximately normally distributed.

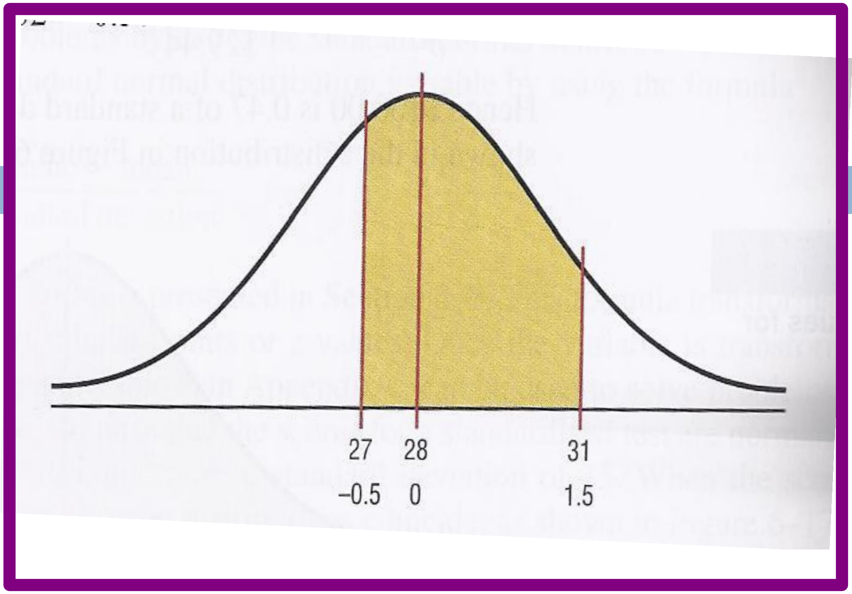
Solution (a) :

**Step 1** : Find the two z value .

$$Z_1 = \frac{X - \mu}{\sigma} = \frac{27 - 28}{2} = -0.5$$

$$Z_2 = \frac{X - \mu}{\sigma} = \frac{31 - 28}{2} = 1.5$$

Step 2 :Draw the figure




Step 3 :Find the area ,using table E.

$$P(-0.5<Z<1.5)=P(Z<1.5)-P(Z<-0.5) = 0.9332-0.3085$$

$$= 0.6247$$

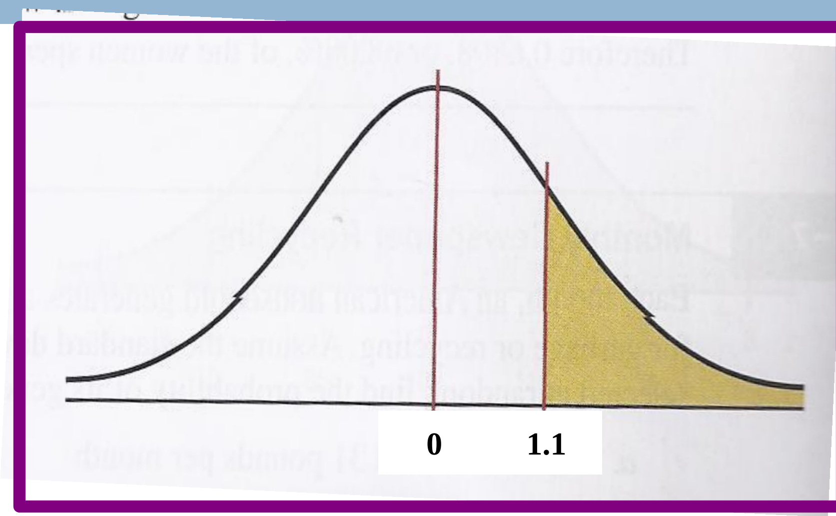
The probability that household generates between 27 and 31 pounds of newspapers per month is 62.47%

## Solution (b) :

**Step 1** : Find the z value .

**Step 2** :Draw the figure

$$Z = \frac{X - \mu}{\sigma} = \frac{30.2 - 28}{2} = 1.1$$




**Step 3** :Find the area ,using table E.

P(Z>1.1)= 1-P(Z<1.1) =1-0.8643 =0.1375

The probability that household generates more than 30.2 pounds of newspapers is 0.1375 or 31.75%

**Example 6-8:**

The American Automobile Association reports that the average time it takes to respond to an emergency call is 25 minutes. Assume the variable is approximately normally distributed and the standard deviation is 4.5 minutes. If 80 calls are randomly selected, approximately how many will be responded to in less than 15 minutes?

0

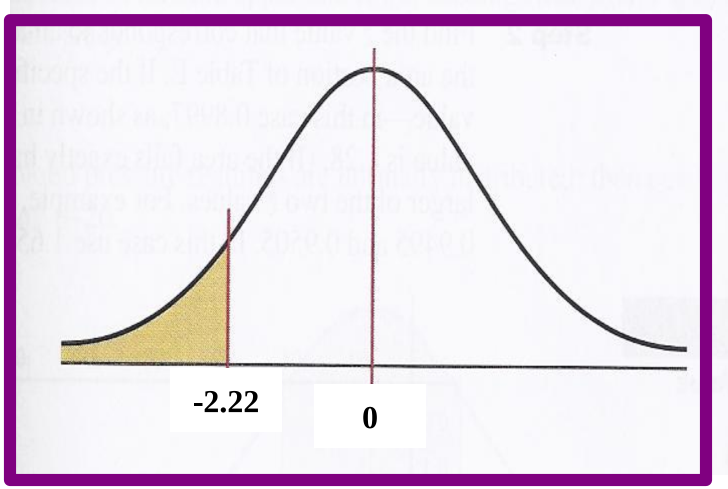
## Example 6-8:

**Solution:**

**Step 1** : Find the z value .

$$Z = \frac{X - \mu}{\sigma} = \frac{15 - 25}{4.5} = -2.22$$

**Step 2** :Draw the figure




**Step 3** :Find the area ,using table E.

P(Z<-2.22)= 0.0132

**Step 4** :to find how many calls .

$$0.0132 \times 80 = 1.056 \approx 1$$

# Finding Data Values Given Specific Probabilities

Formula for Finding X:

$$X = z \cdot \sigma + \mu$$

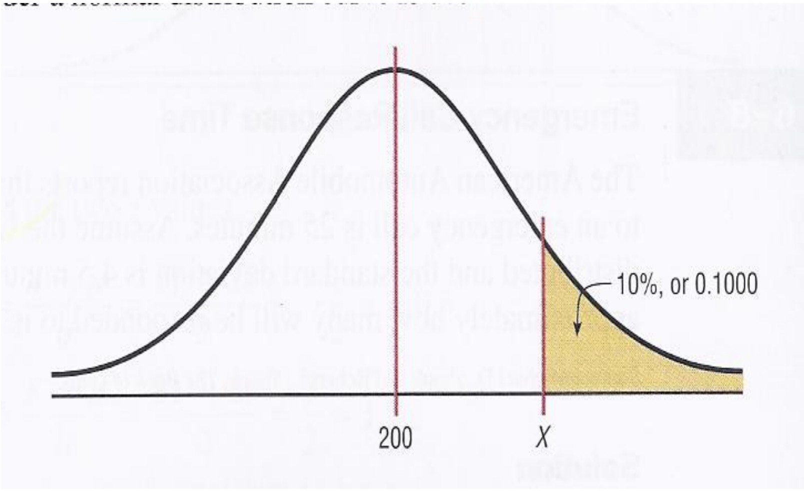
Note: This PowerPoint is only a summary and your main source should be the book.

## Example 6-9:

To qualify for a police academy, candidates must score in the top 10% on a general abilities test. The test has a mean of 200 and standard deviation of 20. Find the lowest possible score to qualify. Assume the test scores is normally distributed.

Solution:

**Step 1** :Draw the figure




**Step 2** : Find the z value .

1-0.10 = 0.9000

Z=1.28

**Step 3** :Find the x.

$$X = (1.28)(20) + 200$$

$$X = 226$$

Any body scoring 226 or higher qualify

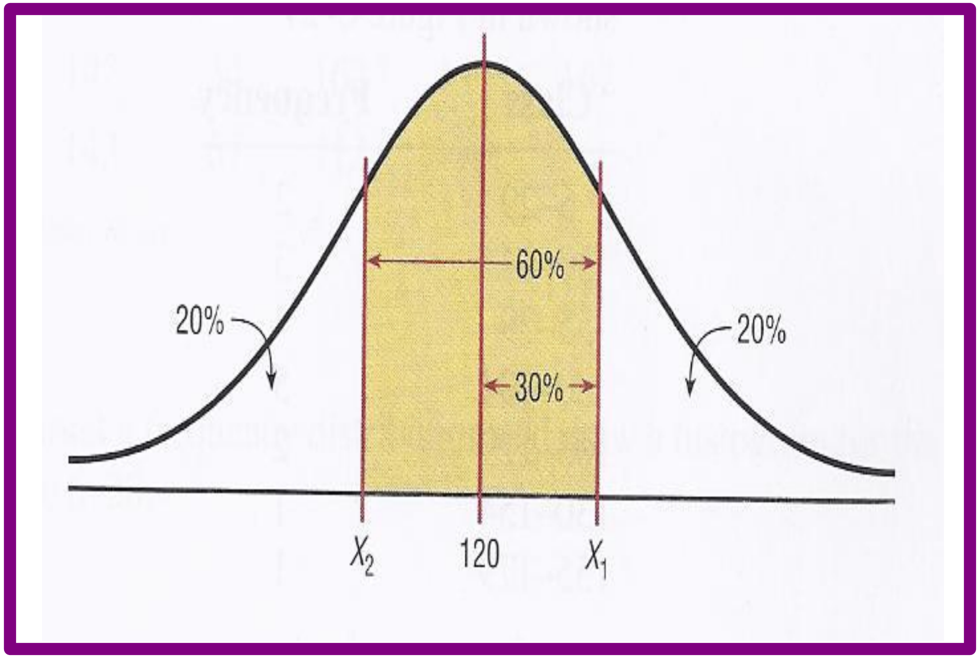
Note: This PowerPoint is only a summary and your main source should be the book.

## Example 6-10:

For a medical study, a researcher wishes to select people in the middle 60% of the population based on blood pressure. If the mean systolic blood pressure is 120 and the standard deviation is 8, find the upper and lower readings that would qualify people to participate in the study. Assume that blood pressure readings is normally distributed.

## Solution:

**Step 1** :Draw the figure



**Step 2** : Find the two z values .

0.5000+0.3000=0.8000

$$Z_1 = 0.84$$

0.5000-0.3000= 0.2000

$$Z_2 = -0.84$$

**Step 3** :Find the two x.

$$X_1 = Z_1 \cdot \sigma + \mu$$

$$X_1 = (0.84)(8) + 120 = 126.72$$

$$X_2 = Z_2 \cdot \sigma + \mu$$

$$X_2 = (-0.84)(8) + 120 = 113.28$$

The middle 60% will have blood pressure reading of 113.28<X<126.72

# 6-3The Central Limit Theorem

# Distribution of Sample Means

- **A sampling distribution of sample means:** is a distribution using the means computed from all possible random samples of a specific size taken from a population.
- **Sampling error:** is the difference between the sample measure and corresponding population measure due to the fact that the sample is not a perfect representation of the population.

# Distribution of Sample Means

- **Properties of the distribution of sample means:**

1- The mean of the sample means will be the same as the population mean.

2- The standard deviation of the sample means will be smaller than the standard deviation of the population, and it will be equal to the population standard deviation divided by the square root of the sample size.

$$\mu_{\overline{X}} = \mu$$

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

## • The Central Limit Theorem:

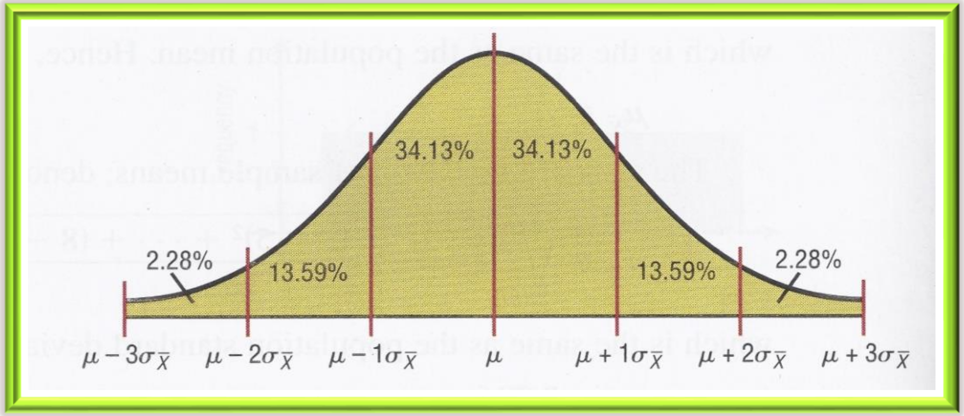
- As the sample size *n* increase without limit, the shape of the distribution of the sample means taken with replacement from a population with mean $\mu$ and standard deviation $\sigma$ will approach a normal distribution. This distribution will have a mean $\mu$ and a standard deviation $\sigma / \sqrt{n}$
- **The formula for *z* values is:**

$$Z = \frac{\overline{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} \quad \Rightarrow \quad z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

# Two important things when we use Central limit theorem :

1- when the original variable is normally distributed , the distribution of the sample means will be normally distributed , for any sample size *n*.

*2*- when the original variable might not be normally distributed, a *sample size must be 30 or more* to approximate the distribution of sample means to normal distribution .

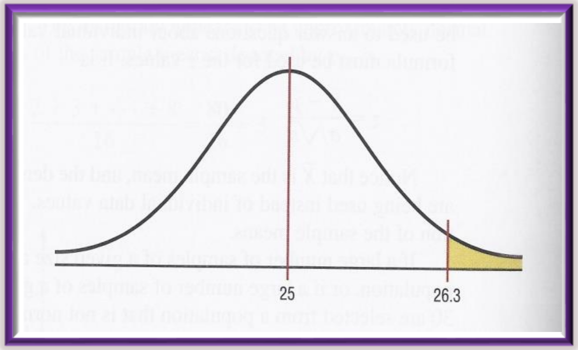


- # Example 6-13:

A. C. Neilsen reported that children between the ages of 2 and 5 watch an average of 25 hours of television per week. Assume the variable is normally distributed and the standard deviation is 3 hours. If 20 children between the ages 2 and 5 are randomly selected, find the probability that **the mean** of the number of hours they watch television will be greater than 26.3 hours.

- # Solution: $\mu = 25, \sigma = 3, n = 20, \overline{X} = 26.3$

$$\mu_{\overline{X}} = \mu = 25 \qquad \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{20}} = 0.671$$




$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{26.3 - 25}{0671} = 1.94$$

$$P(Z > 1.96) = 1 - P(Z < 1.96)$$

$$= 1 - 0.9738 = 0.0262$$

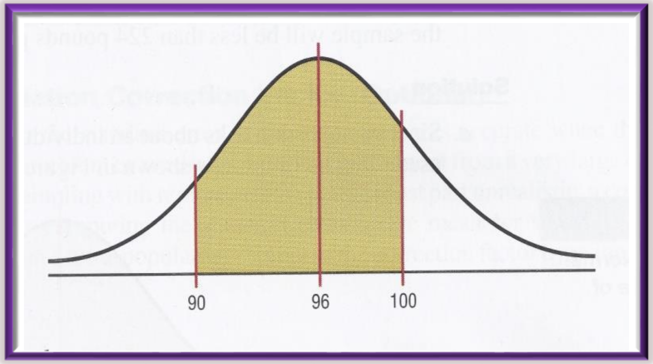
# Example 6-14:

The average age of a vehicle registered in the United States is 8 years, or 96 months. Assume the standard deviation is 16 months. If a random sample of 36 vehicles is selected, find the probability that **the mean** of their age is between 90 and 100 months.

# Solution:

$$\mu = 96, \sigma = 16, n = 36, \overline{X}_1 = 90, \overline{X}_2 = 100$$

$$Z_1 = \frac{90 - 96}{16/\sqrt{36}} = -2.25$$




$$Z_2 = \frac{100 - 96}{16/\sqrt{36}} = 1.50$$

$$P(-2.25 < Z < 1.50) = P(Z < 1.50) - P(Z < -2.25)$$

=0.9332-0.0122=0.9210

## Example 6-15:

The average number of pounds of meat that a person consumes per year is 218.4 pounds. Assume that the standard deviation is 25 pounds and the distribution is approximately normal.

a. Find the probability that a person selected at random consumes less than 224 pounds per year.

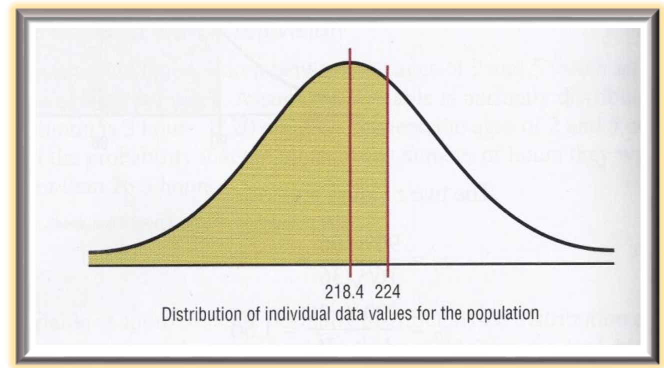
b. If a sample of 40 individuals is selected, find the probability that **the mean of the sample** will be less than 224 pounds per year.

- Solution:

$$\mu = 218.4\ , \sigma = 25$$

$$a.\ \ Z = \frac{X-\mu}{\sigma} = \frac{224-218.4}{25} = 0.22$$

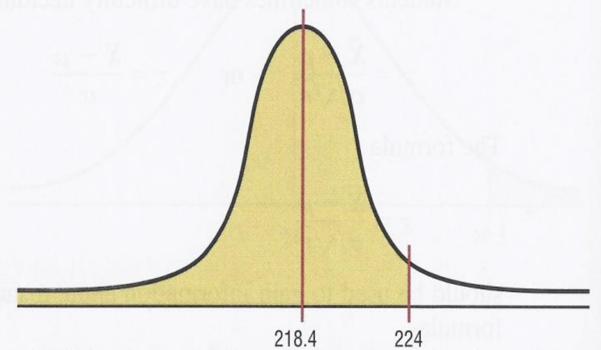
$$P(Z < 0.22) = 0.5871$$



b. If a sample of 40 individuals is selected, find the probability that **the mean** of the sample will be less than 224 pounds per year.

$$b. Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{224 - 218.4}{25 / \sqrt{40}} = 1.42$$

$$P(Z < 1.42) = 0.9222$$



Distribution of means for all samples of size 40 taken from the population

# Study Hard and Good Luck