3.2 : Equations of Lines and Linear Models

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معادلات الخط المستقيم .

(x, y) , (x_1, y_1)

سؤال

slope
الميل

$$m = \frac{y - y_1}{x - x_1}$$

$(x_1, y_1), (x_2, y_2) \Longrightarrow$ slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

point-slope Form:-

let m slope , point (x_1, y_1)

$$\boxed{y - y_1 = m(x - x_1)}$$

معادلة الخط المستقيم بدلالة نقطة وميل .

Example 1. p(99) :~

Write an equation of line through $(-4,1)$, having slope -3

أوجدي معادلة الخط المستقيم المار بالنقطة (-4,1) وميله -3 ،

$x_1 = -4$, $y_1 = 1$, $m = -3$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -3(x - (-4))$$

$$y - 1 = -3(x + 4)$$

$$y - 1 = -3x - 12 \Longrightarrow \boxed{y = -3x - 11}$$

HW 1 P. (99)

Write an equation of the line through $(-3,2)$ and $(2,-4)$. Write the result in standard form

$$Ax + By = C$$

اكتبي معادلة الخط المستقيم المار بالنقطتين .

* هنا في السؤال الميل غير موجود .

أولاً نوجد الميل :-

Find slope , $\overset{x_1 \;\; y_1}{(-3,2)}$, $\overset{x_2 \;\; y_2}{(2,-4)}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{2-(-3)} = \frac{-6}{5}$$

ثانياً : نعوض في معادلة الخط المستقيم

$$y - y_1 = m(x - x_1)$$

باختيار أي نقطة من النقاط .

Let the point $(-3,2) \Rightarrow x_1 = -3$, $y_1 = 2$

$$m = -\frac{6}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-6}{5}(x - (-3))$$

$$y - 2 = -\frac{6}{5}(x + 3)$$

$$y - 2 = -\frac{6}{5}x - \frac{18}{5}$$

$$y = -\frac{6}{5}x - \frac{18}{5} + 2$$

$$y = -\frac{6}{5}x - \frac{18}{5} + \frac{10}{5}$$

$$y = -\frac{6}{5}x - \frac{8}{5}$$

ملاحظة :- عند التعويض في معادلة المستقيم يكون عندنا حرية اختيار إحدى النقطتين

لو اخترنا النقطة الثانية $(2,-4)$ نطلع بنفس الناتج

standard form: $y + \frac{6}{5}x = -\frac{8}{5}$

$$5y + 6x = -8$$

Slope - Intercept Form:

$$y = mx + b$$

$b \rightarrow$ y-intercept
الجزء الذي يقطع محور y.

$m \rightarrow$ slope الميل

Example 2 p. (100) :- Find the slope and y-intercept of the line with equation

$$4x + 5y = -10$$

أوجدي الميل والجزء الذي يقطع محور y.

$$5y = -10 - 4x$$

$$y = -\frac{4}{5}x - \frac{10}{5}$$

$$y = -\frac{4}{5}x - 2$$

$-\frac{4}{5} \rightarrow$ slope

$-2 \rightarrow$ y-intercept

$\therefore$ Slope $m = -\frac{4}{5}$, y-intercept -2

HW2 p. (100) :- Write an equation of the line through (1,1) and (2,4). Then graph (ارسمي) the line using the slope intercept form.

أوجدي معادلة المستقيم المار بالنقطتين وارسميه.

نوجد الميل
① Find the slope: $\overset{x_1, y_1}{(1,1)}, \overset{x_2\, y_2}{(2,4)}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{2-1} = \frac{3}{1} = 3$$

② $y - y_1 = m(x - x_1)$ at point (1,1)

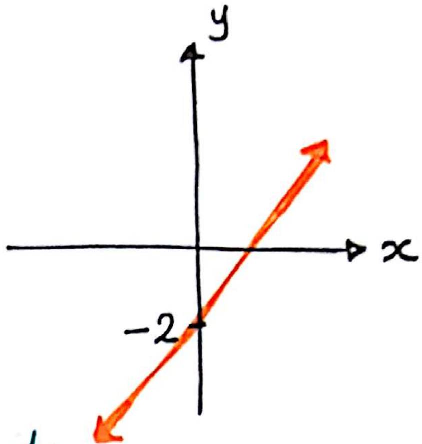
$y-1 = 3(x-1)$

$y-1 = 3x-3$

$y = 3x-3+1$

$y = 3x - 2$ → y-intercept

slope



Example 3 p. (100) :~ use the graph استخدي الرسم .

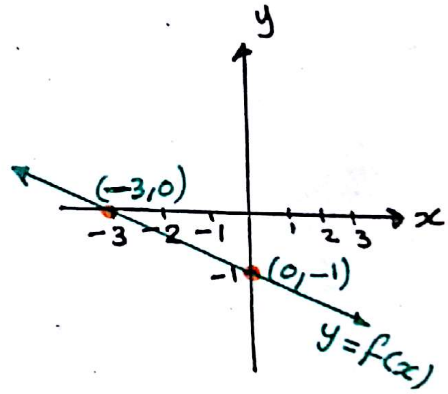
ⓐ Find the slope , y-intercept , x-intercept .

ⓑ Write the equation that defines f .

ⓐ We have the points $\underset{x_1\ y_1}{(-3,0)}, \underset{x_2\ y_2}{(0,-1)}$

Slope $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{-1-0}{0-(-3)}$

$= \dfrac{-1}{3}$



الجزء الذي يقطع محور y y-intercept -1

الجزء الذي يقطع محور x x-intercept -3

ⓑ $m = -\frac{1}{3}$, point $(-3,0)$

$y - y_1 = m(x-x_1)$

$y - 0 = -\frac{1}{3}(x-(-3))$

$y = -\frac{1}{3}(x+3) = -\frac{1}{3}x - 1$

$\boxed{f(x) = -\frac{1}{3}x - 1}$

General Form : الصيغة العامة

$$Ax + By = C$$

$$By = -Ax + c$$

$$y = \frac{-A}{B}x + \frac{C}{B}$$

$\frac{-A}{B}$: slope

$\frac{C}{B}$: y-intercept

$$Ax = -By + c$$

$$x = \frac{-B}{A}y + \frac{C}{A}$$

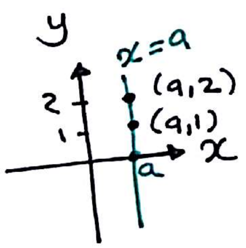
$\frac{C}{A}$: x-intercept

Equations of Vertical and Horizontal Line

Vertical Line الخط الرأسي

(a,b)

$x = a$



سؤال

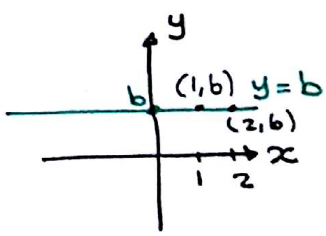
slope: undefined

$m = \frac{2-1}{a-a} = \frac{1}{0}$ undefined

Horizontal Line خط أفقي

(a,b)

$y = b$



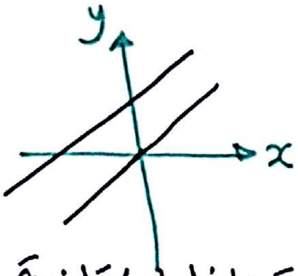
Slope = 0

$m = \frac{b-b}{2-1} = \frac{0}{1} = 0$

Parallel and Perpendicular Lines

خطوط متوازية

Parallel Lines //



→ slope of parallel line

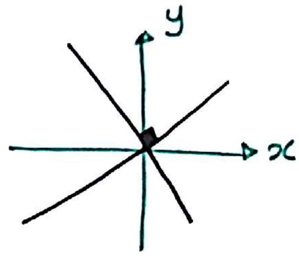
- equal

- ميزة الخطوط المتوازية :.
لها نفس الميل

Same slope.

خطوط متعامدة

Perpendicular Lines ⊥



ميزة الخطوط المتعامدة :
حاصل ضرب ميلهم = -1

$m_1 m_2 = -1$

HW3 . p. (102) :~

Write the equation of line that passes through the point (3, 5)

أوجدي معادلة الخط المستقيم المار بالنقطة (3,5).

ⓐ Parallel to the line $2x + 5y = 4$

- الموازي للخط ⇐ الخطوط المتوازية لها نفس الميل :

① Find the slope.

نوجد الميل.

$$2x + 5y = 4 \implies 5y = -2x + 4$$

$$y = \frac{-2}{5}x + \frac{4}{5}$$

$$\therefore \text{slope } m = \frac{-2}{5}$$

② Find the equation of line , $\overset{x_1\ y_1}{(3,5)}$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{2}{5}(x - 3)$$

$$y - 5 = -\frac{2}{5}x + \frac{6}{5}$$

$$y = -\frac{2}{5}x + \frac{6}{5} + 5$$

$$y = -\frac{2}{5}x + \frac{6+25}{5}$$

$$\boxed{y = -\frac{2}{5}x + \frac{31}{5}}$$

Standard form : $\boxed{5y + 2x = 31}$

ⓑ perpendicular to the line $2x + 5y = 4$

العمودي على المستقيم

من الفقرة a وجدنا أن الميل .

$$m_1 = -\frac{2}{5}$$

في المستقيمات المتعامدة ← حاصل ضرب ميلهم = -1

$$\underbrace{\left(-\frac{2}{5}\right)}_{m_1}\underbrace{\left(\frac{5}{2}\right)}_{m_2} = -1$$

$$\therefore m_2 = \frac{5}{2} \rightarrow \text{slope}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{5}{2}(x - 3)$$

$$y - 5 = \frac{5}{2}x - \frac{15}{2}$$

$$y = \frac{5}{2}x - \frac{15}{2} + 5$$

$$y = \frac{5}{2}x \frac{-15+10}{2}$$

$$\boxed{y = \frac{5}{2}x - \frac{5}{2}}$$

Standard form : $\boxed{2y - 5x = -5}$

Exercises 3.2 p. (103)

Write an equation for the Line .

7 through $(-1,3)$ and $(3,4)$

1 Find the Slope : $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{3-(-1)} = \frac{1}{4}$

2 Find the equation : $y - y_1 = m(x - x_1)$

نختار نقطه من النقاط ← $(-1,3)$

$$y - 3 = \frac{1}{4}(x - (-1))$$

$$y - 3 = \frac{1}{4}(x+1)$$

$$y - 3 = \frac{1}{4}x + \frac{1}{4}$$

$$y = \frac{1}{4}x + \frac{1}{4} + 3$$

$$\boxed{y = \frac{1}{4}x + \frac{13}{4}}$$

Standard form:

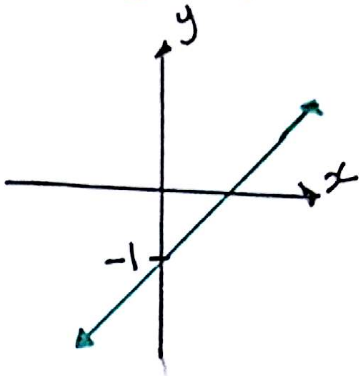
$\boxed{4y - x = 13}$

Give the slope and y-intercept and graph it ارسمي

أوجدي الميل ، الجزء الذي يقطع محور y ، ارسمي .

(15) $y = 3x - 1 \to$ y-intercept

slope

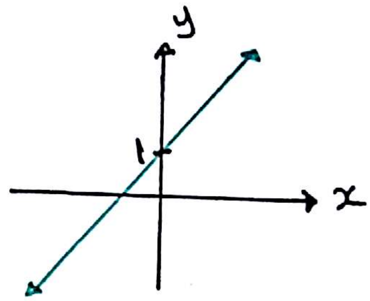


Slope : $m = 3$, y-intercept -1

(19) $y - \frac{3}{2}x - 1 = 0$

$y = \frac{3}{2}x + 1 \to$ y-intercept

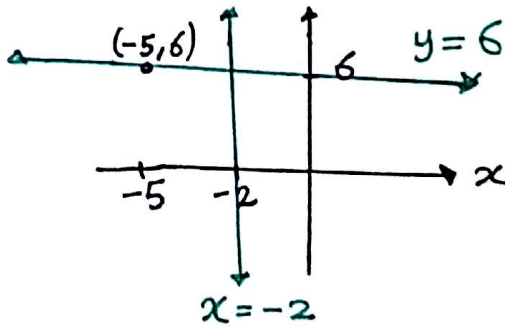
slope



Slope : $m = \frac{3}{2}$, y-intercept 1

(26) Write an equation of line through $(-5, 6)$,

متعامد Perpendicular to $x = -2 \to$ خط رأسي



المستقيم المتعامد للخط الرأسي هو الخط الأفقي .

المستقيم العمودي على المستقيم $x = -2$ ومار بالنقطة $(-5, 6)$ هو الخط الأفقي $\boxed{y = 6}$

3.3 Function Operations and Composition

العمليات على الدوال وتركيب الدوال.

مراجعة مجال الدوال

كثيرة حدود

$\text{Domain} = (-\infty, \infty)$

$f(x) = x^2 - 2x + 1$

$Df = (-\infty, \infty)$

مجال كثيرات الحدود دائماً يساوي جميع الأعداد الحقيقية.

دالة كسرية

المقام ≠ الصفر

$f(x) = \frac{1}{x+1}$

$x + 1 = 0$

$x \neq -1$

$Df = (-\infty, -1) \cup (-1, \infty)$

جميع الأعداد الحقيقية ماعدا صفر المقام.

دالة جذرية

البسط

ماتحت الجذر ≥ 0

$f(x) = \sqrt{2x+1}$

$2x + 1 \geq 0$

$x \geq -\frac{1}{2}$

$Df = [-\frac{1}{2}, \infty)$

المقام.

ماتحت الجذر > 0

$f(x) = \frac{1}{\sqrt{2x+1}}$

$2x + 1 > 0$

$x > -\frac{1}{2}$

$Df = (-\frac{1}{2}, \infty)$

Operation on functions and Domains:

مجال

العمليات على الدوال ومجالها.

Let $f(x)$, $g(x)$ defined functions.

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad , g(x) \neq 0$$

Domains of $f+g$, $f-g$, fg is $Df \cap Dg \Rightarrow$ تقاطع المجالات

Domain of $\frac{f}{g}$ is $Df \cap Dg$ which $g \neq 0 \Rightarrow$ استبعاد اصفار المقام.

Example 1 p.(105): $f(x) = x^2+1$, $g(x) = 3x+5$

Find:

ⓐ $(f+g)(1) = f(1) + g(1) = \left((1)^2+1\right) + \left(3(1)+5\right) = 10$

ⓑ $(f-g)(-3) = f(-3) - g(-3) = \left((-3)^2+1\right) - \left(3(-3)+5\right) = 10-(-4)$
$= 14$

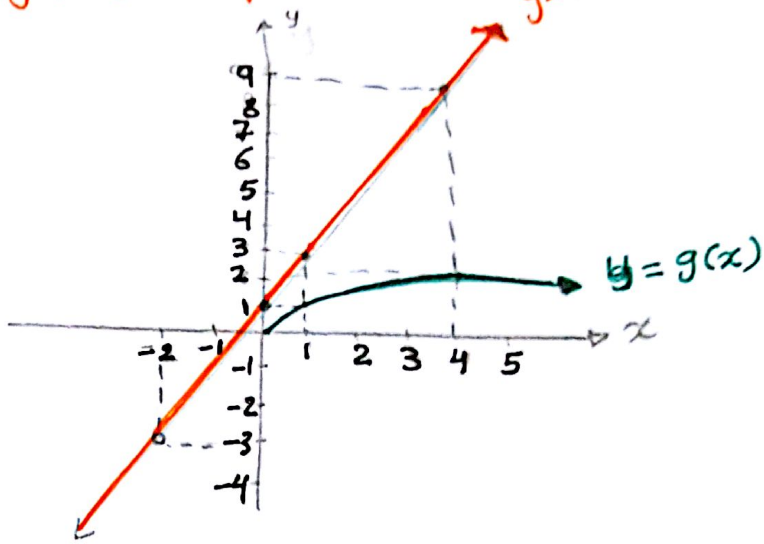
ⓒ $(fg)(5) = f(5) \cdot g(5) = (5^2+1)(3(5)+5) = (26)(20) = 520$

ⓓ $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{0^2+1}{2(0)+5} = \frac{1}{5}$

Example 2 p.(106)

$f(x) = 2x+1$, $g(x) = \sqrt{x}$

x	$f(x)$	$g(x)$
-2	-3	undefined
0	1	0
1	3	1
4	9	2



احسبي :

Evaluate :~

$$(f+g)(4) = f(4) + g(4) = 9 + 2 = 11$$

$$(f-g)(-2) = f(-2) - g(-2) = \text{undefined.}$$ غير معرف

$$(fg)(1) = f(1) \cdot g(1) = 3(1) = 3$$

$$\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{1}{0} \quad \text{undefined.}$$ غير معرف

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The Difference Quotient حاصل الفرق

$$\frac{f(x+h) - f(x)}{h}$$

HW2 p. (108) Let $f(x) = 2x^2 - 3x$

Find $\dfrac{f(x+h) - f(x)}{h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$$

$$= \frac{2(x^2 + 2hx + h^2) - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \frac{\cancel{2x^2} + 4hx + 2h^2 \cancel{- 3x} - 3h \cancel{- 2x^2} \cancel{+ 3x}}{h}$$

$$= \frac{4hx + 2h^2 - 3h}{h} = \frac{h(4x + 2h - 3)}{h}$$

$$= 4x + 2h - 3$$

ملاحظة :

$$f(x+h) \neq f(x) + f(h)$$

– Composition of Functions and Domain:

تركيب الدوال ومجالها.

$$(f\circ g)(x) = f(g(x))$$

$$(g\circ f)(x) = g(f(x))$$

Example 3 p.(109): $f(x) = 2x-1$, $g(x) = \frac{4}{x-1}$

Find: (a) $(f\circ g)(2)$, (b) $(g\circ f)(-3)$

(a) $(f\circ g)(2) = f(g(2))$

$$\boxed{g(2) = \frac{4}{2-1} = 4}$$

$\therefore (f\circ g)(2) = f(g(2)) = f(4) = 2(4)-1$

$= 8-1 = 7$

(b) $(g\circ f)(-3) = g(f(-3))$

$$\boxed{f(-3) = 2(-3)-1 = -7}$$

$\therefore (g\circ f)(-3) = g(f(-3)) = g(-7) = \frac{4}{-7-1}$

$= \frac{4}{-8} = -\frac{1}{2}$

Example 4 p. (110) : $f(x)=\dfrac{6}{x-3}$, $g(x)=\dfrac{1}{x}$

Find : (a) $(f \circ g)(x)$ and its domain

(b) $(g \circ f)(x)$ and its domain

(a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right)$

domain
$x \neq 0$
$(-\infty,0)\cup(0,\infty)$

$$= \frac{6}{\frac{1}{x}-3} = \frac{6}{\frac{1-3x}{x}} = 6 \cdot \frac{x}{1-3x}$$

$$= \frac{6x}{1-3x} \rightarrow$$

دالة كسرية
مجالها المقام ≠ صفر.

$$1-3x=0 \Rightarrow -3x=-1 \Rightarrow x \neq \frac{1}{3}$$

$(-\infty,\frac{1}{3})\cup(\frac{1}{3},\infty)$

∴ Domain of $(f \circ g)(x) = (-\infty,0)\cup(0,\frac{1}{3})\cup(\frac{1}{3},\infty)$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{6}{x-3}\right)$

domain
$x-3 \neq 0$
$x \neq 3$
$(-\infty,3)\cup(3,\infty)$

$$= \frac{1}{\frac{6}{x-3}} = 1 \cdot \frac{x-3}{6} = \frac{x-3}{6}$$

مجالها
جميع الأعداد الحقيقية.

∴ Domain of $(g \circ f)(x) = (-\infty,3)\cup(3,\infty)$

ملاحظة هامة جداً :-

$(f \circ g)(x) \neq (g \circ f)(x)$

Example 5 p.(111): Find f and g

$(f \circ g)(x) = (x^2-5)^3 - 4(x^2-5) + 3$

$g(x) = x^2-5 \quad , \quad f(x) = x^3 - 4x + 3$

$(f \circ g)(x) = f(g(x)) = f(x^2-5)$
$= (x^2-5)^3 - 4(x^2-5) + 3$

Or

$g(x) = x^2 \quad , \quad f(x) = (x-5)^3 - 4(x-5) + 3$

$(f \circ g)(x) = f(g(x)) = f(x^2) = (x^2-5)^3 - 4(x^2-5) + 3$

Exercises 3.3 p.(111)

$f(x) = x^2 + 3 \quad , \quad g(x) = -2x + 6$

Find:

① $(f+g)(3) = f(3) + g(3)$

$= (3^2+3) + (-2(3)+6)$

$= (9+3) + (-6+6)$

$= 12 + 0 = 12$

② $(f-g)(-1) = f(-1) - g(-1)$

$= ((-1)^2+3) - (-2(-1)+6)$

$= (1+3) - (2+6)$

$= 4 - 8 = -4$

③ $(fg)(4) = f(4)\,g(4)$

$= (4^2+3)(-2(4)+6)$

$= (16+3)(-8+6) = 19(-2) = -38$

④ $\left(\frac{f}{g}\right)(-1) = \frac{f(-1)}{g(-1)} = \frac{((-1)^2+3)}{-2(-1)+6} = \frac{4}{8} = \frac{1}{2}$

Find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$

Give the Domain. مجالاتها عبارة عن تقاطع مجال f و g.

(5) $f(x) = 3x+4$, $g(x) = 2x-5$

لأنها كثيرات حدود ← مجالها $(-\infty,\infty)$

Domain of $f(x)$: $(-\infty,\infty)$

$g(x)$: $(-\infty,\infty)$

* $(f+g)(x) = f(x) + g(x)$

$= (3x+4) + (2x-5)$

$= 3x+4+2x-5$

$= 5x-1$

Domain $(f+g)(x)$: $(-\infty,\infty)$ تقاطع مجالي f, g

* $(f-g)(x) = f(x) - g(x)$

$= (3x+4) - (2x-5)$

$= 3x+4-2x+5$

$= x+9$

Domain $(f-g)(x)$: $(-\infty,\infty)$

* $(fg)(x) = f(x)\,g(x) = (3x+4)(2x-5)$

$= 6x^2 - 15x + 8x - 20$

$= 6x^2 - 7x - 20$

Domain $(fg)(x)$: $(-\infty,\infty)$

* $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x+4}{2x-5} \longrightarrow$ المقام $\neq$ الصفر .

$$2x-5 \neq 0 \Rightarrow x \neq \frac{5}{2}$$

Domain $\left(\frac{f}{g}\right)(x)$: $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$

(6) $f(x) = 2x^2 - 3x$, $g(x) = x^2 - x + 3$

Domain $f(x), g(x)$: $(-\infty, \infty)$ كثيرات حدود

$$(f+g)(x) = f(x) + g(x)$$
$$= 2x^2 - 3x + x^2 - x + 3$$
$$= 3x^2 - 4x + 3$$

Domain $(f+g)(x)$: $(-\infty, \infty)$

$$(f-g)(x) = f(x) - g(x)$$
$$= (2x^2 - 3x) - (x^2 - x + 3)$$
$$= 2x^2 - 3x - x^2 + x - 3$$
$$= x^2 - 2x - 3$$

Domain $(f-g)(x)$: $(-\infty, \infty)$

$$(fg)(x) = f(x)\,g(x)$$

$$= (2x^2-3x)(x^2-x+3)$$

$$= 2x^4-2x^3+6x^2-3x^3+3x^2-9x$$

$$= 2x^4-5x^3+9x^2-9x$$

Domain $(fg)(x)$: $(-\infty,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2-3x}{x^2-x+3}$$

$$x^2-x+3 \neq 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2-4(1)(3)}}{2}$$

$$= \frac{1 \pm \sqrt{1-12}}{2} = \frac{1}{2} \pm \frac{\sqrt{-11}}{2} = \frac{1}{2} \pm \frac{\sqrt{11}}{2}\, i$$

أصفار المقام عبارة عن أعداد مركبة ـــك جميع الأعداد الحقيقية مسموحة .

$\therefore$ Domain $\left(\frac{f}{g}\right)(x)$: $(-\infty,\infty)$

⑦ $f(x) = \sqrt{4x-1}$

ماتحت الجذر أكبر من أو يساوي الصفر.

$4x - 1 \geq 0$

$x \geq \frac{1}{4}$

Domain $f(x)$: $[\frac{1}{4}, \infty)$

$g(x) = \frac{1}{x}$

المقام ≠ الصفر

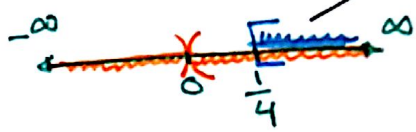
$x \neq 0$

Domain $g(x)$: $(-\infty, 0) \cup (0, \infty)$

$(f+g)(x) = f(x) + g(x)$

$= \sqrt{4x-1} + \frac{1}{x}$

التقاطع هنا هو العناصر المشتركة ،،



Domain $(f+g)(x)$: $[\frac{1}{4}, \infty) \cap \left((-\infty, 0) \cup (0, \infty)\right)$

$= [\frac{1}{4}, \infty)$

التقاطع عبارة عن الفترة الأصغر.

$(f-g)(x) = f(x) - g(x)$

$= \sqrt{4x-1} - \frac{1}{x}$

Domain $(f-g)(x)$: $[\frac{1}{4}, \infty)$

تقاطع مجالي f, g

$(fg)(x) = f(x)\,g(x) = \left(\sqrt{4x-1}\right)\left(\frac{1}{x}\right) = \frac{\sqrt{4x-1}}{x}$

تقاطع المجالين $[\frac{1}{4}, \infty)$

Domain $(fg)(x)$: $[\frac{1}{4}, \infty)$

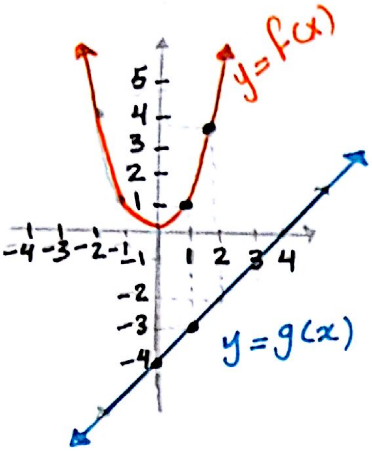
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{4x-1}}{\frac{1}{x}} = x\sqrt{4x-1}$

Domain $\left(\frac{f}{g}\right)(x)$: $[\frac{1}{4}, \infty)$

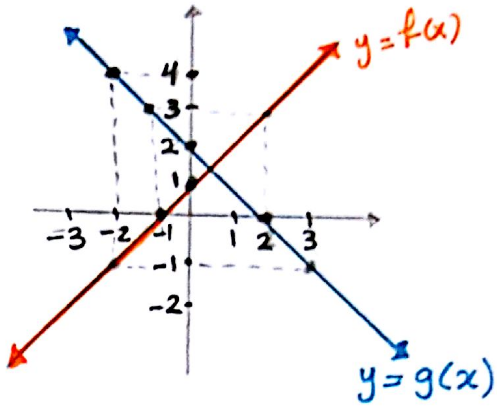
Use the graph to evaluate :-

الرسم

(12)



(13)



12

(a) $(f+g)(2) = f(2) + g(2)$
$= 4 + (-2) = 2$

(b) $(f-g)(1) = f(1) - g(1)$
$= 1 - (-3)$
$= 1 + 3 = 4$

(c) $(fg)(0) = f(0)\,g(0)$
$= 0(-4) = 0$

(d) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1}{-3} = -\frac{1}{3}$

13

(a) $(f+g)(-1) = f(-1) + g(-1)$
$= 0 + 3 = 3$

(b) $(f-g)(-2) = f(-2) - g(-2)$
$= -1 - 4 = -5$

(c) $(fg)(0) = f(0)\,g(0)$
$= 1(2) = 2$

(d) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{3}{0}$ undefined
غير معرف .

(14)

x	$f(x)$	$g(x)$
-2	0	6
0	5	0
2	7	-2
4	10	5

(a) $(f+g)(2) = f(2) + g(2) = 7 + (-2) = 5$

(b) $(f-g)(4) = f(4) - g(4) = 10 - 5 = 5$

(c) $(fg)(-2) = f(-2)\,g(-2) = 0(6) = 0$

(d) $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{5}{0}$ undefined.

x	3	4	6
$f(x)$	1	3	9

x	2	7	1	9
$g(x)$	3	6	9	12

Find :-

(28) $(f \circ g)(2) = f(g(2)) = f(3) = 1$

(29) $(g \circ f)(3) = g(f(3)) = g(1) = 9$

(30) $(f \circ f)(4) = f(f(4)) = f(3) = 1$

– Find $(f \circ g)(x)$ and its domain.
$(g \circ f)(x)$ and its domain.

(36) $f(x) = \frac{2}{x}$, $g(x) = x+1$

$(f \circ g)(x) = f(g(x)) = f(x+1)$ → Domain $(-\infty, \infty)$

$= \frac{2}{x+1}$, $x+1 \neq 0 \Rightarrow x \neq -1$

Domain $(f \circ g)(x)$: $(-\infty, -1) \cup (-1, \infty)$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{2}{x}\right)$ → domain $x \neq 0$ $(-\infty, 0) \cup (0, \infty)$

$= \frac{2}{x} + 1 = \frac{2+x}{x}$ → $x \neq 0$

Domain $(g \circ f)(x)$: $(-\infty, 0) \cup (0, \infty)$

(39) $f(x) = \dfrac{1}{x-2}$, $g(x) = \dfrac{1}{x}$

domain $x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right)$$

$$= \frac{1}{\frac{1}{x} - 2} = \frac{1}{\frac{1-2x}{x}} = \frac{x}{1-2x}$$

مقام المقام بسط

$$1 - 2x \neq 0 \Rightarrow x \neq \frac{1}{2}$$

$(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

$\therefore$ Domain $(f \circ g)(x)$: $(-\infty, 0) \cup (0, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

domain $x \neq 2$

$(-\infty, 2) \cup (2, \infty)$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x-2}\right)$$

$$= \frac{1}{\frac{1}{x-2}} = x - 2$$

$\therefore$ Domain $(g \circ f)(x)$: $(-\infty, 2) \cup (2, \infty)$

Find f, g such that $(f \circ g)(x) = h(x)$.

أوجدي الدوال f, g.

(44) $h(x) = (6x-2)^2$

$g(x) = 6x - 2$, $f(x) = x^2$

اشاكر $(f \circ g)(x) = f(g(x)) = f(6x-2) = (6x-2)^2$

(45) $h(x) = \sqrt{x^2-1}$

$g(x) = x^2 - 1$, $f(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(x^2-1) = \sqrt{x^2-1}$

(46) $h(x) = \sqrt{6x} + 12$

$g(x) = 6x$, $f(x) = \sqrt{x} + 12$

$(f \circ g)(x) = f(g(x)) = f(6x) = \sqrt{6x} + 12$