CHAPTER-7 LINEAR PROGRAMMING PROBLEMS

INTRODUCTION: Many management decisions involve trying to make the most effective use of an organization's resources. Resources typically include machinery, labor, money, time, warehouse space and row materials. These resources may be used to make products (such as machinery, furniture, food or clothing) or services (such as schedules for airlines or production, advertising policies or investment decisions). Linear programming (LP) is a widely used mathematical modeling technique designed to help managers in planning and decision making relative to resource allocation.

REQUIREMENTS OF A LINEAR PROGRAMMING PROBLEM

In the past 60 years, LP has been applied extensively to military, industrial, financial, marketing, accounting and agricultural problems. Even though these applications are diverse, all LP problems have several properties and assumptions in common.

All problems seek to maximize or minimize some quantity, usually profit or cost. We refer to this property as the **Objective Function** of an LP problem.

The second property that LP problems have in common is the presence of restrictions, or **constraints**, that limit the degree to which we can pursue our objective. For example, deciding how many units of each product in a firm's product line to manufacture is restricted by available personnel and machinery. Selection of an advertising policy or a financial portfolio is limited by the amount of money available to be spent or invested. We want, therefore, to maximize or minimize a quantity (the objective function) subject to limited resources (the constraints).

There must be alternative courses of action to choose from. For example, if a company produces 3 different products, management may use LP to decide how to allocate among them its limited production resources (of personnel, machinery and so on). Should it devote all manufacturing capacity to make only the first product, should it produce equal amounts of each product, or should it allocate the resources in some other ratio? If there were no alternatives to select from, we would not need L.P.

The objective and constraints in LP problems must be expressed in terms of linear equations or inequalities. Linear mathematical relationships just mean that all terms used in the objective function and constraints are of the first degree (i.e., not squared, or to the third or higher power or appearing more than once).

The term **linear** implies both proportionality and additivity. Proportionality means that if production of 1 unit of a product uses 3 hours, production of 10 units would use 30 hours. Additivity means that the total of all activities equals the sum of the individual activities. If the production of one product generated $3 profit and the production of another product generated $8 profit, the total profit would be the sum of these two, which would be $11.

We assume that conditions of **certainty** exit: that is, number in the objective and constraints are known with certainty and do not change during the period being studied.

We make the **divisibility** assumption that solutions need not to be in whole numbers (integers). Instead, they are divisible and may take any fractional value. In production problems, we often define variables as the number of units produced per week or per month, and a fractional value (i.e., 0.3 chairs) would simply mean that there is work in process. Something that was started in one week can be finished in the next. However, in other types of problems, fractional values do not make sense. If a fraction of a product cannot be purchased (for example, one-third of a submarine), an integer programming problem exists.

Finally, we assume that all answers or variables are **nonnegative.** Negative values of physical quantities are impossible; you simply cannot produce a negative number of chairs, shirts, lamps or computers. Table 7.1 summarizes these properties and assumptions.

TABLE 7.1
LP Properties and Assumptions

PROPERTIES OF LINEAR PROGRAMS
1. One objective function
2. One or more constraints
3. Alternative courses of action
4. Objective function and constraints are linear—proportionality and divisibility
5. Certainty
6. Divisibility
7. Nonnegative variables

FORMULATING LP PROBLEMS

Formulating a linear program involves developing a mathematical model to represent the managerial problem. Thus, in order to formulate a linear program, it is necessary to completely understand the managerial problem being faced. The steps in formulating a linear program follow:

(1) Completely understand the managerial problem being faced.
(2) Identify the objective and the constraints.
(3) Define the decision variables.
(4) Use the decision variables to write mathematical expressions for the objective function and the constraints.

One of the most common LP applications is the Product Mix Problem. Two or more products are usually produced using limited resources such as personnel, machines, raw materials, and so on. The profit that the firm seeks to maximize is based on the profit contribution per unit of each product. The company would like to determine how many units of each product it should produce so as to maximize overall profit given its limited resources.

Example : Flair Furniture Company

The Flair Furniture Company produces inexpensive tables and chairs. The production process for each is similar in that both require a certain number of hours of carpentry work and a certain number of labor hours in the painting and varnishing department. Each table takes 4 hours of carpentry and 2 hours in the painting and varnishing shop. Each chair requires 3 hours in carpentry and 1 hour in painting and varnishing. During the current production period, 240 hours of carpentry time are available and 100 hours in painting and varnishing time are available. Each table sold yields a profit of $70; each chair produced is sold for $50 profit.

Flair Furniture's problem is to determine the best possible combination of tables and chairs to manufacture in order to reach the maximum profit. The firm would like this production mix situation formulated as an LP problem.

We begin by summarizing information needed to formulate and solve this problem (see Table 7.2)

TABLE 7.2 Flair Furniture Company Data

	HOURS REQUIRED TO PRODUCE 1 UNIT		
DEPARTMENT	TABLES (*T*)	CHAIRS (*C*)	AVAILABLE HOURS THIS WEEK
Carpentry	**4**	**3**	**240**
Painting & varnishing	**2**	**1**	**100**
Profit per unit	**$70**	**$50**	

Formulation:

The decision variables that represent the actual decisions we will make are defined as:

T = number of tables to be produced per week.

C = number of chairs to be produced per week.

Now we can create the LP objective function in terms of T and C:

Maximize profit = $\$70T + \$50C$

Our next step is to develop mathematical relationships for the two constraints:

For carpentry, total time used is:

(4hours per table)(Number of tables produced)

+ (3 hours per chair)(Number of chairs produced).

So the first constraint may be stated as follows:

Carpentry time used ≤ Carpentry time available.

$4T + 3C \leq 240$ (hours of carpentry time)

Similarly, the second constraint is as follows:

Painting and varnishing time used ≤ Painting and varnishing time available.

$2\,T + 1C \leq 100$ (hours of painting and varnishing time)

Both of these constraints represent production capacity restrictions and, of course, affect the total profit.

To obtain meaningful solutions, the values for T and C must be nonnegative numbers. That is, all potential solutions must represent real tables and real chairs. Mathematically, it means that

$T \geq 0$ (number of tables produced is greater than or equal to 0)

$C \geq 0$ (number of chairs produced is greater than or equal to 0)

The complete problem may now be restated mathematically as

Maximize profit $= \$70T + \$50C$

Subjects to the constraints

$4T + 3C \leq 240$ (carpentry constraint)

$2T + 1C \leq 100$ (painting and varnishing constraint)

$T, C \geq 0$ (nonnegativity constraints)

GRAPHICAL SOLUTION TO AN LP PROBLEM

The easiest way to solve a small LP problem such as that of the Flair Furniture Company is with the graphical solution approach. The graphical procedure is useful only when there are two decision variables (such as no. of tables, T and no. of Chairs, C) in the problem. When there are more than two variables, it is not possible to plot the solution on a two-dimensional graph and we must turn to more complex approaches.

GENERAL LINEAR PROGRAMMING PROBLEM IN TWO VARIABLES:

Find the values of x_1 & x_2 that optimize (either maximize or minimize)

$z = c_1x_1 + c_2x_2$ [Linear Objective Function]

Subject to Linear Constraints $a_{11}x_1 + a_{12}x_2 \ (\leq, \geq or =) b_1$

$a_{21}x_1 + a_{22}x_2 \ (\leq, \geq or =) b_2$

……………………………………………………

$a_{m1}x_1 + a_{m2}x_2 \ (\leq, \geq or =) b_m$

And $x_1 \geq 0, \ x_2 \geq 0$ [Non-Negative Constraints]

NOTE (1) A pair of values (x_1, x_2) that satisfy all the constraints is called a **Feasible Solution**. The set of all feasible solutions determines a subset of x_1x_2-plane called the feasible region. A feasible solution that optimizes the objective function is called an **Optimal Solution**.

NOTE (2) The feasible region of an LPP has a boundary consisting of a finite number of straight line segments. If the feasible region can be enclosed in a sufficiently large circle, it is called **Bounded**; otherwise it is called Unbounded.

If the feasible region is empty (contains no points), then the constraints are **Inconsistent** and the LPP has no solution.

Those boundary points of a feasible region that are intersections of two of the straight line boundary segments are called **Extreme points (or Corner points)**.

THEOREM: If the feasible region of an LPP is non-empty and bounded, then the objective function attains both a maximum and a minimum value and these occur at extreme points of the feasible region. If the feasible region is Unbounded, then the objective function may or may not attain a maximum or minimum value; however, if it attains a maximum or minimum value, it does so at an extreme point.

Example: Solve the following LPP by Graphical method-

$$\text{Maximize profit} = \$70T + \$50C$$

Subjects to the constraints

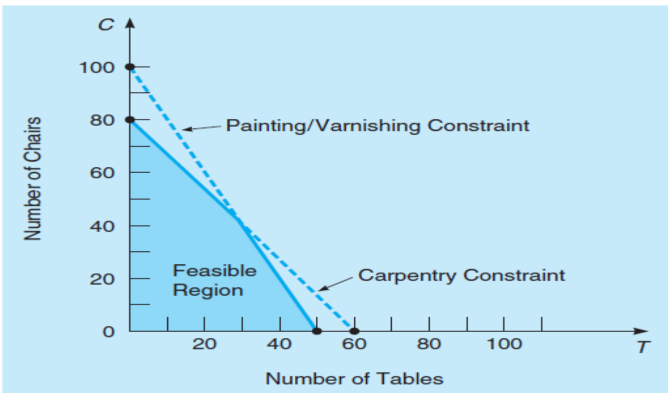
$4T + 3C \leq 240$ (carpentry constraint)

$2T + 1C \leq 100$ (painting and varnishing constraint)

$T, C \geq 0$ (nonnegativity constraints)

Solution: In Fig, we have drawn the feasible region of this problem.

FIGURE 7.5
Feasible Solution Region for the Flair Furniture Company Problem



Since the feasible region is bounded, the maximum value of z is attained at one of the extreme points. For this example, the coordinates of three of the corner points are obvious from observing the graph. These are (0, 0), (50, 0) and (0, 80). The fourth corner point is where the two constraint lines intersect and the coordinates must be found algebraically by solving the two equations simultaneously for two variables.

Therefore solving the equations $4T + 3C = 240$

$$2T + C = 100$$

We get $T = 30$ and $C = 40$ so the intersection point is (30, 40).

The values of objective function at four extreme points are given in the following table:

Extreme Points (T, C)	(0, 0)	(50, 0)	(30, 40)	(0,80)
$z = 70T + 50C$	0	3500	4100	4000

From the Table, the maximum value of z is 4100 which is attained at $T = 30$ & $C = 40$.

CHAPTER-8 LINEAR PROGRAMMING APPLICATIONS

The graphical method of linear programming (LP) discussed in Chapter 7 is useful for understanding how to formulate and solve small LP problems. The purpose of this chapter is to show how a large no of real-life problems can be modeled using LP. We do this by presenting examples of models in the areas of marketing research, media selsection, production mix, labor scheduling, production scheduling etc. We will solve many of these LP problems using Excel's Solver and QM for windows.

MARKETING APPLICATIONS

Media Selection: Linear programming models have been used in the advertising field as a decision aid in selecting an effective media mix. Sometimes the technique is employed in allocating a fixed or limited budget across various media, which might include radio or television commercials, newspaper ads, direct mailings, magazine ads and so on. In other applications, the objective is the maximization of audience exposure. Restrictions on the allowable media mix might arise through contract requirements, limited media availability or company policy. An example follows:

The Win Big Gambling Club promotes gambling junkets from a large Midwestern city to casinos in the Bahamas. The club has budgeted up to $8,000 per week for local advertising. The money is to be allocated among four promotional media: TV spots, newspaper ads, and two types of radio advertisements. Win Big's goal is to reach the largest possible high-potential audience through the various media. The following table presents the number of potential gamblers reached by making use of an advertisement in each of the four media. It also provides the cost per advertisement placed and the maximum number of ads that can be purchased per week.

Medium	Audience reached per Ad	Cost per Ad ($)	Maximum Ads per week
TV spot (1 minute)	5,000	800	12
Daily newspaper(full-page ad)	8,500	925	5
Radio spot (30 seconds, prime time)	2,400	290	25
Radio spot (1 minute, afternoon)	2,800	380	20

Win Big's contractual arrangements require that at least five radio spots be placed each week. To ensure a broad-scoped promotional campaign, management also insists that no more than $1,800 be spent on radio advertising every week.

Formulation:

Let X_1 = number of 1-minute TV spots each week
X_2 = number of daily paper ads each week
X_3 = number of 30-second radio spots each week
X_4 = number of 1-minute radio spots each week

Objective :

Maximize audience coverage = $5{,}000X_1 + 8{,}500X_2 + 2{,}400X_3 + 2{,}800X_4$

Subject to

X_1	≤ 12	(max TV spots/wk)
X_2	≤ 5	(max newspaper ads/wk)
X_3	≤ 25	(max 30-sec radio spots /wk)
X_4	≤ 20	(max 1-minute radio spots/wk)
$800X_1 + 925X_2 + 290X_3 + 380X_4$	$\leq \$8{,}000$	(weekly advertising budget)
$X_3 + X_4$	≥ 5	(min radio spots contracted)
$290X_3 + 380X_4$	$\leq \$1{,}800$	(max dollars spent on radio)
X_1, X_2, X_3, X_4	≥ 0	

The Transportation Problem

The transportation problem deals with the distribution of goods from several points of supply (origins or sources) to a number of points of demand (destinations). Usually we are given a capacity (supply) of goods at each source, a requirement (demand) for goods at each destination and the shipping cost per unit from each source to each destination.

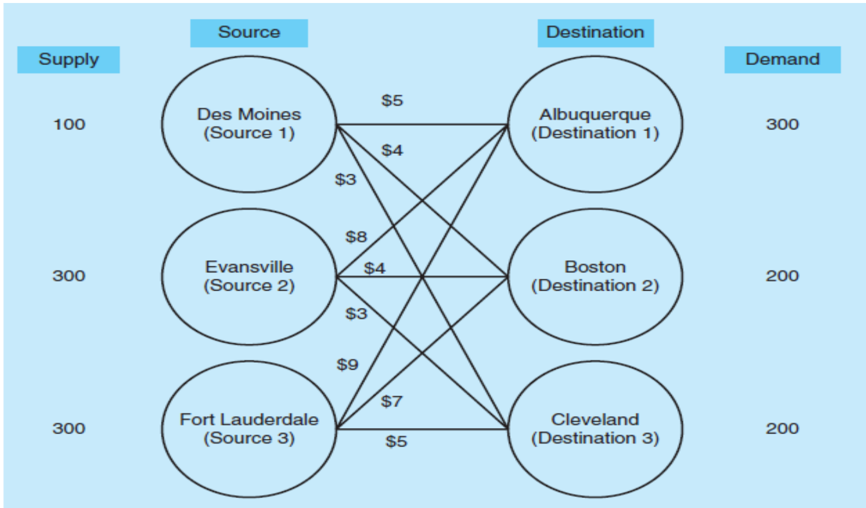
An example is shown in Figure 9.1. The objective of such a problem is to schedule shipments so that total transportation costs are minimized. At times, production costs are included also.

Transportation models can also be used when a firm is trying to decide where to locate a new facility. Before opening a new warehouse, factory or sales office, it is good practice to consider a number of alternative sites. Good financial decisions concerning the facility location also attempt to minimize total transportation and production costs for the entire system.

Linear Program for the Transportation Example

The Executive Furniture Corporation is faced with the transportation problem shown in Figure 9.1.

Figure 9.1Network Representation of a Transportation Problem, with Costs, Demands and Supplies



The company would like to minimize the transportation costs while meeting the demand at each destination and not exceeding the supply at each source. In formulating this as a linear program, there are three supply constraints (one for each source) and three demand constraints (one for each destination). The decision to be made are the number of units to ship on each route, so there is one decision variable for each arc (arrow) in the network.

Let X_{ij} =number of units shipped from source i to destination j

Where i =1, 2, 3, with 1 = Des Moines, 2 = Evansville and 3 = Fort Lauderdale

j = 1, 2, 3, with 1 = Albuquerque, 2 = Boston and 3 = Cleveland

Minimize total cost = $5X_{11} + 4X_{12} + 3X_{13} + 8X_{21} + 4X_{22} + 3X_{23} + 9X_{31} + 7X_{32} + 5X_{33}$

Subject to:

$X_{11} + X_{12} + X_{13} \leq 100$ (Des Moines supply)

$X_{21} + X_{22} + X_{23} \leq 300$ (Evansville supply)

$X_{31} + X_{32} + X_{33} \leq 300$ (Fort Lauderdale supply)

$X_{11} + X_{21} + X_{31} = 300$ (Albuquerque demand)

$X_{12} + X_{22} + X_{32} = 200$ (Boston demand)

$X_{13} + X_{23} + X_{33} = 200$ (Cleveland demand)

$X_{ij} \geq 0$ for all i and j.

A General LP Model for Transportation Problems: In this example, there were 3 sources and 3 destinations. The LP had 3×3 = 9 variables and 3 +3 = 6 constraints. In general, for a transportation problem with m sources and n destinations, the number of variables is mn and the number of constraints is $m + n$. For example, if there are 5 (i.e. $m = 5$) constraints and 8 (i.e., $n = 8$) variables, the linear program would have 5(8) = 40 variables and 5 + 8 = 13 constraints. Let

x_{ij} = number of units shipped from source i to destination j.
c_{ij} = cost of one unit from source i to destination j.
s_i = supply at source i.
d_j = demand at destination j.

The linear programming model is

Minimize cost $= \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij} x_{ij}$

Subject to

$\sum_{j=1}^{n} x_{ij} \leq s_i \qquad i = 1, 2, \ldots\ldots, m$

$\sum_{i=1}^{m} x_{ij} = d_j \qquad j = 1, 2, \ldots\ldots, n$

$x_{ij} \geq 0 \qquad$ for all i and j

The Transportation Algorithm

This is an iterative procedure in which a solution to a transportation problem is found and evaluated using a special procedure to determine whether the solution is optimal. When the solution is optimal, the process stops. If it is not optimal, then a new solution is generated. This new solution is at least as good as the previous one and it is usually better. This new solution is then evaluated and if it is not optimal, another solution is generated. The process continues until the optimal solution is found.

We will illustrate this process using the Executive Furniture Corporation example shown in Figure 9.1. This is presented again in a special format in Table 9.2

TABLE 9.2 Transportation Table for Executive Furniture Corporation

FROM \ TO	WAREHOUSE AT ALBUQUERQUE	WAREHOUSE AT BOSTON	WAREHOUSE AT CLEVELAND	FACTORY CAPACITY
DES MOINES FACTORY	\$5	\$4	\$3	100
EVANSVILLE FACTORY	\$8	\$4	\$3	300
FORT LAUDERDALE FACTORY	\$9	\$7	\$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Des Moines capacity constraint

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

Cleveland warehouse demand

Total demand and total supply

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

We see in Table 9.2 that the total factory supply available is exactly equal to the total ware house demand. When this situation of equal demand and supply occurs (something that is rather unusual in real life), a balanced problem is said to exist. Later in this chapter we take a look at how to deal with unbalanced problems, namely, those in which destination requirements may be gteater than or less than origin capacities.

Developing an Initial Solution: Northwest Corner Rule

Once we have arranged the data in a tabular form, we must establish an initial feasible solution to the problem. One systematic procedure, known as the *northwest corner rule*, requires that we start in the upper left-hand cell (or northwest corner) of the table and allocate units to shipping routes as follows:

1. Exhaust the supply (factory capacity) of each row before moving down to the next row.
2. Exhaust the demand (warehouse) requirements of each column before moving to the right to the next column.
3. Check that all supply and demand requirements are met.

Table 9.3 Initial Solution to Executive Furniture Problem Using the Northwest Corner Method

FROM \ TO	ALBUQUERQUE (A)	BOSTON (B)	CLEVELAND (C)	FACTORY CAPACITY
DES MOINES (D)	100 | $5	| $4	| $3	100
EVANSVILLE (E)	200 | $8	100 | $4	| $3	300
FORT LAUDERDALE (F)	| $9	100 | $7	200 | $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Means that the firm is shipping 100 units along the Fort Lauderdale–Boston route

The cost of this shipping assignment:

ROUTE FROM	TO	UNITS SHIPPED	×	PER-UNIT COST ($)	=	TOTAL COST ($)
D	*A*	100		5		500
E	*A*	200		8		1,600
E	*B*	100		4		400
F	*B*	100		7		700
F	*C*	200		5		1,000
					Total	4,200

Example: The three blood banks in Franklin County are coordinated through a central office that facilitates blood delivery to four hospitals in the region. The cost to ship a standard container of blood from each bank to each hospital is shown in the table below. Also given are the biweekly number of containers available at each bank and the biweekly number of containers of blood needed at each hospital.

a) How many shipments should be made biweekly from each blood bank to each hospital so that total shipment costs are minimized? (Solve using north west corner method)

b) Formulate the Franklin County Blood Bank situation as a linear program problem.

FROM \ TO	HOSPITAL 1	HOSPITAL 2	HOSPITAL 3	HOSPITAL 4	SUPPLY
BANK 1	$8	$9	$11	$16	50
BANK 2	12	7	5	8	80
BANK 3	14	10	6	7	120
DEMAND	90	70	40	50	250

Solution: (a) Using North West Corner Rule,

FROM \ To	Hospital 1	Hospital 2	Hospital 3	Hospital 4	Supply
Bank 1	8 50	9	11	16	50
Bank 2	12 40	7 40	5	8	80
Bank 3	14	10 30	6 40	7 50	120
Demand	90	70	40	50	250

Transportation Cost = 50($8) + 40($12) + 40($7) + 30($10) + 40($6) + 50($7) = $2,050

b) Let B1H1, B1H2, B1H3, B1H4, B2H1, B2H2, B2H3, B2H4, B3H1, B3H2, B3H3, and B3H4 represent the containers of blood shipped from blood banks 1, 2, and 3 to hospitals 1, 2, 3, and 4 respectively.

Minimize cost: 8B1H1 + 9B1H2 +11B1H3 + 16B1H4 +12B2H1 + 7B2H2 + 5B2H3 + 8B2H4 + 14B3H1 + 10B3H2 + 6B3H3 + 7B3H4

subject to:

B1H1 + B1H2 + B1H3 + B1H4 $\leq$ 50
B2H1 + B2H2 + B2H3 + B2H4 $\leq$ 80
B3H1 + B3H2 + B3H3 + B3H4 $\leq$ 120
B1H1 + B2H1 + B3H1 = 90
B1H2 + B2H2 + B3H2 = 70
B1H3 + B2H3 + B3H3 = 40
B1H4 + B2H4 + B3H4 = 50
All variables $\geq$ 0

Example: The table below describes a transportation problem:

	To==>	D	E	F	Supply
From	A	2	5	2	40
	B	1	4	2	30
	C	4	3	2	30
Demand		20	30	50	100

(a) Check whether the problem is balance or not.
(b) Use the northwest corner method to get an initial solution.
(c) What is the cost of the initial solution?
(d) Formulate the transportation problem as a linear programming problem:

Solution: **(a)** Since total supply equal to total demand, problem is balanced.

b) Using North West Corner Rule,

	To==>	D	E	F	Supply
From	A	20 (cost 2)	20 (cost 5)	(cost 2)	40
	B	(cost 1)	10 (cost 4)	20 (cost 2)	30
	C	(cost 4)	(cost 3)	30 (cost 2)	30
Demand		20	30	50	100

c) Cost = 20×2+20×5+10×4+20×2+30×2=280

d) Let

X_{11} = number of units shipped from A to D
X_{12} = number of units shipped from A to E
X_{13} = number of units shipped from A to F
X_{21} = number of units shipped from B to D
X_{22} = number of units shipped from B to E
X_{23} = number of units shipped from B to F
X_{31} = number of units shipped from C to D
X_{32} = number of units shipped from C to E
X_{33} = number of units shipped from C to F

Minimize $2X_{11} + 5X_{12} + 2X_{13} + X_{21} + 4X_{22} + 2X_{23} + 4X_{31} + 3X_{32} + 2X_{33}$

Subject to:

$X_{11} + X_{12} + X_{13} \leq 40$
$X_{21} + X_{22} + X_{23} \leq 30$
$X_{31} + X_{32} + X_{33} \leq 30$
$X_{11} + X_{21} + X_{31} = 20$
$X_{12} + X_{22} + X_{32} = 30$
$X_{13} + X_{23} + X_{33} = 50$
All $X_{ij} \geq 0$

Unbalanced Transportation Problems

A situation occurring quite frequently in real-life problems is the case in which total demand **is** not equal to total supply. These *unbalanced problems* can be handled easily by introducing *dummy sources* or *dummy destinations.*

- If total supply is greater than total demand, a dummy destination (warehouse), with demand exactly equal to the surplus, is created.
- If total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand over supply.

In either case, shipping cost coefficients of zero are assigned to each dummy location or route as no goods will actually be shipped. Any units assigned to a dummy destination represent excess capacity. Any units assigned to a dummy source represent unmet demand.

Degeneracy in Transportation Problems

Degeneracy occurs when the number of occupied squares or routes in a transportation table solution is less than the number of rows plus the number of columns minus 1.

i.e., Number of occupied cells are less than $m + n - 1$

Such a situation may arise in the initial solution or in any subsequent solution. Degeneracy requires a special procedure to correct the problem since there are not enough occupied squares to trace a closed path for each unused route and it would be impossible to apply the stepping-stone method.

More Than One Optimal Solution

It is possible for a transportation problem to have multiple optimal solutions. This happens when one or more of the improvement indices is zero in the optimal solution. This means that it is possible to design alternative shipping routes with the same total shipping cost. The alternate optimal solution can be found by shipping the most to this unused square using a stepping-stone path. In the real world, alternate optimal solutions provide management with greater flexibility in selecting and using resources.

Unacceptable Or Prohibited Routes

At times there are transportation problems in which one of the sources is unable to ship to one or more of the destinations. The problem is said to have an *unacceptable* or *prohibited route.*

- In a minimization problem, such a prohibited route is assigned a very high cost to prevent this route from ever being used in the optimal solution.
- In a maximization problem, the very high cost used in minimization problems is given a negative sign, turning it into a very bad profit.

CHAPTER-6 INVENTORY CONTROL MODELS

INTRODUCTION

Inventory is one of the most expensive and important assets to many companies, representing as much as 50% of total invested capital. Managers have long recognized that good inventory control is crucial. On one hand, a firm can try to reduce costs by reducing on-hand inventory levels. On the other hand, customers bocome dissatisfied when frequent inventory outages, called stockouts, occur. Thus companies must make the balance between low and high inventory levels. As you would expect, cost minimization is the major in obtaining this delicate balance.

Inventory is any stored resource that is used to satisfy a current or future need. Raw materials, work-in-process, and finished goods are examples of inventory.

All organizations have some type of inventory planning and control system. A bank has methods to control its inventory of cash. A hospital has methods to control blood supplies and other important items.

Figure 6.1 illustrates the basic components of an inventory planning and control system. The *planning* phase is concerned primarily with what inventory is to be stocked and how it is to acquired (whether it is to be manufactured or purchased). This information is then used in *forecasting* demand for the inventory and in *controlling* inventory levels. The feedback loop in Figure 6.1 provides a way of revising the plan and forecast based on experiences and observation.

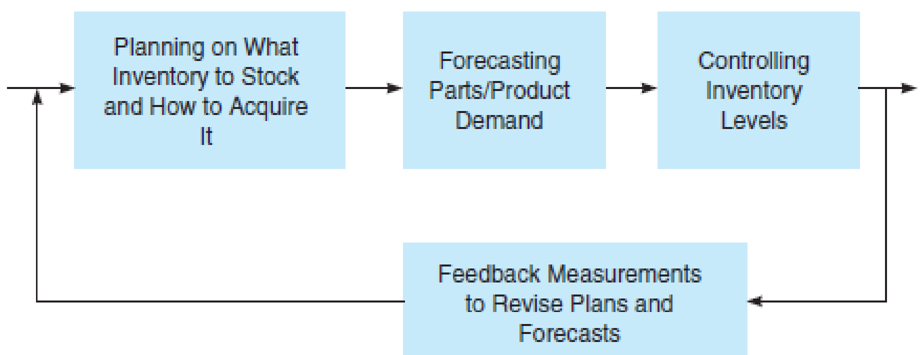


Figure 6.1 Inventory Planning and Control

Through inventory planning, an organization determines what goods and/or services are to be produced. In case of physical products, the organization must also determine whether to produce these goods or to purchase them from another manufacturer. When this has been determined, the next step is to forecast the demand. The emphasis in this chapter is on inventory control, that is, how to maintain adequate inventory levels within an organization.

IMPORTANCE OF INVENTORY CONTROL

Inventory control serves several important functions and adds a great deal of flexibility to the operation of the firm. Consider the following five uses of inventory:

1. The decoupling function
2. Storing resources
3. Irregular supply and demand
4. Quantity discounts
5. Avoiding stockouts and shortages

Decoupling Function

One of the major functions of inventory is to decouple manufacturing processes within the organization. If you did not store inventory, there could be many delays and inefficiencies. For example, when one manufacturing activity has to be completed before a second activity can be started, it could stop the entire process. If, however, you have some stored inventory between processes, it could act as a buffer.

Storing Resources

Agricultural and seafood products often have definite seasons over which they can be harvested or caught but the demand for these products is somewhat constant during the year. In these and similar cases, inventory can be used to store these resources.

In a manufacturing process, raw materials can be stored by themselves, in work-in-process, or in the finished product.

Irregular Supply and Demand

When the supply or demand for an inventory item is irregular, storing certain amounts in inventory can be important. If the greatest demand for Diet-Delight beverage is during the summer, you will have to make sure that there is enough supply to meet this irregular demand. This might require that you produce more of the soft drink in the winter than is actually needed to meet the winter demand. The inventory levels of Diet-Delight will gradually build up over the winter, but this inventory will be needed in the summer. The same is true for irregular *supplies*.

Quantity Discounts

Another use of inventory is to take advantage of **quantity discounts**. Many suppliers offer discounts for large orders. Purchasing in larger quantities can substantially reduce the cost of products. There are, however, some disadvantages of buying in larger quantities. You will have higher storage costs due to spoilage, damaged stock, theft, insurance and so on.

Avoiding Stockouts and Shortages

Another important function of inventory is to avoid shortages or stockouts. If you are repeatedly out of stock, customers are likely to go elsewhere to satisfy their needs. Lost goodwill can be an expansive price to pay for not having the right item at the right time.

INVENTORY DECISIONS

Even though there are literally millions of different types of products produced in our society, there are only two fundamental decisions that you have to make when controlling inventory:

1. How much to order
2. When to order

A major objective in controlling inventory is to minimize total inventory costs. Some of the most significant inventory costs follow:

1. Cost of the items (purchase cost or material cost)
2. Cost of ordering
3. Cost of carrying, or holding, inventory
4. Cost of stockouts

The most common factors associated with ordering cost and holding costs are shown in Table 6.1

TABLE 6.1 Inventory Cost Factors

ORDERING COST FACTORS	CARRYING COST FACTORS
Developing and sending purchase orders	Cost of capital
Processing and inspecting incoming inventory	Taxes
Bill paying	Insurance
Inventory inquiries	Spoilage
Utilities, phone bills, and so on for the purchasing department	Theft
Salaries and wages for purchasing department employees	Obsolescence
Supplies such as forms and paper for the purchasing department	Salaries and wages for warehouse employees
	Utilities and building costs for the warehouse
	Supplies such as forms and paper for the warehouse

Notice that the ordering costs are generally independent of the size of the order and many of these involve personnel time. An ordering cost is incurred each time an order is placed, whether the order is for 1 unit or 1,000units. The time to process the paperwork, pay the bill, and so forth does not depend on the number of units ordered.

On the other hand, the holding cost varies as the size of the inventory varies. If 1,000 units are placed into inventory, the taxes, insurance, cost of capital and other holding costs will be higher than if only 1 unit was put into inventory.

The cost of the items, or the purchase cost, is what is paid to acquire the inventory. The stockout cost indicates the lost sales and goodwill (future sales) that result from not having the items available for the customers.

ECONOMIC OREDER QUANTITY(EOQ): DETERMINING HOW MUCH TO ORDER

The **economic order quantity (EOQ)** is one of the oldest and most commonly known inventory control techniques. This technique is used by a large no. of organizations. It is relatively easy to use, but it does make a number of assumptions. Some of the most important assumptions follow:

1. Demand is known and constant.
2. Lead time is known and constant.
3. Receipt of inventory is instantaneous.
4. Purchase cost per unit is constant throughout the year.
5. The only variable costs are the cost of placing an order, *ordering cost*, and the cost of holding or storing inventory over time, *holding* or *carrying cost*, and these are constant throughout the year.
6. Orders are placed so that stockouts or shortages are avoided completely.

With these assumptions, inventory usage has a sawtooth shape, as in Fig 6.2. In Figure 6.2, Q represents the amount that is ordered. An inventory level increases from 0 to Q units when an order arrives.

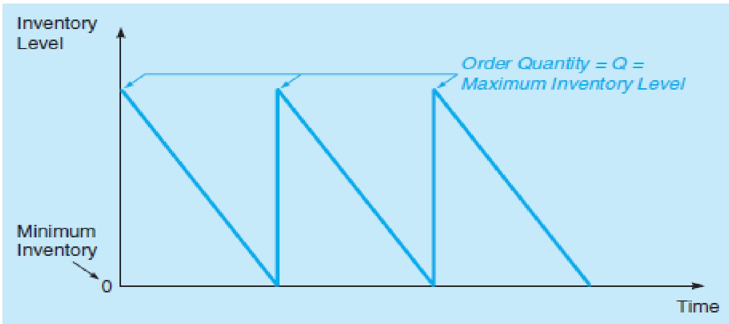


Fig 6.2 Inventory Usage over Time

Because demand is constant over time, inventory drops at a uniform rate over time. (Refer to the sloped line in Fig 6.2) Another order is placed such that when the inventory level reaches 0, the new order is received and the inventory level again jumps to Q units, represented by the vertical lines. This process continues indefinitely over time.

Inventory Costs in the EOQ Situation

The objective of most inventory models is to minimize the total costs. With the assumptions just given, the relevant costs are the ordering cost and the carrying (or holding cost). All other costs, such as the cost of the inventory itself (the purchase cost) are constant. Thus, if we minimize the sum of the ordering and carrying costs, we are also minimizing the total costs.

Using the following variables, we can develop mathematical expressions for the annual ordering and carrying costs:

Q = number of pieces to order
EOQ = Q^* = optimal number of pieces to order
D = annual demand in units for the inventory item
C_o = ordering cost of each order
C_h = holding or carrying cost per unit per year

Average Inventory Level $= \frac{Q}{2}$

Annual ordering cost = (Number of orders placed per year)×(Ordering cost per order)

$$= \frac{\text{Annual demand}}{\text{Number of units in each order}} \times (\text{Ordering cost per order})$$

$$= \frac{D}{Q} C_o$$

Annual holding or carrying cost = (Average inventory)×(Carrying cost per unit per year)

$$= \frac{\text{Order Quantity}}{2} \times (\text{Carrying cost per unit per year})$$

$$= \frac{Q}{2} C_h$$

The graph of the holding cost, the ordering cost and the total of these two is shown in Figure 6.3

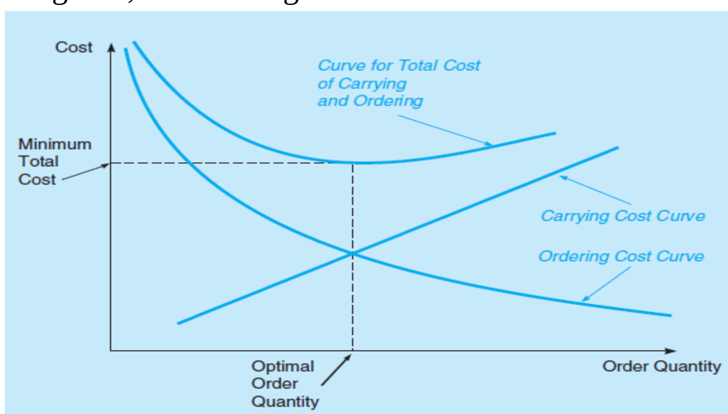


Figure 6.3 Total Cost as a function of Order Quantity

The lowest point on the total cost curves occurs where the ordering cost is equal to the carrying cost. Thus, to minimize total costs given this situation, the order quantity should occur where these two costs are equal.

Finding the EOQ

When the EOQ assumptions are met, total cost is minimized when:

Annual holding cost = Annual ordering cost

$$\frac{Q}{2} C_h = \frac{D}{Q} C_o$$

$$Q^2 C_h = 2 D C_o$$

$$Q = \sqrt{\frac{2 D C_o}{C_h}}$$

This optimal order quantity is often denoted by Q^* . Thus, the economic order quantity is given by the following formula:

$$\text{EOQ} = Q^* = \sqrt{\frac{2 D C_o}{C_h}}$$

Sumco Pump Company Example

Sumco, a company that sells pump housings to other manufacturers, would like to reduce its inventory cost by determining the optimal number of pump housings to obtain per order. The annual demand is 1,000 units, the ordering cost is $10 per order, and the average carrying cost per unit per year is $0.50. Using these figures, if the EOQ assumptions are met, we can calculate the optimal number of units per order:

$$Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

$$= \sqrt{\frac{2(1{,}000)(10)}{0.50}} = 200 \text{ units}$$

The relevant total annual inventory cost is the sum of the ordering costs and the carrying costs:

Total annual cost = Order cost + Holding cost

$$= \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

The total annual inventory cost for Sumco is computed as follows:

$$TC = \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

$$= \frac{1{,}000}{200}(10) + \frac{200}{2}(0.5) = \$100$$

The number of orders per year $= \frac{D}{Q} = \frac{1{,}000}{200} = 5$

And the average inventory $= \frac{Q}{2} = \frac{200}{2} = 100$

As you might expect, the ordering cost is equal to the carrying cost. You may wish to try different values for Q, such as 100 or 300 pumps. You will find that the minimum total cost occurs when Q is 200 units. The EOQ is 200 PUMPS.

REORDER POINT (ROP): DETERMINING WHEN TO ORDER

Once the order quantity is determined, the next decision is *when to order*. The time between placing an order and its receipt, called the *lead time* (L) or *delivery time*, is often a few days or even a few weeks.

When to order decision is usually expressed in terms of a *reorder point* (*ROP*), the inventory position at which an order should be placed. The ROP is given as

$$\text{ROP} = (\text{Demand per day}) \times (\text{Lead time for a new order in days})$$
$$= d \times L$$

Figure 6.4 has two graphs showing the ROP. One of these has a relatively small reorder point, while the other has a relatively large reorder point.

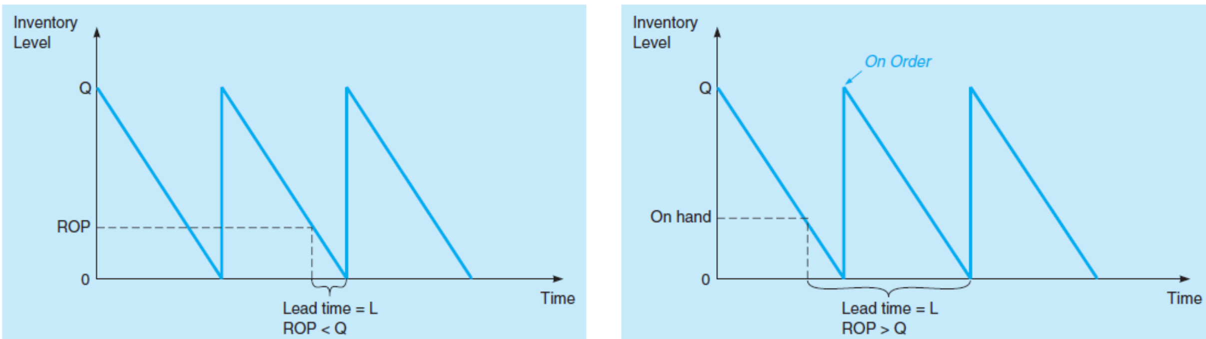


Figure 6.4 Reorder Point Graphs

When the inventory position reaches the ROP, a new order should be placed. While waiting for that order to arrive, the demand will be met with either inventory currently on hand or with inventory that already has been ordered but will arrive when the on-hand inventory falls to zero. Let's look at an example:

PROCOMP'S COMPUTER CHIP EXAMPLE: Procomp's demand for computer chips is 8,000 per year. The firm has a daily demand of 40 units and the order quantity is 400 units. Delivery of an order takes three working days. The reorder point for chips is calculated as follows:

$$\text{ROP} = d \times L = 40 \text{ units per day} \times 3 \text{ days} = 120 \text{ units}$$

Hence, when the inventory stock of chips drops to 120, an order should be placed. The order will arrive three days later, just as the firm's stock is depleted to 0. Since the order quantity is 400 units, the ROP is simply the on-hand inventory. This is the situation in the first graph in Fig .6.4

Suppose the lead time for Procomp Computer Chips was 12 days instead of 3 days. The reorder point would be:

$$\text{ROP} = d \times L = 40 \text{ units per day} \times 12 \text{ days} = 480 \text{ units}$$

Since the maximum on-hand inventory level is the order quantity of 400, an inventory position of 480 would be:

Inventory position = (Inventory on hand) + (Inventory on order)

$$480 = 80 + 400$$

Thus, a new order would have to be placed when the on-hand inventory fell to 80 while there was one other order in-transit. The second graph in Fig 6.4 illustrates this type of situation.

Example: The West Valve sells industrial valves and fluid control devices. One of the most popular valves is the Western, which has an annual demand of 4,000 units. The cost of each valve is $90, and the inventory carrying cost is 10% of the cost of each valve. The average ordering cost is $25 per order. Furthermore, it takes about two weeks for an order to arrive and during this time, the demand per week is approximately 80.

a) What is the EOQ?
b) What is the ROP?
c) What is the average inventory?
d) What is the annual holding cost?
e) How many orders is placed per year?
f) What is the annual ordering cost?

Solution: We are given

D = 4,000 units

$C_h = 10\%$ of $\$90 = \$9$

$C_0 = \$25$

a) $Q = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2(4000)(25)}{9}} = 149\ units$

b) ROP = (2 weeks) × (80 per week) = 160

c) Average inventory= Q/ 2 =149/2 =74.5 units

d) Total holding cost = $\left(\frac{Q}{2}\right) C_h$ =74.5(9) = $670.50 per year

e) Number of orders per year = D/Q= 4,000/149=26.85

f) Total ordering cost = $\left(\frac{D}{Q}\right) C_o$ =26.85($25) = $671.25 per year

Example: The F. W. Harris Company sells an industrial cleaner to a large number of manufacturing plants in the Houston area. An analysis of the demand and costs has resulted in a policy of ordering 300 units of this product every time an order is placed. The demand is constant, at 25 units per day. In an agreement with the supplier, F. W. Harris is willing to accept a lead-time of 20 days since the supplier has provided an excellent price. What is the reorder point? How many units are actually in inventory when an order should be placed?

Solution:

The reorder point is $ROP = d \times L = 25(20) = 500$ units

This means that an order should be placed when the inventory position is 500.

Since the ROP is greater than the order quantity, an order must have been placed already but not yet delivered. Therefore,

Inventory position = (inventory on hand) + (inventory on order)

Therefore, Inventory on hand = inventory position –inventory on order = 500-300=200.

There would be 200 units on hand and an order of 300 units in transit.

EOQ Without The Instantaneous Receipt Assumption

When a firm receives its inventory over a period of time, a new model is needed that does not require the **instantaneous inventory receipt** assumption. This new model is applicable when inventory continuously flows or builds up over a period of time after an order has been placed or when units are produced and sold simultaneously. Under these circumstances, the daily demand rate must be taken into account. Figure 6.5 shows inventory levels as a function of time. Because this model is especially suited to the production environment, it is commonly called the **production run model**.

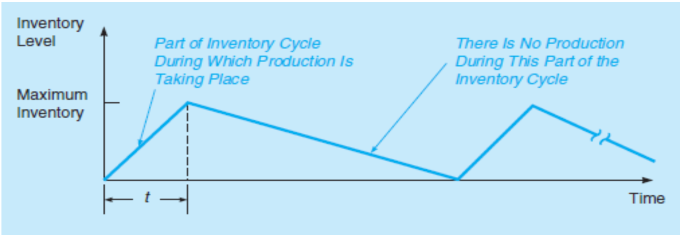


Figure 6.5 Inventory Control and the Production Process

In the production process, instead of having an ordering cost, there will be a *setup cost.* This is the cost of setting up the production facility to manufacture the desired product. It normally includes the salaries and wages of employees who are responsible for setting up the equipment, engineering and design costs of making the setup, paperwork, supplies, utilities and so on. The carrying cost per unit is composed of the same factors as the traditional EOQ model, although the annual carrying cost equation changes due to a change in average inventory.

The optimal production quantity can be derived by setting setup costs equal to holding or carrying costs and solving for the order quantity. Lets start by developing the expression for carrying cost.

Annual Carrying Cost for Production Run Model : We can develop the annual carrying cost or holding cost expression using the following variables:

Q = number of pieces per order, or production run
C_s = Setup cost
C_h = holding or carrying cost per unit per year
p = daily production rate
d = daily demand rate
t = length of production run in days

Annual holding or carrying cost = (Average inventory)×(Carrying cost per unit per year)

$$= \frac{Q}{2}\left(1 - \frac{d}{p}\right) C_h$$

Annual Setup Cost or Annual Ordering Cost

When a product is produced over time, setup cost replaces ordering cost. Both of these are independent of the size of the order and the size of the production run.

Annual setup cost = (number of production runs)× (setup cost)

$$= \frac{D}{Q} C_s$$

Determining the Optimal Production Quantity

When the assumptions of the production run model are met, costs are minimized when the setup cost equals the holding cost. Thus

Annual holding cost = Annual setup cost

$$\frac{Q}{2}\left(1-\frac{d}{p}\right)C_h = \frac{D}{Q}C_s$$

Solving for Q, we get the optimal production quantity:

$$Q^* = \sqrt{\frac{2DC_s}{C_h\left(1-\frac{d}{p}\right)}}$$

Example: Apple Inc. produces smartphones. Annual of smartphones is 4000 units and is constant throughout the year. Apple Inc. produces the smartphones in batches. On average, Apple Inc. can manufacture 100 smartphones per day. Demand for these smartphones during the production process is 20 per day. The setup cost for the equipment necessary to produce the smartphones is $20. Carrying costs are $1 per smartphones per year.

(a) How many smartphones should Apple Inc. manufacture in each batch?
(b) How many orders per year are needed with the optimal policy?
(c) What is the average inventory if costs are minimized?
(d) If lead time is 2 days, what should be the re-order point?
(e) What will the minimum total annual inventory cost?

Solution: The data for Apple Inc. are summarized as follows:

$$D = 4000\,units,\ C_s = \$20,\ C_h = \$1,\ p = 100\ \ and\ \ d = 20$$

This production run problem involves a daily production rate and a daily demand rate. The appropriate calculations are shown here:

(a). Optimal production quantity $Q^* = \sqrt{\dfrac{2DC_s}{C_h\left(1-\dfrac{d}{p}\right)}} = \sqrt{\dfrac{2*4000*20}{1\left(1-\dfrac{20}{100}\right)}} = 447.2\,units$

(b). Number of orders per year = $\dfrac{D}{Q} = \dfrac{4000}{447.2} = 8.95\ orders\ per\ year$

(c). Average inventory $= \dfrac{Q}{2}\left(1-\dfrac{d}{p}\right) = \dfrac{447.2}{2}\left(1-\dfrac{20}{100}\right) = 178.88\,Units$

(d). ROP= $d*L = 20*2 = 40\,units$

(e). Total annual cost $= \dfrac{Q}{2}\left(1-\dfrac{d}{p}\right)C_h + \dfrac{D}{Q}C_s = 178.88\times 1 + 8.95\times 20 = 357.88$

Quantity Discount Models

In developing the EOQ model, we assumed that quantity discounts were not available. However, many companies do offer quantity discounts. If such a discount is possible but all of the other EOQ assumptions are met, it is possible to find the quantity that minimizes the total inventory cost by using the EOQ model and making some adjustments.

When quantity discounts are available, the purchase cost or material cost becomes a relevant cost, as it changes based on the order quantity. The total relevant costs are as follows:

Total cost = Material cost + Ordering cost + Carrying cost

Total cost = $DC + \frac{D}{Q}C_o + \frac{Q}{2}C_h$

Where

D = annual demand in units
C_o = ordering cost of each order
C = cost per unit
C_h = holding or carrying cost per unit per year

Since holding cost per unit per year is based on the cost of the items, it is convenient to express this as

$$C_h = IC$$

Where I =holding cost as a percentage of the unit cost (C)

For a specific purchase cost (C), given the assumptions we have made, ordering the EOQ will minimize total inventory costs. However, in the discount situation, this quantity may not be large enough to qualify for the discount, so we must also consider ordering this minimum quantity for the discount. A typical quantity discount schedule is shown in Table 6.3

DISCOUNT NUMBER	DISCOUNT QUANTITY	DISCOUNT (%)	DISCOUNT COST ($)
1	0 to 999	0	5.00
2	1,000 to 1,999	4	4.80
3	2,000 and over	5	4.75

Use of Safety Stock

When the EOQ assumptions are met, it is possible to schedule orders to arrive so that stockouts are completely avoided. However, if the damand or the lead time is uncertain, the exact demand during the lead time will not be known with certainty. Therefore, to prevent stockouts, it is necessary to carry additional inventory called **safety stock**.

When demand is unusually high during the lead time, you dip into the safety stock instead of encountering a *stockout*. Thus, the main purpose of safety stock is to avoid stockouts when the demand is higher than expected. Its use is shown in Figure 6.7

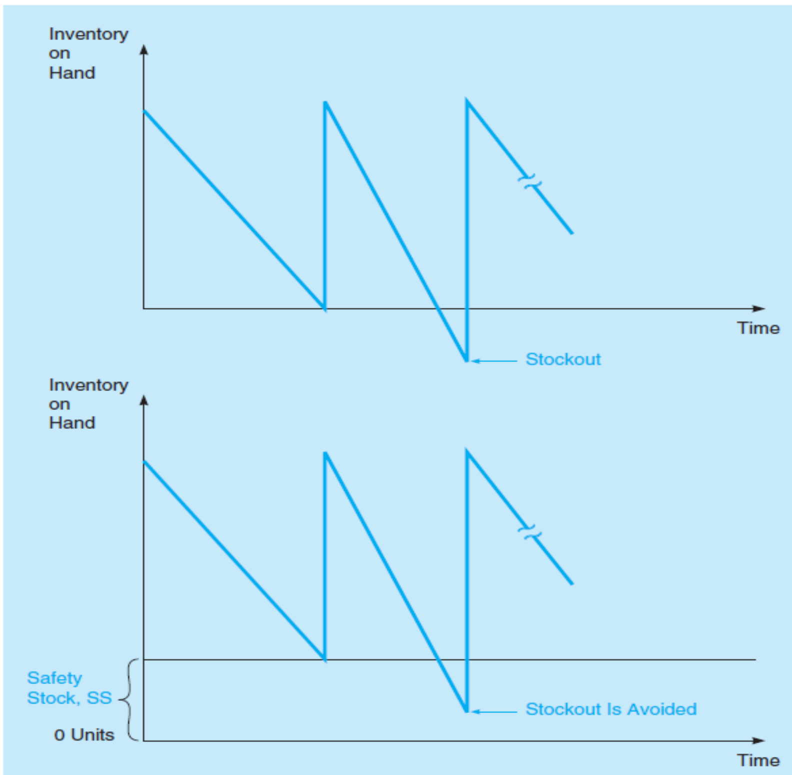


Figure 6.7 Use of Safety Stock

One of the best ways to implement a safety stock policy is to adjust the reorder point. The average inventory usage during the lead time should be computed and some safety stock should be added to this to avoid stockouts. The reorder point becomes

ROP = (Average demand during lead time) + (Safety stock)

ROP = (Average demand during lead time) + SS

Safety Stock with the Normal Distribution

When demand during the lead time is normally distributed, the reorder point becomes

$$ROP = (\text{Average demand during lead time}) + Z\sigma_{dLT}$$

Where Z = number of standard deviations for a given service level

σ_{dLT} = standard deviation of demand during the lead time

Thus, the amount of safety stock is simply $Z\sigma_{dLT}$. The following example looks at how to determine the appropriate safety stock level when demand during the lead time is normally distributed and the mean and standard deviation are known.

Hinsdale Company Example: The Hinsdale Company carries a variety of electronic inventory items and these are typically identified by SKU. One particular item, SKU A3378, has a demand that is normally distributed during the lead time, with a mean of 350 units and a standard deviation of 10. Hinsdale wants to follow a policy that results in stockouts occurring only 5% of the time on any order. How much safety stock should be maintained and what is the reorder point? Figure 6.8 helps visualize this example.

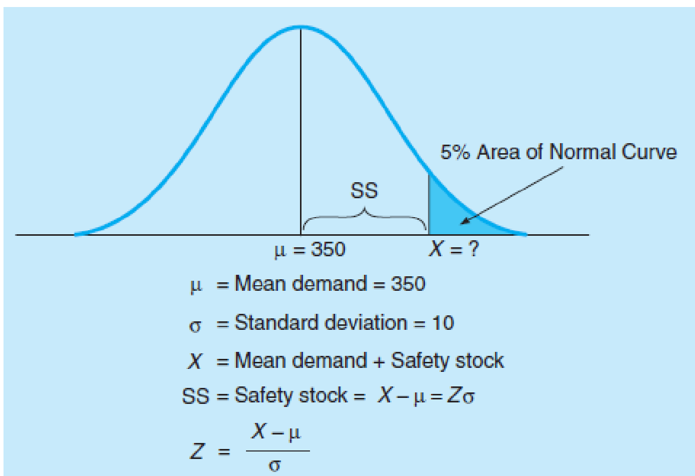


Fig 6.8 Safety Stock and Normal Distribution

From the normal distribution table (Appendix A) we have Z = 1.65:

$$ROP = (\text{Average demand during lead time}) + Z\sigma_{dLT}$$
$$= 350 + 1.65(10)$$
$$= 350 + 16.5 = 366.5 \text{ units}$$

So the reorder point is 366.5 and the safety stock is 16.5 units.

Annual Holding Cost with Safety Stock

Total annual holding cost = holding cost of regular inventory + holding cost of safety stock

$$THC = \frac{Q}{2}C_h + (SS)C_h$$

Where THC = total annual holding cost
Q = order quantity
C_h = holding cost per unit per year
SS = safety stock

Example: A Computer Company sells a desktop computer that is popular among gaming enthusiasts. In the past few months, demand has been relatively consistent, although it does fluctuate from day to day. The company orders the computer cases from a supplier. It places an order for 5,000 cases at the appropriate time to avoid stockouts. The demand during lead-time is normally distributed with a mean of 1,000 units and a standard deviation of 200 units. The holding cost per unit per year is estimated to be $4.

a) How much safety stock should the company carry to maintain a 96% service level? (Using the table for normal distribution, the Z value for a 96% service level is about 1.75)
b) What is the reorder point?
c) What would the total annual holding cost be if this policy is followed?

Solution: Given that Z = 1.75 , $\sigma = 200$, Q = 5000, $C_h = 4$
The safety stock is calculated as $SS = z\sigma = 1.75(200) = 350\ units$

a) For a normal distribution with a mean of 1,000, the reorder point is

ROP = (Average demand during lead time) + SS = 1000+350=1,350 units

b) The total annual holding cost is

$$THC = \frac{Q}{2}C_h + (SS)C_h$$
$$= \frac{5000}{2}4 + (350)4 = \$11{,}400$$

Single-Period Inventory Models

So far, we have considered inventory decisions in which demand continues in the future and future orders will be placed for the same product. There are some products for which a decision to meet the demand for a single time period is made and items that do not sell during this time period are of no value or have a greatly reduced value in the future. For example, a daily news-paper is worthless after the next paper is available. Other examples include weekly magazines, certain prepared foods that have a short life and some seasonal clothes that have greatly reduced value at the end of the season. This type of problem is often called the news vendor problem or a single-period inventory model.

A decision-making approach using marginal profit and marginal loss is called **marginal analysis**. **Marginal profit (MP)** is the additional profit achieved if one additional unit is stocked and sold. **Marginal loss (ML)** is the loss that occurs when an additional unit is stocked but cannot be sold.

When there are a manageable no of alternatives and states of nature and we know the probabilities for each state of nature, marginal analysis with discrete distributions can be used. When there are a very large no of possible alternatives and states of nature and the probability distribution of the states of nature can be described with a normal distribution, marginal analysis with the normal distribution are appropriate.

Steps of Marginal Analysis with Discrete Distributions

1. Determine the value of $\frac{ML}{ML+MP}$ for the problem.
2. Construct a probability table and add a cumulative probability column.
3. Keep ordering inventory as long as the probability (P) of selling at least one additional unit is greater than $\frac{ML}{ML+MP}$.

Café du Donut Example

Café du Donut is a popular New Orleans dining spot on the edge of the French Quarter. Its specialty is coffee and doughnuts; it buys the doughnuts fresh daily from a large industrial bakery. The café pays $4 for each carton (containing two dozen doughnuts) delivered each morning. Any cartons not sold at the end of the day are thrown away, for they would not be fresh enough to meet the café's standards. If a carton of doughnuts is sold, the total revenue is $6. Hence, the marginal profit per carton of doughnuts is

$$\text{MP} = \text{Marginal profit} = \$6 - \$4 = \$2$$

The marginal loss is ML = $4 since the doughnuts cannot be returned or salvaged at day's end.

From past sales, the café's manager estimates that the daily demand will follow the probabilty distribution shown in Table 6.6

DAILY SALES (CARTONS OF DOUGHNUTS)	PROBABILITY (P) THAT DEMAND WILL BE AT THIS LEVEL
4	0.05
5	0.15
6	0.15
7	0.20
8	0.25
9	0.10
10	0.10
	Total 1.00

Table 6.6 Café du Donut's Probability Distribution

The manager then follows three steps to find the optimal number of cartons of doughnuts to order each day:

Step 1. Determine the value of $\frac{\text{ML}}{\text{ML} + \text{MP}}$ for the decision rule

$$P \geq \frac{\text{ML}}{\text{ML} + \text{MP}} = \frac{\$4}{\$4 + \$2} = \frac{4}{6} = 0.67$$

$$P \geq 0.67$$

So the inventory stocking decision rule is to stock an additional unit if $P \geq 0.67$.

Step 2. Add a new column to the table to reflect the probability that doughnut sales will be at each level or greater. This is shown in the right-hand column of Table 6.7. For example, the probability that demand will be 4 cartons or greater is $1.00\,(= 0.05 + 0.15 + 0.15 + 0.20 + 0.25 + 0.10 + 0.10)$. Similarly, the probability that sales will be 8 cartons or greater is $0.45\,(= 0.25 + 0.10 + 0.10)$: namely, the sum of probabilities for sales of 8, 9, or 10 cartons.

Step 3. Keep ordering additional cartons as long as the probability of selling at least one additional carton is greater than P, which is the indifference probability. If Café du Donut orders 6 cartons, marginal profits will still be greater than marginal loss since

$$P \text{ at 6 cartons} = 0.80 > 0.67$$

Dependent Demand: The Case for Material Requirements Planning

In all the inventory models discussed earlier, we assume that the demand for one item is independent of the demand for other items. For example, the demand for refrigerators is usually independent of the demand for toaster ovens. Many inventory problems, however, are interrelated; the demand for one item is dependent on the demand for another item. Consider a manufacturer of small power lawn mowers. The demand for lawn mower wheels and spark plugs is dependent on the demand for lawn mowers. Four wheels and one spark plug are needed for each finished lawn mower. Usually when the demand for different items is dependent, the relationship between the items is known and constant. Thus, you should forecast the demand for the final products and compute the requirements for component parts.

In these situations, *Material Requirements Planning (MRP)* can be employed effectively.

Although most MRP systems are computerized, the analysis is straightforward and similar from one computerized system to the next. Here is the typical procedure:

Material Structure Tree

We begin by developing a **bill of materials (BOM)**. The BOM identifies the components, their descriptions, and the no required in the production of one unit of the final product. From the BOM, we develop a material structure tree. Let's say that demand for product A is 50 units. Each unit of A requires 2 units of B and 3 units of C. Now, each unit of B requires 2 units of D and 3 units of E. Furthermore, each unit of C requires 1 unit of E and 2 units of F. Thus, the demand for B, C, D, E, and F is completely dependent on the demand for A. Given this information, a material structure tree can be developed for the related inventory items (see Fig 6.12)

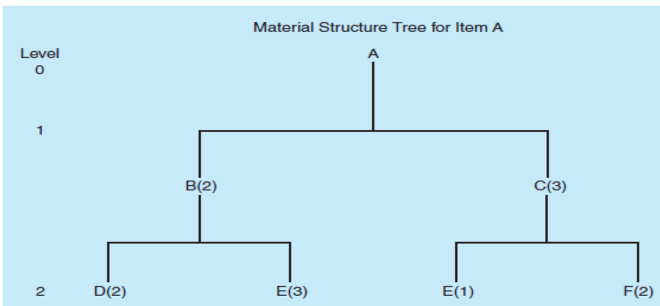


Fig 6.12 Material Structure Tree for item A

The structure tree has three levels: 0, 1, and 2. Items above any level are called *parents*, and items below any level are called *components*. There are three parents: A, B, and C. Each parent item has at least one level below it. Items B, C, D, E, and F are components because each item has at least one level above it. In this structure tree, B and C are both parents and components.

Note that the number in the parentheses in Figure 6.12 indicates how many units of that particular item are needed to make the item immediately above it. Thus, B (2) means that it takes 2 units of B for every unit of A, and F (2) means that it takes 2 units of F for every unit of C.

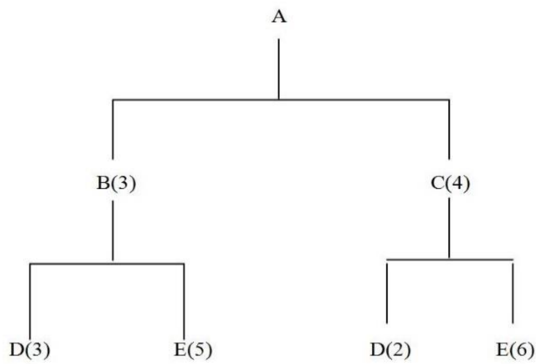
After the material structure tree has been developed, the number of units of each item required to satisfy demand can be determined.

Part B: 2 × number of A's = 2 × 50 = 100.
Part C: 3 × number of A's = 3 × 50 = 150.
Part D: 2 × number of B's = 2× 100 = 200.
Part E: 3 × number of B's + 1 × number of C's = 3 × 100 + 1× 150 = 450.
Part F: 2 × number of C's = 2 × 150 = 300.

Thus, for 50 units of A, we need 100 units of B, 150 units of C, 200 units of D, 450 units of E, and 300 units of F. Of course, the numbers in this table could have been determined directly from the material structure tree by multiplying the numbers along the branches times the demand for A, which is 50 units for this problem.

For example, the number of units of D needed is simply $2 \times 2 \times 50 = 200$ units.

Example: Consider the material structure tree for item A below. Assume 15 units of A are needed.



(a) How many units of B are needed?
(b) How many units of C are needed?
(c) How many units of D are needed?
(d) How many units of E are needed?

Solution:

(a) The no of units B needed$=3 \times 15 = 45$.
(b) The no of units C needed$=4 \times 15 = 60$.
(c) The no of units D needed$=3 \times 3 \times 15 + 2 \times 4 \times 15 = 255$.
(d) The no of units E needed$=5 \times 3 \times 15 + 6 \times 4 \times 15 = 585$.