CHAPTER 1: INTRODUCTION TO QUANTITATIVE ANALYSIS

Quantitative Analysis (definition):

Scientific approach to managerial decision making in which raw data are processed and manipulated to produce meaningful information

Raw Data → Quantitative Analysis → Meaningful Information

Differentiate:

Quantitative Factors	Qualitative Factors
Data that can be accurately calculated (e.g. demand)	More difficult to quantify but affect the decision process (e.g. weather)

The Quantitative Analysis Approach:

Seq.	Step	General Info
1	Define the problem	Most difficult step It is essential to go beyond the symptoms and identify the root causes Specific and measurable objectives may have to be developed
2	Develop a model	**Types of models**: Scale & Schematic **Models usually contain variables:** • **Controllable & uncontrollable ones**: decision variables and are generally unknown • **Parameters**: known quantities that are a part of the model
3	Obtain input data	**GIGO Rule: (Input Data)** Garbage In → **Process** → Garbage Out
4	Develop a solution	**Common Techniques when developing a model:** • **Solving** equations • **Trial and error –** trying various approaches • **Complete enumeration –** trying all possible values • **Using an algorithm –** series of repeating steps to reach a solution
5	Test it	Results should be logical, consistent and represent the real situation
6	Analyze the result	**Sensitivity Analysis (definition):** Determines how much the results will change if the model or input data changes
7	Implement it	• It can be very difficult • People may be resistant to change • May efforts have failed because a good solution wasn't implemented properly

Calculating Profit:

Profit = Revenue – Expenses
= Revenue – (Fixed cost + Variable cost)
= sX – (f+vX)

s = selling price per unit
X = number of units sold
f = fixed cost
v = variable cost

E.g. the company buys, sells and repairs old clocks. Rebuilt springs sell for $10 per unit. Fixt cost of equipment to build springs is $1,000. Variable cost for spring material is $5 per unit.

s = 10 f = 1,000 v = 5 Number of spring sets sold = X

Calculating Break-Even Point (definition): the number of units sold that will result in $0 profit.

$$\text{BEP} = \frac{\text{Fixed Cost}}{(Selling\ price\ per\ unit) - (Variable\ cost\ per\ unit)} = \frac{f}{s-v} = \frac{1,000}{10-5} = 200\ units$$

Advantages of Mathematical Modeling:

1. Can accurately represent reality
2. Can help the decision maker formulate problems
3. Can give us insight and information
4. Can save time and money
5. May be the only way to solve complex problems in timely fashion
6. Can be used to communicate problems and solutions to others

Models Categorized by Risk:

- **Deterministic**: does not involve risk
- **Probabilistic**: involve risk, chance, or uncertainty

CHAPTER 3: DECISION ANALYSIS

List the 6 steps of the decision-making process.

1. Define the problem.
2. List alternatives.
3. Identify possible outcomes.
4. List profit of each alternatives and outcomes.
5. Select one of the mathematical decision theory models.
6. Apply the model and make your decision.

Types of decision-making environments:

1. **Under certainty**

2. **Under uncertainty**
   - Maximax (optimistic) > goes with the best possible payoff
   - Maximin (pessimistic) > goes with the best of the worst payoff
   - Certain of realism
   - Equally likely
   - Minimax regret > goes with the minimum of all maximum regrets

3. **Under risk**
   - The most popular method is to choose the alternative with the highest Expected Monetary Value (EMV).

     EMV = (payment of 1st outcome) x (probability of it)
     + (payment of 2nd outcome) x (probability of it)
     + (payment of 3rd outcome) x (probability of it)
     …. etc
   - Expected Value of Perfect Information (EVPI)

     Maximum amount you would pay for additional information
     EVPI = EVwPI – EVwoPI
     EVwPI = highest number in each outcome (state of nature) x probability
     EVwoPI = maximum EMV
   - Expected Value of Sample Information (EVSI)

     EVSI = EVwSI – EVwoSI

Decision Trees

Calculating Posterior Probabilities

CHAPTER 5: FORECASTING

Common Qualitative Techniques:

- **Delphi method:** respondents provide input to decision makers
- **Jury of executive opinion:** collects opinions of small group of high-level managers
- **Sales force composite**: allows individual salespersons to estimate the sales in their region
- **Consumer market surveys:** input asked from customers or potential ones regarding their purchasing plans

Measures of Forecast Accuracy:

- **Forecast Error**
  = actual value – forecast value
- **Mean Absolute Deviation (MAD):**

  $$= \frac{\text{Sum of Forecast Errors}}{n}$$

- **Mean Squared Error (MSE):**

  $$= \frac{\text{Sum of } (forecast\ errors)^2}{n}$$

- **Mean Absolute Percent Error (MAPE):**

  $$= \frac{\text{Sum of } (\text{Errors} \div Actual)}{n}\ x\ 100\%$$

- **Bias (average error):**
  Not preferable

Moving Averages Calculation:

$$= \frac{\text{Sum of Demands in a specific period}}{n}$$

Weighted Moving Averages Calculation:

$$= \frac{\text{Sum of (weight in period i) } x\ (actual\ value\ in\ period)}{Sum\ of\ Weights}$$

Exponential Smoothing:

Type of moving average that is easy to use and requires little record keeping of data

($0 \leq \alpha \leq 1$)

CHAPTER 4: REGRESSION MODELS

Regression Analysis:

- Very valuable tool for managers
- **Can be used to:**
  1. Understand the relationship between variables
  2. Predict the value of one variable based on another variable

Purpose

- the variable to be predicted is called **Dependent Variable** (aka. Response variable)
- the value of this variable depends on the value of the independent variable

Scatter Diagram (or plot):

- Used to investigate the relationship between variables
- Independent variable is normally plotted on the X axis
- And the dependent on the Y axis

Regression Models:

- Used to test if there is a relationship between variables
- There is some random error that cannot be predicted (Error = actual value – predicted value)

VERY IMPORTANT FORMULA:

Used to calculate the values of the intercept and slope in a line regression model

$$b_0 = \overline{Y} - b_1 \overline{X}$$

Three measures of variability:

- **SST:** total variability about the mean $SST = \sum (Y - \overline{Y})^2$
- **SSE:** variability about the regression line $SSE = \sum e^2 = \sum (Y - \hat{Y})^2$
- **SSR:** total variability that is explained by the model $SSR = \sum (\hat{Y} - \overline{Y})^2$

SST = SSR + SSE

Coefficient of Determination (r^2):

Proportion of the variability in Y explained by the regression equation

$$r^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

Correlation of Coefficient:

An expression of the strength of the linear relationship

It is always between +1 and -1

Correlation coefficient is r

$$r = \sqrt{r^2}$$

Errors characteristics:

- Independent
- Normally distributed
- Have a mean of zero
- Have a constant variance

Estimating the Variance:

- Errors are assumed to have constant variance (σ^2) but we usually don't know this
- It can be estimated using the mean squared error (MSE), s^2
- $s^2 = MSC = \frac{SSE}{n-k-1}$
- $n = number\ of\ observations\ in\ the\ sample\ \&\ k = number\ of\ independent\ variables$

Testing the model for significance:

- Helps determine if the values are meaningful
- We do this by performing a statistical hypothesis test

Steps in hypothesis test:

1. Specify null and alternative hypotheses
2. Select level of significance
3. Calculate the value of the test statistic using the formula
   F = $\frac{MSR}{MSE}$
4. Make a decision

Multiple regression models:

Extensions to the simple linear model and allow the creation of models with more than one independent variable

Below videos explain it:

https://www.youtube.com/watch?v=zPG4NjIkCjc

https://www.youtube.com/watch?v=JvS2triCgOY

CHAPTER 13: WAITING LINES AND QUEUING THEORY MODELS

Queuing theory:

- Study of waiting lines.
- One of the oldest and most widely used quantitative analysis techniques
- Three basic components of a queuing process:
  - ✓ Arrivals
  - ✓ Service facilities
  - ✓ Actual waiting lines
- Analytical models can help managers evaluate the cost and effectiveness of service systems
- Most are focused on finding the ideal level of service a firm should provide
- In most cases, this service level is something management can control

Parts of a queuing system:

Part	Characteristics
Arrival or inputs to the system	**Size**: either unlimited or limited **Pattern**: could be known or random (known as Poisson distribution) **Behavior**: most assume customers are patient and will wait till they are served • **Balking** refers to customers who refuse to join the queue • **Reneging** customers enter the queue but become impatient and leave without receiving their service
Queuing or waiting line itself	Can be either **limited** or **unlimited** Queue discipline refers to the rule by which customers are being served: • FIFO; first in, first out • LIFO; last in, first out
Service facility	Basic queuing system configurations: • Service systems are classified in terms of number of channels, servers, and number or phases or stops ✓ **Single-channel system**: one server is quite common ✓ **Multi-channel system**: multiple servers are fed by one common waiting line ✓ **Single-phase system**: customer receives service from one server ✓ **Multi-phase system**: customer has to go thru more than one server

Characteristics of queuing system:

- Patterns can be either constant or random
- Constant service times are often machine controlled
- Service times are randomly distributed according to a **negative exponential probability distribution**
- Analysts should collect and plot service time data to ensure that the observations fit the assumed distributions when applying these models

Identifying models using Kendall Notation

Arrival Distribution / Service time distribution / number of service channels open

M = Poisson distribution for number of occurrences
D = constant rate
G = general distribution with known mean and variance

M/M/1 • Single channel model • With Poisson distribution	**M/M/2** • Second channel is added • With Poisson distribution
M/D/3 • Three channel system • With Poisson distribution • Constant service time	**M/G/4** • Four channel system • With Poisson distribution • Normally distributed service times

CHAPTER 7: LINEAR PROGRAMMING MODELS: GRAPHICAL AND COMPUTER METHODS

Linear Programming (LP):
Widely used mathematical modeling technique designed to help managers in planning and decision making relative to resource allocation

All Problems have 4 properties in common:
1. Seek to **maximize** or **minimize** some quantity (**Objective Function**)
2. **Constraints** that limit the degree to which we can pursue our objective are present
3. There must be alternative courses of action from which to choose
4. Objective and constraints in problems must be expressed in terms of linear equations or inequalities

Basic Assumptions of LP:
- We assume conditions of **certainty** exist
- We assume **proportionality** exist in the objectives and constrains
- We assume **additivity** in that the total of all activities equals the sum of the individual activities
- We assume **divisibility** in that solutions need not be whole numbers
- All answers or variables and **non-negative**

Steps in formulating a linear program:
1. Completely understand the managerial problem being faced
2. Define decision variables
3. Identify objectives and constraints
4. Use the decision variables to write mathematical expressions for objective functions and constraints

Slack (definition):
Amount of a resource that is not used. For a less-than-or-equal constraint
For a less-than-or-equal constraint

Surplus (definition):
Amount by which the right-hand side of the constraint is exceeded
Used with a greater-than-or-equal constraint
= Actual amount – minimum amount

CHAPTER 9: TRANSPORTATION AND ASSIGNMENT MODELS

Transportation Poblem:

- Deals with the distribution of goods from several points of supply (sources) to a number of points of demand (destinations)
- Typically the objective is to minimize total transportation and production costs

Transshipment point:

- When the items are being moved from a source to a destination through an intermediate point
- The problem is called transshipment problem

Degeneracy:

Occurs when the number of occupied squares or routes in a transportation table solution is less than the number of rows plus the number of columns minus 1

It requires a special procedure to correct the problem since there are not enough occupied squares to trace a closed path for each unused route and it would be impossible to apply the stepping-stone method

Unacceptable or Prohibited Routes:

At times there are transportation problems in which one of the sources is unable to ship to one or more of the destinations

CHAPTER 6: INVENTORY CONTROL MODELS

Five uses of inventory:

- Decoupling function
  - Inventory is used as a buffer between stages in a manufacturing process
  - This reduces delays and improves efficiency
- Storing resources
  - Seasonal products may be stored to satisfy off-season demand
  - Materials can be stored as raw materials, work-in-process, or finished goods
  - Labor can be stored as a component of partially completed subassemblies
- Irregular supply and demand
  - Demand and supply may not be constant over time
  - Inventory can be used to buffer the variability
- Quantity discounts
  - Lower prices may be available for larger orders
  - Extra costs associated with holding more inventory must be balanced against lower purchase price
- Avoiding stockouts and shortages
  - Stockouts may result in lost sales
  - Dissatisfied customers may choose to buy from another supplier

Two Fundamental Decisions in Controlling Inventory:

- How much to order
- When to order

Major objective:

To minimize total inventory costs

Common inventory costs are:

- Cost of items
- Cost of ordering
- Cost of carrying or holding, inventory
- Cost of stockouts

Economic order quantity (EOQ) model:

- One of the oldest and most commonly known inventory control techniques
- Easy to use but has a number of assumptions
- Objective is to minimize total cost of inventory

EOQ Assumptions:

- Demand is known and constant
- Lead time is known and constant
- Receipt of inventory is instantaneous
- Purchase cost per unit is constant throughout the year
- Only variable costs are the cost of placing an order
- Orders are placed so that stockouts or shortages are avoided completely

Equations:

Annual ordering cost = $\frac{D}{Q}\,C_0$

Annual holding cost = $\frac{Q}{2}\,C_h$

EOQ = Q^* = $\sqrt{\frac{2DC_0}{C_h}}$

Total annual cost = order cost + holding cost

TC = $\frac{D}{Q}\,C_0 + \frac{Q}{2}\,C_h$

Sensitivity Analysis:

- EOQ model assumes all values are known and fixed over time
- However, values are estimated or may change
- So determining the effects of these changes is called sensitivity analysis

Reorder Point (ROP):

ROP = demand per day x lead time for a new order in days
= d x L

To prevent stockouts:

It is necessary to carry extra inventory called safety stock
which can be implemented by adjusting the ROP

Total annual holding cost = holding cost of regular inventory + holding cost of safety stock

THC = $\frac{Q}{2}\,C_h + (\boldsymbol{SS})\,C_h$

THC	= total annual holding cost
Q	= order quantity
C_h	= holding cost per unit per year
SS	= safety stock

CHAPTER 12: PROJECT MANAGEMENT

First step in planning and scheduling a project:
Develop the work breakdown structure

Program Evaluation and Review Technique (PERT) & Critical Path Method (CPM)
Two popular quantitative analysis techniques to help plan, schedule, monitor and control projects

Six steps of PERT/CPM:
1. Define the project and all of its significant activities or tasks
2. Develop the relationships among the activities and decide which activities must precede others
3. Draw the network connecting all of the activities
4. Assign time and/or cost estimates to each activity
5. Compute the longest time path through the network; called the **critical path**
6. Use the network to help plan, schedule, monitor and control the project

Critical Path:
Important since any delay in these activities can delay the completion of the project
Longest path through the network

There are two common techniques for drawing PERT networks:
- **Activity-on-node (AON):** nodes represent activities
  Easier and more commonly found in software packages
- **Activity-on-arc (AOA):** where arcs are used to represent the activities
  Used to show the predecessors for each activity

Time Estimates in PERT are:
- **Optimistic Time (a) =** time an activity will take if everything goes as well as possible
- **Pessimistic Time (b) =** time an activity would take assuming very unfavorable conditions
- **Most Likely Time (m) =** most realistic time estimate to complete the activity

Expected Activity Time (t):

$$t = \frac{a + 4m + b}{6}$$

Variance of Activity Completion Time:

$$\text{Variance} = \left[\frac{b - a}{6}\right]^2$$

How to find the critical path:
Need to determine the following quantities for each activity in the network..

Earliest Start Time	(ES)	Earliest time an activity can begin without violation of immediate predecessor requirements
Earliest Finish Time	(EF)	Earliest time at which an activity can end
Latest Start Time	(LS)	Latest time an activity can begin without delaying the entire project
Latest Finish Time	(LF)	Latest time an activity can end without delaying the entire project

Why PERT uses the variance of critical path activities:
To help determine the variance of the overall project
Project variance = $\sigma_t = \sqrt{variances\ of\ activities\ on\ the\ critical\ path}$

Probability of Project Completion

Z = $\frac{Due\ date - Expected\ date\ of\ completion}{\sigma_t}$

Predecessor Activity:

One that must be accomplished before the given activity can be started

Successor Activity:

One that can be started only after the given activity is finished

PERT/Cost:

Modification of PERT that allows a manager to plan, schedule, monitor, and control cost as well as time
Using this can help accomplish the sixth and final step of PERT

Four Basic Budgeting Steps:

1. Identify all costs associated with each of the activities then add these costs together to get one estimated budget for each activity
2. In large projects, activities can be combined into larger work packages. A work package is simply a logical collection of activities
3. Convert the budgeted cost per activity into cost per time period by assuming that the cost of completing any activity is spent at a uniform rate over time
4. Using the ES and LS times, find out how much money should be spent during each week or month to finish the project by the date desired

Monitoring and controlling Project Costs:

To ensure the project is progressing on schedule and that cost overruns are kept to a minimum

Value of work completed = (% of work complete) x (total activity budget)
Activity difference = actual cost – value of work completed
A negative activity difference is a cost underrun and a positive activity difference is a cost overrun

Crashing:

- Reducing a project's completion time
- Starts with using the normal time to create the critical path
  Normal Cost: cost for completing the activity using normal procedures

  - **Crash time:** shortest possible activity time and will require additional resources
  - **Crash cost:** price of completing the activity in the earlier-than-normal time

Crash cost/Time period = $\frac{crash\ cost - normal\ cost}{normal\ time - crash\ time}$

Milestones:

They are major events in a project
May be reflected in Gantt Charts and PERT charts – to highlight the importance of reaching these events

Resource Leveling:

Adjusts the activity start away from the early start so that resource utilization is more evenly distributed over time

CHAPTER 14: SIMULATION MODELING

Simulation:

One of the most widely used quantitative analysis tools

- To simulate is to try to duplicate the features, appearances and characteristic of a real system
- Does not give any optimal solution – only experiments

Using simulation, a manager should:

1. Define problem
2. Introduce important variables
3. Construct simulation model
4. Specify values to be tested
5. Conduct the simulation
6. Examine the results
7. Select best course of action

Main advantages of simulation:

- Relatively straightforward and flexible
- Recent advances in computer software make simulation models very easy to develop
- Can be used to analyze large and complex real-world situations
- Allow "what-if" questions
- Does not interfere with real world system
- Enable study of interactions between components
- Enable time compression
- Enables the inclusion of real world complications

Main Disadvantages are:

- Often expensive as it may require long, complicated process to develop the model
- Does not generate optimal solutions; it is a trial-and-error approach
- Requires managers to generate all conditions and constraints of real world problems
- Each model is unique and the solutions and inferences are not usually transferable to other problems

Monte Carlo Simulation:

This method is used when systems contain elements that exhibit chance in their behavior

e.g. inventory demand, service times, # of employees absent, etc

Basis of Monte Carlo simulation:

Experimentation on the probabilistic elements through random sampling

Steps of Monte Carlo Simulation:

1. Establish a probability distribution for important variables
2. Build a cumulative probability distribution for each variable
3. Establish an interval of random numbers for each variable
4. Generate random numbers
5. Actually simulate a series of trials

Simulation models are often broken into three categories:

- Monte Carlo method
- Operational gaming

  Refers to simulation involving two or more competing players
- Systems simulation
  - ✓ Similar in that allows users to test various managerial policies and decisions to evaluate their effect on the operating environment
  - ✓ Models dynamics of large systems

Verification:

Process involves determining that the computer model is internally consistent and following the logic of conceptual model

Validation:

Process of comparing a simulation model to the real system it represents to make sure it is accurate