Chapter 1 Introduction to Quantitative Analysis

1.1 Introduction

People have been using mathematical tools to help solve problems for thousands of years; however, the formal study and application of quantitative techniques to practical decision making is largely a product of the twentieth century.

Quantitative analysis can be applied to a wide variety of problems in business, government, health care, education and many other areas.

It's not enough, though, just to know the mathematics of how a particular quantitative technique works; you must also be familiar with the limitations, assumptions, and specific applicability of the technique.

Examples of Quantitative Analysis

- In the mid 1990s, Taco Bell saved over $150 million using forecasting and scheduling quantitative analysis models.
- NBC television increased advertising revenues by over $200 million between 1996 and 2000 by using quantitative analysis to develop better sales plans for advertisers.
- Continental Airlines saved over $40 million in 2001 using quantitative analysis models to quickly recover from weather delays and other disruptions.

1.2 What is Quantitative Analysis?

Quantitative analysis is a scientific approach to managerial decision-making in which raw data are processed and manipulated to produce meaningful information.

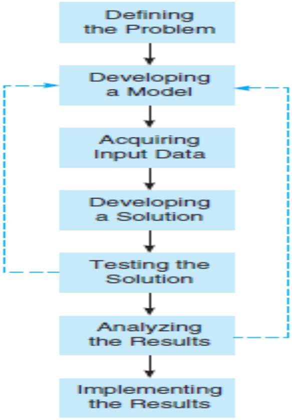
In solving a problem, managers must consider both qualitative and quantitative factors.

- ***Quantitative factors*** are data that can be accurately calculated. Examples include:
  - Different investment alternatives
  - Interest rates
  - Inventory levels
  - Demand
  - Labor cost
- ***Qualitative factors*** are more difficult to quantify but affect the decision process. Examples include:
  - The weather
  - State and federal legislation
  - Technological breakthroughs.

1.3 The Quantitative Analysis Approach

The quantitative analysis approach consists of defining a problem, developing a model, acquiring input data, developing a solution, testing the solution, analyzing the results and implementing the results. (see Figure 1.1)

Figure 1.1 Quantitative Analysis Approach



Defining the Problem

The first step in the quantitative approach is to develop a clear, concise statement of the problem. This statement will give direction and meaning to the following steps. In many cases, defining the problem is the most important and the most difficult step.

Developing a Model

Once we select the problem to be analyzed, the next step is to develop a model. Simply stated, a model is a representation (usually mathematical) of a situation. There are many types of models.

- **Physical Model:** Architects sometimes make a physical model of a building that they construct.
- **Scale Model:** Engineers develop scale models of chemical plants, called pilot plants.
- **Schematic Model:** A schematic model is a picture, drawing, or chart of reality. Automobiles, lawn mowers, gears, fans, typewriters and numerous other devices have schematic models (drawings and pictures) that reveal how these devices work.
- **Mathematical Model:** A mathematical model is a set of mathematical relationships. In most cases, these relationships are expressed in equations and inequalities.

What sets quantitative analysis apart from other techniques is that the models that are used in quantitative analysis are mathematical.

Although there is considerable flexibility in the development of models, most of the models contain one or more variables and parameters.

A **variable**, is a measurable quantity that may vary or is subject to change. Variables can be *controllable or uncontrollable*. A controllable variable is also called a *decision variable*. An example would be how many inventory items to order.

A **parameter**, is a measurable quantity that is inherent in the problem. The cost of placing an order for more inventory items is an example of a parameter.
In most cases, variables are unknown quantities while parameters are known quantities.

Acquiring Input Data

Once we have developed a model, we must obtain the data that are used in the model (*input data*). Obtaining accurate data for the model is essential; even if the model is a perfect representation of reality, improper data will result in misleading results. This situation is called *garbage in, garbage out* (GIGO rule).

There are number of sources that can be used in collecting data such as company reports, company documents, interviews, on-site direct measurement, or statistical sampling.

Developing a Solution

Developing a solution involves manipulating the model to arrive at the best (optimal) solution to the problem.

Common techniques are

- *Solving* equations—this requires that an equation be solved for the best decision.
- *Trial and error* – trying various approaches and picking the best result.
- *Complete enumeration* – trying all possible values for the variables in the model to arrive at the best decision.
- Using an *algorithm* – a series of repeating steps to reach a solution.

Testing the Solution

Before a solution can be analyzed and implemented, it needs to be tested completely. Because the solution depends on the input data and the model, both require testing.

Testing the input data and the model includes determining the accuracy and completeness of the data. One method of testing the data is to collect additional data from a different source. These additional data can then be compared with the original data and statistical tests can be employed to determine whether there are differences between the original data and the additional data. If there are significant differences, more effort is required to obtain accurate input data. If the data are accurate but the results are inconsistent with the problem, the model may not be appropriate. The model can be checked to make sure that it is logical and represents the real situation.

Analyzing the Results and Sensitivity Analysis

Analyzing the results starts with determining the implications of the solution. In most cases, a solution to a problem will result in some kind of action or change in the way an organization is operating. The implications of these actions or changes must be determined and analyzed before the results are implemented.

Because a model is only an approximation of reality, the sensitivity of the solution to changes in the model and input data is a very important part of analyzing the results. This type of analysis is called **Sensitivity Analysis** or *Postoptimality Analysis*. It determines how much the solution will change if there were changes in the model or the input data.

Implementing the Results

The final step is to implement the results. This is the process of incorporating the solution into the company. This can be much more difficult than you would imagine. Experience has shown that a large number of quantitative analysis teams have failed in their efforts because they have failed to implement a good, workable solution properly.

After the solution has been implemented, it should be closely monitored. Over time, there may be numerous changes that call for modifications of the original solution.

Modeling in the Real World

Quantitative analysis models are used extensively by real organizations to solve real problems.

- In the real world, quantitative analysis models can be complex, expensive, and difficult to sell.
- Following the steps in the process is an important component of success.

1.4 How To Develop a Quantitative Analysis Model

Developing a model is an important part of the quantitative analysis approach. Let's see how we can use the following mathematical model, which represents profit:

$$\text{Profit} = \text{Revenue} - \text{Expenses}$$

$$\text{Profit} = \text{Revenue} - (\text{Fixed cost} + \text{Variable cost})$$

$$\text{Profit} = \text{Selling price per unit} \times \text{Number of units sold}$$
$$- [\text{Fixed cost} + \text{Variable cost per unit} \times \text{Number of units sold}]$$

$$\text{Profit} = sX - [f + vX]$$

$$\text{Profit} = sX - f - vX \qquad \ldots\ldots\ldots.(1.1)$$

Where

s = selling price per unit
f = fixed cost
v = variable cost per unit
X = number of units sold

The parameters in this model are f, v and s, as these are inputs that are inherent in this model. The number of units sold (X) is the decision variable of interest.

Break Even Point (BEP)

Companies are often interested in the ***break-even point*** **(BEP)**. The BEP is the number of units sold that will result in 0 profit.

From (1.1), $0 = sX - f - vX$,

or $0 = (s - v)X - f$

or $f = (s - v)X$,

or $\boldsymbol{X = \frac{f}{s-v}}$

i.e., $\textbf{BEP} = \dfrac{\textbf{Fixed Cost}}{\textbf{Selling Price Per Unit} - \textbf{Variable Cost Per Unit}}$

EXAMPLE: PRITCHETT'S PRECIOUS TIME PIECES We will use Bill Pritchett clock repair shop example to demonstrate the use of mathematical models.

Bill's company, Pritchett's Precious Time Pieces buys, sells and repairs old clocks and clock parts. Bill sells rebuilt springs for a price per unit of \$10. The fixed cost of the equipment to build the springs is \$1,000. The variable cost per unit is \$5 for spring material.

In this example, we have $s = 10,\ f = 1{,}000\ \ \&\ v = 5$

The number of springs sold is X and our profit model becomes

$$\text{Profit} = \$10X - \$1{,}000 - \$5X$$

If sales = 0, profits = $10(0) - 1{,}000 - 5(0)$ = –\$1,000 i.e. Bill will realize a loss of \$ 1000.

If sales = 1,000, profits = $[(10)(1{,}000) - 1{,}000 - (5)(1{,}000)] = \$4{,}000$.

BEP for Pritchett's Precious Time Pieces

BEP = $1,000/($10 – $5) = 200 units

Sales of less than 200 units of rebuilt springs will result in a loss.

Sales of over 200 units of rebuilt springs will result in a profit.

Example: Sport Xpert company manufactures sport bikes. It assesses its fixed costs at 81,000 SAR for a year. Each mountain bike carries on average a variable cost of 300 SAR. Sport Xpert sells every bike at a price of 1,200 SAR.

a) Determine the number of bikes that company must sell to reach its break-even point.
b) Assuming that the company sells 180 sport bikes per year (regular activity over the year). What will be its benefit?

Solution:

a) We have $f = 81{,}000$ SAR, $v = 300$ SAR & $s = 1{,}200$ SAR

$$\text{BEP} = \frac{f}{s-v} = \frac{81{,}000}{1200-300} = 90 \text{ Bikes}$$

b) Bikes sold by company per year, $X = 180$.

Benefit = Revenue – Expenses

$$\begin{aligned} &= s.X - (f + v.X) \\ &= (1200)(180) - [81000 + (300)(180)] \\ &= 81{,}000 \text{ SAR} \end{aligned}$$

Example: A manufacturing company manufactures T-Shirts. The fixed cost for a year is 8100 SAR. Each T-Shirt carries on average a variable cost of 30 SAR and the selling price of 120 SAR.

a. Determine the number of T-Shirts that the company must sell to reach its break-even point.
b. What will be its profit if Company sells 120 T-Shirt per year?

Solution:

a) We have $f = 81{,}00$ SAR, $v = 30$ SAR & $s = 1{,}20$ SAR

$$BEP = \frac{f}{s-v} = \frac{81{,}00}{120-30} = 90 \text{ T-Shirts}$$

b) T-Shirts sold by company per year, $X = 120$.

Profit = Revenue – Expenses

$$\begin{aligned} &= s.X - (f + v.X) \\ &= (120)(120) - [8100 + (30)(120)] \\ &= 27{,}00 \text{ SAR} \end{aligned}$$

Advantages of Mathematical Modeling

1. Models can accurately represent reality.
2. Models can help a decision maker formulate problems.
3. Models can give us insight and information.
4. Models can save time and money in decision making and problem solving.
5. A model may be the only way to solve large or complex problems in a timely fashion.
6. A model can be used to communicate problems and solutions to others.

Mathematical Models Categorized by Risk

Some mathematical models, like the profit and break-even models previously discussed, do not involve risk or chance. We assume that we know all values used in the model with complete certainty. These are called **Deterministic Models**.

Other models involve risk or chance. For example, the market for a new product might be "good" with a chance of 60% (a probability of 0.6) or "not good" with a chance of 40% (a probability of 0.4). Models that involve chance or risk, are called **Probabilistic Models**.

1.6 Possible Problems in the Quantitative Analysis Approach

Defining the problem

- Problems may not be easily identified.
- There may be conflicting viewpoints
- There may be an impact on other departments.
- Beginning assumptions may lead to a particular conclusion.
- The solution may be outdated.

Developing a model

- Manager's perception may not fit a textbook model.
- There is a trade-off between complexity and ease of understanding.

Acquiring accurate input data

- Accounting data may not be collected for quantitative problems.
- The validity of the data may be suspect.

Developing an appropriate solution

- The mathematics may be hard to understand.
- Having only one answer may be limiting.

Testing the solution for validity

Analyzing the results in terms of the whole organization

Chapter 3 Decision Analysis

3.1 Introduction

To a great extent, the successes or failures that a person experiences in life depend on the decision that he or she makes. The person who managed the ill-fated space shuttle Challenger is no longer working for NASA. The person who designed the top selling Mustang became president of Ford. Why and how did these people make their respective decisions? In general, what is involved in making good decisions?

Decision theory is an analytic and systematic approach to the study of decision making. In this Chapter, we present the mathematical models useful in helping managers make the best possible decisions.

- What makes the difference between good and bad decisions? A good decision is one that is based on logic, considers all available data and possible alternatives, and the quantitative approach described here.

3.2 The Six Steps in Decision Making

Whether you are deciding about getting a haircut today, building a multimillion-dollar plant, or buying a new camera, the steps in making a good decision are basically the same:

1. Clearly define the problem at hand.
2. List the possible alternatives.
3. Identify the possible outcomes or states of nature.
4. List the payoff (typically profit) of each combination of alternatives and outcomes.
5. Select one of the mathematical decision theory models.
6. Apply the model and make your decision.

Thompson Lumber Company: We use the Thompson Lumber Company case as an example to illustrate these decision theory steps.

Step 1 – Define the problem. The problem that John Thompson identifies is whether to expand his product line by manufacturing and marketing a new product, backyard storage sheds

Step 2 – List the alternatives. In decision theory, an **alternative** is defined as a course of action or a strategy that the decision maker can choose.

John decides that his alternatives are-

(1) To construct a large new plant.
(2) To construct a small new plant.
(3) No plant at all (i.e., Do not develop the new product line at all).

Step 3 – Identify possible outcomes. In decision theory, those outcomes over which that decision maker has little or no control, are called **states of nature**.

Thompson determines that there are only two possible outcomes:

(1) The market for the storage sheds could be favourable, meaning that there is a high demand for the product, or
(2) The market could be unfavourable, meaning that there is a low demand for the sheds.

Once the alternatives and states of nature have been identified, the next step is to express the payoffs resulting from each possible combination of alternatives and outcomes. In decision theory, we call such payoffs or profits as **conditional values**.

Step 4 – List the payoffs. Because Thompson wants to maximize his profits, he can use profit to evaluate each consequence. John has already evaluated the potential profits associated with the various outcomes.

With a favorable market, he thinks a large plant would result in a net profit of $200,000 to his firm and with unfavorable market, the large plant would result in a net loss of $180,000. A small plant would result in a net profit of $100,000 in a favorable market but a net loss of $20,000 would occur if the market was unfavorable.

Table 3.1 Decision Table (or Payoff Table) with Conditional Values for Thompson Lumber

	STATE OF NATURE	
ALTERNATIVE	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)
Construct a large plant	200,000	−180,000
Construct a small plant	100,000	−20,000
Do nothing	0	0

Step 5 – Select the decision model.

- This depends on the environment and amount of risk and uncertainty.

Step 6 – Apply the model to the data.

- Solution and analysis are then used to aid in decision-making.

3.3 Types of Decision-Making Environments

The types of decisions people make depend on how much knowledge or information they have about the situation. There are three decision-making environments:

Type 1: Decision Making Under Certainty: In the environment of **decision making under certainty,** the decision makers *know with certainty* the consequences of every alternative or decision choice. Naturally, they will choose the alternative that will maximize their well-being or will result in the best outcome.

Type 2: Decision Making Under Uncertainty: In **decision making under uncertainty,** there are several possible outcomes for each alternative and the decision maker does not know the probabilities of the various outcomes.

Type 3: Decision Making Under Risk: In **decision making under risk**, there are several possible outcomes for each alternative and the decision maker knows the probability of occurrence of each outcome.

3.4 Decision Making Under Uncertainty

When several states of nature exit and a manager cannot assess the outcome probability with confidence or when virtually no probability data are available, the environment is called decision making under uncertainty. Several criteria exist for making decisions under these conditions:

1. **Optimistic (maximax)**
2. **Pessimistic (maximin)**
3. **Criterion of realism (Hurwicz)**
4. **Equally likely (Laplace)**
5. **Minimax regret**

The first four criteria can be computed directly from the decision (payoff) table, whereas the minimax regret criterion requires use of the opportunity loss table.

The presentation of the criteria for decision making under uncertainty (and also for decision making under risk) is based on the assumption that the payoff is something in which larger values are better and high values are desirable. For payoffs such as profit, total sales, total return on investment and interest returned, the best decision would be one that resulted in some type of maximum payoff. However, there are situations in which lower payoff values (e.g., cost) are better and these payoffs would be minimized rather than maximized. Let's take a look at each of five models and apply them to the Thompson Lumber example.

Optimistic (or Maximax) It is used to find the alternative that maximizes the maximum payoff.

- Locate the maximum payoff for each alternative.
- Select the alternative with the maximum number.

Table 3.2 Thompson's Maximax Decision

ALTERNATIVE	STATE OF NATURE FAVORABLE MARKET ($)	STATE OF NATURE UNFAVORABLE MARKET ($)	MAXIMUM IN A ROW ($)
Construct a large plant	200,000	−180,000	200,000
Construct a small plant	100,000	−20,000	100,000
Do nothing	0	0	0

From the last column of Table 3.2, it is clear that

Maximax = maximum of (200,000 ; 100,000 and 0) = 200,000 which is corresponding to first alternative, "construct a large plant".

So Thompson's optimistic choice is the first alternative "construct a large plant".

Pessimistic (or Maximin) It is used to find the alternative that maximizes the minimum payoff.

- Locate the minimum payoff for each alternative.
- Select the alternative with the maximum number.

Table 3.3 Thompson's Maximin Decision

ALTERNATIVE	STATE OF NATURE FAVORABLE MARKET ($)	STATE OF NATURE UNFAVORABLE MARKET ($)	MINIMUM IN A ROW ($)
Construct a large plant	200,000	−180,000	−180,000
Construct a small plant	100,000	−20,000	−20,000
Do nothing	0	0	0

From the last column of Table 3.3, it is clear that

Maximin = maximum of (−180,000; −20,000 and 0) = 0 which is corresponding to third alternative, "do nothing".

So Thompson's pessimistic choice is third alternative "do nothing".

Criterion of Realism (Hurwicz Criterion)

Often called the **weighted average**, the **criterion of realism** (the **Hurwicz criterion**) is a compromise between an optimistic and a pessimistic decision. To begin with, a coefficient of realism, α, is selected; which measures the degree of optimism of the decision maker. This coefficient lies between 0 and 1. When α is 1 , the decision maker is 100% optimistic about the future. When α is 0, the decision maker is 100% pessimistic about the future. The weighted average is computed as follows:

$$\text{Weighted average} = \alpha(\text{best in row}) + (1-\alpha)(\text{worst in row})$$

For a maximization problem, the alternative with the highest weighted average is chosen as best decision.

If we assume that John Thompson sets his coefficient of realism, α, to be 0.80,
The weighted average for the large plant = (0.8)(200,000) + (1 – 0.8)(–180,000)
$= 160{,}000 - 36{,}000 = 124{,}000$
The weighted average for the small plant = (08)(100,000) + (1 – 0.8)(–20,000)
$= 80{,}000 - 4{,}000 = 76{,}000$
The weighted average for doing nothing = (0.8)(0) + (1 – 0.8)(0) = 0

Table 3.4 Thompson's Criterion of Realism Decision

ALTERNATIVE	STATE OF NATURE		CRITERION OF REALISM OR WEIGHTED AVERAGE
	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	For $\alpha = 0.8$ ($)
Construct a large plant	200,000	−180,000	(0.8)(200,000) + (1 – 0.8)(–180,000) =124,000
Construct a small plant	100,000	−20,000	(08)(100,000) + (1 – 0.8)(–20,000) =76,000
Do nothing	0	0	0

From the last column of Table 3.4, it is clear that highest weighted average is 124,000 corresponding to the first alternative, " construct a large plant". Hence, according to Criterion of Realism, the best decision would be the first alternative " construct a large plant".

Equally Likely (Laplace)

Considers all the payoffs for each alternative

- Find the average payoff for each alternative.
- Select the alternative with the highest average.

Table 3.5 Thompson's Equally Likely Decision

ALTERNATIVE	STATE OF NATURE		ROW AVERAGE ($)
	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	
Construct a large plant	200,000	−180,000	$= \frac{200{,}000+(-180{,}000)}{2} = 10{,}000$
Construct a small plant	100,000	−20,000	$= \frac{100{,}000+(-20{,}000)}{2} = 40{,}000$
Do nothing	0	0	0

From the last column of Table 3.5, it is clear that the maximum average payoff is 40,000 corresponding to second alternative, "construct a small plant". Hence, according to Equally Likely Criteria, the best decision is the second alternative "construct a small plant".

Minimax Regret

Based on *opportunity loss* or *regret*, this is the difference between the optimal profit and actual payoff for a decision.

- Create an opportunity loss table by determining the opportunity loss from not choosing the best alternative.
- Opportunity loss is calculated by subtracting each payoff in the column from the best payoff in the column.
- Find the maximum opportunity loss for each alternative and pick the alternative with the minimum number.

Table 3.7 Opportunity Loss Table For Thompson Lumber

ALTERNATIVE	STATE OF NATURE	
	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)
Construct a large plant	200,000−200,000 = 0	0−(−180,000) =180,000
Construct a small plant	200,000−100,000= 100,000	0 − (−20,000) =20,000
Do nothing	200,000−0=200,000	0−0 =0

Using the opportunity loss (regret) table, the **minimax regret** criterion finds the alternative that minimizes the maximum opportunity loss within each alternative.

Table 3.8 Thompson's Minimax Decision Using Opportunity Loss

ALTERNATIVE	STATE OF NATURE		MAXIMUM IN A ROW ($)
	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	
Construct a large plant	0	180,000	180,000
Construct a small plant	100,000	20,000	100,000
Do nothing	200,000	0	200,000

From the last column of Table 3.8, it is clear that
Minimax= minimum of (180,000; 100,000 and 200,000) =100,000 which is corresponding to the second alternative, "construct a small plant". Hence according to minimax regret criteria, the best decision is the second alternative "construct a small plant".

Example: Consider the following payoff (profit) table :

	State of nature		
Action	**1**	**2**	**3**
A	10	20	50
B	5	30	40

a) Which action would an optimistic (maximax) decision maker choose?
b) Which action would a pessimistic (maximin) decision maker choose?
c) What decision would be taken using equally likely method?

Solution:

a) **Optimistic (maximax) Criteria:** **Table 1**

	State of nature			
Action	1	2	3	Max in a row
A	10	20	50	50 (maximax)
B	5	30	40	40

From the above Table 1, it is clear that Maximax = maximum of 50 & 40
= 50 (corresponding to the action A)
Therefore, the optimistic decision maker will choose the action 'A'.

b) Pessimistic (Maximin) Criteria: **Table 2**

	State of nature			
Action	1	2	3	Min in a row
A	10	20	50	10 (maximin)
B	5	30	40	5

From the above Table 2, it is clear that Maximin = maximum of 10 & 5
= 10 (corresponding to the action A)
Therefore, the pessimistic decision maker will choose the action 'A'.

c) Equally Likely Criteria: **Table 3**

	State of nature			
Action	1	2	3	Row Average
A	10	20	50	$\frac{10+20+50}{3} = 26.67$
B	5	30	40	$\frac{5+30+40}{3} = 25$

From the Table 3, it is clear that the maximum average payoff is 26.67 corresponding to action A. Hence, according to Equally Likely Criteria, the best decision is the action 'A'.

3.5 Decision Making Under Risk

In this Section, we consider one of the most popular methods of making decisions under risk: selecting the alternative with the highest expected monetary value (or simply expected value). We also use the probabilities with the opportunity loss table to minimize the expected opportunity loss.

Expected Monetary Value (EMV)

Given a decision table with conditional values (payoffs) that are monetary values, and probability assessments for all states of nature, it is possible to determine the **expected monetary value** (EMV) for each alternative.

$$\text{EMV (alternative)} = \sum X_i \, P(X_i) \qquad \ldots\ldots(3.1)$$

Where

X_i = payoff for the alternative in state of nature i

$P(X_i)$ = probability of achieving payoff X_i (i.e., probability of state of nature i)

If this were expanded, it would become

EMV (alternative) = (payoff in first state of nature)×(probability of first state of nature)
+(payoff in second state of nature)×(probability of second state of nature)
+…..+(payoff in last state of nature)×(probability of last state of nature)

The alternative with the maximum EMV is then chosen.

Suppose that John Thompson now believes that the probability of a favorable market is exactly the same as the probability of an unfavorable market; that is, each state of nature has a 0.50 probability. Which alternative would give the greatest expected monetary value?

To determine this, John has expanded the decision table, as shown in Table 3.9

$$\text{EMV (large plant)} = (\$200{,}000)(0.50) + (-\$180{,}000)(0.50)$$
$$= \$100{,}000 - \$90{,}000 = 10{,}000$$
$$\text{EMV (small plant)} = (\$100{,}000)(0.50) + (-\$20{,}000)(0.50)$$
$$= \$50{,}000 - \$10{,}000 = 40{,}000$$
$$\text{EMV (do nothing)} = (\$0)(0.50) + (\$0)(0.50) = \$0$$

Table 3.9 Decision Table with Probabilities and EMVs for Thompson Lumber

ALTERNATIVE	STATE OF NATURE		EMV ($)
	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	
Construct a large plant	200,000	−180,000	$(200{,}000)(0.50) + (-180{,}000)(0.50)$ =10,000
Construct a small plant	100,000	−20,000	$(100{,}000)(0.50) + (-20{,}000)(0.50)$ =40,000
Do nothing	0	0	$(0)(0.50) + (0)(0.50)$ =0
Probabilities	0.50	0.50	

From the last column of Table 3.9, the largest expected value ($40,000) results from the second alternative, “construct a small plant”. Thus, Thompson should proceed with the project and put up a small plant to manufacture storage sheds.

Expected Value of Perfect Information (EVPI)

John Thompson has been approached by Scientific Marketing, Inc., a firm that proposes to help John make the decision about whether to build the plant to produce storage sheds. Scientific Marketing claims that its technical analysis will tell John with certainty whether the market is favorable for his proposed product. In other words, it will change his environment from one of decision making under risk to one of decision making under certainty. This information could prevent John from making a very expensive mistake. Scientific Marketing would charge Thompson $65,000 for the information. What would you recommend to John? Should he hire the firm to make the marketing study? Even if the information from the study is perfectly accurate, is it worth $65,000? What would it be worth? Although some of these questions are difficult to answer, determining the value of such perfect information can be very useful. In this section, two related terms are investigated: the **expected value of perfect information (EVPI)** and the **expected value with perfect information (EVwPI)**. These techniques can help John make his decision about hiring the marketing firm.

The expected value *with* perfect information is the expected or average return, in the long run, if we have perfect information before a decision has to be made.

$$\begin{aligned} \text{EVwPI} = {} & (\text{best payoff in first state of nature})\times(\text{probability of first state of nature}) \\ & +(\text{best payoff in second state of nature})\times(\text{probability of second state of nature}) \\ & +\ldots\ldots+(\text{best payoff in last state of nature})\times(\text{probability of last state of nature}) \end{aligned}$$

The expected value of perfect information (EVPI) is the expected value *with* perfect information minus the expected value *without* perfect information (i.e., the best or maximum EMV). Thus,

$$\text{EVPI} = \text{EVwPI} - \text{Best EMV} \qquad \ldots\ldots.(3.3)$$

By referring to Table 3.9, Thompson can calculate the maximum that he would pay for information, that is, the expected value of perfect information, or EVPI. He follows a three-stage process. First, the best payoff in each state of nature is found. These values are shown in the "with perfect information" row in Table 3.10. Second, the expected value *with* perfect information is computed. Then using this result, EVPI is calculated.

Table 3.10 Decision Table with Perfect Information

ALTERNATIVE	STATE OF NATURE FAVORABLE MARKET ($)	 UNFAVORABLE MARKET ($)	EMV ($)
Construct a large plant	200,000	−180,000	10,000
Construct a small plant	100,000	−20,000	40,000
Do nothing	0	0	0
With perfect information	200,000	0	(200,000)(0.50) + (0)(0.50)=100,000 (EVwPI)
Probabilities	0.50	0.50	

The expected value with perfect information is

$$\text{EVwPI} = (\$200{,}000)(0.50) + (\$0)(0.50) = \$100{,}000$$

Thus, if we had perfect information, the payoff would average $100,000.

The maximum EMV without additional information is $40,000 (from Table 3.9). Therefore, the increase in EMV is

$$\begin{aligned} \text{EVPI} &= \text{EVwPI} - \text{maximum EMV} \\ &= \$100{,}000 - \$40{,}000 = \$60{,}000 \end{aligned}$$

Thus, the *most* Thompson would be willing to pay for perfect information is $60,000. Therefore, Thompson should not pay $65,000 for this information. This, of course, is again based on the assumption that the probability of each state of nature is 0.50.

Example: Consider the following payoff (profit) table :

	State of nature		
Action	**1**	**2**	**3**
A	10	20	50
B	5	30	40
Probability	0.5	0.3	0.2

a) What is the best alternative using the expected monetary value analysis?
b) Compute the Expected Value of Perfect Information.

Solution:

a) **Table 1**

	State of nature			
Action	1	2	3	EMV
A	10	20	50	$(0.5)10 + (0.3)20 + (0.2)50 = 21$
B	5	30	40	$(0.5)5 + (0.3)30 + (0.2)40 = 19.5$
Probability	0.5	0.3	0.2	

EMV (for alternative A) = $(0.5)10 + (0.3)20 + (0.2)50 = 21$
& EMV(for alternative B) = $(0.5)5 + (0.3)30 + (0.2)40 = 19.5$

So Max EMV=21 corresponding to alternative 'A' so using expected monetary value (EMV) analysis, the best alternative is 'A'.

b) **Table 2**

	State of nature			
Action	1	2	3	
A	10	20	50	
B	5	30	40	
Max in column	10	30	50	EVwPI = (0.5)(10)+(0.3)(30)+(0.2)(50) = 24
Probability	0.5	0.3	0.2	

Now, EVPI = EVwPI – Max(EMV)

= 24 – 21 = 3.

Expected Opportunity Loss (EOL): (EOL is the cost of not picking the best solution.) An alternative approach to maximizing EMV is to minimize expected opportunity loss (EOL). First, an opportunity loss table is constructed. Then the EOL is computed for each alternative by multiplying the opportunity loss by the probability and adding these together.

Using opportunity loss Table 3.7 for Thompson Lumber example, we compute the EOL for each alternative as follows:

EOL (construct large plant) = (0.5)($0) + (0.5)($180,000)
= $90,000
EOL (construct small plant) = (0.5)($100,000) + (0.5)($20,000)
= $50,000 + $10,000 = $60,000
EOL (do nothing) = (0.5)($200,000) + (0.5)($0)
= $100,000

Table 3.11 (see Table 3.7)EOL Table for Thompson Lumber

ALTERNATIVE	STATE OF NATURE		EOL
	FAVORABLE MARKET ($)	UNFAVORABLE MARKET ($)	
Construct a large plant	0	180,000	(0.5)(0) + (0.5)(180,000)=90,000
Construct a small plant	100,000	20,000	(0.5)(100,000) + (0.5)(20,000)=60,000
Do nothing	200,000	0	(0.5)(200,000) + (0.5)(0) =100,000
Probabilities	0.50	0.50	

From the last column of Table 3.11, it is clear that minimum EOL is 60,000 corresponding to the second alternative, “construct a small plant.” Hence using minimum EOL criterion, the best decision would be the second alternative, “construct a small plant.”

NOTE: It is important to note that minimum EOL will always result in the same decision as maximum EMV and the EVPI will always equal to the minimum EOL.

Example: Consider the following payoff (profit) table. What is best alternative using the expected opportunity loss analysis?

	State of nature		
Action	**1**	**2**	**3**
A	10	20	50
B	5	30	40
Probability	0.5	0.3	0.2

Solution: Opportunity Loss Table:

	State of nature			**EOL**
Action	**1**	**2**	**3**	
A	10−10=0	30−20=10	50−50=0	(0.5)(0)+(0.3)(10)+(0.2)(0)=3
B	10−5=5	30−30=0	50−40=10	(0.5)(5)+(0.3)(0)+(0.2)(10)=4.5
Probability	0.5	0.3	0.2	

EOL(for alternative A) = (0.5)(0)+(0.3)(10)+(0.2)(0) = 3
& EOL(for alternative B) = (0.5)(5)+(0.3)(0)+(0.2)(10) = 4.5

Now Min EOL = 3 corresponding to the alternative A, so using expected opportunity table (EOL) analysis, the best alternative is ‘A’.

Sensitivity Analysis: Sensitivity analysis examines how the decision might change with different input data.

In previous sections we determined that the best decision (with the probabilities known) for Thompson Lumber was to construct the small plant, with an expected value of \$40,000. This conclusion depends on the values of the economic consequences and the two probability values of a favorable and an unfavorable market. *Sensitivity analysis* investigates how our decision might change given a change in the problem data. In this section, we investigate the impact that a change in the probability values would have on the decision facing Thompson Lumber. We first define the following variable:

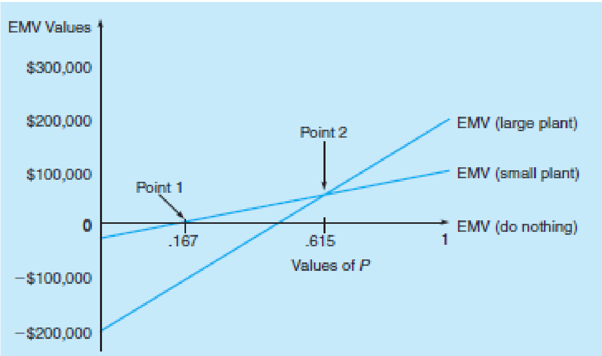
$$P = \text{probability of a favorable market}$$

Because there are only two states of nature, the probability of an unfavorable market must be $1 - P$.

We can now express the EMVs in terms of P, as shown in the following equations. A graph of these EMV values is shown in Figure 3.1.

$$\begin{aligned}
\text{EMV(large plant)} &= \$200{,}000P - \$180{,}000(1 - P) \\
&= \$200{,}000P - \$180{,}000 + 180{,}000P \\
&= \$380{,}000P - \$180{,}000 \\
\text{EMV(small plant)} &= \$100{,}000P - \$20{,}000(1 - P) \\
&= \$100{,}000P - \$20{,}000 + 20{,}000P \\
&= \$120{,}000P - \$20{,}000 \\
\text{EMV(do nothing)} &= \$0P + \$0(1 - P) = \$0
\end{aligned}$$

FIGURE 3.1
Sensitivity Analysis



As you can see in Figure 3.1, the best decision is to do nothing as long as P is between 0 and the probability associated with point 1, where the EMV for doing nothing is equal to the EMV for the small plant. When P is between the probabilities for points 1 and 2, the best decision is to build the small plant. Point 2 is where the EMV for the small plant is equal to the EMV for the large plant. When P is greater than the probability for point 2, the best decision is to construct the large plant. Of course, this is what you would expect as P increases. The value of P at points 1 and 2 can be computed as follows:

Point 1: EMV (do nothing) = EMV (small plant)

$$0 = \$120{,}000P - \$20{,}000 \qquad P = \frac{20{,}000}{120{,}000} = 0.167$$

Point 2: EMV (small plant) = EMV (large plant)

$$\$120{,}000P - \$20{,}000 = \$380{,}000P - \$180{,}000$$

$$260{,}000P = 160{,}000 \qquad P = \frac{160{,}000}{260{,}000} = 0.615$$

The results of this sensitivity analysis are displayed in the following table:

BEST ALTERNATIVE	RANGE OF P VALUES
Do nothing	Less than 0.167
Construct a small plant	0.167 – 0.615
Construct a large plant	Greater than 0.615

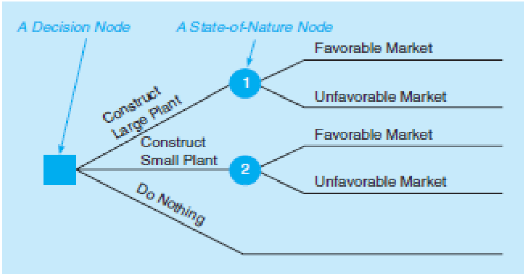
3.6 Decision Trees

Any problem that can be presented in a decision table can also be graphically illustrated in a **decision tree**. All decision trees are similar in that they contain *decision points* or **decision nodes** and *state-of-nature points* or **state-of-nature nodes**:

- A decision node from which one of several alternatives may be chosen
- A state-of-nature node out of which one state of nature will occur

In drawing the tree, we begin at the left and move to the right. Thus, the tree presents the decisions and outcomes in sequential order. Lines or branches from the squares (decision nodes) represent alternatives and branches from the circles represent the states of nature. Figure 3.2 gives the basic decision tree for the Thompson Lumber example.

FIGURE 3.2
Thompson's Decision Tree



Five Steps of Decision Tree Analysis: Analyzing problems with decision trees involves five steps:

1. Define the problem.
2. Structure or draw the decision tree.
3. Assign probabilities to the states of nature.
4. Estimate payoffs for each possible combination of alternatives and states of nature.
5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node. This is done by working backward, that is, starting at the right of the tree and working back to decision nodes on the left. Also, at each decision node, the alternative with the best EMV is selected.

The final decision tree with the payoffs and probabilities for John Thompson's decision situation is shown in Figure 3.3. Beginning with the payoffs on the right of the figure, the EMVs for each state-of-nature node are then calculated and placed by their respective nodes. The EMV of the first node, to construct a large plant is $10,000. The EMV of the second node, to construct a small plant is $40,000. Building no plant or doing nothing has, of course, a payoff of $0. The branch leaving the decision node leading to the state-of-nature node with the highest EMV should be chosen. In Thompson's case, a small plant should be built.

FIGURE 3.3
Completed and Solved Decision Tree for Thompson Lumber