CHAPTER-12 PROJECT MANAGEMENT

INTRODUCTION

Most realistic projects that organizations like Microsoft, General Motors or the U.S. Defense Department undertake are large and complex. A builder putting up an office building must complete thousands of activities costing millions of dollars. NASA must inspect countless components before it launches a rocket. Almost every industry worries about how to manage similar large-scale, complicated projects effectively. It is a difficult problem and the stakes are high.

The first step in planning and scheduling a project is to develop the *work breakdown structure*. This involves identifying the activities that must be performed in the project. An **activity** is a job or task that is a part of a project. The beginning or end of an activity is called an **event**. There may be varying levels of detail and each activity may be broken into its most basic components. The time, cost, resource requirements, predecessors and person(s) responsible are identified for each activity. When this has been done, a schedule for the project can be developed.

The **program evaluation and review technique (PERT)** and the **critical path method (CPM)** are two popular quantitative analysis techniques to help plan, schedule, monitor, and control large and complex projects.

When they were first developed, PERT and CPM were similar in their basic approach but they differed in the way activity times were estimated. For every PERT activity, three time estimates are combined to determine the expected activity completion time. Thus, PERT is a probabilistic technique. On the other hand, CPM is a deterministic technique since it is assumed that the times are known with certainty. They have become so similar that they are commonly considered one technique, PERT/CPM.

Six Steps of PERT/CPM

1. Define the project and all of its significant activities or tasks.
2. Develop the relationships among the activities and decide which activities must precede others.
3. Draw the **network** connecting all of the activities.
4. Assign time and/or cost estimates to each activity.
5. Compute the longest time path through the network; this is called the ***critical path***.
6. Use the network to help plan, schedule, monitor, and control the project.

Finding the critical path is a major part of controlling a project. The activities on the critical path represent tasks that will delay the entire project if they are delayed. Managers derive flexibility by identifying noncritical activities and replanning, rescheduling and reallocating resources such as personnel and finances.

PERT/CPM: Almost any large project can be subdivided into a series of smaller activities or tasks that can be analyzed with PERT/CPM. When you recognize that projects can have thousands of specific activities, you see why it is important to be able to answer questions such as the following:

1. When will the entire project be completed?
2. What are the *critical* activities or tasks in the project, that is, the ones that will delay the entire project if they are late?
3. Which are the *non-critical* activities, that is, the ones that can run late without delaying the entire project's completion?
4. If there are three time estimates, what is the probability that the project will be completed by a specific date?
5. At any particular date, is the project on schedule, behind schedule, or ahead of schedule?
6. On any given date, is the money spent equal to, less than, or greater than the budgeted amount?
7. Are there enough resources available to finish the project on time?

General Foundry Example of PERT/CPM

General Foundry, Inc., a metalworks plant in Milwaukee, has long been trying to avoid the expense of installing air pollution control equipment. The local environmental protection group has recently given the foundry 16 weeks to install a complex air filter system on its main smoke stack. General Foundry was warned that it will be forced to close unless the device is installed in the allotted period. Lester Harky, the managing partner, wants to make sure that installation of the filtering system progresses smoothly and on time.

When the project begins, the building of the internal components for the device (activity A) and the modifications that are necessary for the floor and roof (activity B) can be started. The construction of the collection stack (activity C) can begin once the internal components are completed and pouring of the new concrete floor and installation of the frame (activity D) can be completed as soon as the roof and floor have been modified. After the collection stack has been constructed, the high temperature burner can be built (activity E) and the installation of the pollution control system (activity F) can begin. The air pollution device can be installed (activity G) after the high-temperature burner has been built, the concrete floor has been poured and the frame has been installed. Finally, after the control system and pollution device have been installed, the system can be inspected and tested (activity H).

All of these activities seem rather confusing and complex until they are placed in a network. First, all of the activities must be listed. This information is shown in Table 12.1 We see in the table that before the collection stack can be constructed (activity C), the internal components must be built (activity A). Thus, activity A is the immediate predecessor of activity C. Similarly, both activities D and E must be performed just prior to installation of the air pollution device (activity G).

ACTIVITY	DESCRIPTION	IMMEDIATE PREDECESSORS
A	Build internal components	—
B	Modify roof and floor	—
C	Construct collection stack	*A*
D	Pour concrete and install frame	*B*
E	Build high-temperature burner	*C*
F	Install control system	*C*
G	Install air pollution device	*D, E*
H	Inspect and test	*F, G*

Table 12.1 Activities and Immediate Predecessors for General Foundry, Inc.

Drawing the PERT/CPM Network

There are two common techniques for drawing PERT networks. The first is called *Activity-on-node* (*AON*) because the nodes represent activities. The second is called *Activity-on-arc* (*AOA*) because the arcs are used to represent the activities. The AON approach is easier and more commonly used in software packages.

In constructing an AON network, there should be one node representing the start of the project, and one node representing the finish of the project. There will be one node (represented as a rectangle in this chapter) for each activity. Figure 12.1 gives the entire network for General Foundry. The arcs (arrows) are used to show the predecessors for the activities. For example, the arrows leading into activity G indicate that both D and E are immediate predecessors for G

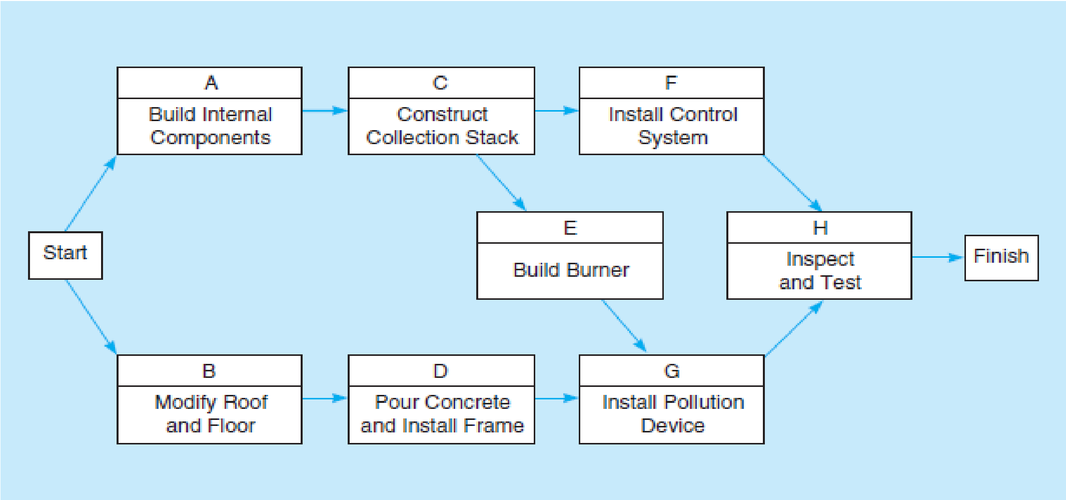


Figure 12.1 Network for General Foundry Inc.

Activity Times

The next step in both CPM and PERT is to assign estimates of the time required to complete each activity. For some projects, such as construction projects, the time to complete each activity may be known with certainty. The developers of CPM assigned just one time estimate to each activity. These times are then used to find the critical path.

In many projects, there is uncertainty about activity times. For this reason, the developers of PERT employed a probability distribution based on three time estimates for each activity. A weighted average of these estimates is used with PERT in place of the single time estimate used with CPM and these averages are used to determine the critical path. The time estimates in PERT are

> ***Optimistic time* (*a*)** = time an activity will take if everything goes as well as possible. There should be only a small probability (say, $^{1}/_{100}$) of this occurring.
>
> ***Pessimistic time* (*b*)** = time an activity would take assuming very unfavorable conditions. There should also be only a small probability that the activity will really take this long.
>
> ***Most likely time* (*m*)** = most realistic time estimate to complete the activity

PERT often assumes that time estimates follow *beta probability distribution* (See Fig 12.2). This continuous distribution has been found to be appropriate, in many cases, for determining an expected value and variance for activity completion times.

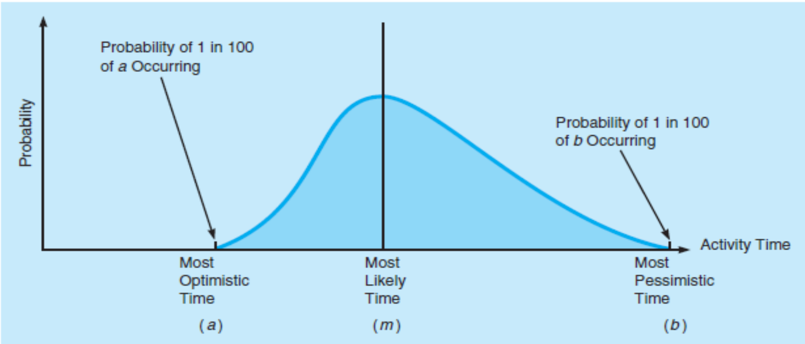


Fig 12.2 Beta Probability Distribution with Three Time Estimates

To find the **expected activity time** (t), the beta distribution weights the estimates as follows:

$$t = \frac{a + 4m + b}{6} \qquad \ldots\ldots(12.1)$$

To compute the dispersion or **variance of activity completion time**, we use this formula

$$Variance = \left(\frac{b - a}{6}\right)^2 \qquad \ldots\ldots(12.2)$$

Table 12.2 shows General Foundry's optimistic, most likely and pessimistic time estimates for each activity. It also reveals the expected time (t) and variance for each of the activities, as computed with Equations 12.1 and 12.2.

TABLE 12.2 Time Estimates (Weeks) for General Foundry, Inc.

ACTIVITY	OPTIMISTIC, a	MOST PROBABLE, m	PESSIMISTIC, b	EXPECTED TIME, $t = [(a + 4m + b)/6]$	VARIANCE, $[(b - a)/6]^2$
A	1	2	3	2	$\left(\frac{3-1}{6}\right)^2 = \frac{4}{36}$
B	2	3	4	3	$\left(\frac{4-2}{6}\right)^2 = \frac{4}{36}$
C	1	2	3	2	$\left(\frac{3-1}{6}\right)^2 = \frac{4}{36}$
D	2	4	6	4	$\left(\frac{6-2}{6}\right)^2 = \frac{16}{36}$
E	1	4	7	4	$\left(\frac{7-1}{6}\right)^2 = \frac{36}{36}$
F	1	2	9	3	$\left(\frac{9-1}{6}\right)^2 = \frac{64}{36}$
G	3	4	11	5	$\left(\frac{11-3}{6}\right)^2 = \frac{64}{36}$
H	1	2	3	2	$\left(\frac{3-1}{6}\right)^2 = \frac{4}{36}$
				25	

How to Find the Critical Path

Once the expected completion time for each activity has been determined, we accept it as the actual time of that task. Variability in times will be considered later.

Although Table 12.2 indicates that the total expected time for all eight of General Foundry's activities is 25 weeks, it is obvious in Fig 12.3 that several of the tasks can be taking place simultaneously. To find out just how long the project will take, we perform the critical path analysis for the network.

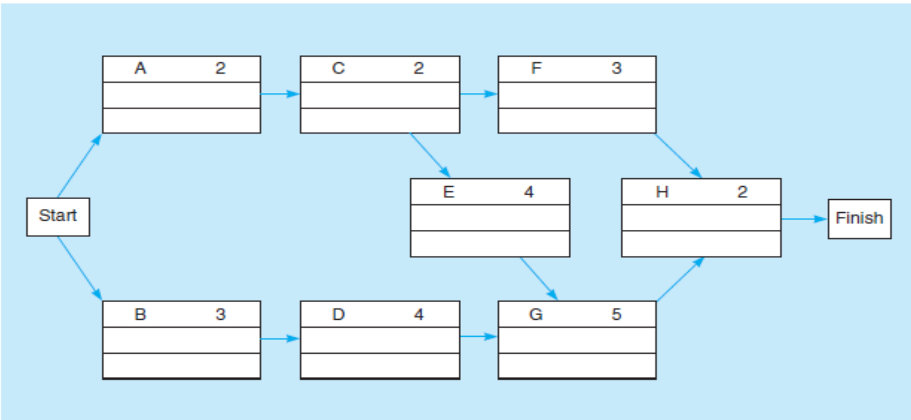


Figure 12.3 General Foundry's Network with Expected Activity Times

The *critical path* is the longest time path route through the network. If Lester Harky wants to reduce the total project time for General Foundry, he will have to reduce the length of some activity on the critical path. Conversely, any delay of an activity on the critical path will delay completion of the entire project.

To find the critical path, we need to determine the following quantities for each activity in the network:

1. *Earliest start time* (*ES*): the earliest time an activity can begin without violation of immediate predecessor requirements.
2. *Earliest finish time* (*EF*): the earliest time at which an activity can end.
3. *Latest start time* (*LS*): the latest time an activity can begin without delaying the entire project.
4. *Latest finish time* (*LF*): the latest time an activity can end without delaying the entire project.

In the network, we represent these times as well as activity times (*t*) in the nodes, as seen here:

ACTIVITY	*t*
ES	EF
LS	LF

We first show how to determine the earliest times. When we find these, the latest times can be computed.

EARLIEST TIMES

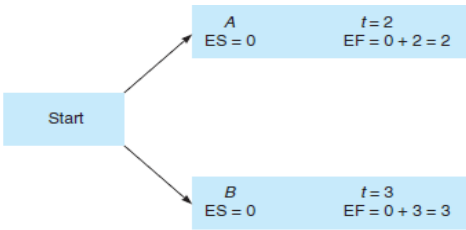
Earliest finish time = Earliest start time + Expected activity time

$$EF = ES + t \qquad (12.3)$$

Earliest start = Largest of the earliest finish times of immediate predecessors

ES = Largest EF of immediate predecessors

The start of the whole project will be set at time zero. Therefore, any activity that has no predecessors will have an earliest start time of zero. So ES = 0 for both A and B in the General Foundry problem, as seen here:



The rest of the earliest times for General Foundry are shown in Figure 12.4. These are found using a **forward pass** through the network.

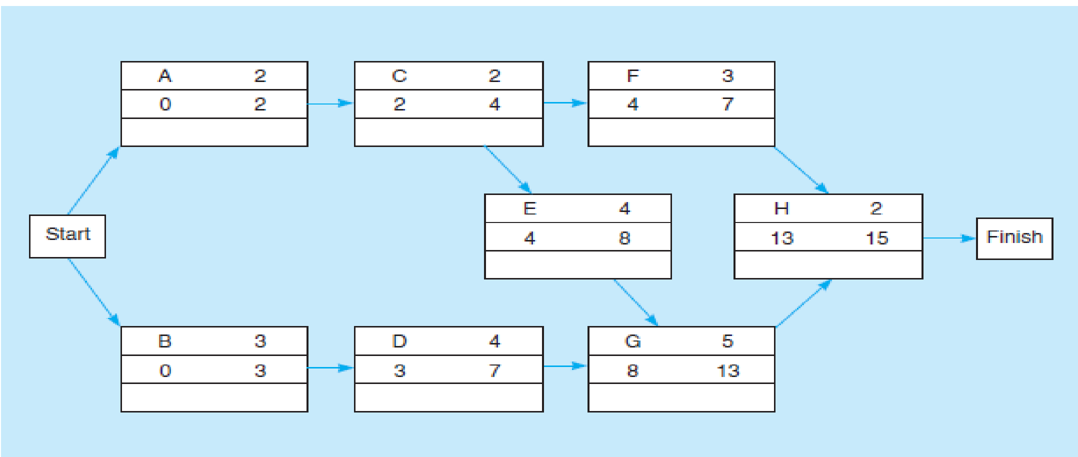


Fig.12.4 General Foundry's Earliest Start (ES) and Earliest Finish (EF) Times

LATEST TIMES

The next step in finding the critical path is to compute the latest start time (LS) and the latest finish time (LF) for each activity. We do this by making a **backward pass** through the network, that is, starting at the finish and working backward.

Latest start time = Latest finish time – Expected activity time

LS = LF – t (12.4)

Latest finish time = Smallest of latest start times for following activities

LF = Smallest LS of following activities

All the latest times are shown in Figure 12.5

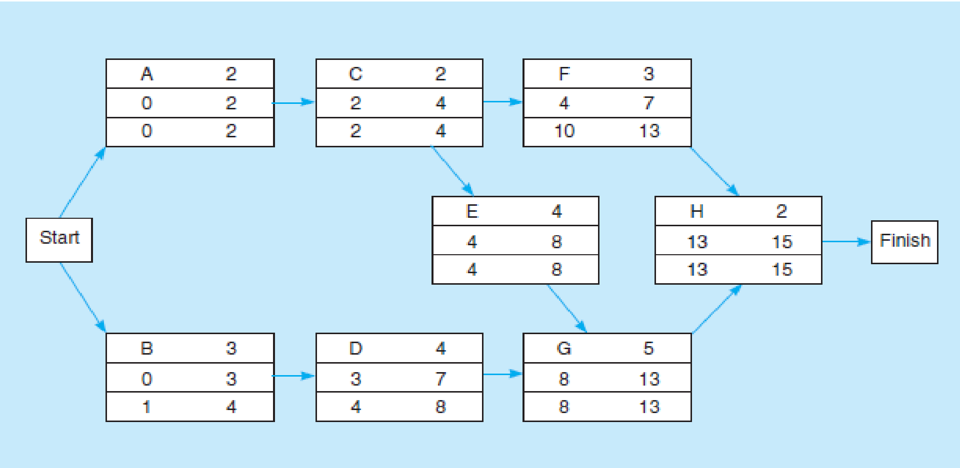


Fig 12.5 General Foundry's Latest Start (LS) and Latest Finish (LF) Times

CONCEPT OF SLACK IN CRITICAL PATH COMPUTATIONS

When ES, LS, EF and LF have been determined, it is a simple matter to find the amount of **slack time** or free time, that each activity has. Slack is the length of time an activity can be delayed without delaying the whole project.

Mathematically,

Slack = LS−ES, or Slack = LF−EF ….(12.5)

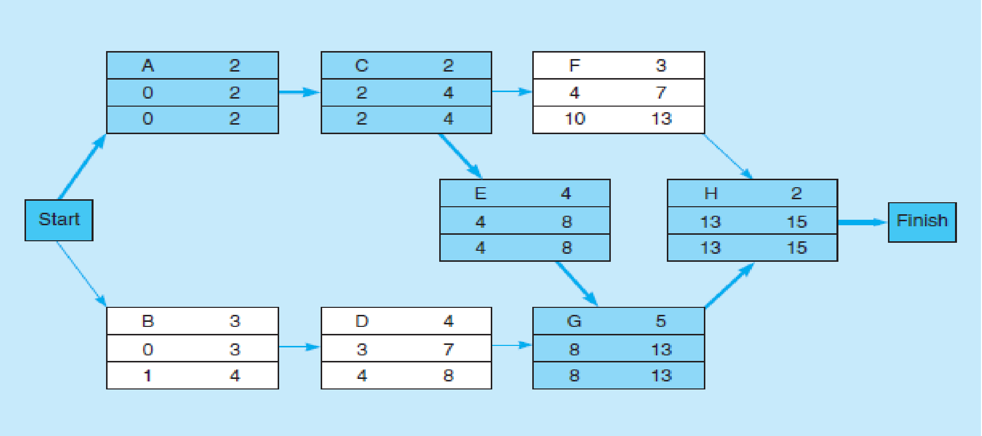
Table 12.3 summarizes ES, EF, LS, LF and slack times for all of General Foundry's activities. Activity B, for example, has 1 week of slack time. This means that it can be delayed up to 1 week without causing the project to run any longer than expected.

Table 12.3 General Foundry's Schedule and Slack Times

ACTIVITY	EARLIEST START, ES	EARLIEST FINISH, EF	LATEST START, LS	LATEST FINISH, LF	SLACK, LS − ES	ON CRITICAL PATH?
A	0	2	0	2	0	Yes
B	0	3	1	4	1	No
C	2	4	2	4	0	Yes
D	3	7	4	8	1	No
E	4	8	4	8	0	Yes
F	4	7	10	13	6	No
G	8	13	8	13	0	Yes
H	13	15	13	15	0	Yes

On the other hand, activities A, C, E, G and H have no slack time; this means that none of them can be delayed without delaying the entire project. Because of this, they are called *critical activities* and are said to be on the *critical path*. The General Foundry's critical path is shown in network form in Figure 12.6. The total project completion time (T), 15 weeks is seen as the largest number in the EF or LF columns of Table 12.3

Figure 12.6 General Foundry's Critical Path (A→C→E→G→H)



Probability of Project Completion

The **critical path analysis** helped us determine that the foundry's expected project completion time is 15 weeks. Harky knows, however, that if the project is not completed in 16 weeks, General Foundry will be forced to close by environmental controllers. He is also aware that there is significant variation in the time estimates for several activities. Variation in activities that are on critical path can affect overall project completion- possibly delaying it.

PERT uses the variance of critical path activities to help determine the variance of the overall project. If the activity times are statistically independent, the project variance is computed by summing the variances of the critical activities:

$$\text{Project variance} = \sum \text{variances of activities on the critical path} \qquad \text{....(12.6)}$$

From Table 12.2 we know that

CRITICAL ACTIVITY	VARIANCE
A	$^4/_{36}$
C	$^4/_{36}$
E	$^{36}/_{36}$
G	$^{64}/_{36}$
H	$^4/_{36}$

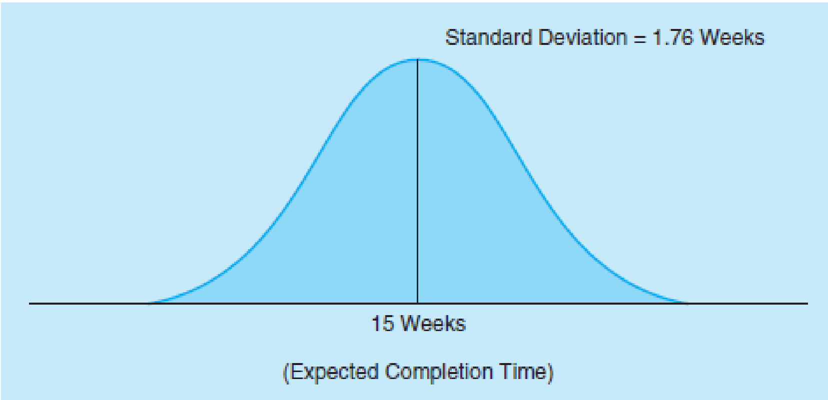
Hence, the Project Variance $= \frac{4}{36} + \frac{4}{36} + \frac{36}{36} + \frac{64}{36} + \frac{4}{36} = 3.111$

We know that the standard deviation is just the square root of the variance, so

$$\text{Project standard deviation, } \sigma_T = \sqrt{\text{Project variance}}$$
$$= \sqrt{3.111} = 1.76\text{weeks}$$

With the assumption that the activity times are independent and total project completion time follows a normal probability distribution, the bell-shaped curve shown in Figure 12.7 can be used to represent project completion dates. It also means that there is a 50% chance that the entire project will be completed in less than the expected 15 weeks and a 50% chance that it will exceed 15 weeks.

Figure 12.7 Probability Distribution for Project Completion Times



For Harky to find the probability that his project will be finished on or before the 16-week deadline, he needs to determine the appropriate area under the normal curve. The standard normal equation can be applied as follows:

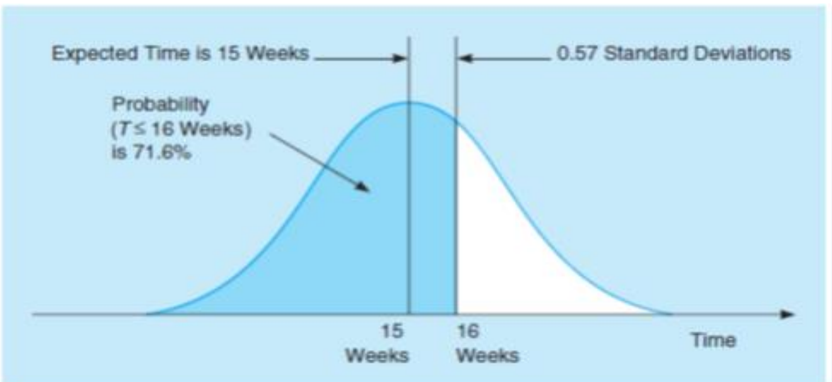
$$z = \frac{Due\ date - Expected\ date\ of\ completion}{\sigma_T} \qquad \text{....(12.7)}$$

$$= \frac{16-15}{1.76} = 0.57$$

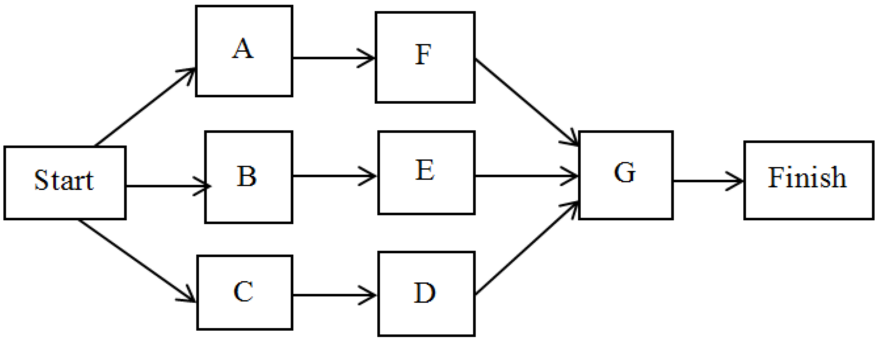
Where z is the no of standard deviations the due date lies from the mean or expected date.

Referring to the normal table, we find a probability of 0.71566. Thus, there is a 71.6% chance that the pollution control equipment can be put in place in 16 weeks or less. This is shown in Figure 12.8

Figure 12.8 Probability of General Foundry' Meeting the 16 Week Deadline



Example: Given the following project network and table.

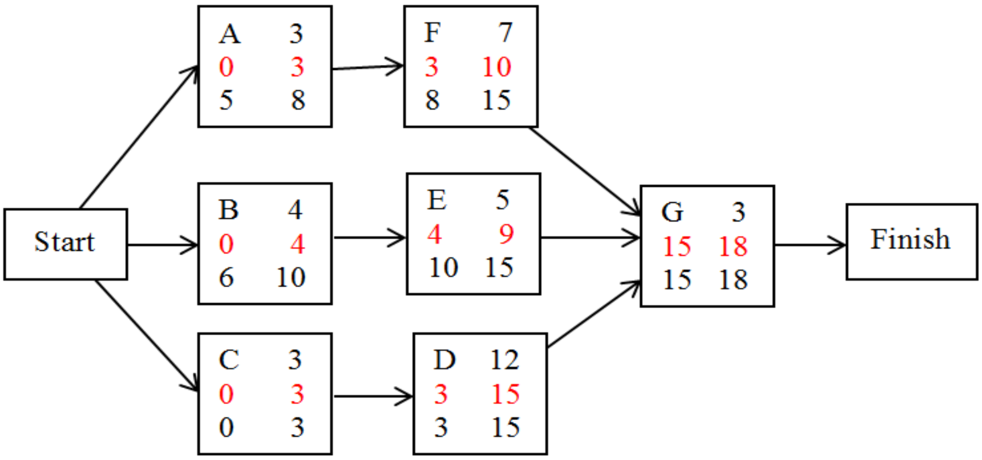


Activity	Estimated Activity Time (weeks)	Immediate Predecessors
A	3	-
B	4	-
C	3	-
D	12	C
E	5	B
F	7	A
G	3	D, E, F

a) Fill in the table to give the immediate predecessor(s) for each activity.
b) What will be the project's estimated completion time?
c) Identify Critical activities and the critical path.

Solution:

a. See the table



b. The project's estimated completion time =18

c. Since activities C, D and G have 0 slack, there C, D and G are critical activities. The path through these activities will be a critical path i.e. C → D → G

Example: In a project following are the activities' ES, LS, EF and LF times

Activity	Earliest start (ES)	Latest start (LS)	Earliest finish (EF)	Latest finish (LF)	Slack	Activities are on Critical path **Yes or No**
A	0	0	5	5		
B	0	6	6	12		
C	5	8	9	12		
D	5	7	8	10		
E	5	5	6	6		
F	6	6	10	10		
G	10	10	24	24		

a. Fill the blank space in the table
b. What are the critical activities?

Solution:

a.

Activity	Earliest start (ES)	Satest start (LS)	Earliest finish (EF)	Latest finish (LF)	Slack (LS-ES)	Activities are on Critical path **Yes or No**
A	0	0	5	5	0	Yes
B	0	6	6	12	6	No
C	5	8	9	12	3	No
D	5	7	8	10	2	No
E	5	5	6	6	0	Yes
F	6	6	10	10	0	Yes
G	10	10	24	24	0	Yes

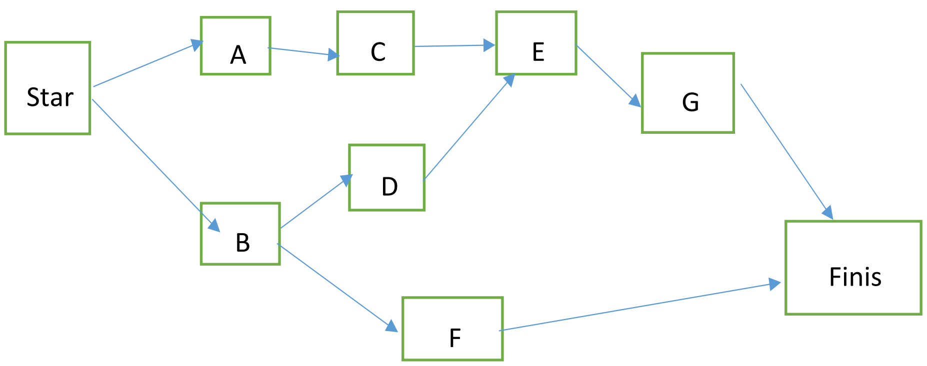
b. A, E, F and G are critical activities.

Example: Tom Schreiber, a director of personal of Management Resources, Inc., is in the process of designing a program that its customers can use in the job finding process. Some of the activities including preparing resume, writing letters, making appointments to see prospective employers, researching companies and industries, and so on. Some of the information on the activities is shown in the following table:

Activity	Immediate Predecessor	Optimistic time "a"	Most likely time "m"	Pessimistic time "b"
A	--	1	2	3
B	--	2	3	4
C	A	4	5	6
D	B	8	9	10
E	C, D	2	5	8
F	B	4	5	6
G	E	1	2	3

Draw the PERT network associated with the activity and calculate activities expected time.

Solution: The PERT diagram for the given problem is as follows:



Expected time(t) = $\frac{a+4m+b}{6}$

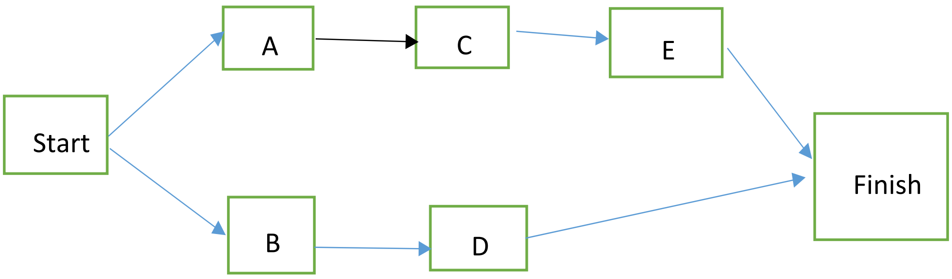
Activity	a	m	b	Expected time (t)
A	1	2	3	2
B	2	3	4	3
C	4	5	6	5
D	8	9	10	9
E	2	5	8	5
F	4	5	6	5
G	1	2	3	2

Example: Consider the following project schedule, the times are estimated and provided in the following table:

Activity	Immediate Predecessor	Optimistic time "a"	Most likely time"m"	Pessimistic time "b"
A	--	8	10	12
B	--	6	7	9
C	A	3	3	4
D	B	10	20	30
E	C	6	7	8

Draw the PERT and calculate the variances.

Solution: The PERT diagram for the given problem is as follows:



Variance = $(\frac{b-a}{6})^2$

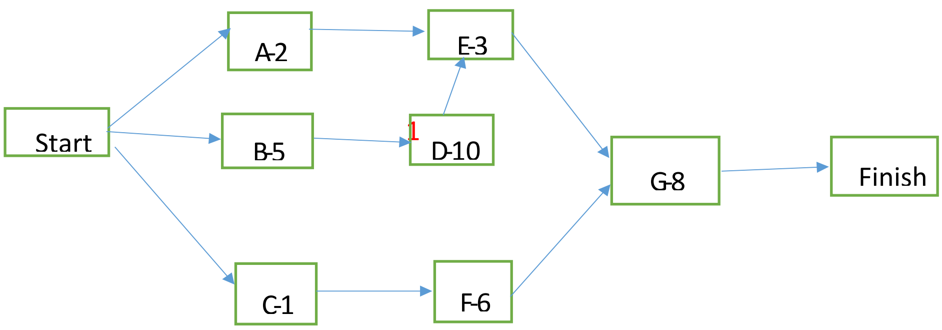
Activity	a	m	b	Variance
A	8	10	12	4/9
B	6	7	9	¼
C	3	3	4	1/36
D	10	20	30	100/9
E	6	7	8	1/9

Example: Sid Davidson is the personal director of Babson and Willcount, a company that specializes in consulting and research. One of the training programs that Sid is considering for the middle level managers of Babson and Willcount is leadership training. Sid has listed a number of activities that must be completed before a training program of this nature could be conducted. The activities, immediate predecessors and activity times appear in the following table :

Activity	Immediate predecessors	Time (days)
A	-	2
B	-	5
C	-	1
D	B	10
E	A,D	3
F	C	6
G	E,F	8

Construct the network for this problem and determine the earliest start, earliest finish, latest start, latest finish and slack time for each activity.

Solution: The PERT diagram for the given problem is as follows:



Activity	Earliest start time (ES)	Earliest finish time (EF)	Latest start time (LS)	Latest Finish time (LF)	Slack LS - ES
A	0	2	13	15	13
B	0	5	0	5	0
C	0	1	11	12	11
D	5	15	5	15	0
E	15	18	15	18	0
F	1	7	12	18	11
G	18	26	18	26	0

CHAPTER-14 SIMULATION MODELING

INTRODUCTION

We are all aware to some extent of the importance of simulation models in our world. Boeing Corporation and Airbus Industries, for example, commonly build **simulation** models of their proposed jet aircraft and then test the aerodynamic properties of the models. The U.S. Army simulates enemy attacks and defense strategies in war games played on a computer. Business students take courses that use management games to simulate realistic competitive business situations. And thousands of business, government and service organizations develop simulation models to assist in making decisions concerning inventory control, maintenance scheduling, plant layout, investments and sales forecasting.

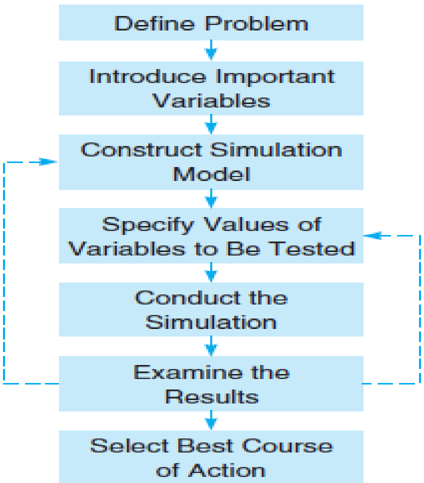
As a matter of fact, simulation is one of the most widely used quantitative analysis tools. Let's begin our discussion of simulation with a simple definition.

To *simulate* is to try to duplicate the features, appearance and characteristics of a real system. In this chapter, we show how to simulate a business or management system by building a *mathematical model* that comes as close as possible to representing the reality of the system. We won't build any physical models, as might be used in airplane wind tunnel simulation tests. But just as physical model airplanes are tested and modified under experimental conditions, our mathematical models are used to experiment and to estimate the effects of various actions. The idea behind simulation is to imitate a real-world situation mathematically, then to study its properties and operating characteristics and finally, to draw conclusions and make action decisions based on the results of the simulation.

Using simulation, a manager should:

1. Define a problem.
2. Introduce the variables associated with the problem.
3. Construct a simulation model.
4. Set up possible courses of action for testing.
5. Run the simulation experiment.
6. Consider the results.
7. Decide what courses of action to take.

These steps are illustrated in **Fig 14.1**



Advantages and Disadvantages of Simulation

The main advantages of simulation are:

1. It is relatively straightforward and flexible.
2. Recent advances in computer software make simulation models very easy to develop.
3. Can be used to analyze large and complex real-world situations.
4. Allows “what-if?” type questions.
5. Does not interfere with the real-world system.
6. Enables study of interactions between components.
7. Enables time compression.
8. Enables the inclusion of real-world complications.

The main disadvantages of simulation are:

1. It is often expensive as it may require a long, complicated process to develop the model.
2. It does not generate optimal solutions; it is a trial-and-error approach.
3. It requires managers to generate all conditions and constraints of real-world problem.
4. Each model is unique and the solutions and inferences are not usually transferable to other problems.

MONTE CARLO SIMULATION

When systems contain elements that exhibit chance in their behavior, the Monte Carlo method of simulation can be applied.

The basic idea in **Monte Carlo Simulation** is to generate values for the variables making up the model being studied. There are a lot of variables in real-world systems that are probabilistic in nature and that we might want to simulate. A few examples of these variables follow:

1. Inventory demand on a daily or weekly basis.
2. Lead time for inventory orders to arrive.
3. Times between machine breakdowns.
4. Times between arrivals at service facility.
5. Service times.
6. Times to complete project activities.
7. Number of employees absent from work each day.

Some of these variables, such as the daily demand and the number of employees absent, are discrete and must be integer valued. Other variables, such as those related to time, are continuous and are not required to be integers because time can be any value. When selecting a method to generate values for the random variable, this characteristic of the random variable should be considered.

The basis of Monte Carlo simulation is experimentation on the chance (or probabilistic) elements through random sampling. The technique breaks down into five simple steps:

Five Steps of Monte Carlo Simulation

1. Establishing a probability distribution for important variables.
2. Building a cumulative probability distribution for each variable.
3. Establishing an interval of random numbers for each variable.
4. Generating random numbers.
5. Actually simulating a series of trials.

We will examine each of these steps and illustrate them with the following example:

Harry’s Auto Tire Example

Harry’s Auto Tire sells all types of tires, but a popular radial tire accounts for a large portion of Harry’s overall sales. Recognizing that inventory costs can be quite significant with this product,
Harry wishes to determine a policy for managing this inventory. To see what the demand would look like over a period of time, he wants to simulate the daily demand for a number of days.

Step 1: Establishing Probability Distributions. One common way to establish a *probability distribution* for a given variable is to examine historical outcomes. The probability, or relative frequency, for each possible outcome of a variable is found by dividing the frequency of observation by the total no. of observations. The daily demand for radial tires at Harry’s Auto Tire over the past 200 days is shown in Table 14.1

DEMAND FOR TIRES	FREQUENCY (DAYS)	PROBABILITY OF OCCURRENCE
0	10	10/200 = 0.05
1	20	20/200 = 0.10
2	40	40/200 = 0.20
3	60	60/200 = 0.30
4	40	40/200 = 0.20
5	30	30/200 = 0.15
	200	200/200 = 1.00

Table 14.1 Historical Daily Demand for Radial Tires at Harry’s Auto Tire and Probability Distribution

We can convert these data to a probability distribution, if we assume that past demand rates will hold in the future, by dividing each demand frequency by the total demand, 200.

Probability distributions, we should note, need not be based solely on historical observations. Often, managerial estimates based on judgement and experience are used to create a distribution.

Step 2:Building a cumulative probability distribution for each variable. The Conversion from a regular probability distribution, such as in the right-hand column of Table 14.1, to a cumulative distribution is an easy job. A cumulative probability is the probability that a variable (demand) will be less than or equal to a particular value. A cumulative distribution lists all of the possible values and the probabilities, as shown in Table 14.2.

DAILY DEMAND	PROBABILITY	CUMULATIVE PROBABILITY
0	0.05	0.05
1	0.10	0.15
2	0.20	0.35
3	0.30	0.65
4	0.20	0.85
5	0.15	1.00

Table 14.2 Cumulative Probabilities for Radial Tires

The cumulative probability, graphed in Figure 14.2, is used in step 3 to help assign random numbers.

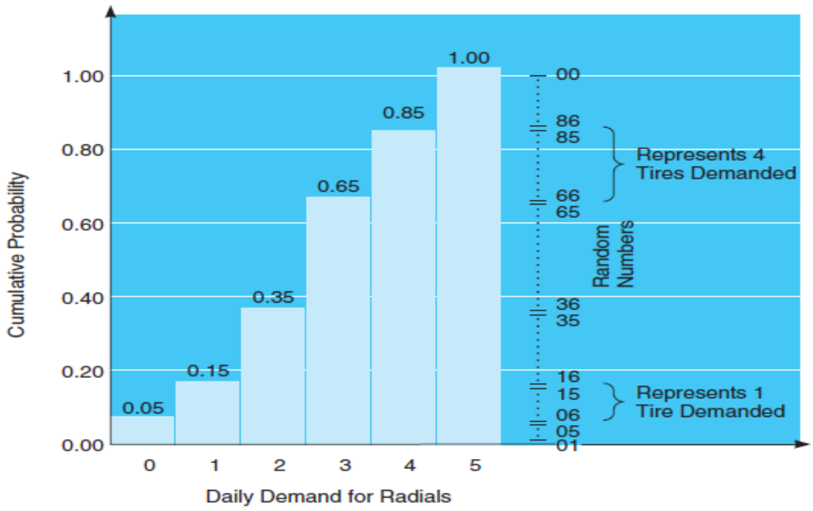


Figure 14.2 Graphical Representation of the Cumulative Probability Distribution for Radial Tires

Step 3: Setting random number intervals. After we have established a cumulative probability distribution for each variable included in the simulation, we must assign a set of numbers to represent each possible value or outcome. These are referred to as *random number intervals.* Basically, a *random number* is a series of digits (say, two digits from 01, 02, …..,98, 99, 00) that have been selected by a totally random process.

In general, using the cumulative probability distribution computed and graphed in step 2, we can set the interval of random numbers for each level of demand in a very simple fashion. You will note in Table 14.3 that the interval selected to represent each possible daily demand is very closely related to the cumulative probability on its left. The top end of each interval is always equal to the cumulative probability percentage.

DAILY DEMAND	PROBABILITY	CUMULATIVE PROBABILITY	INTERVAL OF RANDOM NUMBERS
0	0.05	0.05	01 to 05
1	0.10	0.15	06 to 15
2	0.20	0.35	16 to 35
3	0.30	0.65	36 to 65
4	0.20	0.85	66 to 85
5	0.15	1.00	86 to 00

Table 14.3 Assignment of Random Number Intervals for Harry's Auto Tire

Note: Alternatively, we could have assigned the random numbers 00, 01, 02, 03, 04 to represent a demand of 0 units. The two digits 00 can be thought of as either 0 or 100. As long as 5 numbers out of 100 are assigned to be 0 demand, it doesn't make any difference which 5 they are.

Similarly, we can see in Figure 14.2 and in Table 14.3 that the length of each interval on the right corresponds to the probability of one of each of the possible daily demands. Hence, in assigning random numbers to the daily demand for three radial tires, the range of the random number interval (36 to 65) corresponds exactly to the probability (or proportion) of that outcome. A daily demand for three radial tires occurs 30% of the time. Any of the 30 random numbers greater than 35 up to and including 65 are assigned to that event.

Step 4: Generating random numbers. Random numbers may be generated for simulation problems in several ways. If the problem is very large and the process being studied involves thousands of simulation trials, computer program are available to generate the random numbers needed.

If the simulation is being done by hand, the numbers may be selected by the spin of a roulette wheel that has 100 slots, by blindly grabbing numbered chips out of a hat. The most commonly used means is to choose numbers from a table of random digits such as Table 14.4

52	06	50	88	53	30	10	47	99	37	66	91	35	32	00	84	57	07
37	63	28	02	74	35	24	03	29	60	74	85	90	73	59	55	17	60
82	57	68	28	05	94	03	11	27	79	90	87	92	41	09	25	36	77
69	02	36	49	71	99	32	10	75	21	95	90	94	38	97	71	72	49
98	94	90	36	06	78	23	67	89	85	29	21	25	73	69	34	85	76
96	52	62	87	49	56	59	23	78	71	72	90	57	01	98	57	31	95
33	69	27	21	11	60	95	89	68	48	17	89	34	09	93	50	44	51
50	33	50	95	13	44	34	62	64	39	55	29	30	64	49	44	30	16
88	32	18	50	62	57	34	56	62	31	15	40	90	34	51	95	26	14
90	30	36	24	69	82	51	74	30	35	36	85	01	55	92	64	09	85
50	48	61	18	85	23	08	54	17	12	80	69	24	84	92	16	49	59
27	88	21	62	69	64	48	31	12	73	02	68	00	16	16	46	13	85
45	14	46	32	13	49	66	62	74	41	86	98	92	98	84	54	33	40
81	02	01	78	82	74	97	37	45	31	94	99	42	49	27	64	89	42
66	83	14	74	27	76	03	33	11	97	59	81	72	00	64	61	13	52
74	05	81	82	93	09	96	33	52	78	13	06	28	30	94	23	37	39
30	34	87	01	74	11	46	82	59	94	25	34	32	23	17	01	58	73
59	55	72	33	62	13	74	68	22	44	42	09	32	46	71	79	45	89
67	09	80	98	99	25	77	50	03	32	36	63	65	75	94	19	95	88
60	77	46	63	71	69	44	22	03	85	14	48	69	13	30	50	33	24
60	08	19	29	36	72	30	27	50	64	85	72	75	29	87	05	75	01
80	45	86	99	02	34	87	08	86	84	49	76	24	08	01	86	29	11
53	84	49	63	26	65	72	84	85	63	26	02	75	26	92	62	40	67
69	84	12	94	51	36	17	02	15	29	16	52	56	43	26	22	08	62
37	77	13	10	02	18	31	19	32	85	31	94	81	43	31	58	33	51

Table 14.4 Table of Random Numbers

Table 14.4 was itself generated by a computer program. It has the characteristic that every digit or number in it has an equal chance of occurring. Because *everything* is random, we can select numbers from anywhere in the table to use in our simulation procedures in step 5.

Step 5: Simulating the experiment. We can simulate outcomes of an experiment by simply selecting random numbers from Table 14.4. Beginning anywhere in the table, we note the interval in Table 14.4 or Figure 14.2 into which each number falls. For example, if the random number chosen is 81 and the interval 66 to 85 represents a daily demand for four tires, we select a demand of four tires.

We now illustrate the concept further by simulating 10 days of demand for radial tires at Harry's Auto Tire (see Table 14.5). We select the random numbers needed from Table 14.4, starting in the upper left-hand corner and continuing down the first column.

DAY	RANDOM NUMBER	SIMULATED DAILY DEMAND
1	52	3
2	37	3
3	82	4
4	69	4
5	98	5
6	96	5
7	33	2
8	50	3
9	88	5
10	90	5
		39 = total 10-day demand
		3.9 = average daily demand for tires

Table 14.5 Ten-Day Simulation of Demand for Radial Tires

It is interesting to note that the average demand of 3.9 tires in this 10-day simulation differs significantly from the expected daily demand, which we can compute from the data in Table 14.2

$$\text{Expected daily demand} = \sum_{i=0}^{5}(\text{Probability of } i \text{ tires}) \times (\text{Demand of } i \text{ tires})$$

$$= (0.05)(0) + (0.10)(1) + (0.20)(2) + (0.30)(3) + (0.20)(4) + (0.15)(5)$$
$$= 2.95 \text{ tyres}$$

If this simulation were repeated hundreds or thousands of times, it is much more likely that the average *simulated* demand would be nearly the same as the *expected* demand.

Naturally, it would be risky to draw any hard and fast conclusions regarding the operation of a firm from only a short simulation. However, this simulation by hand demonstrates the important principles involved. It helps us to understand the process of Monte Carlo simulation that is used in computerized simulation models.

The simulation for Harry's Auto Tire involved only one variable. The true power of simulation is seen when several random variables are involved and the situation is more complex. In Section 14.4, we see a simulation of an inventory problem in which both the demand and the lead time may vary.

Example: The demand for a STAT201 textbook from an online bookstore is observed to be the following during the historical data of last semesters:

Demand (per week)	Frequency
0	5
1	3
2	2
3	1
4	2
5	2
6	1
Total	16

a) Set up the probability and cumulative probability distribution for the textbook demand (round the cumulative probabilities to 2 decimal digits).
b) Establish random number intervals for the variable and calculate the average demand over 5 week period using the random numbers 15, 84, 23, 42, 67.

Solution:

Table 1 for Probability, Cumulative probability and Interval for Random Numbers

Demand (per week)	Frequency	Probability of occurrence	Cumulative probability	Interval for random numbers
0	5	0.3125	0.31	01 – 31
1	3	0.1875	0.50	32 – 50
2	2	0.1250	0.63	51 – 63
3	1	0.0625	0.69	64 – 69
4	2	0.1250	0.81	70 – 81
5	2	0.1250	0.94	82 – 94
6	1	0.0625	1	95 – 00
Total	16	1	–	–

Table 2 for Simulated Demand

Week	Random Number	Simulated Demand
1	15	0
2	84	5
3	23	0
4	42	1
5	67	3
		Total = 9

Average demand = Total / 5 weeks
=9/5 = 1.8 books per week

Example: The head of emergency department in a hospital wants to study the arrival of patients needing urgent care during the 48 hours of the weekends. The probability of the urgent patients' arrival per hour is observed to be the following:

No. of urgent patients arriving	Probability
0	0.35
1	0.30
2	0.15
3	0.15
4	0.05

a) Establish random number intervals for the variable of number of urgent patients arriving per hour during the weekends.
b) Simulate the arrival of urgent patients during 10 hours, using the following double digit random numbers: 52, 37, 82, 69, 98, 96, 33, 50, 88, and 90. Then compute the average simulated arrival rate.

Solution:

Table 1 for Probability, Cumulative probability and Interval for Random Numbers

No. of urgent patients arriving	Probability	Cumulative Probability	Intervals of Random Numbers
0	0.35	0.35	01- 35
1	0.30	0.65	36- 65
2	0.15	0.80	66- 80
3	0.15	0.95	81- 95
4	0.05	1.00	96- 00

Table 2 for Simulated Arrivals

Hour	Random Number	Simulated Arrivals
1	52	1
2	37	1
3	82	3
4	69	2
5	98	4
6	96	4
7	33	0
8	50	1
9	88	3
10	90	3
		Total = 17

Average arrival= 17/10 = 1.7

Example: A grocery store has only one checkout counter. Customers arrive at this checkout at random from 1 to 8 minutes apart each possible value of inter-arrival time has the same probability of occurrence as given below. Analyze the system by simulating the arrival of 20 customers using the random numbers 913, 727, 15, 948, 309, 922, 753, 235, 302, 109, 93, 607, 738, 359, 888, 106, 212, 493 and 535. Also, calculate the average time between arrival.

Distribution of time between arrivals

Time Arrivals (Min.)	Probability
1	0.125
2	0.125
3	0.125
4	0.125
5	0.125
6	0.125
7	0.125
8	0.125

Solution: **Distribution of time between arrivals**

Time arrivals (Minutes)	Probability	Cumulative Probability	Random Digits Assignments
1	0.125	0.125	001- 125
2	0.125	0.250	126- 250
3	0.125	0.375	251- 375
4	0.125	0.500	376- 500
5	0.125	0.625	501- 625
6	0.125	0.750	626- 750
7	0.125	0.875	751- 875
8	0.125	1.00	876- 000

Time between arrival determinations:

Customer	Random Digits	Time between arrivals (minutes)
1	---	---
2	913	8
3	727	6
4	015	1
5	948	8
6	309	3
7	922	8
8	753	7
9	235	2
10	302	3
11	109	1
12	093	1
13	607	5
14	738	6
15	359	3
16	888	8
17	106	1
18	212	2
19	493	4
20	535	5
		82

Average time between arrivals $= \frac{82}{20} = 4.1$ minutes