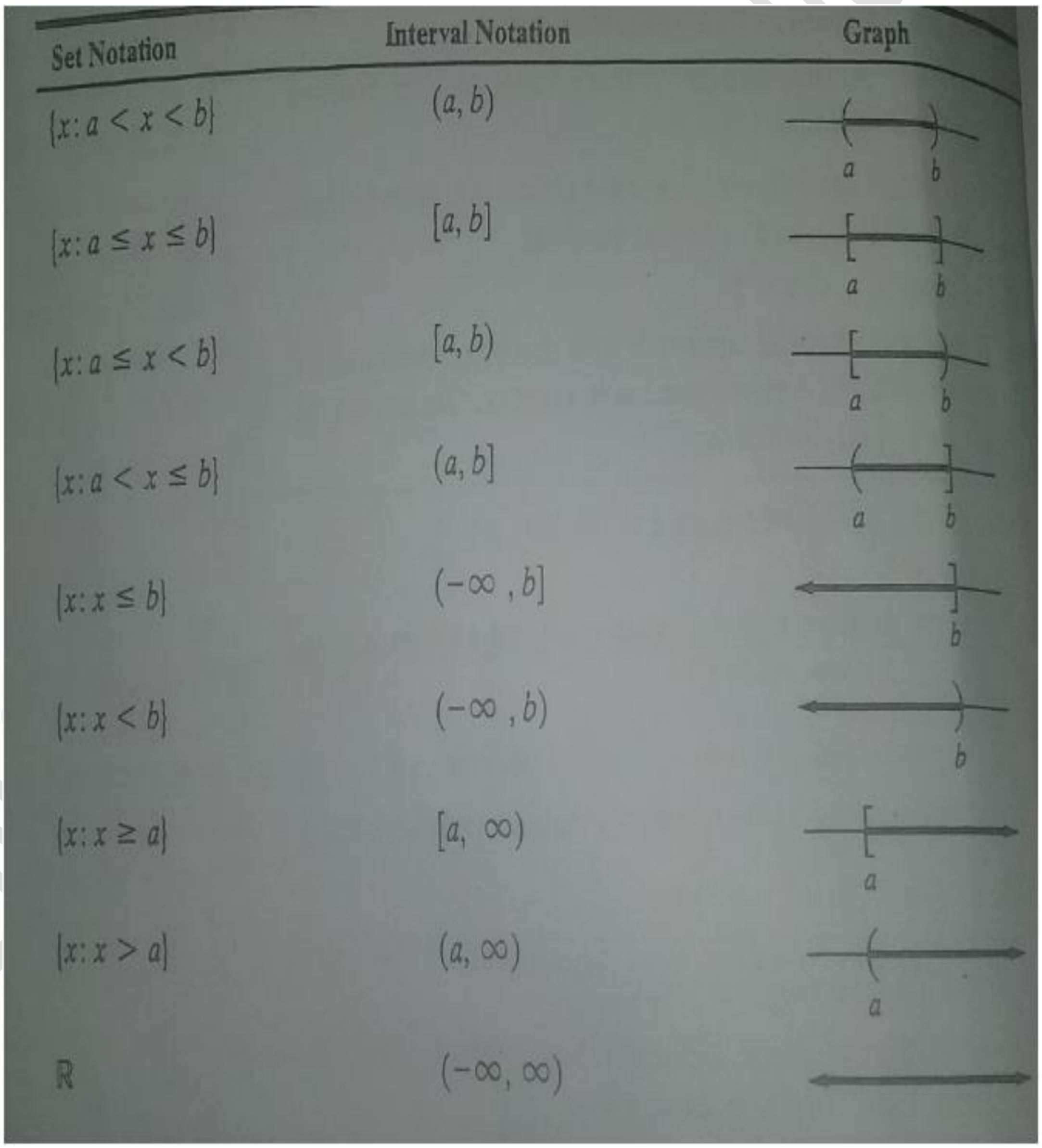
ENG. Mahdi Alsiddig
0563938879

0.2 Inequalities and Absolute Values

- Intervals:



Set Notation	Interval Notation	Graph
$\{x: a < x < b\}$	(a, b)	a b
$\{x: a \le x \le b\}$	$[a, b]$	a b
$\{x: a \le x < b\}$	$[a, b)$	a b
$\{x: a < x \le b\}$	$(a, b]$	a b
$\{x: x \le b\}$	$(-\infty, b]$	b
$\{x: x < b\}$	$(-\infty, b)$	b
$\{x: x \ge a\}$	$[a, \infty)$	a
$\{x: x > a\}$	(a, ∞)	a
$\mathbb{R}$	$(-\infty, \infty)$	

ENG. Mahdi Alsiddig
0563938879

- **Inequalities :**

Example 1 : solve the inequality : $2x - 7 < 4x - 2$

Solution

$$2x - 7 < 4x - 2$$

$$2x - 4x < -2 + 7$$

$$-2x < 5$$

$$\frac{-2x}{-2} < \frac{5}{-2}$$

$$x > -\frac{5}{2}$$

$\therefore$ ***The solution set :*** $\left(-\frac{5}{2}, \infty\right)$

Example 2 : solve the inequality: $-5 \leq 2x + 6 < 4$

Solution

$$-5 - 6 \leq 2x + 6 - 6 < 4 - 6$$

$$-11 \leq 2x < -2$$

$$\frac{-11}{2} \leq \frac{2x}{2} < \frac{-2}{2}$$

$$\frac{-11}{2} \leq x < -1$$

$\therefore$ ***The solution set :*** $\left[-\frac{11}{2}, -1\right)$

➢ حل معادلات من الدرجة الثانية:

$$a\,x^2 + b\,x + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 = \frac{-b + \sqrt{b^2-4ac}}{2a} \quad , \quad x_2 = \frac{-b - \sqrt{b^2-4ac}}{2a}$$

Example 3 : solve the quadratic inequality: $x^2 - x < 6$

Solution

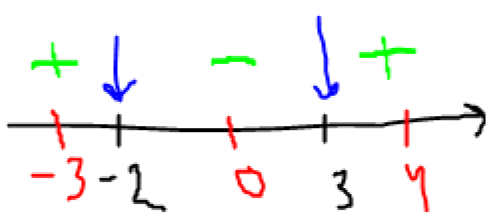
$$x^2 - x - 6 < 0$$

$$a = 1 , \quad b = -1, \quad c = -6$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+24}}{2}$$

$$x_1 = \frac{1+\sqrt{1+24}}{2} = \frac{1+\sqrt{25}}{2} = \frac{1+5}{2} = \frac{6}{2} = \mathbf{3}$$

$$x_2 = \frac{1-\sqrt{1+24}}{2} = \frac{1-\sqrt{25}}{2} = \frac{1-5}{2} = \frac{-4}{2} = -\mathbf{2}$$



نأخذ الفترة السالبة؛ لأن الاشارة في السؤال هي اصغر من الصفر

$\therefore$ ***The solutoin set*** : $(-2\,,3)$

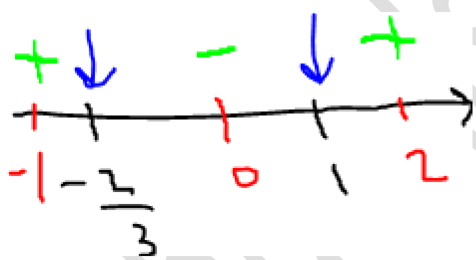
Example 4 : solve $3x^2 - x - 2 > 0$

$$a = 3\,,\, b = -1\,,\, c = -2$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+24}}{6}$$

$$x_1 = \frac{1+\sqrt{1+24}}{6} = \frac{1+\sqrt{24}}{6} = \frac{1+5}{6} = \frac{6}{6} = \mathbf{1}$$

$$x_2 = \frac{1-\sqrt{1+24}}{6} = \frac{1-\sqrt{24}}{6} = \frac{1-5}{6} = \frac{-4}{6} = -\frac{\mathbf{2}}{\mathbf{3}}$$

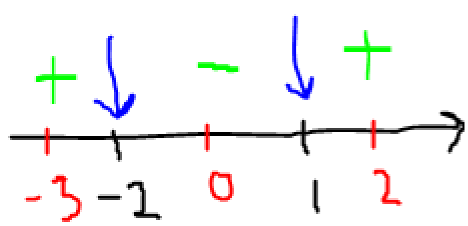


نأخذ الفترات الموجبة ؛ لأن الاشارة في السؤال هي اكبر من الصفر

$\therefore$ ***The solutoin set :*** $\left(-\infty\,,-\frac{\mathbf{2}}{\mathbf{3}}\right) \cup (\,\mathbf{1}, \infty\,)$

Example 5 : solve $\frac{x-1}{x+2} \geq 0$

Solution

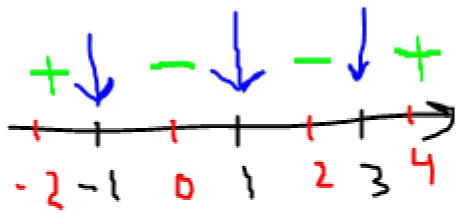


$\therefore$ ***The solution set :*** $(\,-\infty, -\mathbf{2}\,) \cup [\mathbf{1}, \infty)$

ENG. Mahdi Alsiddig
0563938879

Example 6 : solve $(x+1)(x-1)^2(x-3) \leq 0$

Solution



$\therefore$ ***The solution set*** : $[-1, 1] \cup [1, 3]$ ***or*** $[-1, 3]$

- **Absolute value** (Inequalities involving Absolute Values)

$$|u| < a \rightarrow -a < u < a$$
$$|u| > a \rightarrow u < -a \quad or \quad u > a$$

Example 8 : solve the inequality: $|x-4| < 2$

Solution

$$|x-4| < 2 = -2 < x-4 < 2$$
$$-2+4 < x-4+4 < 2+4$$
$$2 < x < 6$$

$\therefore$ ***The solution set*** : $(2, 6)$

Example 9 : solve the inequality: $|3x-5| \geq 1$

Solution

$$3x-5 \leq -1 \quad or \quad 3x-5 \geq 1$$
$$3x \leq 4 \quad or \quad 3x \geq 6$$
$$x \leq \frac{4}{3} \quad or \quad x \geq 2$$

$\therefore$ *The solution set* : $\left(-\infty, \frac{4}{3}\right] \cup [2, \infty)$

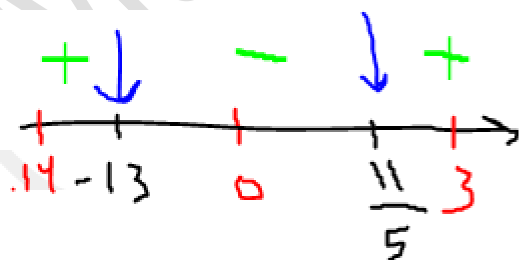
$$|u| < |z| \;\rightarrow\; u^2 < z^2$$

Example 9 : solve the inequality: $|3x+1| < 2|x-6|$

Solution

$$|3x+1| < |2x-12|$$
$$(3x+1)^2 < (2x-12)^2$$
$$9x^2+6x+1 < 4x^2-48x+144$$
$$9x^2+6x+1-4x^2+48x-144 < 0$$
$$5x^2+54x-143 < 0$$

$$x_1 = \frac{-54+\sqrt{2916+2860}}{10} = \frac{-54+\sqrt{5776}}{10} = \frac{-54+76}{10} = \frac{22}{10} = \mathbf{\frac{11}{5}}$$
$$x_2 = \frac{-54-\sqrt{2916+2860}}{10} = \frac{-54-\sqrt{5776}}{10} = \frac{-54-76}{10} = \frac{-130}{10} = \mathbf{-13}$$



$\therefore$ ***The solution set :*** $\left(-13, \frac{11}{5}\right)$