Let $[\Omega, \mathcal{A}, P]$ be the probability space with $\Omega = \{a, b, c, d, e\}, \mathcal{A} = 2^{\Omega}, P(A) = \frac{|A|}{|\Omega|}, \forall A \in \mathcal{A}$, and we consider X as a map on Ω defined as follow:

$$X: \Omega \longrightarrow \mathbb{R}, \quad \omega \longmapsto X(\omega) = \begin{cases} -5 & for\ \omega = d \\ 1 & for\ \omega = a, c \\ 0 & otherwise \end{cases}$$

Then:

a) Determine Ω^* the values set of X.

$\Omega^* = \{-5, 0, 1\}$

b) Complete the relation:

$$\{\boldsymbol{\omega} \in \boldsymbol{\Omega} |\ \boldsymbol{X}(\boldsymbol{\omega}) \leq \boldsymbol{x}\} = \begin{cases} \emptyset & for\ x < -5 \\ \{d\} & for\ -5 \leq x < 0 \\ \{b, d, e\} & for\ 0 \leq x < 1 \\ \Omega & for\ x \geq 1 \end{cases}$$

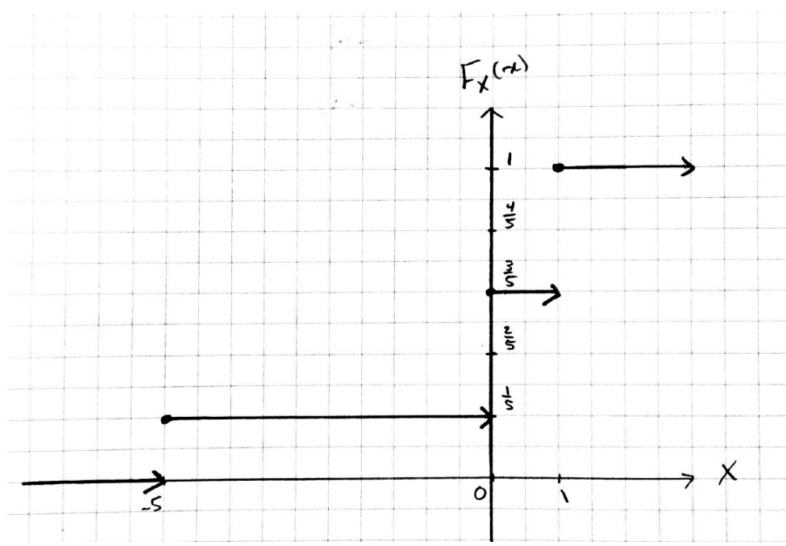
c) Is given map X is a random variable on $[\boldsymbol{\Omega}, \boldsymbol{\mathcal{A}}, \boldsymbol{P}]$, and why?

Yes, it is because $X^{-1}(B) \in 2^{\Omega}, \forall B \in \mathfrak{R}$ **<u>or</u>** each $\emptyset, \{d\}, \{b, d, e\}, \Omega \in 2^{\Omega}$ **<u>or</u>**

$$X^{-1}(\{-5\}) = \{d\} \in 2^{\Omega}$$
$$X^{-1}(\{1\}) = \{a, c\} \in 2^{\Omega}$$
$$X^{-1}(\{0\}) = \{b, e\} \in 2^{\Omega}$$

d) Determine the distribution F_X and draw its graph.

$$F_X(x) = P(\{\omega \in \Omega |\ X(\omega) \leq x\}) = \begin{cases} 0 & for\ x < -5 \\ \frac{1}{5} & for\ -5 \leq x < 0 \\ \frac{3}{5} & for\ 0 \leq x < 1 \\ 1 & for\ x \geq 1 \end{cases}$$



e) Calculate $P(X = -3), P(X = 0)$ *and* $P(X \leq 7)$.

$$P(X = -3) = P(\{\omega \in \Omega | \, X(\omega) = -3\}) = P(\emptyset) = 0$$
$$P(X = 0) = P(\{\omega \in \Omega | \, X(\omega) = 0\}) = P(\{b, e\}) = \frac{2}{5}$$
$$P(X \leq 7) = P(\{\omega \in \Omega | \, X(\omega) \leq 7\}) = P(X = 1) + P(X = 0) + P(X = -5) = 1$$

f) As studied in this course, is this random variable of famous random variables (has a special name)? If yes, what is it?

No, it is not famous.

g) Calculate $E(X), E(X^2)$ and the variance of X.

$$E(X) = \sum x_i \cdot p_i = x_1 \cdot p_1 + x_2 \cdot p_2 + x_3 \cdot p_3$$
$$= (-5) \cdot \left(\frac{1}{5}\right) + (1) \cdot \left(\frac{2}{5}\right) + 0 \cdot \left(\frac{2}{5}\right) = -\frac{3}{5}$$

$$E(X^2) = \sum {x_i}^2 \cdot p_i = {x_1}^2 \cdot p_1 + {x_2}^2 \cdot p_2 + {x_3}^2 \cdot p_3$$
$$= (-5)^2 \cdot \left(\frac{1}{5}\right) + (1)^2 \cdot \left(\frac{2}{5}\right) + 0^2 \cdot \left(\frac{2}{5}\right) = \frac{27}{5}$$

$$Var(X) = \mathrm{E}(\mathrm{X}^2) - [E(X)]^2 = \frac{27}{5} - \left(-\frac{3}{5}\right)^2 = \frac{126}{25}$$