مدونة المناهج السعودية

https://eduschool40.blog

الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

Math 101

Nada
Altiary

Continuity on open interval

Definition:

A function f is continuous on an open interval (a,b) if it is continuous at each point in the interval.

Remark:- A function f that is continuous on the entire line $(-\infty,\infty)$ is everywhere continuous.

Theorem: The following types of function are continuous at every point in their domains.

Functions الدوال	Forms الشكل	Domain المجال
Polynomial functions	$p(x)=a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0$	D=R
Rational functions	$r(x)=\frac{p(x)}{q(x)}$, $p(x)$ and ,$q(x)$ are polynomials.	D= R-{أصفار المقام}
Radical functions	$f(x)=\sqrt[n]{x}$ n: even	D=ماتحت الجذر≥0
	n: odd	D=R
Trigonometric functions	$\sin x$, $\cos x$, $\tan x$, $\sec x$, $\csc x$, $\cot x$	D=R
Exponential functions	e^x, a^x $a>0$	D= R
Logarithmic functions	$\ln x$, $\log_a x$	D= مابداخل الدالة>0

Example 1: Find the intervals in which each the following function is continuous.

1)- $F(x) = x^2 + 2x + 1$

$F(x)$ is continuous on $\mathbb{R} = (-\infty, \infty)$

2)- $F(x) = \dfrac{x}{x^2 - 6x + 9}$

$F(x)$ is continuous on $\mathbb{R} - \{$أصفار المقام$\}$

$\therefore \quad x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$\Rightarrow \quad x = 3$

$\therefore$ $f(x)$ is continuous on $\mathbb{R} - \{3\} = (-\infty, 3) \cup (3, \infty)$.

3)- $f(x) = \sqrt[5]{x+2}$

$F(x)$ is continuous on $\mathbb{R}$.

4)- $F(x) = \sqrt{x(x-1)}$

F is continuous iff $x(x-1) \geq 0$

$\Rightarrow x \geq 0$ or $x - 1 \geq 0$

$\Rightarrow x \geq 1$

5)- $F(x) = \ln(x+4)$

F is continuous iff $x + 4 > 0$

$x > -4$

$\therefore F$ is continuous on $(-4, \infty)$

6)- $f(x) = \sqrt[4]{x+7}$

f is continuous iff $x+7 \geq 0$

$x \geq -7$

$\therefore$ f is continuous iff $[-7, \infty)$

7)- $F(x) = \dfrac{1}{x^2+1}$

F is continuous on R. why?!

8)- $F(x) = \dfrac{x^2+x-12}{x^2-3x}$

F is continuous iff $x^2 - 3x \neq 0$

$x(x-3) \neq 0$

$\Rightarrow x \neq 0$ or $x - 3 \neq 0$

$x = 3$

$\therefore$ F is continuous on $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

9)- $f(x) = \sqrt{x^2+25}$

f is continuous iff $x^2 + 25 \geq 0$

10) $f(x) = \sin(x^2-4)$

Continuity on a closed interval

A function f is continuous on the closed interval $[a,b]$ iff

1)- f continuous on (a,b).

2) $\lim_{x \to a^+} f(x) = f(a)$

3)- $\lim_{x \to b^-} f(x) = f(b)$

Example 1: Discuss the continuity of

a)- $f(x) = \sqrt{25 - x^2}$

Remember:

$x^2 + y^2 = r^2$

$y = \sqrt{25 - x^2}$

$y^2 = 25 - x^2$

$x^2 + y^2 = 25$

f is continuous if $25 - x^2 \geq 0$

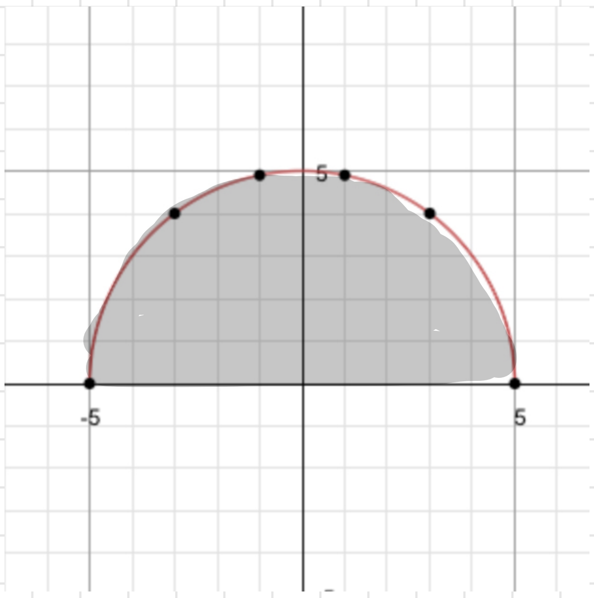
$(5-x)(5+x) \geq 0$

$\Rightarrow 5 - x \geq 0$ or $5 + x \geq 0$

$\Rightarrow 5 \geq x$ or $x \geq -5$

$\Rightarrow x \leq 5$ or $x \geq -5$

$\Rightarrow -5 \leq x \leq 5$



$\therefore D(f) = [-5, 5]$

1)- f is continuous on $(-5, 5)$

2)- $\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} \sqrt{25 - x^2} = \sqrt{25 - x^2} = 0 = f(5)$

3)- $\lim_{x \to -5^+} f(x) = \lim_{x \to -5^+} \sqrt{25 - x^2} = \sqrt{25 - 25} = 0 = f(-5)$

$\therefore f$ is continuous on $[-5, 5]$

b)- $g(x) = \sqrt{1 - x^2}$ H.W

One sided continuity

Right and left continuity:

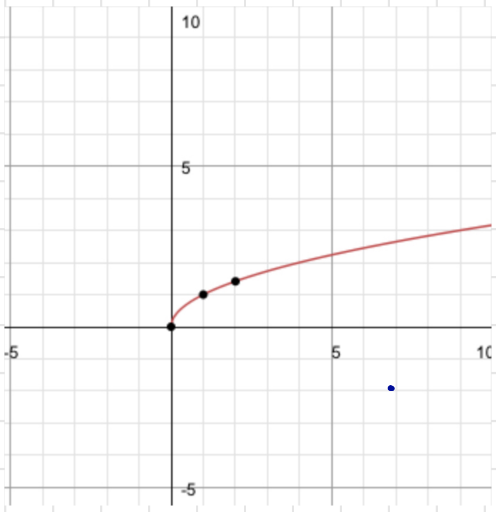
- A function f is continuous from the right at a if
$$\lim_{x \to a^+} f(x) = f(a)$$

- A function f is continuous from the left at a if
$$\lim_{x \to a^-} f(x) = f(a)$$

Example 1: Discuss the continuity of the following function

your Note:

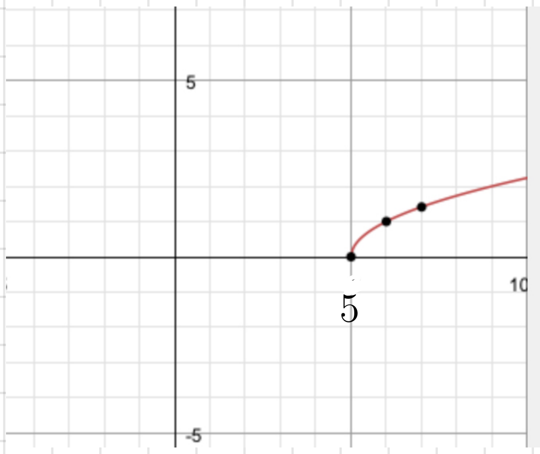
$f(x) = \sqrt{x}$



f is continuous iff $x \geq 0$

$\therefore f$ is continuous on $[0, \infty)$

$g(x) = \sqrt{x-5}$

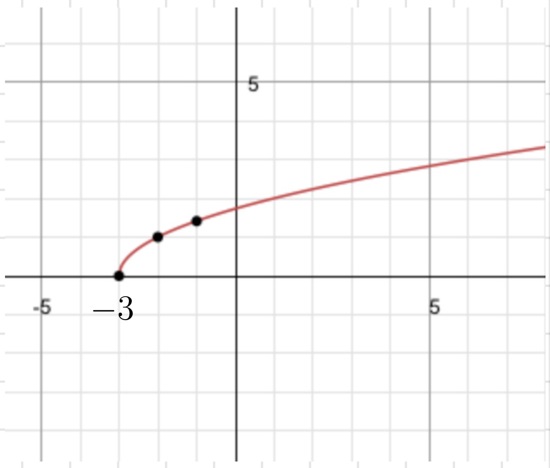


f is continuous iff $x - 5 \geq 0$

$\Rightarrow x \geq 5$

$\therefore f$ is continuous on $[5, \infty)$

$h(x) = \sqrt{x+3}$



f is continuous iff $x + 3 \geq 0$

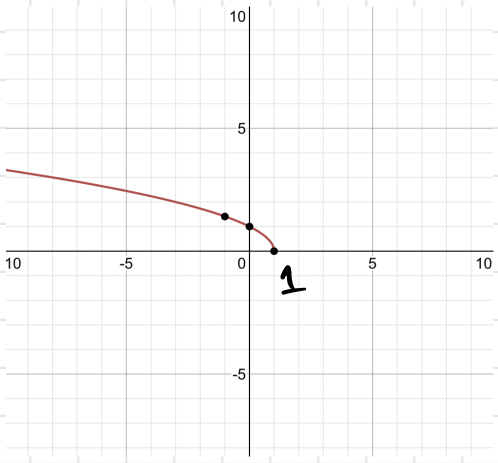
$x \geq -3$

$\therefore f$ is continuous on $[-3, \infty)$

Example 2: Discuss the continuity of the following function

Your Note:

$f(x) = \sqrt{1-x}$

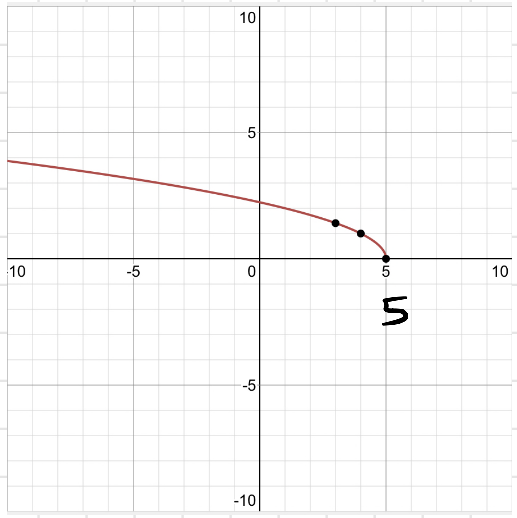


f is continuous iff $1-x \geq 0$

$\Rightarrow 1 \geq x$

$\therefore f$ is continuous on $(-\infty, 1]$

$g(x) = \sqrt{5-x}$

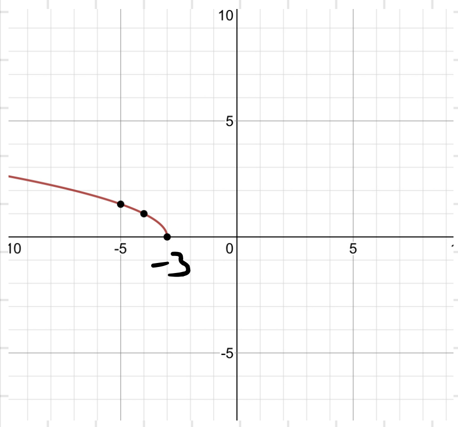


f is continuous iff $5-x \geq 0$

$\Rightarrow 5 \geq x$ or $x \leq 5$

$\therefore f$ is continuous on $(-\infty, 5]$

$h(x) = \sqrt{-3-x}$



f is continuous iff $-3-x \geq 0$

$\Rightarrow -3 \geq x$ or $x \leq -3$

$\therefore f$ is continuous on $(-\infty, -3]$

Example 3: Discuss the continuity of the following function at the given number

Your Note:

$f(x) = \sqrt{x}$ at $a=0$

1)_ $f(0) = \sqrt{0} = 0$

2)_ $\lim_{x \to 0^+} f(x) = \lim_{x \to 0} \sqrt{x}$

$= \sqrt{0} = 0$

3)_ $\because \lim_{x \to 0^+} f(x) = f(0)$

$\therefore f$ is continuous from the right

Example 4: Greatest integer function

The greatest integer function $[x]$ is the largest integer less than or equal to x.

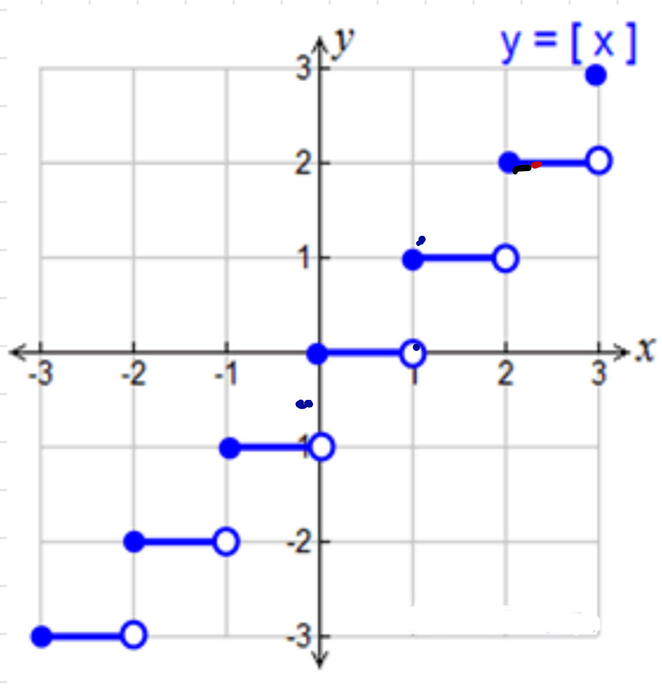
$$[x] = n \iff n \leq x < n+1$$

$[2.9] = 2 \qquad [0] = 0 \qquad [1.4] = 1 \qquad [3] = 3$

$[-2.51] = -3 \qquad [-0.5] = -1 \qquad [-1.01] = -2 \qquad [-2] = -2$

$\lim_{x\to 1^-} [x] = 0$	$\lim_{x\to 1} [x] =$ DNE	$\lim_{x\to 1^+} [x] = 1$
$\lim_{x\to 2^-} [x] = 1$	$\lim_{x\to 2} [x]$ DNE	$\lim_{x\to 2^+} [x] = 2$
$\lim_{x\to 3^-} [x] = 2$	$\lim_{x\to 3} [x]$ DNE	$\lim_{x\to 3^+} [x] = 3$
$\lim_{x\to n^-} [x] = n-1$	$\lim_{x\to n} [x]$ DNE	$\lim_{x\to n^+} [x] = n$

Discuss the continuity of $g(x) = [x]$ at $a = n$ عدد صحيح



$g(n) = [n] = n$

$$\left.\begin{array}{l} \lim_{x\to n^+} [x] = n \\ \lim_{x\to n^-} [x] = n-1 \end{array}\right\} \text{Exist but not equal}$$

$\therefore$ g is discontinuous at $a = n$ or g is continuous from the right

$\therefore$ g has jump discontinuity.

Remark:-

1)- There is a jump at each integer and so

$$\lim_{x \to n^+} [x] \neq \lim_{x \to n^-} [x]$$

2)- What about if a is not integer i.e $a = 1.5$

💡 Does $g(x) = [x]$ is continuous at $a = 1.5$

Theorem 2.4.1: ***[Properties of Continuity]***
If f and g are continuous function at a and k is any real number, then the following functions are continuous at a.

1. Sum and Difference: $f \pm g$
2. Product: fg
3. Quotient: $\dfrac{f}{g}$ provided $g(a) \neq 0$
4. Constant multiple: kf.