KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1431/1432)

Solution Final Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50 Time: Three hours

Marks:

Multiple Choice (1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$\{a,b,c,d\}$	d	d	c	a	d	b	b	d	b	a	c	c	d	a	a	b	a	d	a	d

Q. No: 1 If $\sum_{k=3}^{5}(k+\alpha)=6$, then the value of α is equal to:

(a) $\frac{-9}{5}$ (b) $\frac{9}{5}$ (c) 2 (d) -2

Q. No: 2 The average value of the function $f(x)=\sin x\cos x$ on $[-\pi,\pi]$ is equal to:

(a) $\frac{1}{2\pi}$ (b) $\frac{1}{\pi}$ (c) 1 (d) 0

Q. No: 3 The integral $\int \tan(2x)\,dx$ is equal to:

(a) $\frac{-1}{2}\ln|\sec(2x)|+c$ (b) $\frac{1}{2}\sec^2(2x)+c$ (c) $\frac{-1}{2}\ln|\cos(2x)|+c$ (d) $2\sec^2(2x)+c$

Q. No: 4 If $F(x)=\displaystyle\int_1^{x^2}\ln(t^2)\,dt$, then $F'(2)$ is equal to:

(a) $4\ln 16$ (b) $2\ln 16$ (c) $4\ln 8$ (d) $2\ln 8$

Q. No: 5 The integral $\int \ln\left(2^{\sin x}\right)dx$ is equal to:

(a) $\frac{1}{2}\ln(2)\sin x+c$ (b) $2^{-\sin x}\cos x+c$ (c) $-\sin x+c$ (d) $-(\cos x)\ln 2+c$

Q. No: 6 If $f(x)=\tanh^{-1}(\cos(3x))$ then $f'(x)$ is equal to:

(a) $3\csc(3x)$ (b) $-3\csc(3x)$ (c) $\frac{-3\sin(3x)}{1+\cos^2(3x)}$ (d) 0

Q. No: 7 $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$ is equal to:

(a) ∞ (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 0

Q. No: 8 To evaluate the integral $\displaystyle\int \frac{1}{\sqrt[3]{x}-\sqrt{x}}dx$, we use the substitution:

(a) $u=\sqrt{x}$ (b) $u=\sqrt[4]{x}$ (c) $x=\sqrt[6]{u}$ (d) $u=\sqrt[6]{x}$

Q. No: 9 The integral $\displaystyle\int \frac{\cos x}{1+\sin^2(x)}dx$ is equal to:

(a) $\frac{1}{1+\sin x}+c$ (b) $\tan^{-1}(\sin x)+c$ (c) $\frac{1}{1+\cos x}+c$ (d) $\tanh^{-1}(\sin x)+c$

Q. No: 10 To evaluate the integral $\displaystyle\int \frac{1}{x^4\sqrt{x^2-7}}dx$, we use the substitution:

(a) $x=\sqrt{7}\sec\theta$ (b) $x=7\sec\theta$ (c) $x=7\tan\theta$ (d) $x=\sqrt{7}\tan\theta$

Q. No: 11 The partial fraction decomposition of $\dfrac{x-1}{x^2\left(x^2+1\right)}$ takes the form:

(a) $\frac{A}{x}+\frac{Bx+C}{(x^2+1)}$ (b) $\frac{A}{x}+\frac{B}{(x^2+1)}$ (c) $\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+1}$ (d) $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^2+1}$

Q. No: 12 The improper integral $\displaystyle\int_0^{\pi/2}\frac{\cos x}{\sqrt{\sin x}}dx$

(a) converges to -2 (b) converges to 1 (c) converges to 2 (d) Diverges

Q. No: 13 The area of the region bounded by the graphs of the functions $y=x^2$, $y=2-x^2$ is equal to:

(a) 2 (b) 4 (c) $\frac{3}{8}$ (d) $\frac{8}{3}$

Q. No: 14 The slope of the tangent line at the point corresponding to $t=1$ on the curve given parametrically by the equations $x=2t^3+1$, $y=5t^3-1$, $-2\le t\le 2$ is:

(a) $\frac{5}{2}$ (b) $-\frac{5}{2}$ (c) $\frac{2}{5}$ (d) $-\frac{2}{5}$

Q. No: 15 If a graph has polar equation $r=2\sec\theta$, then its equation in $xy-$system is:

(a) $x=2$ (b) $y=2$ (c) $x+y+1=0$ (d) $y=\frac{1}{2}$

Q. No: 16 The length of the curve $C: x=\cos(2t)$, $y=\sin(2t)$, $0\le t\le\pi$ is equal to:

(a) 2 (b) 2π (c) π (d) 4π

Q. No: 17 The surface area resulting by revolving the graph of the parametric equation $x=3t$, $y=3t$, $0\le t\le 1$ around the $x-$axis is equal to:

(a) $9\sqrt{2}\pi$ (b) $18\sqrt{2}\pi$ (c) $24\sqrt{2}\pi$ (d) $\frac{9}{2}\sqrt{2}\pi$

Q. No: 18 If a point has $xy-$coordinates $(x,y)=(1,1)$ then one of its $(r,\theta)-$coordinates is:

(a) $\left(1,\dfrac{\pi}{2}\right)$ (b) $\left(-1,\dfrac{5\pi}{4}\right)$ (c) $\left(2,\dfrac{\pi}{4}\right)$ (d) $\left(\sqrt{2},\dfrac{\pi}{4}\right)$

Q. No: 19 The slope of the tangent line to the graph of the equation $r=2$ at $\theta=-\dfrac{\pi}{4}$ is:

(a) 1 (b) -1 (c) 0 (d) ∞

Q. No: 20 The graph of the curve C defined by the parametric equations $x=2+\cos(2t)$; $y=-1+\sin(2t)$, $0\le t\le\pi$ is a:

(a) line (b) parabola (c) cardioids (d) circle

Full Questions

Question No: 21 Approximate the integral $\int_0^4 \sqrt{x^3+1}dx$ using the **Simpson's rule** with $n = 4$. [**4**]

Solution:

Let $f(x) = \sqrt{x^3+1}$.

$\Delta x = \frac{4}{4} = 1$

$x_0 = 0, \ \ x_1 = 1, \ \ x_2 = 2, \ \ x_3 = 3 \ \text{ and } \ x_4 = 4.$ (1)

$$\begin{aligned}\int_0^4 \sqrt{x^3+1}dx &\approx \tfrac{4-0}{3\times 4}\{f(0)+4f(1)+2f(2)+4f(3)+f(4)\} && (2)\\ &= \tfrac{1}{3}\left\{1+4\left(\sqrt{2}\right)+2(3)+4(\sqrt{28})+\sqrt{65}\right\} && (1)\\ &= \tfrac{1}{3}\{1+5.6568+6+21.166+8.0623\}\\ &= \tfrac{1}{3}\{41.885\} \approx 13.962\end{aligned}$$

Question No: 22 **Evaluate** $\int \frac{1}{\sqrt{x^2+4x+5}}dx.$ [**4**]

Solution:

$$\begin{aligned}\int \frac{1}{\sqrt{x^2+4x+5}}dx &= \int \frac{1}{\sqrt{(x+2)^2+1}}dx && (1)\\ &= \int \frac{1}{\sqrt{u^2+1}}du \quad (\text{with } u = x+2) && (2)\\ &= \sinh^{-1}(u)+c\\ &= \sinh^{-1}(x+2)+c && (1)\end{aligned}$$

Question No: 23 **Evaluate** $\int \frac{1}{(x^2+9)^2}dx$ **[6]**

Solution:

$$\begin{aligned}
\text{Let } x = 3\tan\theta \quad & \Rightarrow dx = 3\sec^2\theta d\theta && (1)\\
\int \frac{1}{(x^2+9)^2}dx \quad & = \int \frac{3\sec^2\theta}{(9\tan^2\theta+9)^2}d\theta \\
& = \frac{1}{27}\int \frac{1}{\sec^2\theta}d\theta && (1)\\
& = \frac{1}{27}\int \cos^2\theta d\theta \\
& = \frac{1}{27}\int \left(\frac{1+\cos(2\theta)}{2}\right) d\theta && (1)\\
& = \frac{1}{54}(\theta + \sin\theta\cos\theta) + c && (1)\\
& = \frac{1}{54}\left(\tan^{-1}\left(\frac{x}{3}\right) + \frac{3x}{x^2+9}\right) + c && (2)
\end{aligned}$$

Question No: 24 **Evaluate** $\int \frac{\sin^{-1}(\ln x)}{x}dx$ **[5]**

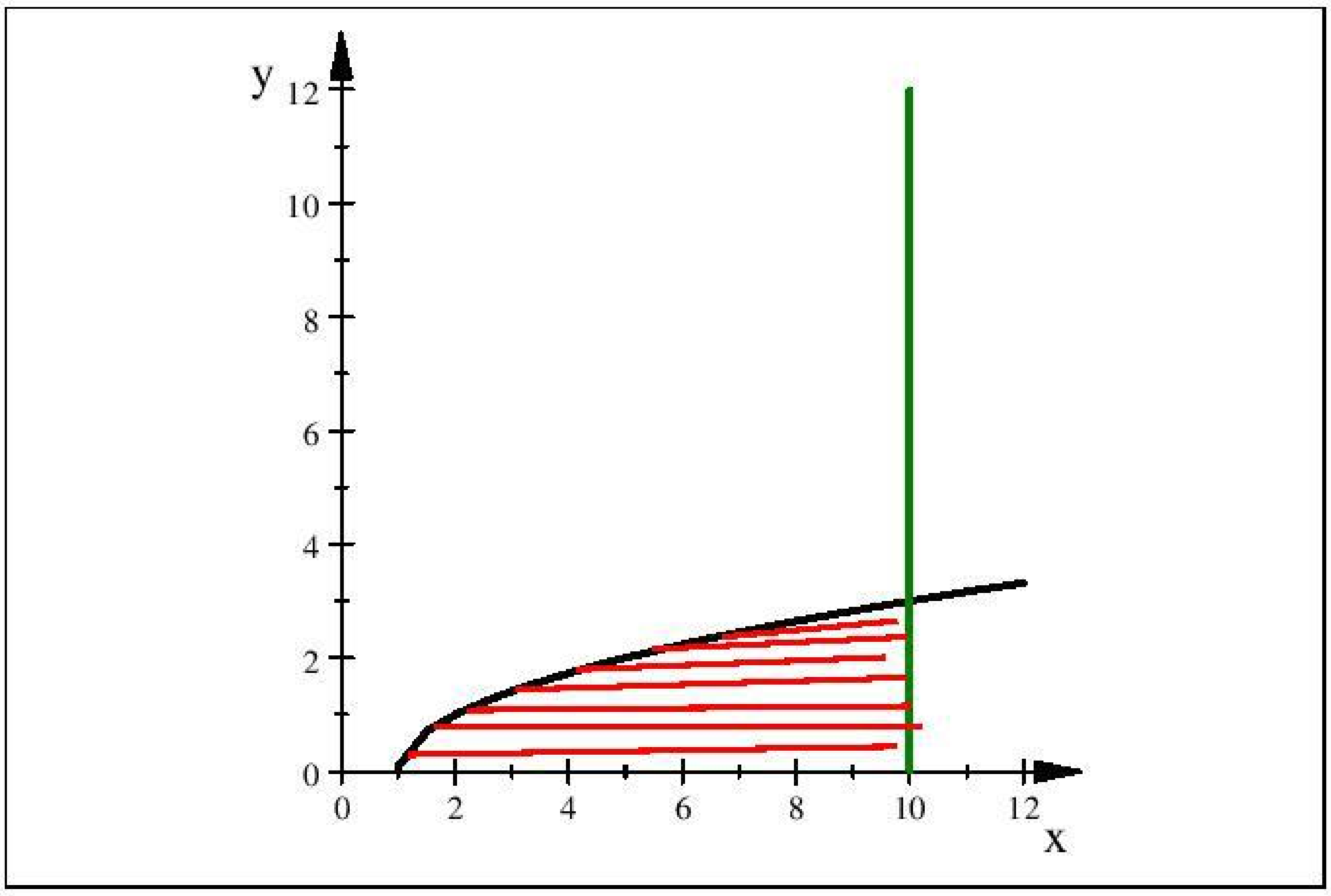
Solution:

$$\begin{aligned}
\text{Let } u = \ln x \quad & \Rightarrow du = \tfrac{1}{x}dx && (1)\\
\int \frac{\sin^{-1}(\ln x)}{x}dx \quad & = \int \sin^{-1}(u)\,du && (1)\\
& = u\sin^{-1}u + \sqrt{1-u^2} + c && (2)\\
& = (\ln x)\sin^{-1}(\ln x) + \sqrt{1-(\ln x)^2} + c && (1)
\end{aligned}$$

Question No: 25 **Sketch** the region R bounded by the graph of the equations $y = \sqrt{x-1}$, $x = 10$ and $y = 0$. **Find** the **volume** of the solid generated by revolving the region R around the $y-axis$. (Use any method) [**6**]

Solution:

Graph: (3)



By Washer Method:

$$V = \pi \int_0^3 \left((10)^2 - (y^2+1)^2 \right) dy = \tfrac{1152}{5}\pi. \qquad (2+1)$$

By Cylindrical shell metod:

$$V = 2\pi \int_1^{10} x\sqrt{x-1}dx = \tfrac{1152}{5}\pi. \qquad (2+1)$$