Dr. George Karraz, Ph. D.

Introduction to Backpropagation

- In 1969 a method for learning in multi-layer network, Backpropagation, was invented by Bryson and Ho.

- The Backpropagation algorithm is a sensible approach for dividing the contribution of each weight.

- Works basically the same as perceptrons

Backpropagation Learning Principles: **Hidden Layers** and Gradients

There are two **differences for the updating rule :**

1) The activation of the hidden unit is used instead of activation of the   input value.

2) The rule contains a term for the gradient of the activation function.

Backpropagation Network training

- 1. Initialize network with random weights
- 2. For all training cases (called examples):
  - **a.** Present training inputs to network and calculate output
  - **b.** For all layers (starting with output layer, back to input layer):
    - i. Compare network output with correct output (error function)
    - ii. Adapt weights in current layer

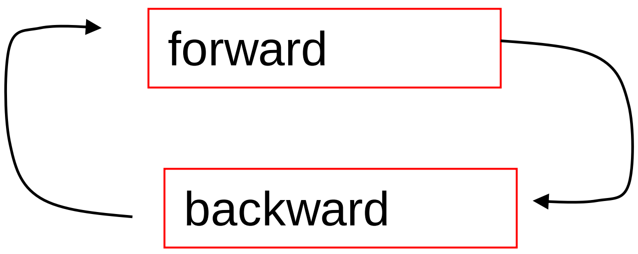
This is what you want

Backpropagation Learning Details

- Method for learning weights in feed-forward (FF) nets

- Can't use Perceptron Learning Rule
  - no teacher values are possible for hidden units

- Use gradient descent to minimize the error
  - propagate deltas to adjust for errors
    backward from outputs
    to hidden layers
    to inputs



Backpropagation Algorithm – Main Idea – error in hidden layers

The ideas of the algorithm can be summarized as follows :

1. Computes the **error term for the output units** using the observed error.

2. From output layer, repeat
   - propagating the error term back to the previous layer and
   - updating the weights between the two layers

   until the earliest hidden layer is reached.

Backpropagation Algorithm

- Initialize weights (typically random!)
- Keep doing epochs
  - For each example **e** in training set do
    - **forward pass** to compute
      - O = neural-net-output(network,e)
      - miss = (T-O) at each output unit
    - **backward pass** to calculate deltas to weights
    - update all weights
  - end
- until tuning set error stops improving

Forward pass explained earlier

Backward pass explained in next slide

Backward Pass

- Compute deltas to weights
  - from hidden layer
  - to output layer

- Without changing any weights (yet), compute the actual contributions
  - within the hidden layer(s)
  - and compute deltas

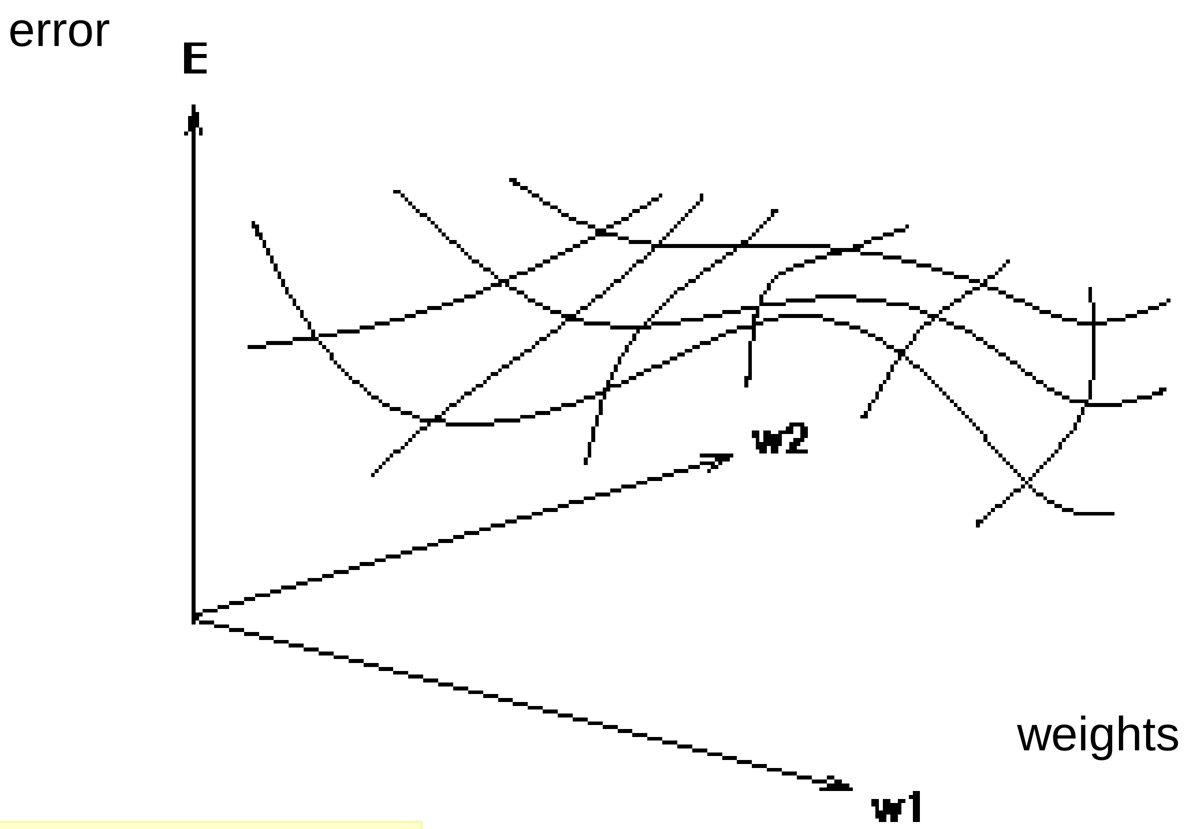
Gradient Descent

- Think of the N weights as a point in an N-dimensional space

- Add a dimension for the observed error

- Try to minimize your position on the “error surface”

Error Surface



Error as function of weights in multidimensional space

Gradient

Compute deltas

- Trying to make error decrease the fastest
- Compute:
  - $Grad_E$ = [dE/dw1, dE/dw2, . . ., dE/dwn]
- Change i-th weight by
  - $delta_{wi}$ = -*alpha* * dE/dwi

Derivatives of error for weights

- We need a derivative!
- Activation function must be **continuous**, differentiable, non-decreasing, and easy to compute

Can’t use LTU

- To effectively assign credit / blame to units in hidden layers, we want to look at the first derivative of the activation function

- Sigmoid function is easy to differentiate and easy to compute forward

~~Linear Threshold Units~~ → Sigmoid function

Updating hidden-to-output

- We have teacher supplied desired values

- $\text{delta}_{wji} = \alpha * a_j * (T_i - Oi) * g'(in_i)$

  $= \alpha * a_j * (T_i - O_i) * O_i * (1 - O_i)$

  – for sigmoid the derivative is, $g'(x) = g(x) * (1 - g(x))$

alpha

Here we have general formula with derivative, next we use for sigmoid

miss

derivative

Updating interior weights

- Layer k units provide values to all layer k+1 units
  - "miss" is ***sum of misses*** from all units on k+1
  - $\text{miss}_j = \Sigma\ [\ a_i(1 - a_i)\ (T_i - a_i)\ w_{ji}\ ]$
  - weights coming into this unit are adjusted based on their contribution

$\text{delta}_{kj} = \alpha * I_k * a_j * (1 - a_j) * \text{miss}_j$

For layer k+1

Compute deltas

How do we pick α?

1. Tuning set, or

2. Cross validation, or

3. Small for slow, conservative learning

How many hidden layers?

- Usually just one (i.e., a 2-layer net)

- How many hidden units in the layer?
  - Too few ==> can't learn
  - Too many ==> poor generalization

How big a training set?

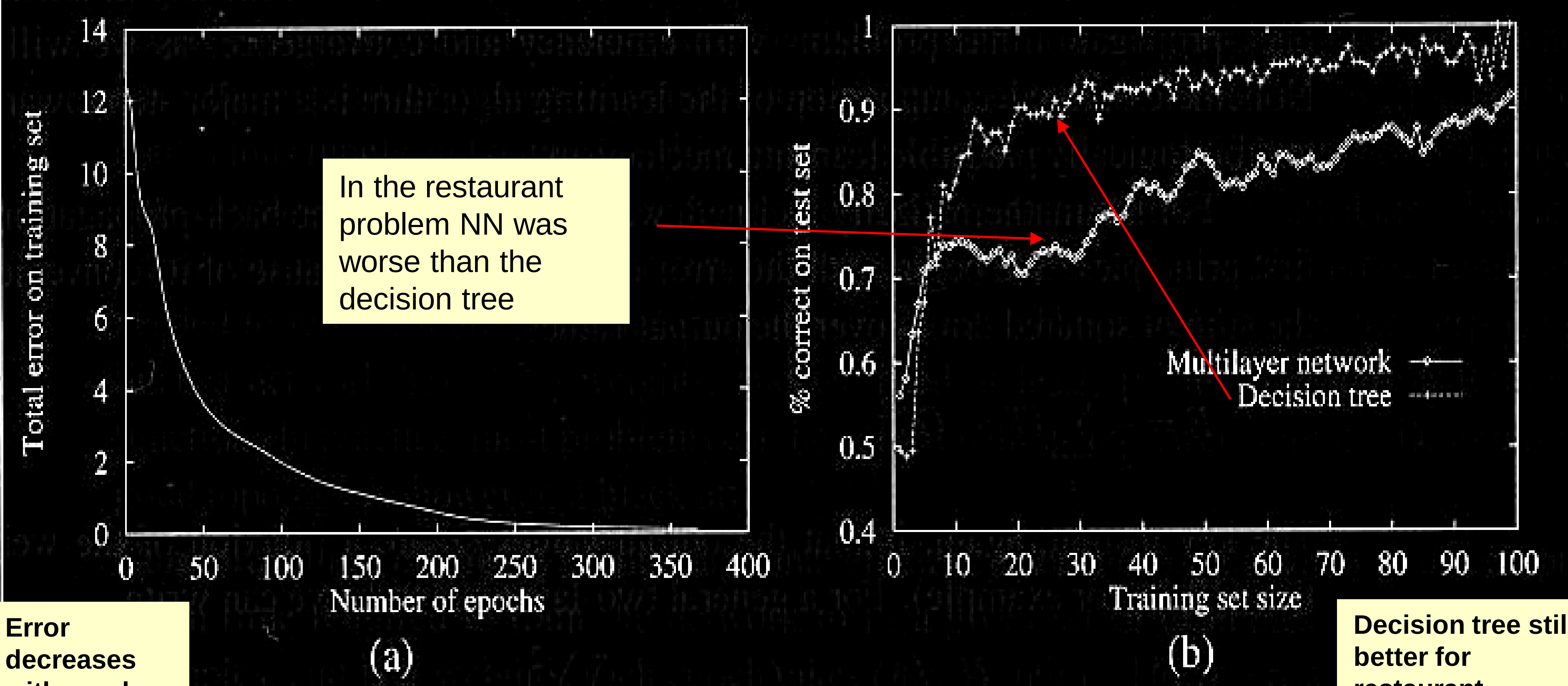
- Determine your target error rate, e
- Success rate is 1- e
- Typical training set approx. n/e, where n is the number of weights in the net
- Example:
  - $e = 0.1$, n = 80 weights
  - training set size 800

    trained until 95% correct training set classification

    should produce 90% correct classification

    on testing set (typical)

Examples of Backpropagation Learning



In the restaurant problem NN was worse than the decision tree

Error decreases with number of epochs

Decision tree still better for restaurant example

Figure 19.15 (a) Training curve showing the gradual reduction in error as weights are modified over several epochs, for a given set of examples in the restaurant domain. (b) Comparative learning curves for a back-propagation and decision-tree learning.

Examples of Backpropagation Learning

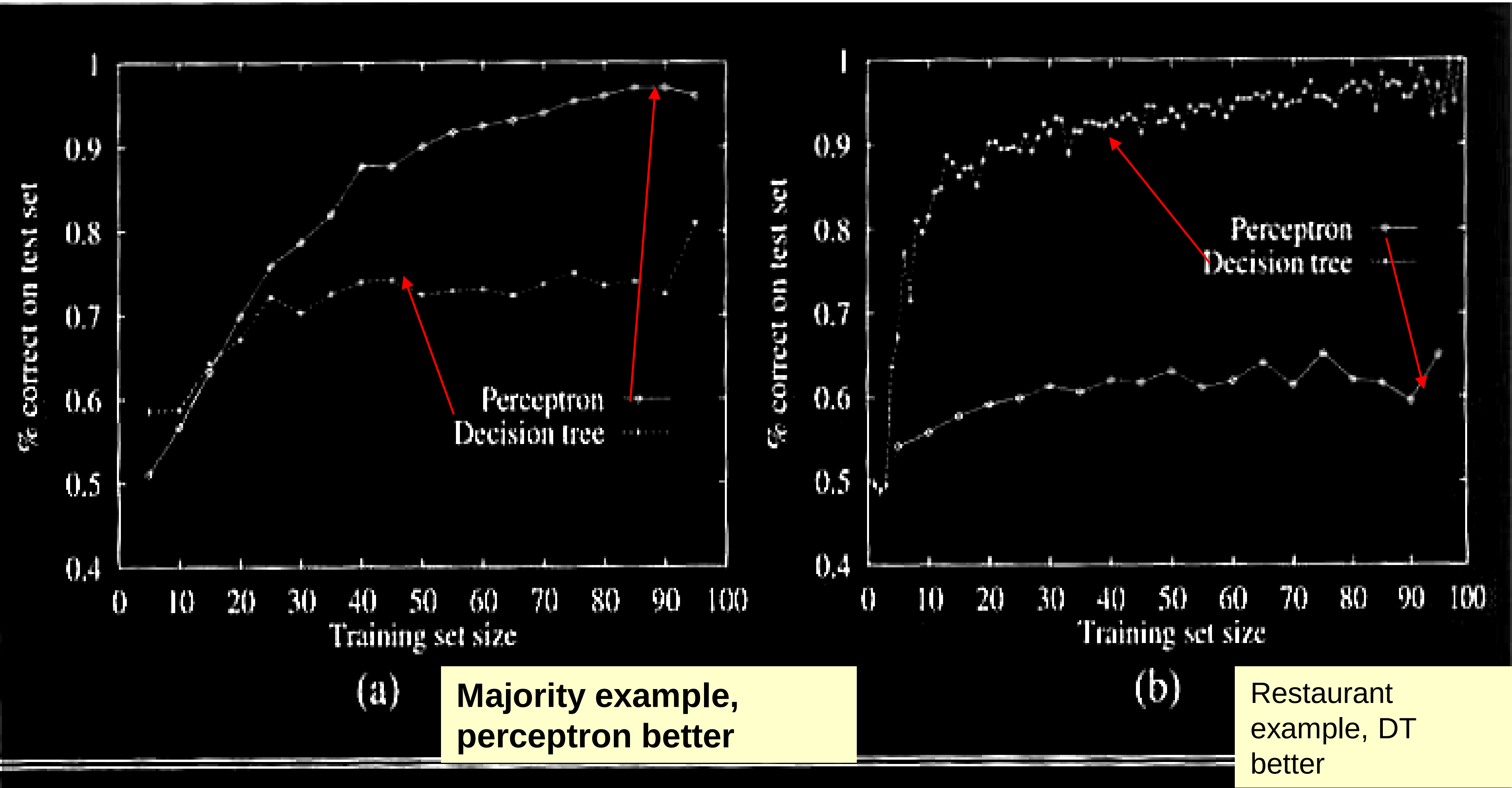


Figure 19.12 Comparing the performance of perceptrons and decision trees. (a) Perceptrons are better at learning the majority function of 11 inputs. (b) Decision trees are better at learning the *WillWait* predicate for the restaurant example.

Backpropagation Learning Math

$$E_{out\ i} = d_{out\ i} - out_i$$

$$E_{total} = \sum_{i=0}^{num(n_{out})} E_{out\ i}^2$$

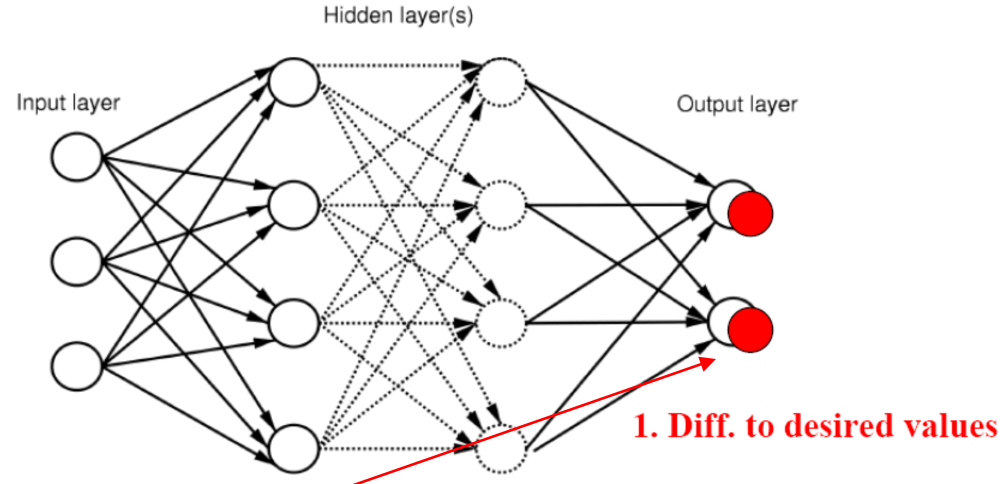
$$E_{hid\ i} = \sum_{k=1}^{num(n_{out})} E_{out\ k} \cdot w_{out\ i,k}$$

$$diff_{hid\ i} = E_{hid\ i} \cdot (1 - o(n_{hid\ i})) \cdot o(n_{hid\ i})$$

See next slide for explanation

Visualization of Backpropagation learning

Backpropagation Learning



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Backpropagation Learning

$$E_{out\,i} = d_{out\,i} - out_i$$

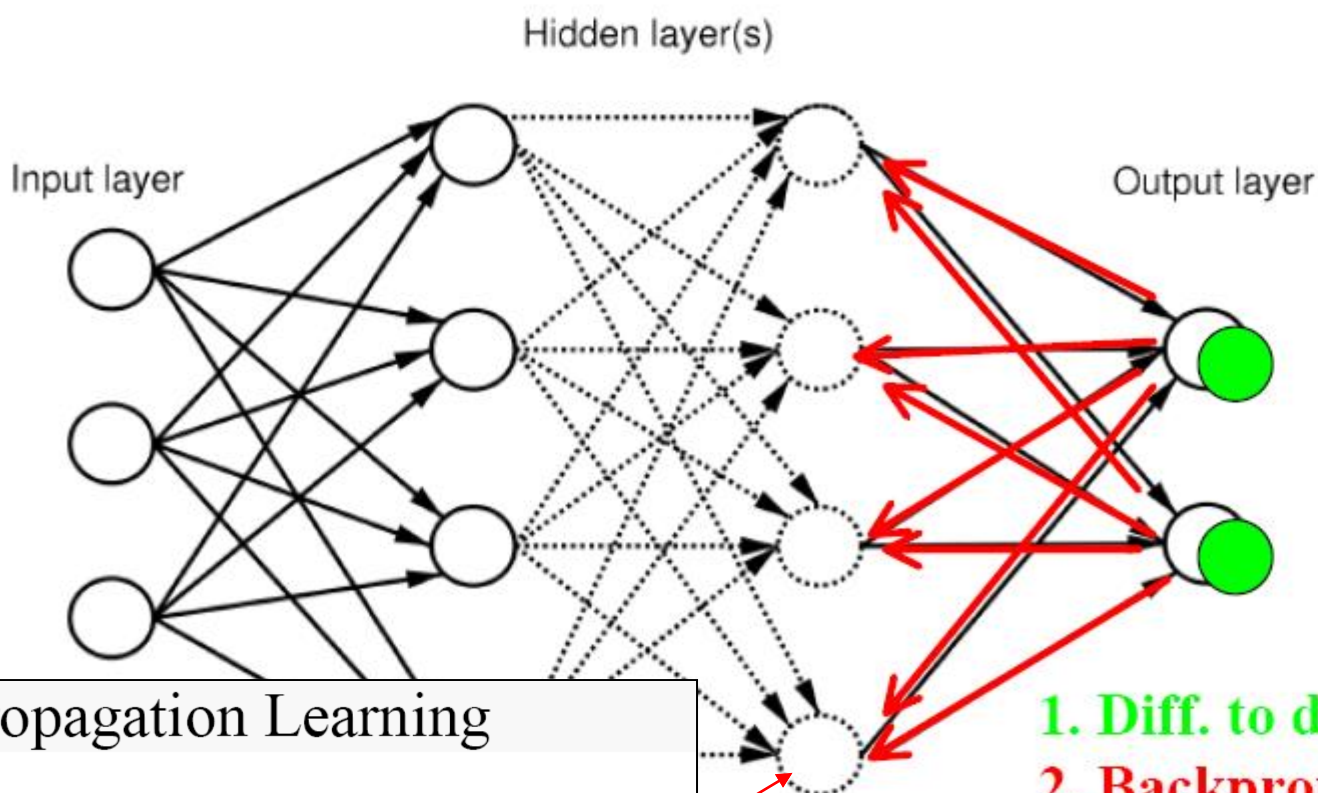
$$E_{total} = \sum_{i=0}^{num(n_{out})} E_{out\,i}^2$$

$$E_{hid\,i} = \sum_{k=1}^{num(n_{out})} E_{out\,k} \cdot w_{out\,i,k}$$

Backprop output layer

$$diff_{hid\,i} = E_{hid\,i} \cdot (1 - o(n_{hid\,i})) \cdot o(n_{hid\,i})$$

Backpropagation Learning



1. Diff. to desired values

2. Backprop output layer

Backpropagation Learning

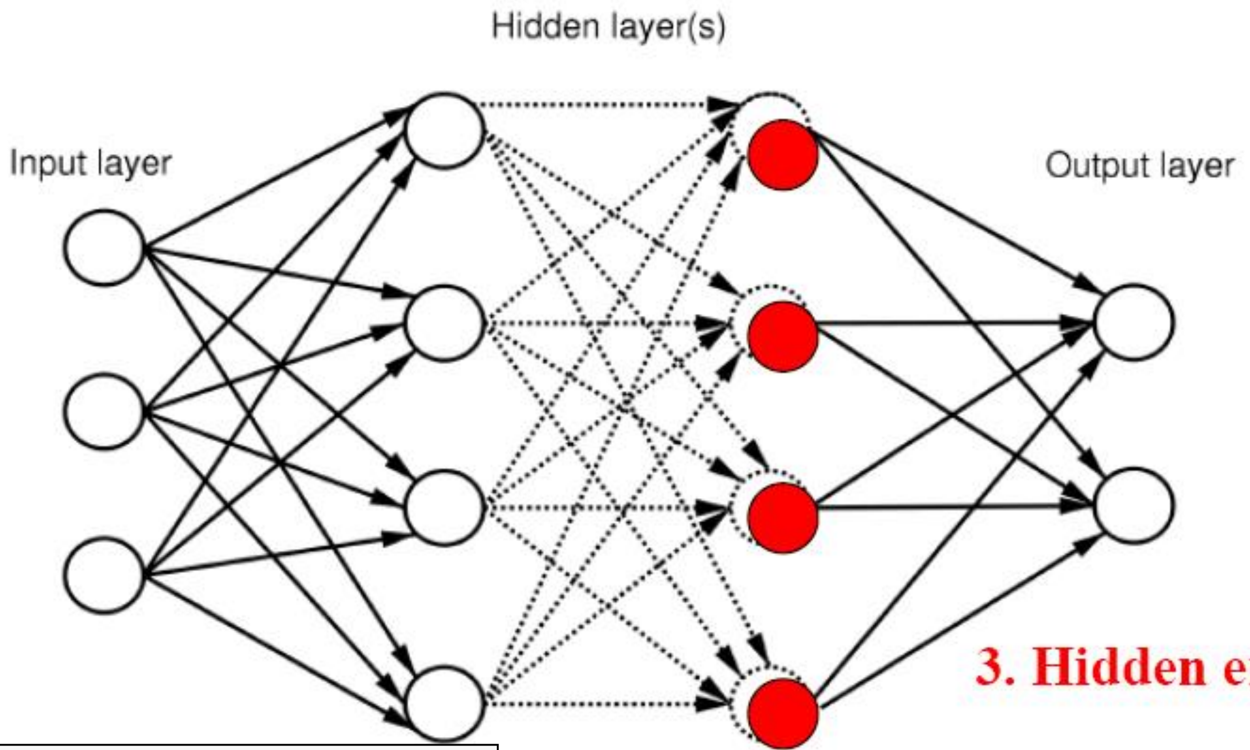
$$E_{out\ i} = d_{out\ i} - out_i$$

$$E_{total} = \sum_{i=0}^{num(n_{out})} E^2_{out\ i}$$

$$E_{hid\ i} = \sum_{k=1}^{num(n_{out})} E_{out\ k} \cdot w_{out\ i,k}$$

$$diff_{hid\ i} = E_{hid\ i} \cdot (1 - o(n_{hid\ i})) \cdot o(n_{hid\ i})$$

Backpropagation Learning



3. Hidden error values

Backpropagation Learning

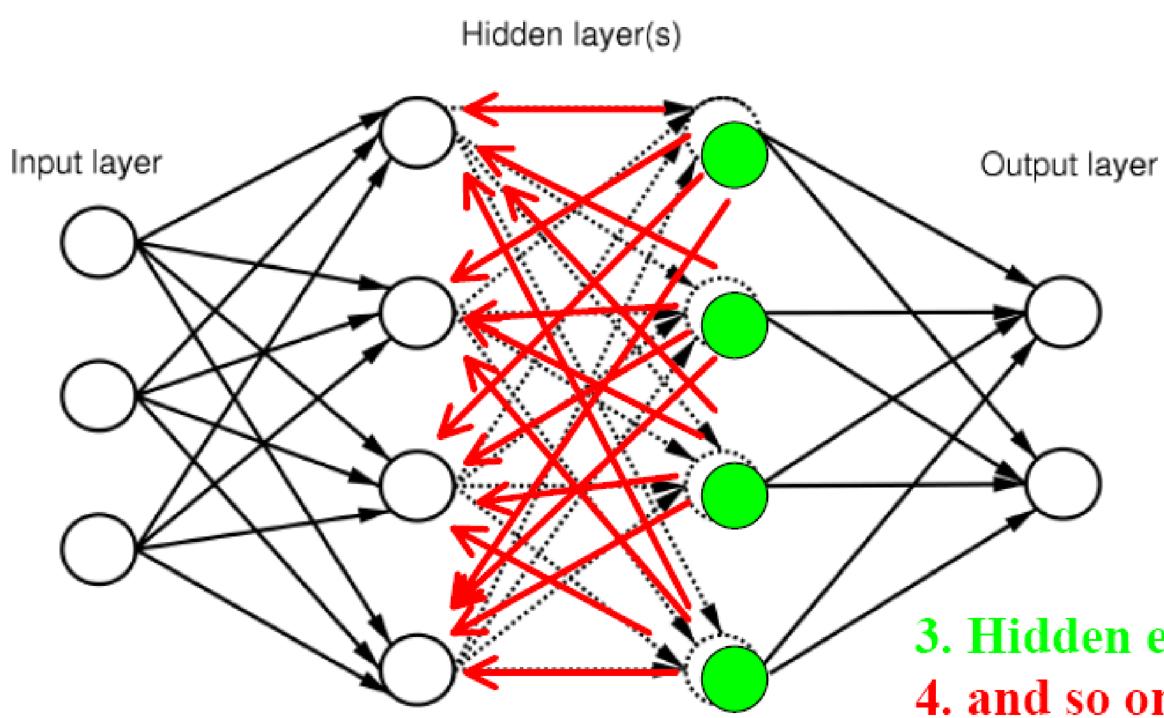
$$E_{out\ i} = d_{out\ i} - out_i$$

$$E_{total} = \sum_{i=0}^{num(n_{out})} E^2_{out\ i}$$

$$E_{hid\ i} = \sum_{k=1}^{num(n_{out})} E_{out\ k} \cdot w_{out\ i,k}$$

$$diff_{hid\ i} = E_{hid\ i} \cdot (1 - o(n_{hid\ i})) \cdot o(n_{hid\ i})$$

Backpropagation Learning



3. Hidden error values

4. and so on …

Backpropagation Learning

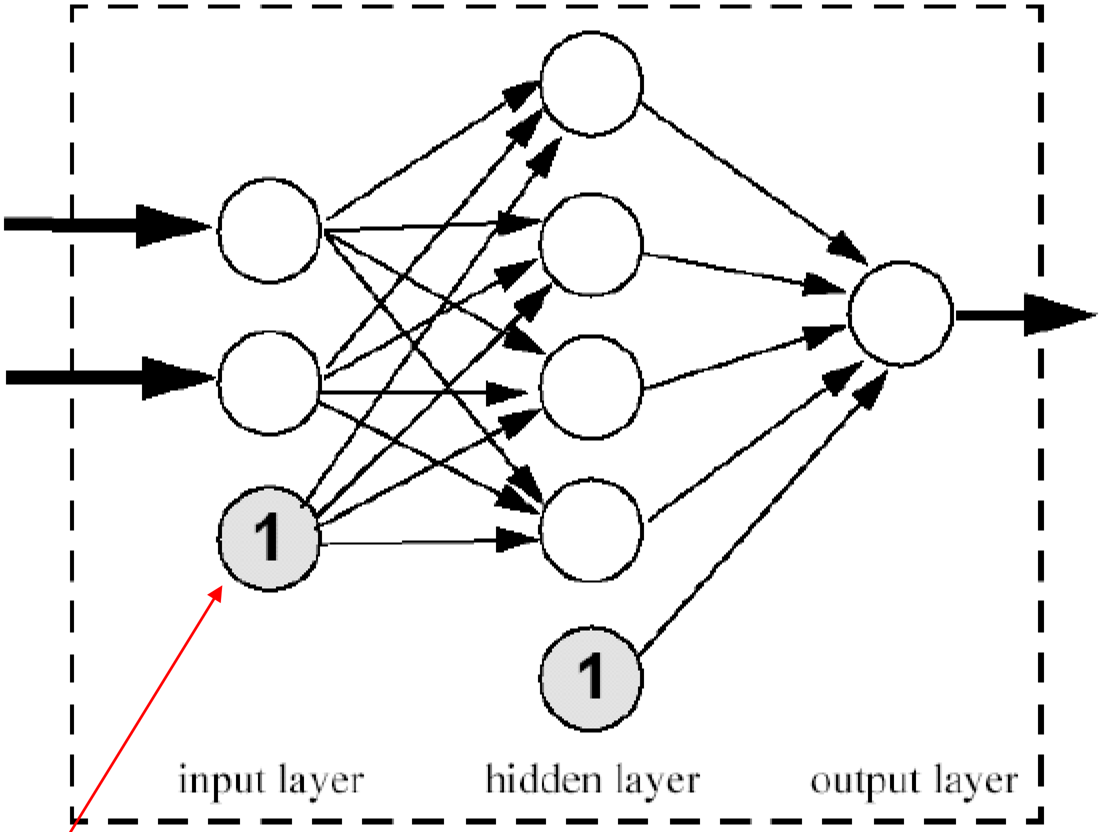
$$E_{out\ i} = d_{out\ i} - out_i$$

$$E_{total} = \sum_{i=0}^{num(n_{out})} E_{out\ i}^2$$

$$E_{hid\ i} = \sum_{k=1}^{num(n_{out})} E_{out\ k} \cdot w_{out\ i,k}$$

$$diff_{hid\ i} = E_{hid\ i} \cdot (1 - o(n_{hid\ i})) \cdot o(n_{hid\ i})$$

Bias Neurons in Backpropagation Learning



Bias neurons

bias neuron in input layer

Software for Backpropagation Learning

This routine calculate error for backpropagation

Training pairs

```
float backprop(float train_in[NIN], float train_out[NOUT])
/* returns current square error value */
{ int i,j;
  float err_total;
  float N_out[NOUT],err_out[NOUT];
  float diff_out[NOUT];
  float N_hid[NHID], err_hid[NHID], diff_hid[NHID];

  //run network, calculate difference to desired output
  feedforward(train_in, N_hid, N_out);
  err_total = 0.0;
  for (i=0; i<NOUT; i++)
  {   err_out[i] = train_out[i]-N_out[i];
      diff_out[i]= err_out[i] * (1.0-N_out[i]) * N_out[i];
      err_total += err_out[i]*err_out[i];
  }
  ...
```

Run network forward. *Was explained earlier*

Calculate difference to desired output

Calculate total error

Software for Backpropagation Learning continuation

Here we do not use alpha, the learning rate

Update output weights

Calculate hidden difference values

Update input weights

Return total error

```
...
  // update w_out and calculate hidden difference values
  for (i=0; i<NHID; i++)
  { err_hid[i] = 0.0;
    for (j=0; j<NOUT; j++)
    { err_hid[i]  += err_out[j] * w_out[i][j];
      w_out[i][j] += diff_out[j] * N_hid[i];
    }
    diff_hid[i] = err_hid[i] * (1.0-N_hid[i]) * N_hid[i];
  }
  // update w_in
  for (i=0; i<NIN; i++)
    for (j=0; j<NHID; j++)
      w_in[i][j] += diff_hid[j] * train_in[i];

  return err_total;
}
```

The general Backpropagation Algorithm for updating weights in a multilayer network

Repeat until convergent

Here we use alpha, the learning rate

```
function BACK-PROP-UPDATE(network, examples, α) returns a network with modified weights
   inputs: network, a multilayer network
           examples, a set of input/output pairs
           α, the learning rate

   repeat
      for each e in examples do
         /* Compute the output for this example */
            O ← RUN-NETWORK(network, I^e)
         /* Compute the error and Δ for units in the output layer */
            Err^e ← T^e − O
         /* Update the weights leading to the output layer */
            W_{j,i} ← W_{j,i} + α × a_j × Err_i^e × g'(in_i)
         for each subsequent layer in network do
            /* Compute the error at each node */
               Δ_j ← g'(in_j) Σ_i W_{j,i} Δ_i
            /* Update the weights leading into the layer */
               W_{k,j} ← W_{k,j} + α × I_k × Δ_j
         end
      end
   until network has converged
   return network
```

Go through all examples

Run network to calculate its output for this example

Compute the error in output

Update weights to output layer

Compute error in each hidden layer

Update weights in each hidden layer

Return learned network

Examples and Applications of ANN

Neural Network in Practice

NNs are used for classification and function approximation or mapping problems which are:

- Tolerant of some imprecision.
- Have lots of training data available.
- Hard and fast rules cannot easily be applied.

NETalk (1987)

- Mapping character strings into phonemes so they can be pronounced by a computer

- Neural network trained how to pronounce each letter in a word in a sentence, given the three letters before and three letters after it in a window

- Output was the correct phoneme

- Results
  - 95% accuracy on the training data
  - 78% accuracy on the ***test set***

Other Examples

- Neurogammon (Tesauro & Sejnowski, 1989)
  - Backgammon learning program
- Speech Recognition (Waibel, 1989)
- Character Recognition (LeCun et al., 1989)
- Face Recognition (Mitchell)

ALVINN

- Steer a van down the road
  - 2-layer feedforward
    - using backpropagation for learning
  - Raw input is 480 x 512 pixel image 15x per sec
  - Color image preprocessed into 960 input units
  - 4 hidden units
  - 30 output units, each is a steering direction

Neural Network Approaches

Learning on-the-fly

- ALVINN learned as the vehicle traveled
  - initially by observing a human driving
  - learns from its own driving by watching for future corrections
  - **<u>never saw bad driving</u>**
    - didn't know what was dangerous, NOT correct
    - computes alternate views of the road (rotations, shifts, and fill-ins) to use as "bad" examples
  - keeps a buffer pool of 200 pretty old examples to avoid overfitting to only the most recent images

Feed-forward vs. Interactive Nets

- Feed-forward
  - activation propagates in one direction
  - We usually focus on this

- Interactive
  - activation propagates forward & backwards
  - propagation continues until equilibrium is reached in the network
  - We do not discuss these networks here, complex training. May be unstable.

Ways of learning with an ANN

- Add nodes & connections
- Subtract nodes & connections
- Modify connection weights
  - current focus
  - can simulate first two

- I/O pairs:
  - given the inputs, what should the output be? ["typical" learning problem]

More Neural Network Applications

- May provide a model for massive parallel computation.

- More successful approach of "parallelizing" traditional serial algorithms.

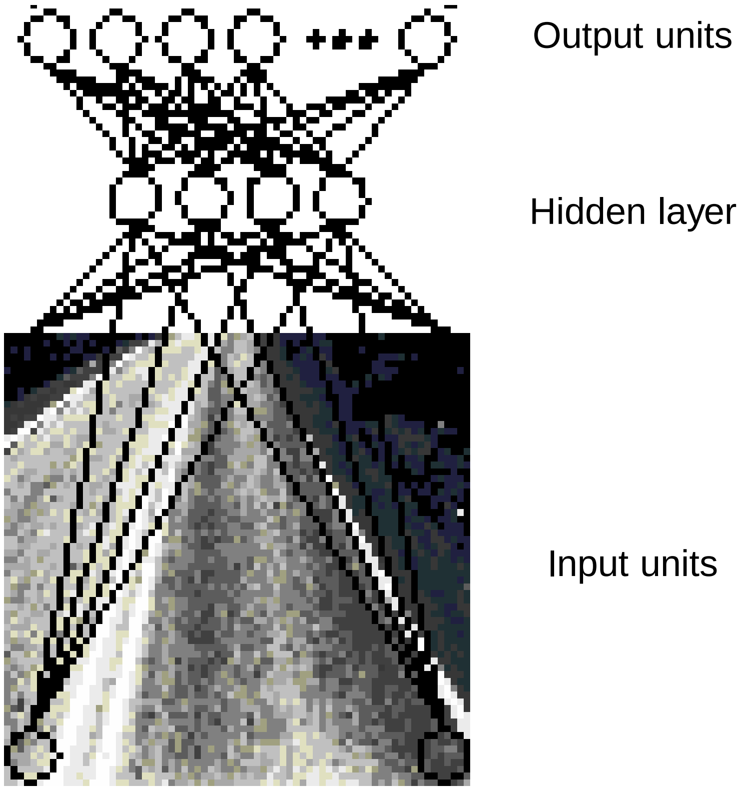
- Can compute any computable function.

- Can do everything a normal digital computer can do.

- Can do even more under some impractical assumptions.

Neural Network Approaches to driving

- Use special hardware
- ASIC
- FPGA
- analog



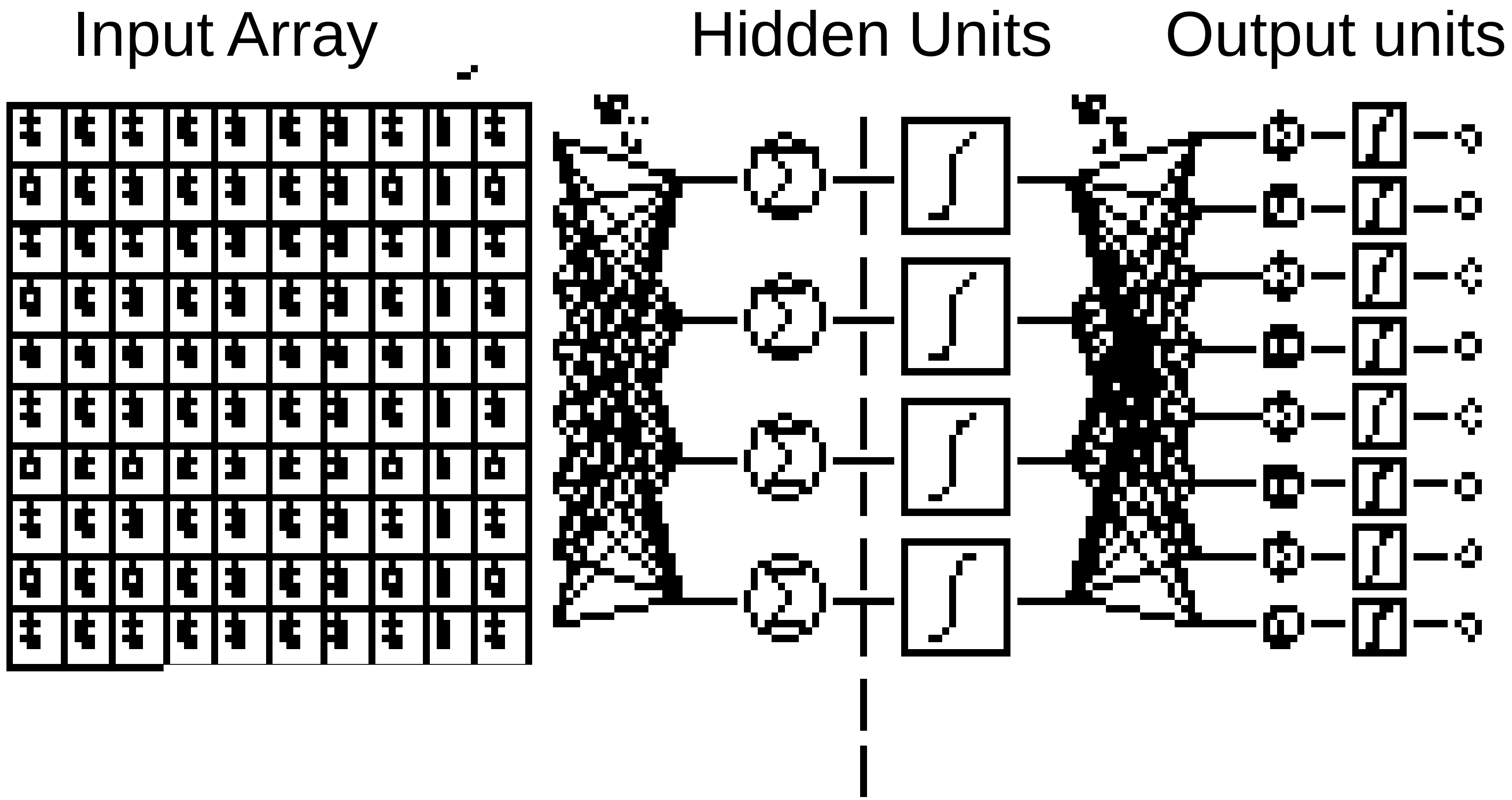
- Developed in 1993.

- Performs driving with Neural Networks.

- An intelligent VLSI image sensor for road following.

- Learns to filter out image details not relevant to driving.

Neural Network Approaches



Actual Products Available

ex1. Enterprise Miner:

- Single multi-layered feed-forward neural networks.
- Provides business solutions for data mining.

ex2. Nestor:

- Uses Nestor Learning System (NLS).
- Several multi-layered feed-forward neural networks.
- Intel has made such a chip - NE1000 in VLSI technology.

Ex1. Software tool - Enterprise Miner

- Based on SEMMA (Sample, Explore, Modify, Model, Access) methodology.

- Statistical tools include :

  Clustering, decision trees, linear and logistic regression and neural networks.

- Data preparation tools include :

  Outliner detection, variable transformation, random sampling, and partition of data sets (into training, testing and validation data sets).

Ex 2. Hardware Tool - Nestor

- With low connectivity within each layer.

- Minimized connectivity within each layer results in rapid training and efficient memory utilization, ideal for VLSI.

- Composed of multiple neural networks, each specializing in a subset of information about the input patterns.

- Real time operation without the need of special computers or custom hardware DSP platforms

  • Software exists.

Problems with using ANNs

1. Insufficiently characterized development process compared with conventional software
   - What are the steps to create a neural network?

2. How do we create neural networks in a repeatable and predictable manner?

3. Absence of quality assurance methods for neural network models and implementations
   - How do I verify my implementation?

Solving Problem 1 – The Steps to create a ANN

Define the process of developing neural networks:

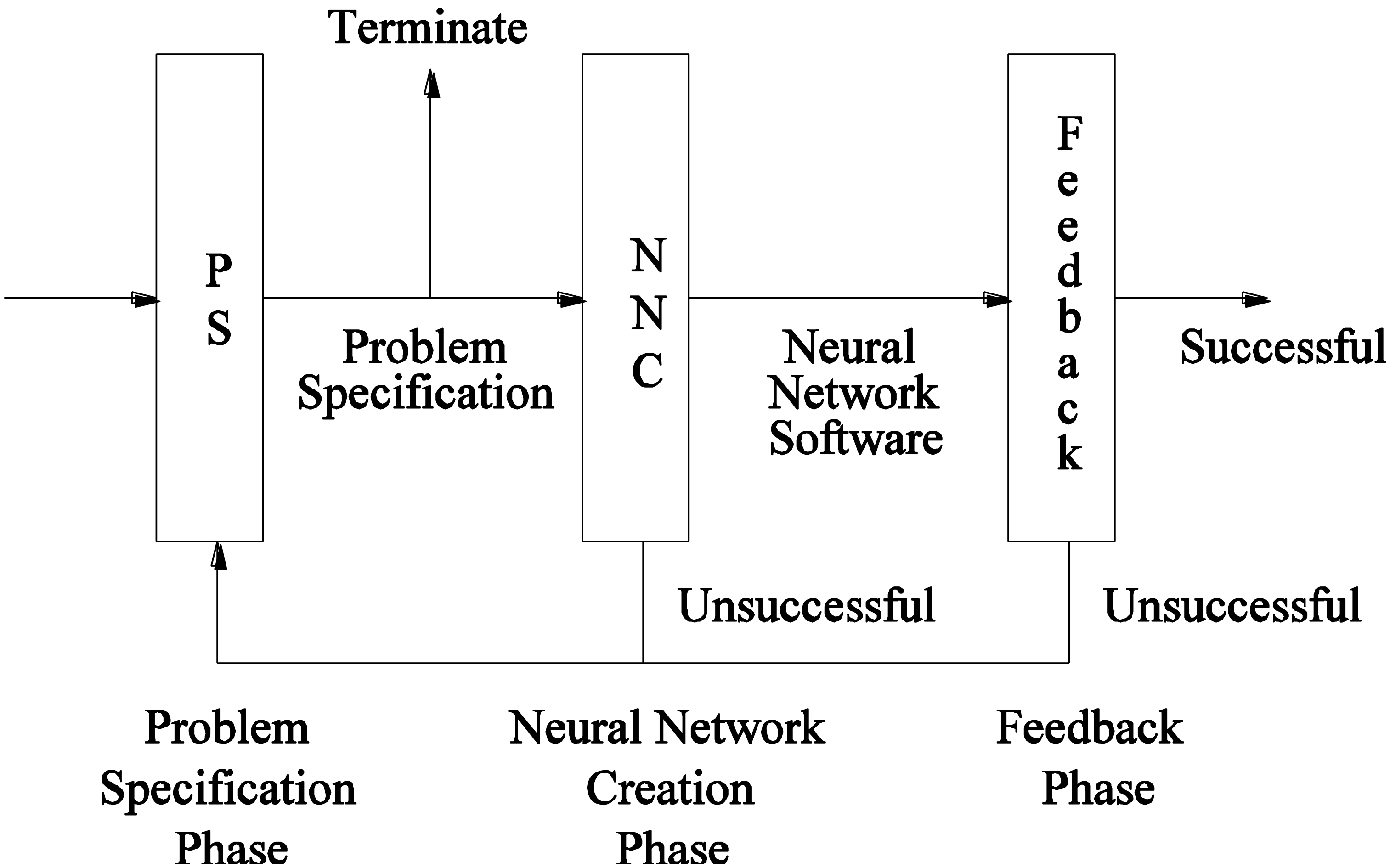
1. Formally capture the specifics of the problem in a document based on a template

2. Define the factors/parameters for creation
   - Neural network creation parameters
   - Performance requirements

3. Create the neural network

4. Get feedback on performance

Neural Network **Development Process**



Problem Specification Phase

- Some factors to define in problem specification:
  - Type of neural networks (based on experience or published results)
  - How to collect and transform problem data
  - Potential input/output representations
  - Training & testing method and data selection
  - Performance targets (accuracy and precision)

- Most important output is the ranked collection of factors/parameters

Problem 2 –
How to create a Neural Network

- Predictability (with regard to resources)
  - Depending on creation approach used, record time for one iteration
  - Use timing to predict maximum and minimum times for all of the combinations specified

- Repeatability
  - Relevant information must be captured in problem specification and combinations of parameters

Problem 3 - Quality Assurance

- Specification of generic neural network software (models and learning)
- Prototype of specification
- Comparison of a given implementation with specification prototype
- Allows practitioners to create arbitrary neural networks verified against models

Two Methods for Comparison

- Direct comparison of outputs:

	20-10-5 (with particular connections and input)
Prototype	<0.123892, 0.567442, 0.981194, 0.321438, 0.699115>
Implementation	<0.123892, 0.567442, 0.981194, 0.321438, 0.699115>

- Verification of weights generated by learning algorithm:

20-10-5	Iteration 100	Iteration 200	……….	Iteration n
Prototype	Weight state 1	Weight state 2	……….	Weight state n
Implementation	Weight state 1	Weight state 2	……….	Weight state n

Further Work on improvements

- Practitioners to use the development process or at least document in problem specification
- Feedback from neural network development community on the content of the problem specification template
- Collect problem specifications and analyse to look for commonalities in problem domains and improve predictability (eg. control)
- More verification of specification prototype

Further Work (2)

- Translation methods for formal specification
- Extend formal specification to new types
- Fully prove aspects of the specification
- Cross discipline data analysis methods (eg. ICA, statistical analysis)
- Implementation of learning on distributed systems
  - Peer-to-peer network systems (farm each combination of parameters to a peer)
- Remain unfashionable

Summary

- Neural network is a computational model that simulate some properties of the human brain.

- The connections and nature of units determine the behavior of a neural network.

- Perceptrons are feed-forward networks that can only represent linearly separable functions.

Summary

- Given enough units, any function can be represented by Multi-layer feed-forward networks.

- Backpropagation learning works on multi-layer feed-forward networks.

- Neural Networks are widely used in developing artificial learning systems.

References

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