Q 1

a) Number of heads in 10 tosses of coin

b) The random variable that recording the number of heads by tossing a coin infinite Times

c) The variable that measures the life. TV

Q 2 a) continuous

b) discrete

c) discrete

d) discrete

e) continuous

Q3 a) $P(X=k)=\frac{1}{k}$; $k=1,2,3,4,5$

k : r.v	1	2	3	4	5	
$P(X=k)$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	

1) $P(X=k) \geq 0$

2) $\sum_{1}^{5} P(X=k) = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$

$= 2.28 \neq 1$

$P(X=k)$ not mass function

b) $P(X=q)=\frac{1}{q}$; $k=-2,1,2,3,4,5$

k : r.v	-2	1	2	3	4	5
$P(X=q)$	$\frac{1}{-2}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

$P(X=-2)=-\frac{1}{2}<0 \rightarrow$ not mass function

Q3 | c)

$P(X=j) = \frac{1}{9}$ $j = 1,2,3,4,5,6,7,8,9$

k n.v	1	2	3	4	5	6	7	8	9
P(X=j)	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

1) $P(X=j) = \frac{1}{9} > 0$

2) $\sum_{1}^{9} P(X=j) = \frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9} = \frac{9}{9} = 1$

$P(X=j) = \frac{1}{9}$ mass function

d) $P(X=x) = -x \quad ; x \in [0,-1]$

not mass function

X not discret random variable

4] a)
$$f(x)=\begin{cases}\frac{8}{3}x & \text{for } x\in[-0.5,1]\\ 0 & \text{otherwise}\end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\,dx=\int_{-\infty}^{-0.5} 0\,dx+\int_{-0.5}^{1} f(x)\,dx+\int_{1}^{\infty} 0\,dx$$

$$=\int_{-0.5}^{1}\frac{8}{3}x\,dx=\frac{8x^2}{6}\Big|_{-0.5}^{1}$$

$$=\frac{8}{6}-\frac{8(-0.5)^2}{6}=1$$

$f(x)\rightarrow$ density function (DF)

b)
$$f(x)=\begin{cases}2x & \text{for } x\in[0,1]\\ 0 & \text{otherwise}\end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\,dx=\int_{-\infty}^{0} 0\,dx+\int_{0}^{1} f(x)\,dx+\int_{1}^{\infty} 0\,dx$$

$$=\int_{0}^{1} 2x\,dx=x^2\Big|_{0}^{1}=1^2-0^2=1$$

$f(x)$ density function (DF)

Q 4) c]

$$f(x) = \begin{cases} x & x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} 0\,dx$$

$$= \int_{0}^{1} x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2} - 0$$

$$= \frac{1}{2} \neq 1$$

$f(x)$ not density function

Q5 a) i) $8! = 40320$

ii) $(8-1)! = 7! = 5040$

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b) i) $_{12}P_5 = \frac{12!}{(12-5)!} = 95040$

ii) $_{12}C_5 = \frac{12!}{5!(12-5)!} = 792$

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c) $_{52}C_7 = 133784560$

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d) $28^2 \times 10^4 + 26^3 \times 10^5 = 17654400000$

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e) $_{16}C_6 \times {}_6C_3 = 160160$

Q 6/a)

$$P(A) = P(A \setminus B) + P(A \cap B)$$
$$= 0.25 + 0.25 = 0.5$$

$$P(B) = P(B \setminus A) + P(A \cap B) =$$
$$= 0.25 + 0.25 = 0.5$$

$$P(C) = P(C \setminus A) + P(A \cap C)$$
$$= 0.25 + 0.25 = 0.5$$

$$P(A \setminus C) = P(A) - P(A \cap C)$$
$$= 0.5 - 0.25 = 0.25$$

$$P(C \setminus B) = P(C) - P(C \cap B)$$
$$= 0.5 - 0.25 = 0.25$$

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(2 6) 9

$$P(B \setminus C) = P(B) - P(B \cap C)$$

$$= 0.5 - 0.25 = 0.25$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.5} = 0.5$$

$$P(C|A \cap B) = \frac{P(C \cap A \cap B)}{P(A \cap B)} = \frac{0.125}{0.25}$$

$$= 0.5$$

b) $P(\bar{A} \cup \bar{B} \cup \bar{C}) = P(\overline{A \cap B \cap C})$

$$= 1 - P(A \cap B \cap C) = 1 - 0.125$$

$$= 0.875$$

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Q6 : b

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = 0.5 + 0.5 + 0.5 - 0.25 - 0.25 - 0.25 + 0.125 = 0.875$$

c) $P(A) \cdot P(B) = 0.5 \times 0.5 = 0.25$

$P(A \cap B) = 0.25$

$P(A \cap B) = P(A) \cdot P(B)$

A and B independent

$P(A) \cdot P(C) = 0.5 \times 0.5 = 0.25$

$P(A \cap C) = 0.25 = P(A) \cdot P(C)$

A and C independent

$P(B) \cdot P(C) = 0.5 \times 0.5 = 0.25$

$P(B \cap C) = 0.25 = P(B) \cdot P(C)$

B and C independent

Q:7 a) S_1 , S_2 , S_3 , S_4

6% 5% 2% 7%

Let E selected student is excellent

$$p(S_1) = p(S_2) = p(S_3) = p(S_4) = 0.25$$

$$P(E) = \sum_{1}^{4} p(S_k) \cdot p(E|S_k)$$

$$P(E) = p(S_1) \cdot p(E|S_1) + p(S_2) \cdot p(E|S_2)$$
$$+ p(S_3) \cdot p(E|S_3) + p(S_4) \cdot p(E|S_4)$$

$$P(E) = 0.25 [0.06 + 0.25 \times 0.05$$
$$+ 0.25 \times 0.02 + 0.25 \times 0.07$$

$$P(E) = 0.05$$

$$p(\bar{E}) = 1 - 0.05 = 0.95$$

تتمة السؤال Q1 (b)

$$P(S_3 \mid \bar{E}) = \frac{P(S_3) \cdot P(\bar{E} \mid S_3)}{P(\bar{E})}$$

$$= \frac{0.25 \times (1 - 0.02)}{0.95}$$

$$= \frac{49}{190} = 0.2579$$

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Q 9 | Let A the car wants forefud

$P(A) = 0.95$, $P(\bar{A}) = 0.05$

a) $P(\bar{A}) = 1 - 0.95 = 0.05$

b) $P(A \cap B) = P(A) \cdot P(B)$

$= 0.95 \times 0.95 = 0.9025$

c) $P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$

$= 0.05 \times 0.05 \times 0.05$

$= 0.000125$

d) $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

$P(X=k) = \binom{10}{k} (0.95)^k (0.05)^{10-k}$

السؤال ٩

أ)

$$P(X \geq 1) = 1 - P(X < 1) =$$

$$= 1 - P(X = 0)$$

$$= 1 - \binom{10}{0} (0.95)^0 (0.05)^{10}$$

$$= 1$$

ب)

$$P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$+ P(X=5) + P(X=6) + P(X=7) =$$

$$1 - [P(X=8) + P(X=9) + P(X=10)]$$

$$1 - \left[\binom{10}{8}(0.95)^8(0.05)^2 + \binom{10}{9}(0.95)^9(0.05)^1 + \binom{10}{10}(0.95)^{10}(0.05)^0\right]$$

$$= 0.01115$$

e) $\lambda = 5 \rightarrow \mu = \frac{1}{5}$

$$f_x(x) = \frac{1}{5} e^{-\frac{x}{5}} \qquad x \geq 0$$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - \left(\int_0^2 \frac{1}{5} e^{-\frac{x}{5}} \right) dx$$

$$= 1 - \frac{1}{5} \int_0^2 e^{-\frac{x}{5}} \, dx$$

$$= 1 - \frac{1}{5}(-5)\left[e^{-\frac{x}{5}} \right]_0^2$$

$$= 1 + e^{-\frac{2}{5}} - e^{0}$$

$$= 1 + e^{-\frac{2}{5}} - 1 = + e^{-\frac{2}{5}}$$

$$= 0.67$$

Q:10 a) $X = 1, 2, 3, 4, 5, 6$

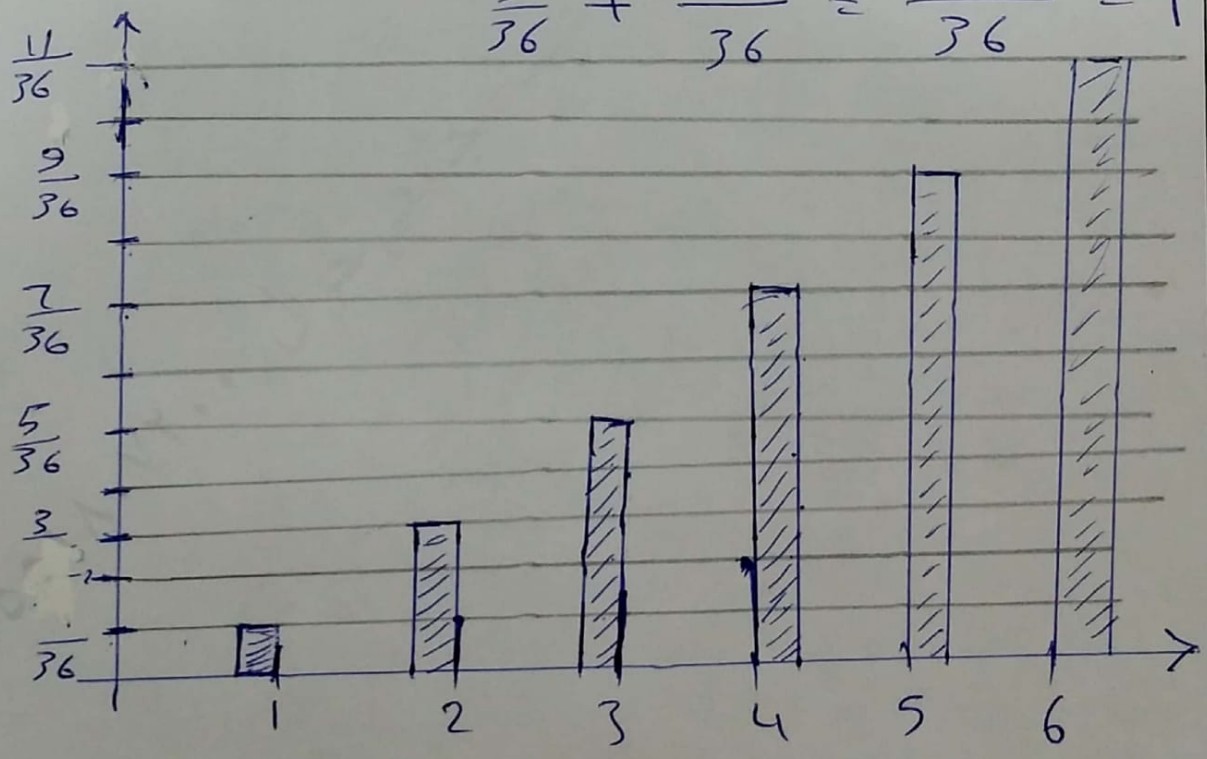
b)

k n_{v_X}	1	2	3	4	5	6
$P(X=\bullet)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

1) $P(X=k) > 0$

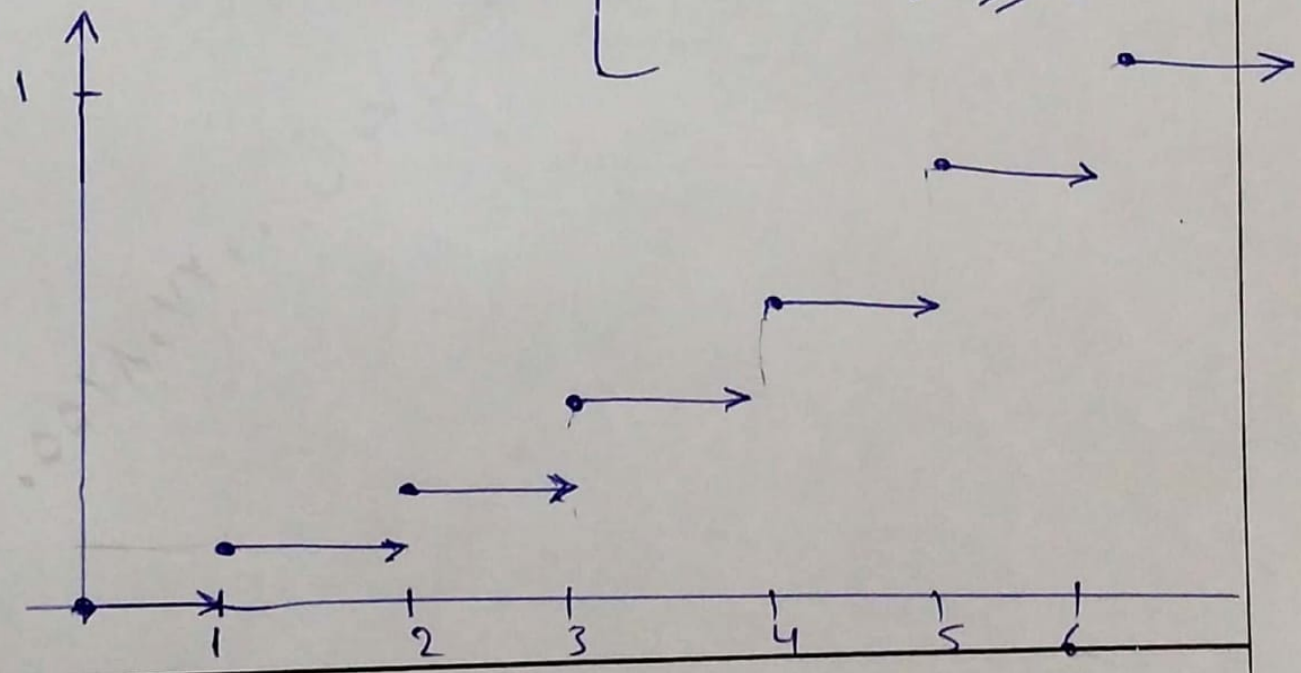
2) $\sum_{1}^{6} P(X=k) = \frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{7}{36} + \frac{9}{36} + \frac{11}{36} = \frac{36}{36} = 1$

c)



d)

$$F(x) = \sum_{1}^{x} P(X=x) = \begin{cases} 0 & x<1 \\ \frac{1}{36} & 1 \le x < 2 \\ \frac{4}{36} & 2 \le x < 3 \\ \frac{9}{36} & 3 \le x < 4 \\ \frac{16}{36} & 4 \le x < 5 \\ \frac{25}{36} & 5 \le x < 6 \\ 1 & x \ge 6 \end{cases}$$



Q2 10| f

X	P(X=.)	X . P(X)	X^2	X^2 . P(X)
1	$\frac{1}{36}$	$\frac{1}{36}$	1	$\frac{1}{36}$
2	$\frac{3}{36}$	$\frac{6}{36}$	4	$\frac{12}{36}$
3	$\frac{5}{36}$	$\frac{15}{36}$	9	$\frac{45}{36}$
4	$\frac{7}{36}$	$\frac{28}{36}$	16	$\frac{112}{36}$
5	$\frac{9}{36}$	$\frac{45}{36}$	25	$\frac{225}{36}$
6	$\frac{11}{36}$	$\frac{66}{36}$	36	$\frac{396}{36}$
Total	1	$\frac{161}{36}$		$\frac{791}{36}$

$$\mu = E(X) = \sum_{1}^{6} x_i \cdot P(X = x_i)$$

$$= \frac{161}{36} = 4.472$$

10) 9

g) $Var(X) = E(X^2) - (E(X))^2$

$= 21.97 - (4.472)^2$

$= 21.97 - 19.99$

$Var(X) = 1.97$

$\sigma = \sqrt{1.97} = 1.40$

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h) $Var(y) = Var(2X + 5)$

$= 2^2 \, Var(X)$

$= 4(1.97) = 7.88$

$\sigma_y = 2.81$

Q11 / a)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{-1} 0\,dx + \int_{-1}^{1} (ax+b)\,dx + \int_{1}^{\infty} 0\,dx = 1$$

$$\int_{-1}^{1} (ax+b)\,dx = 1$$

$$\int_{-1}^{1} (ax+b)\,dx = \frac{ax^2}{2} + bx \Big|_{-1}^{1} =$$

$$\left(\frac{a}{2} + b\right) - \left(\frac{a}{2} - b\right) =$$

$$\frac{a}{2} + b - \frac{a}{2} + b = 2b$$

$$2b = 1 \longrightarrow \boxed{b = \frac{1}{2} = 0.5}$$

$$P(X > 0) = 0.25$$

$$P(X > 0) = 1 - F_X(x) = 0.25$$

تتمة السؤال 11

(2 11) a

$$P(X \le 0) = \int_{-\infty}^{0} f(x)\,dx = \int_{-\infty}^{0} (ax+b)\,dx$$

$$= \int_{-\infty}^{-1} 0\,dx + \int_{-1}^{0} (ax+b)\,dx =$$

$$= \frac{ax^2}{2} + bx \Big|_{-1}^{0} = 0 - \left(\frac{a}{2} - b\right)$$

$$= -\frac{a}{2} + b = b - \frac{a}{2}$$

$$1 - P(X \le 0) = 0.25$$

$$1 - \left(b - \frac{a}{2}\right) = 0.25$$

$$1 - \frac{1}{2} + \frac{a}{2} = 0.25$$

$$\frac{1}{2} + \frac{a}{2} = 0.25 \rightarrow \frac{a}{2} = 0.25 - 0.5$$

$$\frac{a}{2} = -0.25 \longrightarrow \boxed{a = -0.5}$$

(2 11) a)

$$a = -0.5 \quad , \quad b = 0.5$$

$$f(x) = \begin{cases} -0.5x + 0.5 & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

b) $P(-1.2 < X < 0.5) = \int_{-1.2}^{0.5} f(x)\,dx =$

$$\int_{-1.2}^{-1} 0\,dx + \int_{-1}^{0.5} (-0.5x + 0.5)\,dx =$$

$$\left. \frac{-0.5x^2}{2} + 0.5x \right|_{-1}^{0.5} =$$

$$\left[\frac{-0.5(-1)^2}{2} + 0.5(-1)\right] - \left[\frac{-0.5(0.5)^2}{2} + 0.5(0.5)\right]$$

$$= \frac{16}{15} = 0.9375$$

(2 ب)

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$$

$$= \int_{-\infty}^{-1} 0\,dt + \int_{-1}^{x} f(t)\,dt$$

$$= \int_{-1}^{x} (-0.5t + 0.5)\,dt$$

$$= \frac{-0.5t^2}{2} + 0.5t \Big|_{-1}^{x}$$

$$= -0.25t^2 + 0.5t \Big|_{-1}^{x}$$

$$= -0.25x^2 + 0.5x - (-0.5(-1)^2 + 0.5(-1))$$

$$= -0.25x^2 + 0.5x + 0.75$$

$$F(x) = \begin{cases} 0 & x < -1 \\ -0.25x^2 + 0.5x + 0.75 & -1 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

Q11)

$$F(x) = \begin{cases} 0 & x < -1 \\ -0.25x^2 + 0.5x + 0.75 & -1 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

f)

$$\mu = E(x) = \int_{-\infty}^{\infty} x \cdot f(x) = \int_{-\infty}^{-1} 0\,dx +$$

$$\int_{-1}^{1} x f(x)\,dx + \int_{1}^{\infty} 0\,dx = \int_{-1}^{1} x(-0.5x + 0.5)$$

$$= \int_{-1}^{1} (-0.5x^2 + 0.5x)\,dx =$$

$$\frac{-0.5x^3}{3} + \frac{0.5x^2}{2}\Big|_{-1}^{1} =$$

$$\left[\frac{-0.5(1)^3}{3} + \frac{0.5(1)^2}{2}\right] - \left[\frac{-0.5(-1)^3}{3} + \frac{0.5(-1)^2}{2}\right]$$

$$= \frac{-1}{3}$$

Q11] سنة أول

f) $E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$

$= \int_{-1}^{1} x^2(-0.5x + 0.5)\, dx$

$= \int_{-1}^{1} (-0.5x^3 + 0.5x^2)\, dx$

$= -\frac{0.5x^4}{4} + \frac{0.5x^3}{3} \Big|_{-1}^{1}$

$= \left(\frac{-0.5(1)^4}{4} + \frac{0.5(1)^3}{3}\right) - \left(\frac{-0.5(-1)^4}{4} + \frac{0.5(-1)^3}{3}\right)$

$= \frac{1}{3}$

$Var(X) = E(x^2) - (E(x))^2$

$= \frac{1}{3} - \left(-\frac{1}{3}\right)^2 = \frac{2}{9}$

$\sigma = \sqrt{\frac{2}{9}} = 0.471$

Q:12 | $\mu = 175$, $\sigma = 15$

a) $P(X < 155) = P\left(\frac{X-\mu}{\sigma} < \frac{155-175}{15}\right)$

$= P(Z < -1.33) = \Phi(-1.33)$

$= 0.0918$

b) $P(177 < X < 188)$

$P\left(\frac{177-175}{15} < \frac{X-\mu}{\sigma} < \frac{188-175}{15}\right)$

$P(0.13 < Z < 0.87) =$

$P(Z < 0.87) - P(Z < 0.13) =$

$\Phi(0.87) - \Phi(0.13) =$

$0.8078 - 0.5517 =$

0.2561

Q 12

c) $P(X > 195) = 1 - P(X < 195)$

$= 1 - P\left(\frac{X-\mu}{6} < \frac{195-175}{15}\right)$

$= 1 - P(Z < 1.33)$

$= 1 - \Phi(1.33) =$

$= 1 - 0.9082$

$= 0.0918$

13) $P(X=k) = \frac{C}{5k}$; $k = 1, 2, 3, 4, 5$

a) $\sum_{1}^{5} P(X=k) = 1$

$$\sum_{1}^{5} \frac{C}{5k} = \sum_{1}^{5} \frac{1}{5} \cdot \frac{C}{k} = \frac{1}{5} \sum_{1}^{5} \frac{C}{k} = 1$$

$$\frac{1}{5}\left(\frac{C}{1} + \frac{C}{2} + \frac{C}{3} + \frac{C}{4} + \frac{C}{5} \right) = 1$$

$60 \quad 30 \quad 20 \quad 15 \quad 12$

$$\frac{1}{5}\left(\frac{60C + 30C + 20C + 15C + 12C}{60} \right) = 1$$

$$\frac{137C}{5 \times 60} = 1 \rightarrow 137C = 300$$

$$C = \frac{300}{137}$$

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(2) [13] [b]

$$P(X=k) = \frac{C}{5k} = \frac{\frac{300}{137}}{\frac{5k}{1}} = \frac{300}{137} \times \frac{1}{5k}$$

$$= \frac{60}{137k} \qquad k = 1, 2, 3, 4, 5$$

X	1	2	3	4	5
P(X=k)	$\frac{60}{137}$	$\frac{30}{137}$	$\frac{20}{137}$	$\frac{15}{137}$	$\frac{12}{137}$

14)

$$k = 1 - (0.01 + 0.29 + 0.20 + 0.35 + 0.02)$$

$$k = 1 - 0.87 = 0.13$$

$$\boxed{k = 0.13}$$

8) a)

$$P(A) = \frac{2}{13} \times \frac{2}{13} \times \frac{0}{13} \times \frac{0}{13} = 0$$

b) $$P(B) = \frac{7}{13} \times \frac{6}{12} \times \frac{5}{11} \times \frac{4}{10} +$$

$$\frac{4}{13} \times \frac{3}{12} \times \frac{2}{11} \times \frac{1}{10} =$$

$$P(B) = \frac{7}{143} + \frac{1}{715} = 0.0503$$

c) $$P(C) = \frac{{}^{7}C_4 \times {}^{6}C_0}{{}^{13}C_4} + \frac{{}^{4}C_4 \times {}^{9}C_0}{{}^{13}C_4}$$

$$= 0.050$$

c) $$P(D) = \frac{7}{13} \times \frac{4}{12} \times \frac{2}{11} \times \frac{0}{10}$$

$$P(D) = 0$$