KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

Summer Semester (1430/1431)

Final Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50

Time: Three hours

Marks:

Multiple Choice (1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$\{a, b, c, d\}$																				

Q. No: 1 $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sec^2\left(\frac{k}{n}\right)$ is equal to:

(a) ∞ (b) $\tan 1$ (c) 0 (d) $\frac{\pi}{4}$

Q. No: 2 If $\int \frac{e^{\cos^{-1}(x)}}{\sqrt{1-x^2}}dx = f(x) + c$, then $f(x)$ is equal to:

(a) $e^{\cos^{-1}(x)}$ (b) $e^{-\cos^{-1}(x)}$ (c) $-e^{\cos^{-1}(x)}$ (d) $e^{\sin^{-1}(x)}$

Q. No: 3 If $\ln(x^2) = \ln(4x-4)$, then the value of x is equal to:

(a) -2 (b) 2 (c) 1 (d) -1

Q. No: 4 The integral $\int \frac{1}{x(1-\ln x)}dx$ is equal to:

(a) $-\ln|1-\ln x|+c$ (b) $\ln|1-\ln x|+c$ (c) $-\ln(1-\ln x)+c$ (d) $\ln|\ln x|+c$

Q. No: 5 If $\int_0^{x^3} f\left(\sqrt[3]{t}\right) dt = x$, then $f(x)$ is equal to:

(a) $\frac{1}{x^2}$ (b) $\frac{1}{3x}$ (c) $\frac{1}{3x^2}$ (d) $\frac{1}{x}$

Q. No: 6 $\lim_{x\to\infty} x\left(\frac{\pi}{2} - \tan^{-1} x\right)$ is equal to:

(a) ∞ (b) 1 (c) 0 (d) -1

Q. No: 7 If $F(x) = \sinh^{-1}(\tan x)$, then $F'(x)$ is equal to:

(a) $\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$ (b) $\sec x$ (c) $\tan x$ (d) $|\sec x|$

Q. No: 8 The integral $\int \sqrt{x(6-x)}dx$ with a suitable substitution is equal to:

(a) $\int \sqrt{9-u^2}du$ (b) $\int \sqrt{u^2-9}du$ (c) $\int \frac{1}{\sqrt{9-u^2}}du$ (d) $\int \sqrt{3-u^2}du$

Q. No: 9 To evaluate the integral $\int \sec^2 x\sqrt{\tan x}dx$, we use the substitution:

(a) $u = \sec x$ (b) $u = \tan x$ (c) $u = \sqrt{\tan x}$ (d) $u = \sec^2 x$

Q. No: 10 The partial fraction decomposition of $\dfrac{x^2}{x^3-1}$ takes the form:

(a) $\frac{A}{x-1}+\frac{Bx+C}{x^2-x+1}$ (b) $\frac{A}{x-1}+\frac{B}{x^2+x+1}$ (c) $\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$ (d) $\frac{A}{x-1}+\frac{Bx+C}{1+x+x^2}$

Q. No: 11 The substitution $u=\tan\left(\frac{x}{2}\right)$ transforms the integral $\displaystyle\int \frac{1}{\sin x+\cos x}dx$ into:

(a) $\int \frac{2}{-u^2+2u+1}du$ (b) $\int \frac{1}{-u^2+2u+1}du$ (c) $\int \frac{2}{-u^2-2u+1}du$ (d) $\int \frac{2}{-u^2+1}du$

Q. No: 12 To evaluate the integral $\displaystyle\int \frac{\sqrt{x}}{x^{\frac{1}{3}}+x^{\frac{2}{3}}}dx$, we use the substitution:

(a) $u=\sqrt{x}$ (b) $u=x^{\frac{1}{3}}$ (c) $u=x^{\frac{1}{6}}$ (d) $u=x^{\frac{2}{3}}$

Q. No: 13 The improper integral $\displaystyle\int_e^{\infty} \frac{1}{x(\ln x)^2}dx$

(a) converges to 0 (b) converges to 1 (c) diverges (d) converges to -1

Q. No: 14 The area of the region bounded by the graphs of $y=x^2$ and $y=1$ is equal to:

(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

Q. No: 15 If a point has polar coordinates $\left(-1,\dfrac{\pi}{3}\right)$ then its other possible polar coordinates are:

(a) $\left(1,\frac{5\pi}{3}\right)$ (b) $\left(1,\frac{2\pi}{3}\right)$ (c) $\left(1,\frac{4\pi}{3}\right)$ (d) $\left(-1,\frac{2\pi}{3}\right)$

Q. No: 16 If a point has polar coordinates $\left(2,\dfrac{-\pi}{4}\right)$ then its $xy-$coordinates are:

(a) $(\sqrt{2},\sqrt{2})$ (b) $(\sqrt{2},-\sqrt{2})$ (c) $(-\sqrt{2},-\sqrt{2})$ (d) $(-\sqrt{2},\sqrt{2})$

Q. No: 17 The slope of the tangent line to the curve $C: x=\cos t$, $y=2\sin t$ at $t=\dfrac{3\pi}{4}$ is:

(a) 2 (b) -2 (c) $\dfrac{1}{2}$ (d) $\dfrac{-1}{2}$

Q. No: 18 The length of the curve $C: x=\frac{1}{2}\sin t,\ y=2+\frac{1}{2}\cos t;\ 0\le t\le \dfrac{\pi}{2}$ is equal to:

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{8}$ (d) $\dfrac{3\pi}{4}$

Q. No: 19 The surface area resulting by revolving the graph of the equation $y = 1 - x$, $-1 \le x \le 1$ around the $x-$axis is equal to:

(a) $2\pi\sqrt{2}$ (b) $4\pi\sqrt{2}$ (c) $\pi\sqrt{2}$ (d) $3\pi\sqrt{2}$

Q. No: 20 If a graph has polar equation $r = \sec\theta$, then its equation in $xy-$system is:

(a) $y = 1$ (b) $(x-1)^2 + y^2 = 1$ (c) $x = 1$ (d) $x^2 + (y-1)^2 = 1$

Full Questions

Question No: 21 Approximate the integral $\int_0^{\pi} \sqrt{1 + \sin x}\,dx$ using **Simpson's rule** with $n = 4$. **[4]**

Question No: 22 Determine whether the improper integral $\int_{\frac{\pi}{2}}^{\pi} \frac{1}{1 + \cos x}dx$ converges or diverges and if it converges find its value. **[4]**

Question No: 23 **Evaluate** $\int \frac{1}{x\left(x^2+x+1\right)} dx.$ **[6]**

Question No: 24 Let R be the region bounded by the graph of the equations $y = x^2$ and $y = x + 2$.

Sketch the region R and **set up** (Do not evaluate) an integral that can be used to find the **volume** of the solid generated by revolving the region R around the **line** $x = 2$. (**Use Cylindrical Shell**) [**5**]

Question No: 25 Let R be the region that is outside the graph of the equation $r = 1$ and inside the graph of the equation $r = 1 + \sin\theta$.

Sketch the region R and **Evaluate** its **area**. [6]

Question No: 26 Let R be the region **bounded** by the graph of the equations: $r = 2\sec\theta$, $\theta = 0$ and $\theta = \frac{\pi}{4}$.

Sketch the region R and **set up** (Do not evaluate) an integral that can be used to find its **area**. [5]