Question IV:

A. Prove by cases that $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ for every real numbers $x, y \in \mathbb{R}$ and $y \neq 0$.

Case 1: $x \geq 0$ $y < 0$
Case 2: $x < 0$ $y > 0$
Case 3: $y > 0$ $x = 0$
Case 4: $y < 0$ $x = 0$
Case 5: $y > 0$ $x \geq 0$
Case 6: $y < 0$ $x < 0$

$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

$|y| = \begin{cases} y & x > 0 \\ -y & x < 0 \end{cases}$

3.5

Case 1: $x > 0$ $y < 0$

$a \in R$ $a = \frac{|x|}{-|y|}$ $a \ngtr 0$ is true.

Case 2: $x < 0$ $y > 0$

$b \in R$ $b = \frac{-|x|}{|y|}$ $b \ngtr 0$ is true.

Case 3: $y > 0$ $x = 0$

$c \in R$ $C = \frac{|x|}{y}$ $C = 0$ is true

Case 4: $y < 0$ $x = 0$

$D \in R$ $D = \frac{|x|}{-|y|}$ $D = 0$ is true

Case 5: $y > 0$ $x \ngtr 0$

$E \in R$ $E = \frac{+|x|}{+|y|}$ $E \ngtr 0$ is true,

Case 6: $y < 0$ $x < 0$

$F \in R$ $F = \frac{-|x|}{-|y|}$ $F > 0$ is true

Question Number	1	2	3	4	5	6	Total
Answer	b	b	c	X	c	a	2.5

Question I:

A. **Choose the correct answer, then fill in the table above:**

p	T	¬p	¬T	¬p∧¬T	
T	T	F	F	F	F
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	F

(1) The compound proposition $p \wedge \neg(p \vee T)$ is:

(a) a tautology (b) a contradiction (c) a contingency (d) None of the previous.

(2) The proposition "n is odd is a necessary and sufficient condition for n^2 to be odd" is logically equivalent to the proposition:

(a) If n is odd, then n^2 is odd.

(b) n is odd if and only if n^2 is odd.

(c) If n^2 is odd, then n is odd

(d) None of the previous

(3) Let $A = \{\phi\}$. Then $\mathcal{P}(A) =$

(a) $\{\emptyset\}$ (b) $\{\{\emptyset\}\}$ (c) $\{\emptyset, \{\emptyset\}\}$ (d) None of the previous

(4) $\forall x\,(x > 0 \vee x < 0)$ is false if the domain is:

(a) $\mathbb{Z}$ (b) $\mathbb{Z}^+$ (c) $\mathbb{Z}^-$ (d) None of the previous

(5) The converse of the proposition "We go camping whenever it is not raining,":

(a) If it is not raining, then we go camping. (b) If it is raining, then we cannot go camping.

(c) If we go camping, then it is not raining (d) None of the previous

(6) Let $Q(x)$ be the statement" $x + 1 < 2x$" where the domain is the set of all integers, then the truth value of $\exists x \neg Q(x)$ is:

(a) True (b) False (c) None of the previous

Question II:

A. **Without using truth tables** prove the following:

$$p \to ((q \to r) \vee q) \equiv T$$

L.H.S : $p \to ((q \to r) \vee q)$

$\equiv \neg p \vee ((q \to r) \vee q)$

$\equiv \neg p \vee (\neg q \vee r) \vee q$ ✱

$(\neg p \vee r) \vee (\neg q \vee q) \longrightarrow$ [illegible] $(\neg q \vee q) \equiv T$

$\equiv \neg p \vee r \vee T$

$\equiv T$

L.H.S $\equiv$ R.H.S

1,75/2

B. Is the following argument valid or invalid? Justify your answer:

(F)

③ $\neg q \to r$: T
② $\neg q \vee \neg r$: T
① $\neg p \to q$: T
$\therefore \neg p \vee r$: F

① $p \to q$, p:T , q T
② $\neg q \vee \neg r$, r:F , q T
③ $\neg q \to r$, q:T , r:F

-: argument is invalid

2/2

Question III:

(1) Prove that if $\overset{p}{5n+6}$ is odd, then $\overset{q}{n}$ is odd. in postiv intgr.

$5n+6$ is odd $\xrightarrow{??}$ n is odd

prove by Conterpositiv → كونتر بوزيتف

$p \to q$

$\neg q \to p$

n is even $\xrightarrow{??}$ $5n+6$ is even

Let n is even $n = 2k$, $k \in \mathbb{Z}^+$

$n = 2k$

$5n = 10k$

$5n+6 = 10k+6$

$5n+6 = 2\underbrace{(5k+3)}_{m}$

$5n+6 = 2m$, $m = 5k+3 \in \mathbb{Z}^+$

∴ n is even q

$\frac{1.5}{2}$

(2) Show that if x is a rational number, then $3x - 1$ is also rational.

x is a rational $\xrightarrow{??}$ $3x-1$ is rational

prove by Dircte

Let x is a rational, $x = \frac{a}{b}$ $a, b \in \mathbb{Z}$, $b \neq 0$

$x = \frac{a}{b}$

$3x - 1 = 3\left(\frac{a}{b}\right) - 1$

$\frac{3a}{b} - 1$

$\frac{3a}{\cancel{b}} = \frac{-b}{b}$

$a = \frac{b}{3}$ ∴ x is rational

1.5

(3) Prove that the statement "$\forall n\,(n \leq n^3)$ **is false** in the domain of integer number $\mathbb{Z}$.

$n = -2$

$-2 \leq (-2)^3$

$-2 \leq -8$ false

$-2 \nleq -8$ false

by Conter exmple

∴ $\forall n (n \leq n^3)$ is false

1

T → كلها صحيح

F → وحده خطأ

د ,

Conter exmple

Question IV:

$x \geq 0$

A. Prove by cases that $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ for every real numbers $x, y \in \mathbb{R}$ and $y \neq 0$.

$x, y \in R$ $\quad y \neq 0$

$|x| = \begin{cases} x & \geq 0 \\ -x & < 0 \end{cases}$

by 4 cases

$|y| = \begin{cases} y & \geq 0 \\ -y & < 0 \end{cases}$

Cas 1:
$x > 0$

$\left|\frac{x}{y}\right| = \frac{x}{y}$ $\quad y \neq 0 \quad x \in R$

$= \frac{1}{y}$

$\frac{2.25}{3.5}$

Cas 2:
$-x < 0$

$\left|\frac{-x}{y}\right| = \frac{x}{y}$ $\quad y \neq 0$

Cos 3:
$y \geq 0$

$\left|\frac{x}{y}\right| = \frac{x}{y}$ $\quad y \neq 0$

Cose 4
$-y < 0$

$\left|\frac{x}{-y}\right| = \frac{x}{y}$ $\quad y \neq 0$

صحيحه موجبه

B. If the universal set $U = \mathbb{N}$ and $A = \{x \in \mathbb{N} | 3 < x \le 7\}$ and $B = \{y \in \mathbb{N} | y \le 10 \wedge y \text{ is even}\}$. Then answer the following:

$A = \{4, 5, 6, 7\}$ $\frac{1}{2}$

$B = \{2, 4, 6, 8, 10\}$ $\frac{1}{2}$

(1) Find the sets $\overline{A \cup B}$ and $A - B$.

$A \cap B$ and $A - B$

$A \cap B \{4, 6\}$

$A - B \{5, 7\}$ $\frac{1}{2}$

$\boxed{1.5}$

(2) Find $|A \times B| = \{(4,2), (4,4), (4,6), (4,8), (4,10), (5,2), (5,4), (5,6), (5,8), (5,10), (6,2), (6,4), (6,6), (6,8), (6,10), (7,2), (7,4), (7,6), (7,8), (7,10)\}$

$\boxed{0.75}$

20

~~$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1$~~

(3) $P(A \cap B) =$

$(A \cap B) = \{4, 6\}$

$P = 2^2 = 4$

$\boxed{0.3}$

Good Luck☺

Question Number	1	2	3	4	5	6	Total
Answer	a	b	d	c	a	a	3.75

Question I:

A. Choose the correct answer, then fill in the table above:

(1) For the equivalence relation on $\mathbb{Z}$ defined by $aRb \Leftrightarrow a \equiv b\ (mod 4)$

0 4 8
0 -8 -4 0 4 8

(a) $[8] = [0]$ (b) $[8] = [1]$ (c) $[8] = \emptyset$ (d) None of the previous

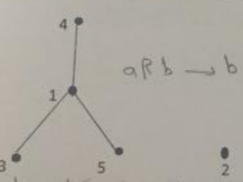
(2) The transitive closure of the relation $R = \{(1,1),(1,2),(3,1)\}$ defined on the set $A = \{1,2,3\}$ equals:

(1,1) 2,1

R o R (1,1), (1,2) (3,1) (2,3)

(a) R (b) R^2 (c) $A \times A$ (d) None of the previous

(3) If S is a partial ordering relation on the set $A = \{1,2,3,4,5\}$ with the Hass diagram Shown below, then S equals to:



aRb → bRa

aRc

(4,1) (1,1)

انعكاسي
انتي متماثل
ترانسيتف

(a) $\{(1,1), (2,2), (3,3), (4,4),(5,5), (3,1), (1,4),(3,4), (5,1), (5,4)\}$ (b) $\{(3,1), (3,4), (5,1), (5,4)\}$

(c) $\{(1,1), (2,2), (3,3), (4,4),(5,5),(3,1), (1,4),(3,4), (5,1)\}$ (d) None of the previous

(4) Number of edges of $K_{4,5}$ is:

(a) 21 (b) 9 (c) 20 (d) None of the previous

(5) If R is an equivalence relation on A. If $\forall a,b \in A, [a] \cap [b] \neq \phi$, then:

aRb → bRa
$[a] \cap [b] \neq 0$

(a) $[a] = [b]$ (b) $a = b$ (c) $a \not R b$ (d) None of the previous

$[b] \cap [a]$

(6) If $G = (V,E)$ be an undirected graph with 3 vertices and 6 edges. If all vertices have the same degree, then $\forall v \in V$:

$r + r + r = 2 \cdot 6$
$\frac{3r}{3} = \frac{12}{3}$
$r = 4$

$\boxed{\deg(v) = 4}$ (b) $\deg(v) = 2$ (c) $\deg(v) = 6$ (d) None of the previous

Question II:

(a) Let $A = \mathbb{Z}$. Prove that the relation R defined on A by

$$xRy \Leftrightarrow x + y \text{ is even}$$

is an equivalence relation. Justify your answer.

$\frac{2.75}{3}$

$X R y \longleftrightarrow x + y$ is even

2k

رفلكسف aRa $\forall a \in A$

Sine ~~aRa~~

$a + a = 2k$

$2a = 2k$ $k = 1 \in \mathbb{Z}$

$a = k$

$A = \mathbb{Z}$

سمترك $\forall a\ \forall b$ aRb $\longrightarrow$ bRa

Sin aRb

$a + b = 2k$

$b + a = 2(k)$

$\therefore$ bRa is symitric

ترانسف

$\forall a \forall a \forall c$ aRb $\land$ bRa $\longrightarrow$ aRc

Let aRb $\land$ bRa

$a + b = 2t \land b + a = 2m$ $t, m \in A$

$a + b + b + a = 2t + 2m$

$2a + 2b = 2(t + m)$

$2(a + b) = 2(t + m)$

$\therefore$ is equivalence

(b) Find the equivalence classes [1] and [2].

$[1] \{ b \in \mathbb{Z} = \{ 1 R a \leftrightarrow a + b = 2k \}$

zero
1

$[2] \{ b \in \mathbb{Z} = \{ 2 R a \longleftrightarrow x + y = 2k \}$

(c) Give a partition of $\mathbb{Z}$ relative to the relation R.

$XRy \longleftrightarrow x + y$ is even

zero
0.5

Question III:

(a) Define the relation R on $\mathbb{Z}$ by $a\,R\,b \leftrightarrow b = ka$ for some integer $k \in \mathbb{Z}^{+}$.

(1) Prove that R is a partial ordering for $\mathbb{Z}$.

$aRb \longleftrightarrow b = ka$

رفلكسف

$aRa \quad \forall a \in Z$

Sine aRa

$a = ka \qquad k = 1 \in Z \quad 0.5$

$a = a$

$\therefore aRa$ is Reflixive

انتيسمترك $\forall a \, \forall b \in A \quad aRb \wedge bRa \xrightarrow{??} a = b$

let $aRb \wedge bRa$ $\quad 0.25$

$a = mb \wedge b = ta \quad m, t \in Z^{+}$

$a = b$

$\therefore aRb \wedge bRa \xrightarrow{why} a = b$ is intysmtric 0.25

ترانسف $\forall a \, \forall b \, \forall c \in Z \quad aRb \wedge bRc \longrightarrow aRc$

let $aRb \wedge bRc$

$a = kb \wedge b = kc$ is trinsitiv

$\therefore R$ is Partial ordering

(2) Is R total ordering for $\mathbb{Z}$, justify your answer.

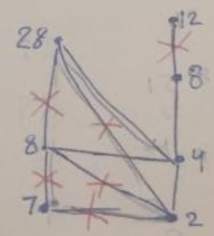
Ex

$aRb \leftrightarrow b \geq ka$

3ero $\frac{}{0.5}$

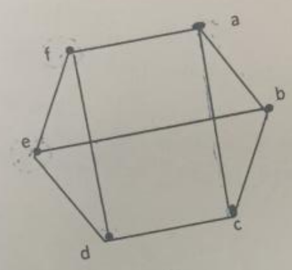
(b) Draw the Hasse diagram for divisibility relation defined by $a\,R\,b \leftrightarrow a \mid b$ on the set $\{2,3,4,7,8,12,28\}$.

3ero $\frac{}{1}$



Question IV

(1) Let G be the graph shown below.



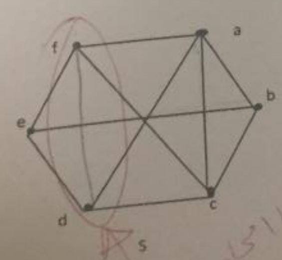
G

(a) Is G bipartite? Justify your answer.

Not bipartite

لان كل راس يرتبط مع كم ضلع وما قدرت
احط كل راس في مجموعه لان من شرط bipartite
انه يكون كل راس في مجموعه والروس الي ما تكون مرتبطه
بنفس الراس

(b) Is the graph G a subgraph of the graph S represented below? Justify your answer.



S

No subgraph

لانه اضافنا ضلع ماصح؟

كيف كتبتها [illegible]

Question V

(1) Let $A = \{2,3,4\}, B = \{2,6,8,9\}$ and let R and S be two relations from A to B defined as follows:

$a\,R\,b \leftrightarrow a|b$ and $a\,S\,b \leftrightarrow b - a$ is odd .

2-2 = 0
2-3 = -1
2-4 = 2
6-2 = 4
6-3 = 3
6-4 = 2
8-2 = 6
8-3 = 5
8-4 = 4
9-2 = 7
9-3 = 6
9-4 = 5

(a) Find R and S.

$R = \{(2,2),(2,6),(2,8),(3,6),(3,9),(4,8)\}$

$S = \{(3,2),(3,6),(3,8),(2,9),(4,9)\}$

(b) Find $R \cap S$ and $R \cup S$.

$R \cap S = \{(3,6)\}$

$R \cup S = \{(2,2),(2,6),(2,8),(3,6),(3,9),(4,8),(3,2),(3,8)$
$(2,9),(4,9)\}$

(c) Find $S \circ R$ $= \{(2,9),\}$

(2) Are the two graphs $K_{3,2}$ and W_4 isomorphic or not? Justify your answer.

K 3,2

W4

الرؤوس = 5

الاضلاع K 32 = 6

الاضلاع W4 = 8

Not isomorphic

لا تحقق شرط من شروطها وهي الاضلاع تتساوى

6

151 Second Quiz. Name:

#:

Question	1	2	3	4	5	6	7	8	9	10	Total
Answer	b	c	b	a	a	b	d	b	a	c	

Choose the correct answer, then fill in the table above:

(1) Consider the congruent relation modulo 5, $x \equiv y \pmod 5$, then:

(a) $6 \in [39]$. (b) $6 \in [51]$. (c) $6 \in [-21]$ (d) None of the previous .

(2) The symmetric closure of the relation $R = \{(a,b): a>b \text{ or } a=b\}$ defined on the set Z is:

(a) R (b) R^2 (c) $Z \times Z$ (d) None of the previous

(3) If $S = \{(x,y) \in R \times R : y^2 = x\}$

$3^2 = 9$ 9,3

(a) $(3,9) \in S$. (b) $(9,3) \in S$.. (c) $(\sqrt{3},3) \in S$. (d) None of the previous

(4) Number of edges of W_{90} is

(a) 180 (b) 90 (c) 89 (d) None of the previous.

---

(5) If R is an equivalence relation on A. If $a, b \in A, a\,R\,b$, then:

(a) $[a] = [b]$ (b) $a = b$ (c) $[a] = [b] = \phi$ (d) None of the previous

(6) If $G = (V, E)$ be an undirected graph with 4 vertices and each of degree 6. Then G has

(a) 8 edges. (b) 12 edges. (c) 6 edges. (d) None of the previous.

(7) Let R be a relation on $\{a,b,c,d\}$ such that $R = \{(a,b),(a,d),(b,c),(c,c),(d,a)\}$. R^2 is

a) $\{(a,c),(a,a),(b,c),(d,b),(d,a),(d,d)\}$.

b) $\{(a,c),(a,a),(b,c),(c,c),(d,b),(d,a),(d,d)\}$

c) $\{(a,c),(a,a),(b,c),(d,b),(d,d)\}$ **d)** None of the previous.

(8) Let $A = \{1,2,3,\cdots,10\}$. Which of the following is <u>not</u> a partition on A

(a) $\{\{1\},\{2\},\cdots,\{10\}\}$ b) $\{\{3,8,10\},\{1,2,5,9\},\{6,7\}\}$ c) $\{\{1,2,3,\cdots,10\}\}$.

) The graph C_6 is

(a) bipartite. (b) complete. (c) multigraph. (d) None of the pr s.

0) Let R be a relation on $\mathbb{Z}$ defined by $aRb \leftrightarrow ab > 0$. R is

(a) an equivalence relation. (b) a partial order relation. (c) None of th

Question IV:

A. Prove by cases that $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ for every real numbers $x, y \in \mathbb{R}$ and $y \neq 0$.

$\{x,y \mid x \geq 0 \ , y > 0\}$

Case 1 : $\left|\frac{x}{y}\right| = \frac{|x|}{|y|} = \frac{x}{y}$

Case 2 : $\left|\frac{x}{y}\right| = \frac{|-x|}{|y|} = \frac{x}{y}$ $\{x,y \mid x \leq 0 \ , y > 0\}$

Case 3 : $\{x,y \mid x \geq 0 \ , y \leq 0\}$

$\left|\frac{x}{y}\right| = \frac{|x|}{|-y|} = \frac{x}{y}$

$\{x,y \mid x \leq 0 \ , y \leq 0\}$

Case 4 : $\left|\frac{x}{y}\right| = \frac{|-x|}{|-y|} = \frac{x}{y}$

$\boxed{\therefore \forall x, y \in R \ \left|\frac{x}{y}\right| = \frac{|x|}{|y|}}$

$\frac{3}{3.5}$

Question III:

(1) Prove that if $5n+6$ is odd, then n is odd. where n is positive integer

$5n+6$ is odd $\rightarrow$ n is odd   we can't do it by direct prove so,

By contra position, positive

$\neg q \rightarrow \neg p$

n is even $\rightarrow$ $5n+6$ is even

Suppose that n is even show that $5n+6$ is even ?

s.t $\exists k \in \mathbb{Z}$, $n=2k$

$5n = 5(2k)$

$5n+6 = 5(2k)+6$

$5n+6 = 10k+6$ $m = 5k+3 \in$?

$5n+6 = 2(5k+3)$

$5n+6 = 2m$ which is even   if $5n+6$ ~~odd~~ then n is odd

1.75/2

(2) Show that if x is a rational number, then $3x-1$ is also rational.

x rational $\rightarrow$ $3x-1$ is also rational

Suppose that x is rational $\exists x \in \mathbb{Z}$ show that $3x-1$ is rational ?

when s.t $x = \frac{a}{b} \in \mathbb{Z}$

$3x-1 = 3(\frac{a}{b})-1$

$3x-1 = 3(\frac{a}{b}) - \frac{1}{b}$

$b.\frac{3x-1}{b} = \frac{3a}{b} - \frac{1.b}{b}$

$\frac{3x-1}{b} = \frac{(3a-1).\cancel{b}}{\cancel{b}}$ which is $3x-1$ is rational.

$\frac{3x-1}{b} = 3a-1$

2

$2 \leq 2^3$

(3) Prove that the statement $\boxed{\text{"}\forall n\,(n \leq n^3)}$ $\boxed{\text{is false}}$ in the domain of integer number $\mathbb{Z}$.

By counter example $\frac{1}{2} \leq (\frac{1}{2})^3$: false   1

Domain $= \{-2, -1, 0, 1, 2\}$

$\boxed{n=-1}$ $-1 \leq (-1)^3$ false $\boxed{n=-2}$ $\boxed{n \leq n^3}$ $-2 \leq (-2)^3$ $n=2$ $2 \leq 2^3$ $2 \leq 8$

$\boxed{n=0}$ $0 \leq (0)^3$ true

$\boxed{n=1}$ $1 \leq (1)^3$ true

we prove that's, $n \leq n^3$ is false.

Question II:

(a) Let $A = \mathbb{Z}$. Prove that the relation R defined on A by

$$xRy \Leftrightarrow x + y \text{ is even}$$

is an <u>equivalence relation</u>. Justify your answer.

$\forall x$ yRy $y+y=2y$

1) since if xRx → $x+x=2x$ $x \in \mathbb{Z}$

∴ R is reflexive.

$\frac{3}{3}$

2) $\forall x \forall y$ $x, y \in \mathbb{Z}$

if $xRy \xrightarrow{?} yRx$

$xRy \to x+y=2m \to$

$\to y+x=2m$ ∴ R is symm.

3) $\forall x \forall y \forall z$ $x, y, z \in \mathbb{Z}$

if $xRy \wedge yRz \to xRz$

$xRy \wedge yRz \to x+y=2m$ $y+z=2t$

$x+y+y+z=2m+2t$ →

$x+y+y+z=2m+2t$

$x+z = 2m+2t+2y$

$x+z=2(m+t+y)$

$x+z=2L$

∴ R is transitive.

∴ is an equivalence relation

(b) Find the equivalence classes [1] and [2].

$[a]=\{b \in \mathbb{Z} \mid bRa\}$

$[1]=\{\ldots -5, -3, -1, 1, 3, 5, 7 \ldots\}$

$[2]=\{\ldots -4, -2, 0, 2, 4, 6, 8 \ldots\}$

$\frac{1}{1}$

$\frac{x+y}{2} = \frac{2m}{2}$

$\frac{x+y}{2} = m$

$2 \mid x+y$: m

$x \equiv y \mod 2$

(c) Give a partition of $\mathbb{Z}$ relative to the relation R.

$[a] = [1] \cup [2] = \mathbb{Z}$

$\{\ldots -5, -3, -1, 1, 3, 5, 7 \ldots\} \cup \{\ldots -4, -2, 0, 2, 4, 6, 8 \ldots\} = \mathbb{Z}$

$\frac{0.5}{0.5}$

Well.