CHAPTER 5

Probability

INTRODUCTION:

- ***PROBABILITY*** *AS A GENERAL CONCEPT CAN BE DEFINED AS THE CHANCE OF AN EVENT OCCURRING.*
- *PROBABILITY ARE USED IN GAMES OF CHANCE, INSURANCE, INVESTMENTS, AND WEATHER FORECASTING, AND IN VARIOUS AREAS.*

BASIC CONCEPTS

- *A **PROBABILITY EXPERIMENT** IS A CHANCE PROCESS THAT LEADS TO WELL-DEFINED RESULTS CALLED OUTCOMES.*
- *AN **OUTCOME** IS THE RESULT OF A SINGLE TRIAL OF A PROBABILITY EXPERIMENT.*
- *A **SAMPLE SPACE** IS THE SET OF ALL POSSIBLE OUTCOMES OF A PROBABILITY EXPERIMENT.*
- *AN **EVENT** CONSISTS OF A SET OF OUTCOMES OF A PROBABILITY EXPERIMENT.*
- *AN EVENT WITH ONE OUTCOME IS CALLED A **SIMPLE EVENT** AND WITH MORE THAN ONE OUTCOME IS CALLED **COMPOUND EVENT**.*

- ***EX: FIND THE SAMPLE SPACE FOR THE GENDER OF THE CHILDREN IF A FAMILY HAS THREE CHILDREN AND GIVE AN EXAMPLE FOR SIMPLE EVENT AND ANOTHER ONE FOR A COMPOUND EVENT. USE B FOR BOY AND G FOR GIRL.***

SOLUTION:

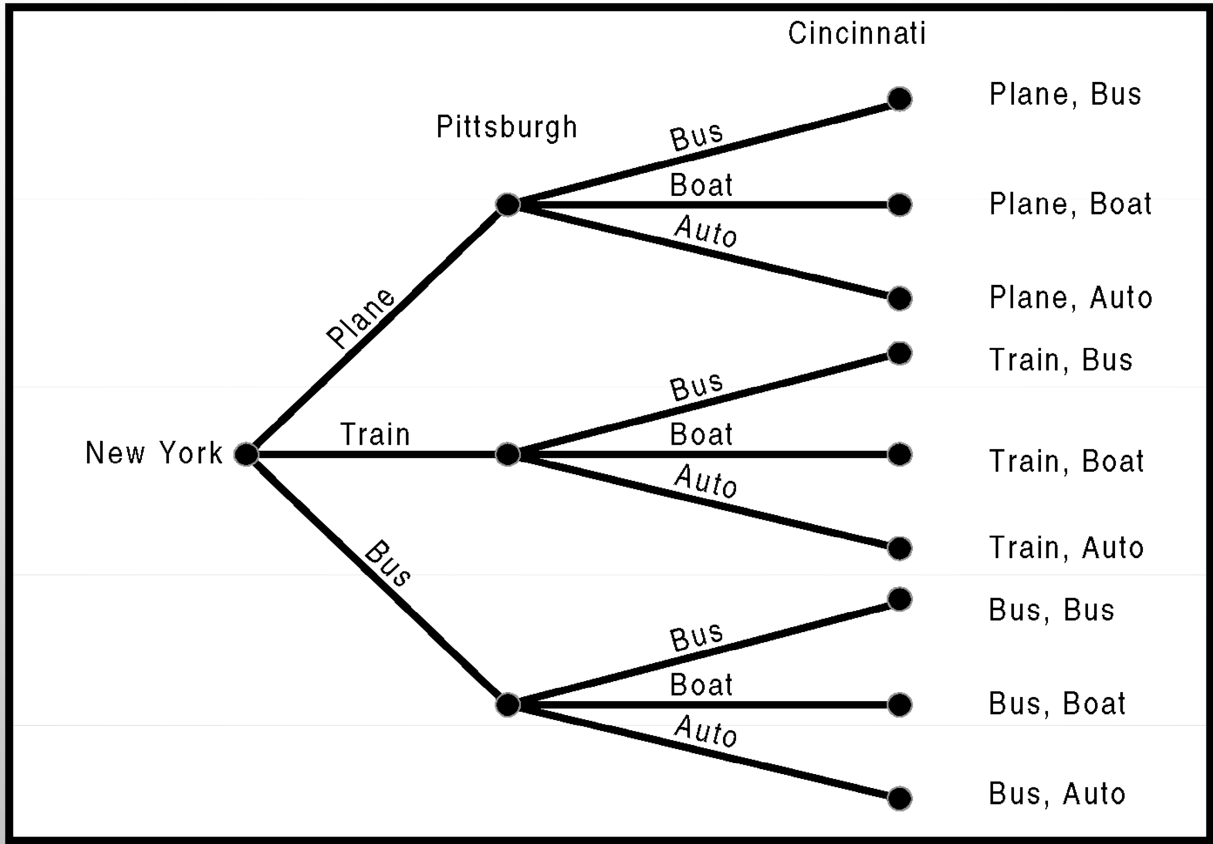
THE SAMPLE SPACE IS

S={ BBB,BBG,BGB,GBB,GGG,GGB,GBG,BGG }

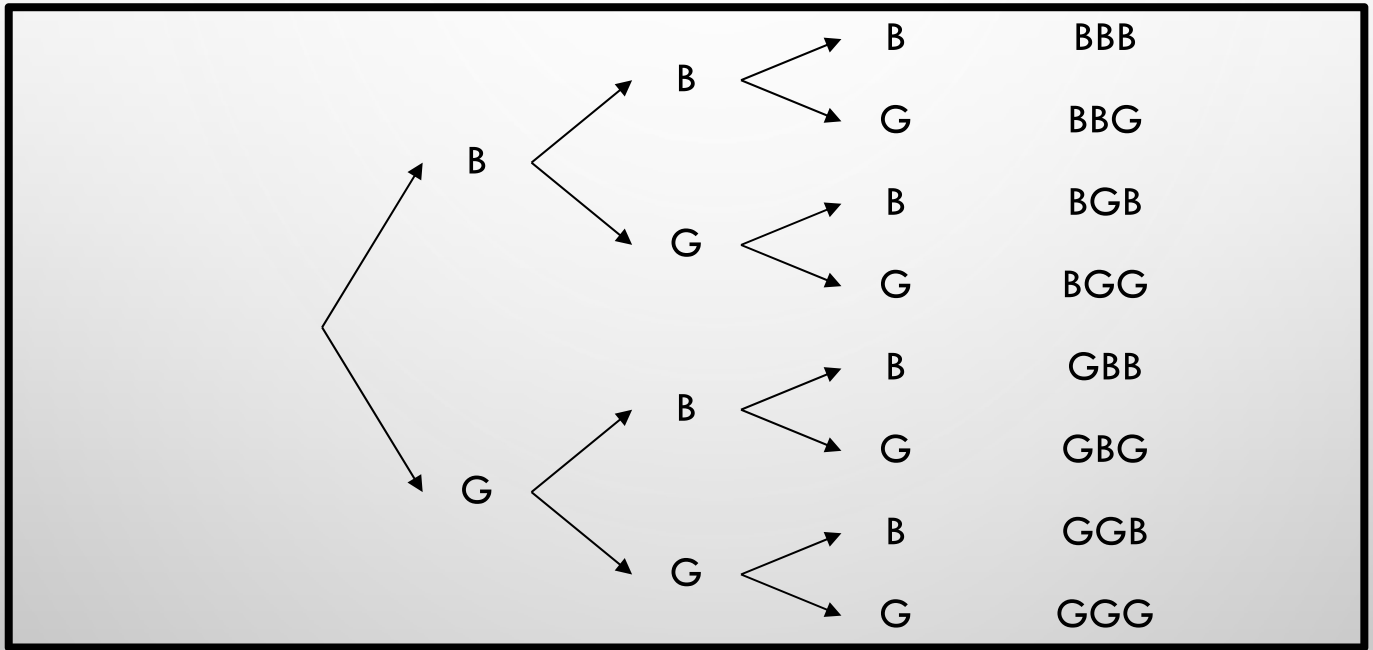
SIMPLE EVENT AS E={BBB}

COMPOUND EVENT AS E={BBG, BGB, GBB}

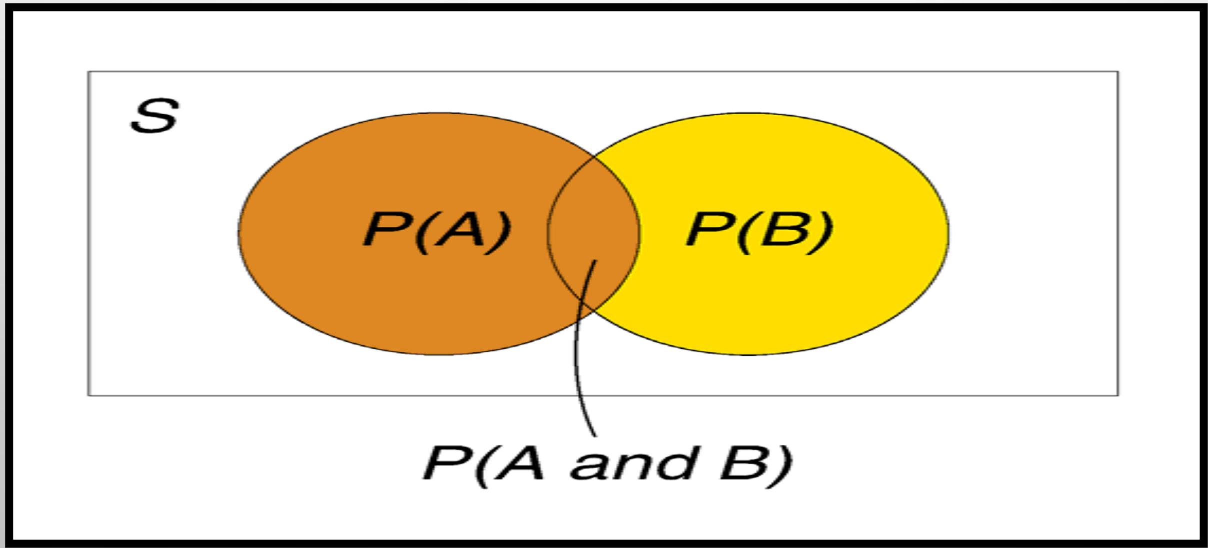
- *A **TREE DIAGRAM** IS A DEVICE USED TO LIST ALL POSSIBILITIES OF A SEQUENCE OF EVENTS IN A SYSTEMATIC WAY.*



- *EX: FIND THE SAMPLE SPACE FOR THE GENDER OF THE CHILDREN IF A FAMILY HAS THREE CHILDREN. USE B FOR BOY AND G FOR GIRL. USE A TREE DIAGRAM TO FIND THE SAMPLE SPACE FOR THE GENDER OF THE THREE CHILDREN.*



- ***<u>EQUALLY LIKELY EVENTS</u>*** *ARE EVENTS THAT HAVE THE SAME PROBABILITY OF OCCURRING.*
- <u>VENN DIAGRAMS</u> ARE USED TO REPRESENT PROBABILITIES PICTORIALLY.



CLASSICAL PROBABILITY

- ***CLASSICAL PROBABILITY*** *USES SAMPLE SPACES TO DETERMINE THE NUMERICAL PROBABILITY THAT AN EVENT WILL HAPPEN. IT ASSUMES THAT ALL OUTCOMES IN THE SAMPLE SPACE ARE EQUALLY LIKELY TO OCCUR.*
- *THE PROBABILITY OF AN EVENT E CAN BE DEFINED AS*

$$\boldsymbol{P(E) = \frac{n(E)}{n(S)} = \frac{\text{Number of outcomes in E}}{\text{Total number of outcomes in the sample space}}}$$

- ***EX: IF A FAMILY HAS THREE CHILDREN, FIND THE PROBABILITY THAT TWO OF THE CHILDREN ARE GIRLS.***

SOLUTION:

THE SAMPLE SPACE IS

S={ BBB,BBG,BGB,GBB,GGG,GGB,GBG,BGG }

N(S)=8

THE EVENT OF TWO GIRLS IS

E={GGB, GBG, BGG}, N(E)=3

THE PROBABILITY THAT TWO OF THE CHILDREN ARE GIRLS IS

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

PROBABILITY RULES

- *THE PROBABILITY OF AN EVENT **E** IS A NUMBER (EITHER A FRACTION OR DECIMAL) BETWEEN AND INCLUDING **0** AND **1**, $0 \leq P(E) \leq 1$.*

- *IF AN EVENT **E** CANNOT OCCUR (I.E., THE EVENT **E** CONTAINS NO MEMBERS IN THE SAMPLE SPACE), THE PROBABILITY IS ZERO.*

- *IF AN EVENT **E** IS CERTAIN, THEN THE PROBABILITY OF E IS ONE.*

- *THE SUM OF THE PROBABILITIES OF THE OUTCOMES IN THE SAMPLE SPACE IS ONE.*

EX: WHEN A SINGLE DIE IS ROLLED, FIND THE PROBABILITY OF GETTING A 9.

SOLUTION:

SINCE THE SAMPLE SPACE IS ***S={1,2,3,4,5, 6}****, IT IS IMPOSSIBLE TO GET A 9,*

$$\boldsymbol{P(9) = \frac{n(9)}{n(S)} = \frac{0}{6} = 0}$$

EX: WHEN A SINGLE DIE IS ROLLED, WHAT IS THE PROBABILITY OF GETTING A NUMBER LESS THAN 7?

SOLUTION:

SINE ALL OUTCOMES IN THE SAMPLE SPACE ARE LESS THAN 7,

$$\boldsymbol{P(number\ less\ than\ 7) = \frac{n(number\ less\ than\ 7)}{n(S)} = \frac{6}{6} = 1}$$

COMPLEMENTARY EVENTS

- THE <u>COMPLEMENT OF AN EVENT *E*</u> IS THE SET OF OUTCOMES IN THE SAMPLE SPACE THAT ARE NOT INCLUDED IN THE OUTCOMES OF EVENT *E*. THE COMPLEMENT OF *E* IS DENOTED BY $\overline{E}$

  - RULE FOR COMPLEMENTARY EVENTS

    $P(E) + P(\overline{E}) = 1$

    $P(\overline{E}) = 1 - P(E)$ or $P(E) = 1 - P(\overline{E})$

  - COMPLEMENTARY EVENTS ARE MUTUALLY EXCLUSIVE.

EX: FIND THE COMPLEMENT OF EACH EVENT.

A- ROLLING A DIE AND GETTING A 4.

GETTING A 1,2,3,5 OR 6

B- SELECTING A MONTH AND GETTING A MONTH THAT BEGINS WITH A J.

GETTING FEBRUARY, MARCH, APRIL, MAY, AUGUST, SEPTEMBER, OCTOBER, NOVEMBER OR DECEMBER

C- SELECTING A DAY OF THE WEEK AND GETTING A WEEKDAY

GETTING THURSDAY OR FRIDAY

EX: IF THE PROBABILITY THAT A PERSON LIVES IN AN INDUSTRIALIZED COUNTRY OF THE WORLD IS 1/5, FIND THE PROBABILITY THAT A PERSON DOES NOT LIVE IN AN INDUSTRIALIZED COUNTRY.

P(LIVING IN AN INDUSTRIALIZED COUNTRY) = 1/5

P(NOT LIVING IN AN INDUSTRIALIZED COUNTRY)

= 1 - P(LIVING IN AN INDUSTRIALIZED COUNTRY)

= 1 - (1/5) = 4/5

EMPIRICAL PROBABILITY

- ***EMPIRICAL PROBABILITY*** *RELIES ON ACTUAL EXPERIENCE TO DETERMINE THE LIKELIHOOD OF OUTCOMES.*

- *GIVEN A FREQUENCY DISTRIBUTION, THE PROBABILITY OF AN EVENT BEING IN A GIVEN CLASS IS:*

$$\boldsymbol{P(E) = \frac{frequency\ of\ the\ class}{total\ frequecies\ in\ the\ distribution} = \frac{f}{n}}$$

EX: THE FREQUENCY DISTRIBUTION OF BLOOD TYPES FOR SAMPLE OF 50 PEOPLE AS FOLLOW:

Blood Type	Frequency
A	22
B	5
O	21
AB	2

FIND THE FOLLOWING PROBABILITIES.

- *A PERSON HAS TYPE O BLOOD.*

$$P(O) = \frac{f_O}{n} = \frac{21}{50}$$

- *B- A PERSON HAS TYPE A OR TYPE B BLOOD.*

$$P(A\ or\ B) = \frac{f_A}{n} + \frac{f_B}{n} = \frac{22}{50} + \frac{5}{50} = \frac{27}{50}$$

- *C- A PERSON HAS NEITHER TYPE A NOR TYPE O BLOOD.*

$$P(neither\ A\ nor\ O) = P(B\ or\ AB) = \frac{f_B}{n} + \frac{f_{AB}}{n} = \frac{5}{50} + \frac{2}{50} = \frac{7}{50}$$

OR

$$P(neither\ A\ nor\ O) = 1 - P(A\ or\ O) = 1 - \left(\frac{f_A}{n} + \frac{f_O}{n}\right)$$

$$= 1 - \left(\frac{22}{50} + \frac{21}{50}\right) = 1 - \frac{43}{50} = \frac{7}{50}$$

- *D- A PERSON DOES NOT HAVE TYPE AB BLOOD.*

$$P(not\ AB) = 1 - P(AB) = 1 - \frac{f_{AB}}{n} = 1 - \frac{2}{50} = \frac{48}{50} = \frac{24}{25}$$

MUTUALLY EXCLUSIVE EVENTS

- *TWO EVENTS ARE* ***MUTUALLY EXCLUSIVE*** *IF THEY CANNOT OCCUR AT THE SAME TIME (THEY HAVE NO INTERSECTION, WHICH MEANS THERE ARE NO OUTCOMES IN COMMON).*
- *TWO EVENTS ARE* ***NOT MUTUALLY EXCLUSIVE*** *IF THEY CAN OCCUR AT THE SAME TIME (THEY HAVE INTERSECTION, WHICH MEANS THERE ARE OUTCOMES IN COMMON).*
- *THE PROBABILITY OF TWO OR MORE EVENTS CAN BE DETERMINED BY THE* ***ADDITION RULES****.*

EX: DETERMINE WHICH EVENTS ARE MUTUALLY EXCLUSIVE AND WHICH ARE NOT, WHEN A SINGLE DIE IS ROLLED.

A- GETTING AN ODD NUMBER AND GETTING AN EVEN NUMBER

THE EVENTS ARE MUTUALLY EXCLUSIVE; SINCE THE FIRST EVENT CAN BE 1, 3 OR 5 AND THE SECOND EVENT CAN BE 2, 4 OR 6.

B- GETTING A 3 AND GETTING AN ODD NUMBER.

THE EVENTS ARE NOT MUTUALLY EXCLUSIVE, SINCE THE FIRST EVENT IS A 3 AND THEN SECOND EVENT CAN BE 1, 3 OR 5. HENCE, 3 IS CONTAINED IN BOTH EVENTS.

C- GETTING AN ODD NUMBER AND GETTING A NUMBER LESS THAN 4.

THE EVENTS ARE NOT MUTUALLY EXCLUSIVE, SINCE THE FIRST EVENT CAN BE 1, 3 OR 5 AND THE SECOND EVENT CAN BE 1, 2 OR 3. HENCE, 1 AND 3 ARE CONTAINED IN BOTH EVENTS.

D- GETTING A NUMBER GREATER THAN 4 AND GETTING A NUMBER LESS THAN 4.

THE EVENTS ARE MUTUALLY EXCLUSIVE, SINCE THE FIRST EVENT CAN BE 5 OR 6 AND THE SECOND EVENT CAN BE 1, 2 OR 3.

- *WHEN TWO EVENTS* $\boldsymbol{A}$ *AND* $\boldsymbol{B}$ *ARE MUTUALLY EXCLUSIVE, THE PROBABILITY THAT* $\boldsymbol{A}$ *OR* $\boldsymbol{B}$ *WILL OCCUR IS:*

$$\boldsymbol{P(A\ or\ B) = P(A) + P(B)}$$

- *WHEN TWO EVENTS* $\boldsymbol{A}$ *AND* $\boldsymbol{B}$ *ARE NOT MUTUALLY EXCLUSIVE, THE PROBABILITY THAT* $\boldsymbol{A}$ *OR* $\boldsymbol{B}$ *WILL OCCUR IS:*

$$\boldsymbol{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

EX: A BOX CONTAINS 3 GLAZED DOUGHNUTS, 4 JELLY DOUGHNUTS AND 5 CHOCOLATE DOUGHNUTS. IF A PERSON SELECTS A DOUGHNUT AT RANDOM, FIND THE PROBABILITY THAT IT IS EITHER A GLAZED DOUGHNUT OR A CHOCOLATE DOUGHNUT.

SOLUTION:

THE TOTAL NUMBER OF DOUGHNUTS IN THE BOX IS 12 AND THE EVENT ARE MUTUALLY EXCLUSIVE, SO

$$\boldsymbol{P(Glazed\ or\ Chocolate) = P(Galzed) + P(Chocolate)}$$

$$\boldsymbol{= \frac{3}{12} + \frac{5}{12} = \frac{8}{12} = \frac{2}{3}}$$

EX: A DAY OF THE WEEK IS SELECTED AT RANDOM. FIND THE PROBABILITY THAT IT IS A WEEKEND DAY (THURSDAY OR FRIDAY)

SOLUTION:

THE TOTAL NUMBER OF DAYS IN WEEK IS 7 AND THE EVENT ARE MUTUALLY EXCLUSIVE, SO

$$\boldsymbol{P(Thursday\ or\ Friday) = P(Thursday) + P(Friday)}$$

$$= \frac{\mathbf{1}}{\mathbf{7}} + \frac{\mathbf{1}}{\mathbf{7}} = \frac{\mathbf{2}}{\mathbf{7}}$$

- *EX: IN A HOSPITAL UNIT THERE ARE 8 NURSES AND 5 PHYSICIANS SHOWN IN THE FOLLOWING TABLE*

Staff	Female	Male	Total
Nurses	7	1	8
Physicians	3	2	5
Total	10	3	13

IF A STAFF IS SELECTED, FIND THE PROBABILITY THAT THE SUBJECT IS A NURSE OR A MALE.

SOLUTION:

THE EVENTS ARE NOT MUTUALLY EXCLUSIVE AND THE SAMPLE SPACE IS

$$\boldsymbol{P(Nurse\ or\ Male) = P(Nurse) + P(Male) - P(Nurse\ and\ Male)}$$

$$= \frac{\mathbf{8}}{\mathbf{13}} + \frac{\mathbf{3}}{\mathbf{13}} - \frac{\mathbf{1}}{\mathbf{13}} = \frac{\mathbf{10}}{\mathbf{13}}$$

INDEPENDENT EVENTS

- *TWO EVENTS **A** AND **B** ARE **INDEPENDENT** IF THE FACT THAT **A** OCCURS DOES NOT AFFECT THE PROBABILITY OF **B** OCCURRING.*
- *MULTIPLICATION RULES*
  - *THE **MULTIPLICATION RULES** CAN BE USED TO FIND THE PROBABILITY OF TWO OR MORE EVENTS THAT OCCUR IN SEQUENCE.*
  - *WHEN TWO EVENTS ARE INDEPENDENT, THE PROBABILITY OF BOTH OCCURRING IS:*

$$P(A\ and\ B) = P(A)P(B)$$

- *EX: A BOX CONTAINS 3 RED BALLS, 2 BLUE BALLS AND 5 WHITE BALLS. A BALL IS SELECTED AND ITS COLOR NOTED. THEN IT IS REPLACED. A SECOND BALL IS SELECTED AND ITS COLOR NOTED. FIND THE PROBABILITY OF EACH OF THESE.*
  - *A. SELECTING 2 BLUE BALLS*

$$P(Blue\ and\ Blue) = P(Blue)P(Blue) = \left(\frac{2}{10}\right)\left(\frac{2}{10}\right) = \frac{4}{100} = \frac{1}{25}$$

  - *B. SELECTING 1 BLUE BALL AND THEN 1 WHITE BALL*

$$P(Blue\ and\ White) = P(Blue)P(White) = \left(\frac{2}{10}\right)\left(\frac{5}{10}\right) = \frac{10}{100} = \frac{1}{10}$$

  - *C. SELECTING 1 RED BALL AND THEN 1 BLUE BALL*

$$P(Red\ and\ Blue) = P(Red)P(Blue) = \left(\frac{3}{10}\right)\left(\frac{2}{10}\right) = \frac{6}{100} = \frac{3}{50}$$

- ***EX: APPROXIMATELY 9% OF MEN HAVE A TYPE OF COLOR BLINDNESS THAT PREVENTS THEM FROM DISTINGUISHING BETWEEN RED AND GREEN. IF 3 MEN ARE SELECTED AT RANDOM, FIND THE PROBABILITY THAT ALL OF THEM WILL HAVE THIS TYPE OF RED-GREEN COLOR BLINDNESS.***

SOLUTION:

LET C DENOTE RED-GREEN COLOR BLINDNESS. THEN

$$P(C \text{ and } C \text{ and } C) = P(C)P(C)P(C) = (0.09)(0.09)(0.09) = 0.000729$$

Counting Rules

- *The Fundamental Counting Rule:*

*a sequence of n events in which the first one has k1 possibilities and the second event has k2 and the third has k3, and so forth, the total number of possibilities of the sequence will be k1 * k2 * k3 * ... *kn*

Note: In this case and means to multiply

EX:The manager of a department store chain wishes to make four-digit identification cards for her employees. How many different cards can be made if she uses the digits 1, 2, 3,4, 5, and 6 and repetitions are permitted?

Solution:

*Since there are 4 spaces to fill on each card and there are 6 choices for each space, the total number of cards that can be made is 6 * 6 * 6 * 6 = 1296.*

- *Factorial Notation:*

These rules use factorial notation. The factorial notation uses the exclamation point.

*5! = 5 * 4 * 3 * 2 * 1*

*9! = 9*8*7*6*5*4*3*2*1*

To use the formulas in the permutation and combination rules, a special definition of 0! is needed. 0! = 1.

Factorial Formulas For any counting n

n! = n(n - 1)(n - 2) ...1

0! = 1

EX:The manager of a department store chain wishes to make four-digit identification cards for her employees. How many different cards can be made if she uses the digits 1, 2, 3,4, 5, and 6 and repetitions are permitted?

Solution:

*Since there are 4 spaces to fill on each card and there are 6 choices for each space, the total number of cards that can be made is 6 * 6 * 6 * 6 = 1296.*

- *Permutations:*

The arrangement of n objects in a specific order using r objects at a time is called a permutation of n objects taking r objects at a time. It is written as nPr, and the formula is

$$_nP_r = \frac{n!}{(n-r)!}$$

EX: The advertising director for a television show has 7 ads to use on the program. If she selects 1 of them for the opening of the show, 1 for the middle of the show, and 1 for the ending of the show, how many possible ways can this be accomplished?

Solution:

Since order is important, the solution is

$$_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 210$$

- *Combinations:*

The number of combinations of r objects selected from n objects is denoted by nCr and is given by the formula

$$_nC_r = \frac{n!}{(n-r)!r!}$$

EX: A newspaper editor has received 8 books to review. He decides that he can use 3 reviews in his newspaper. How many different ways can these 3 reviews be selected?

Solution:

$$_8C_3 = \frac{8!}{(8-3)!3!} = \frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$$