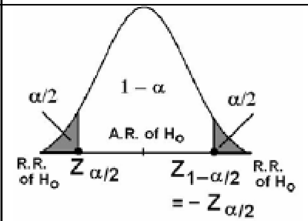
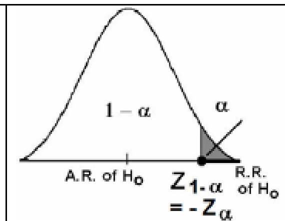
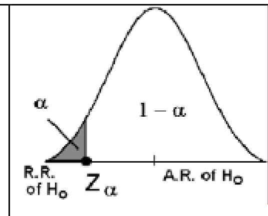
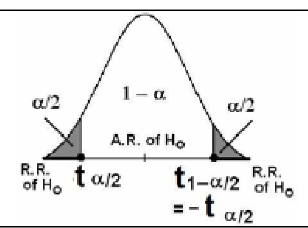
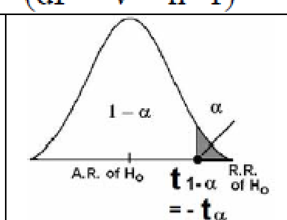
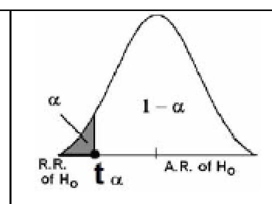
Hypotheses Testing

1-Single Mean

($if\ \sigma\ known$):

Hypotheses	H_o: $\mu = \mu_o$ H_A: $\mu \neq \mu_o$	H_o: $\mu \leq \mu_o$ H_A: $\mu > \mu_o$	H_o: $\mu \geq \mu_o$ H_A: $\mu < \mu_o$
Test Statistic (T.S.)	Calculate the value of: $Z = \dfrac{\overline{X} - \mu_o}{\sigma / \sqrt{n}} \sim N(0,1)$		
R.R. & A.R. of H_o			
Critical value (s)	$Z_{\alpha/2}$ and $-Z_{\alpha/2}$	$Z_{1-\alpha} = -Z_{\alpha}$	Z_{α}
Decision:	We reject H_o (and accept H_A) at the significance level α if:		
	$Z < Z_{\alpha/2}$ or $Z > Z_{1-\alpha/2} = -Z_{\alpha/2}$ Two-Sided Test	$Z > Z_{1-\alpha} = -Z_{\alpha}$ One-Sided Test	$Z < Z_{\alpha}$ One-Sided Test

($if\ \sigma\ unknown$):

Hypotheses	H_o: $\mu = \mu_o$ H_A $\mu \neq \mu_o$	H_o: $\mu \leq \mu_o$ H_A: $\mu > \mu_o$	H_o: $\mu \geq \mu_o$ H_A: $\mu < \mu_o$
Test Statistic (T.S.)	Calculate the value of: $t = \dfrac{\overline{X} - \mu_o}{S / \sqrt{n}} \sim t(n-1)$ $(df = \nu = n-1)$		
R.R. & A.R. of H_o			
Critical value (s)	$t_{\alpha/2}$ and $-t_{\alpha/2}$	$t_{1-\alpha} = -t_{\alpha}$	t_{α}
Decision:	We reject H_o (and accept H_A) at the significance level α if:		
	$t < t_{\alpha/2}$ or $t > t_{1-\alpha/2} = -t_{\alpha/2}$ Two-Sided Test	$t > t_{1-\alpha} = -t_{\alpha}$ One-Sided Test	$t < t_{\alpha}$ One-Sided Test

Question 1:

Suppose that we are interested in estimating the true average time in seconds it takes an adult to open a new type of tamper-resistant aspirin bottle. It is known that the population standard deviation is $\sigma = 5.71$seconds. A random sample of 40 adults gave a mean of 20.6 seconds. Let μ be the population mean, then, to test if the meanμ is 21 seconds at level of significant 0.05 (H_0: $\mu = 21$ vs H_A: $\mu \neq 21$) then:

(1) The value of the test statistic is:

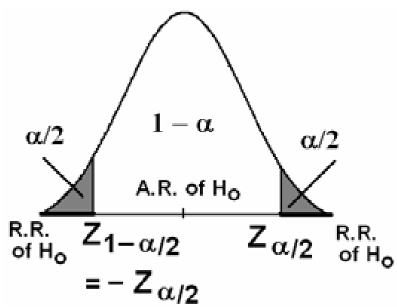
$$\sigma = 5.71 \quad n = 40 \quad \bar{X} = 20.6$$

$$Z = \frac{\bar{X} - \mu_o}{\sigma/\sqrt{n}} = \frac{20.6 - 21}{5.71/\sqrt{40}} = -0.443$$

(A) 0.443 *(B) - 0.012* *<u>(C) -0.443</u>* *(D) 0.012*

(2) The acceptance area is:

$$Z_{\frac{\alpha}{2}} = Z_{\frac{0.05}{2}} = Z_{0.025} = 1.96$$



<u>(A) (-1.96, 1.96)</u> *(B) (1.96, ∞)* *(C) (-∞, 1.96)* *(D) (-∞, 1.645)*

(3) The decision is:

(A) Reject H_0 *<u>(B)</u> Accept H_0* *(C) no decision* *(D) None of these*

Question 2:

If the hemoglobin level of pregnant women (امرأه حامل) is normally distributed, and if the mean and standard deviation of a sample of 25 pregnant women were $\bar{X} = 13$ *(g/dl),*$s = 2$ *(g/dl).Using* $\alpha = 0.05$, *to test if the average hemoglobin level for the pregnant women is greater than 10 (g/dl) [*$H_0 : \mu \leq 10$ *,* $H_A: \mu > 10$*].*

(1) The test statistic is:

(A) $Z = \frac{\bar{X}-10}{\sigma/\sqrt{n}}$ (B) $Z = \frac{\bar{X}-10}{S/\sqrt{n}}$ (C) $t = \frac{\bar{X}-10}{\sigma/\sqrt{n}}$ (D) $t = \frac{\bar{X}-10}{S/\sqrt{n}}$

(2) The value of the test statistic is:

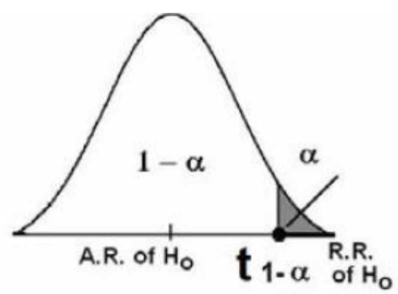
$$s = 2 \quad n = 25 \quad \bar{X} = 13$$

$$t = \frac{\bar{X} - \mu_o}{S/\sqrt{n}} = \frac{13 - 10}{2/\sqrt{25}} = 7.5$$

(A) 10 *(B) 1.5* *(C) 7.5* *(D) 37.5*

(3) The rejection of H_0 is :

$$t_{1-\alpha,n-1} = t_{0.95,24} = 1.711$$

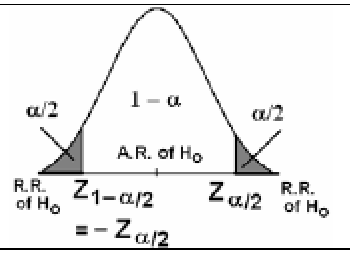
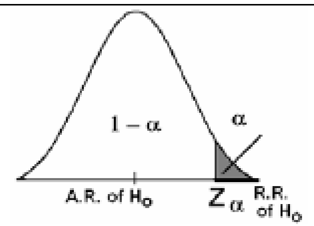
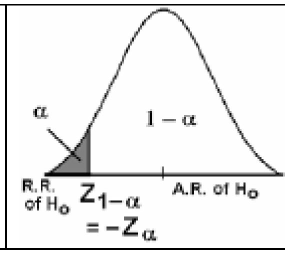
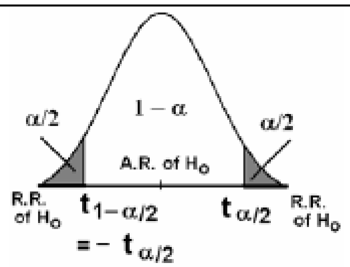
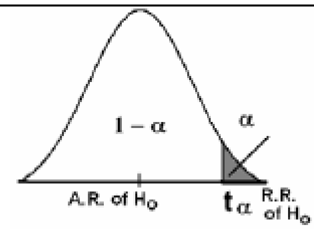
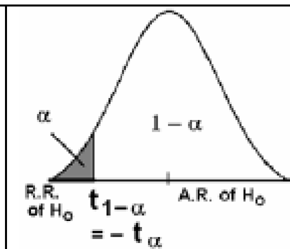


(A) Z< -1.645 *(B) z> 1.645* *(C) t < -1.711* *(D) t > 1.711*

(4) *The decision is:*

(A) Reject H_0 *(B) Do not reject (Accept) H_0*
(C) Accept both H_0 and H_A *(D) Reject both H_0 and H_A*

2-Two Means:

<table>
<tr><td>Hypotheses</td><td>H_o: $\mu_1 - \mu_2 = 0$
H_A: $\mu_1 - \mu_2 \neq 0$</td><td>H_o: $\mu_1 - \mu_2 \leq 0$
H_A: $\mu_1 - \mu_2 > 0$</td><td>H_o: $\mu_1 - \mu_2 \geq 0$
H_A: $\mu_1 - \mu_2 < 0$</td></tr>
<tr><td>Test Statistic For the First Case:</td><td colspan="3">$Z = \dfrac{\overline{X}_1 - \overline{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim N(0,1)$ {if σ_1^2 and σ_2^2 are known}</td></tr>
<tr><td>R.R. and A.R. of H_o (For the First Case)</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Test Statistic For the Second Case:</td><td colspan="3">$T = \dfrac{\overline{X}_1 - \overline{X}_2}{\sqrt{\dfrac{S_p^2}{n_1} + \dfrac{S_p^2}{n_2}}} \sim t(n_1+n_2-2)$ {if $\sigma_1^2=\sigma_2^2=\sigma^2$ is unknown}</td></tr>
<tr><td>R.R. and A.R. of H_o (For the Second Case)</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Decision:</td><td colspan="3">Reject H_o (and accept H_A) at the significance level α if:</td></tr>
<tr><td></td><td>T.S. ∈ R.R.
Two-Sided Test</td><td>T.S. ∈ R.R.
One-Sided Test</td><td>T.S. ∈ R.R.
One-Sided Test</td></tr>
</table>

Question 1:

A standardized chemistry test was given to 50 girls and 75 boys. The girls made an average of 84, while the boys made an average grade of 82. Assume the population standard deviations are 6 and 8 for girls and boys respectively. To test the null hypothesis $H_0: \mu_1 - \mu_2 \leq 0$ *against the alternative hypothesis* $H_A: \mu_1 - \mu_2 > 0$ *at 0.05 level of significance:*

(1) The standard error of $(\bar{X}_1 - \bar{X}_2)$ **is:**

$$\begin{array}{ll} girls: & n_1 = 50,\ \bar{X}_1 = 84,\ \sigma_1 = 6 \\ boys: & n_2 = 75,\ \bar{X}_2 = 82,\ \sigma_2 = 8 \end{array}$$

$$S.E(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{6^2}{50} + \frac{8^2}{75}} = 1.2543$$

(A) 0.2266　　(B) 2　　(C) 1.5733　　(D) 1.2543

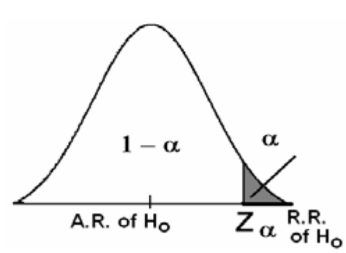
(2) The value of the test statistic is:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(84 - 82)}{\sqrt{\frac{6^2}{50} + \frac{8^2}{75}}} = \frac{2}{1.2543} = 1.5945$$

(A) -1.59　　(B) 1.59　　(C) 1.25　　(D) 4.21

(3) The rejection region (RR) of H_0 **is:**

$$Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.05}{2}} = Z_{0.975} = 1.645$$



(A) $(1.645, \infty)$　　*(B)* $(-\infty, -1.645)$

(C) $(1.96, \infty)$　　*(D)* $(-\infty, -1.96)$

(4) The decision is:

(A) Reject H_0　　*(B) Do not reject (Accept)* H_0

(C) Accept both H_0 *and* H_A　　*(D) Reject both* H_0 *and* H_A

Question 2:

Cortisol level determinations were made on two samples of women at childbirth. Group 1 subjects underwent emergency cesarean section following induced labor. Group 2 subjects natural childbirth route following spontaneous labor. The sample sizes, mean cortisol levels, and standard deviations were $(n_1 = 40, \bar{x}_1 = 575, \sigma_1 = 70), (n_2 = 44, \bar{x}_2 = 610, \sigma_2 = 80)$
If we are interested to test if the mean Cortisol level of group 1 (μ_1) *is less than that of group 2* (μ_2) *at level 0.05 (or* $H_0: \mu_1 \geq \mu_2$ *vs* $H_1: \mu_1 < \mu_2$ *), then:*

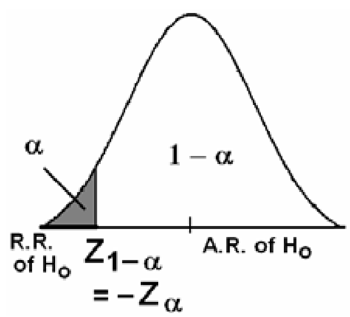
(1) *The value of the test statistic is:*

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(575 - 610)}{\sqrt{\frac{70^2}{40} + \frac{80^2}{44}}} = -2.138$$

(A) -1.326 *(B) -2.138* *(C) -2.576* *(D) -1.432*

(2) *Reject* $\boldsymbol{H_0}$ ***if :***

$$Z_\alpha = Z_{0.05} = -1.645$$



(A) Z >1.645 *(B) T >1.98* *(C) Z< -1.645* *(D) T < -1.98*

(3) *The decision is:*

(A) Reject H_0 *(B) Accept* H_0 *(C) no decision* *(D) none of these*

Question 3:

An experiment was conducted to compare time length (duration time in minutes) of two types of surgeries (A) and (B). 10 surgeries of type (A) and 8 surgeries of type (B) were performed. The data for both samples is shown below.

Surgery type	*A*	*B*
Sample size	*10*	*8*
Sample mean	*14.2*	*12.8*
Sample standard deviation	*1.6*	*2.5*

Assume that the two random samples were independently selected from two normal populations with equal variances. If μ_A and μ_B are the population means of the time length of surgeries of type (A) and type (B), then, to test if μ_A is greater than μ_B at level of significant 0.05 $(H_0: \mu_A \leq \mu_B$ vs $H_A: \mu_A > \mu_B)$ then:

(4) The value of the test statistic is:

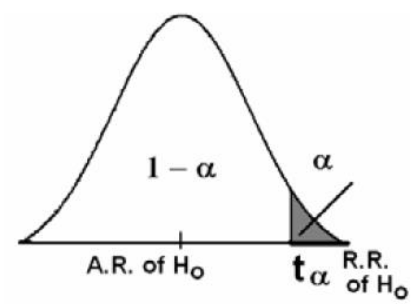
$$Sp^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{1.6^2(10-1)+2.5^2(8-1)}{10+8-2} = 4.174$$

$$t = \frac{(\bar{X}_1-\bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{(14.2-12.8)}{\sqrt{4.174}\sqrt{\frac{1}{10}+\frac{1}{8}}} = 1.44$$

(A) -1.44 *(<u>B</u>) 1.44* *(C) - 0.685* *(D) 0.685*

(5) Reject H_0 if :

$$t_{\alpha,n_1+n_2-2} = t_{0.05,10+8-2} = t_{0.05,16} = 1.746$$



(A) Z >1.645 *(B) Z<-1.645* *(<u>C</u>) T >1.746* *(D) T< -1.746*

(6) The decision is:

(A) Reject H_0 *(<u>B</u>) Accept H_0* *(C) no decision* *(D) none of these*

Question 4:

A researcher was interested in comparing the mean score of female students μ_1, with the mean score of male students μ_2 in a certain test. Assume the populations of score are normal with equal variances. Two independent samples gave the following results:

	Female	*male*
Sample size	$n_1 = 5$	$n_2 = 7$
Mean	$\bar{x}_1 = 82.63$	$\bar{x}_2 = 80.04$
Variance	$s_1^2 = 15.05$	$s_2^2 = 20.79$

Test that is there is a difference between the mean score of female students and the mean score of male students.

(1) The hypotheses are:

(A) H_o: $\mu_1 = \mu_2$; H_A: $\mu_1 \neq \mu_2$

(B) H_o: $\mu_1 = \mu_2$; H_A: $\mu_1 < \mu_2$

(C) H_o: $\mu_1 < \mu_2$; H_A: $\mu_1 > \mu_2$

(D) H_o: $\mu_1 \leq \mu_2$; H_A: $\mu_1 > \mu_2$

(2) The value of the test statistic is:

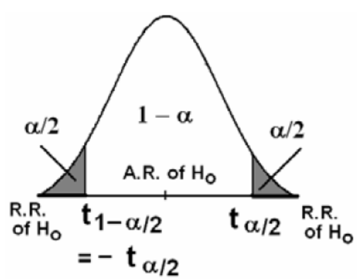
$$Sp^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{15.05(4)+20.79(6)}{5+7-2} = 18.494$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{82.63-80.04}{\sqrt{18.494}\sqrt{\frac{1}{5}+\frac{1}{7}}} = 1.029$$

(A) 1.3 *(B) 1.029* *(C) 0.46* *(D) 0.93*

(4) The acceptance region (AR) of H_0 is:

$$t_{\frac{\alpha}{2},n_1+n_2-2} = t_{\frac{0.05}{2},5+7-2} = t_{0.025,10} = 2.228$$



(A) $(2.2281, \infty)$

(B) $(-\infty, -2.2281)$

(C) (-2.228, 2.228)

(D) (-1.96, 1.96)

Question 5:

A nurse researcher wished to know if graduates of baccalaureate nursing program and graduate of associate degree nursing program differ with respect to mean scores on personality inventory at α = 0.02. A sample of 50 associate degree graduates (sample A) and a sample of 60 baccalaureate graduates (sample B) yielded the following means and standard deviations:

$$\bar{X}_A = 88.12,\ \ S_A = 10.5,\ \ n_A = 50$$
$$\bar{X}_B = 83.25,\ \ S_B = 11.2,\ \ n_B = 60$$

1) ***The hypothesis is:***

A) $H_0: \mu_1 \le \mu_2 \ \ vs \ \ H_1: \mu_1 > \mu_2$ *B)* $H_0: \mu_1 \ge \mu_2 \ \ vs \ \ H_1: \mu_1 < \mu_2$
C) $H_0: \mu_1 = \mu_2 \ \ vs \ \ H_1: \mu_1 \neq \mu_2$ *D) None of the above.*

2) ***The test statistic is:***

A) Z *B) t* *C) F* *D) None of the above.*

3) ***The computed value of the test statistic is:***

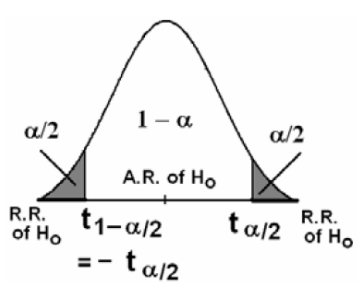
$$Sp^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{10.5^2(50-1)+11.2^2(60-1)}{50+60-2} = 118.55$$

$$t = \frac{(\bar{X}_1-\bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{88.12-83.25}{\sqrt{118.55}\sqrt{\frac{1}{50}+\frac{1}{60}}} = 48.19$$

A) 2.72 *B) 1.50* *C) 1.86* *D) 2.35*

4) ***The critical region (rejection area) is:***

$$t_{\frac{\alpha}{2}, n_1+n_2-2} = t_{\frac{0.02}{2},\ 50+60-2} = t_{0.01,\ 108} =$$

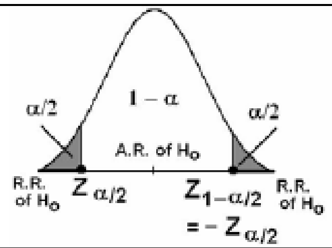
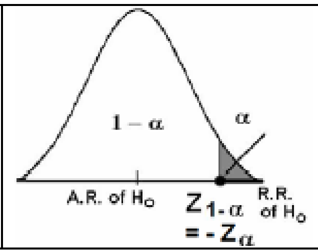
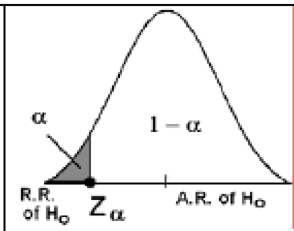


A) 2.60 Or -2.60 *B) 2.06 Or -2.06* *C) 2.33 Or - 2.33* *D) 2.58*

5) ***Your decision is:***

A) accept & reject H_0 *B) accept* H_0 *C) reject* H_0 *D) no decision.*

Single proportion:

Hypotheses	$H_o: p = p_o$ $H_A: p \neq p_o$	$H_o: p \leq p_o$ $H_A: p > p_o$	$H_o: p \geq p_o$ $H_A: p < p_o$
Test Statistic (T.S.)	$Z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} \sim N(0,1)$		
R.R. & A.R. of H_o			
Decision:	Reject H_o (and accept H_A) at the significance level α if:		
	$Z < Z_{\alpha/2}$ or $Z > Z_{1-\alpha/2} = -Z_{\alpha/2}$ Two-Sided Test	$Z > Z_{1-\alpha} = -Z_{\alpha}$ One-Sided Test	$Z < Z_{\alpha}$ One-Sided Test

Question 1:

Toothpaste (معجون الأسنان) company claims thatmorethan75% of the dentists recommend their product to the patients. Suppose that 161 out of 200 dental patients reported receiving a recommendation for this toothpaste from their dentist. Do you suspect that the proportion is actually morethan75%. If we use 0.05 level of significance to test $H_0: P \leq 0.75$, $H_A: P > 0.75$, *then:*

(1) The sample proportion $\hat{p}$ is:

$$n = 200, \;\; \hat{p} = \frac{161}{200} = 0.8050$$

(A) 0.75 *(B) 0.195* *(C) 0.805* *(D) 0.25*

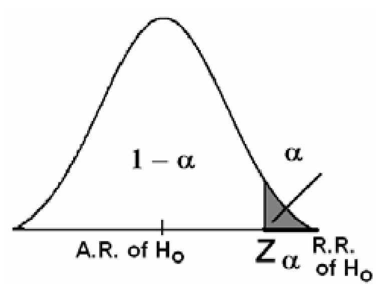
(2) The value of the test statistic is:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.805 - 0.75}{\sqrt{\frac{(0.75)(0.25)}{200}}} = 1.7963$$

(A) 1.963 *(B) 1.796* *(C) -1.796* *(D) -1.963*

(3)The decision is:

$$\alpha = 0.05 \rightarrow Z_{1-\alpha} = Z_{0.95} = 1.645$$



(A) Reject H_0 *(B) Do not reject (Accept)* H_o

(C) Accept both H_o *and* H_A *(D) Reject both* H_o *and* H_A

Question 2:

A researcher was interested in studying the obesity (السمنة) disease in a certain population. A random sample of 400 people was taken from this population. It was found that 152 people in this sample have the obesity disease. If p is the population proportion of people who are obese. Then, to test if p is greater than 0.34 at level 0.05 ($H_0: p \leq 0.34$ vs $H_A: p > 0.34$) then:

(1) The value of the test statistic is:

$$n = 400, \quad \hat{p} = \frac{152}{400} = 0.38$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.38 - 0.34}{\sqrt{\frac{0.34 \times 0.66}{400}}} = 1.69$$

(A) 0.023 *(B) 1.96* *(C) 2.50* *<u>(D)</u> 1.69*

(2) The P-value is

$$P - value = P(Z > 1.96) = 1 - P(Z < 1.96) = 1 - 0.9545 = 0.0455$$

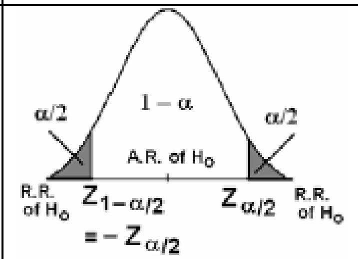
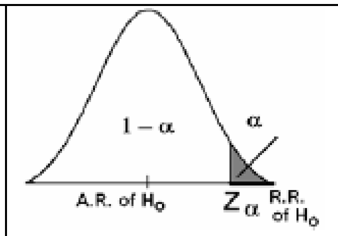
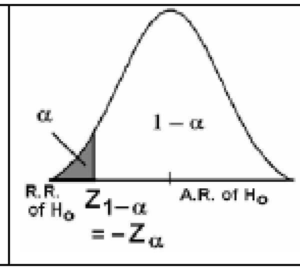
(A) 0.9545 *(B) 0.0910* *<u>(C)</u> 0.0455* *(D)1.909*

(3) The decision is:

$$P - value = 0.0455 < 0.05$$

<u>(A)</u> Reject H_0 *(B) Accept H_0* *(C) no decision* *(D) none of these*

Two proportions:

Hypotheses	$H_o: p_1 - p_2 = 0$ $H_A: p_1 - p_2 \neq 0$	$H_o: p_1 - p_2 \leq 0$ $H_A: p_1 - p_2 > 0$	$H_o: p_1 - p_2 \geq 0$ $H_A: p_1 - p_2 < 0$
Test Statistic (T.S.)	$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}} \sim \mathrm{N}(0,1)$$		
R.R. and A.R. of H_o			
Decision:	Reject H_o (and accept H_1) at the significance level α if $Z \in$ R.R.:		
Critical Values	$Z > Z_{\alpha/2}$ or $Z < -Z_{\alpha/2}$ Two-Sided Test	$Z > Z_{\alpha}$ One-Sided Test	$Z < -Z_{\alpha}$ One-Sided Test

Question 1:

In a first sample of 200 men, 130 said they used seat belts and a second sample of 300 women, 150 said they used seat belts. To test the claim that men are more safety-conscious than women ($H_0: p_1 - p_2 \leq 0, H_1: p_1 - p_2 > 0$), at 0.05 level of significant:

(1) The value of the test statistic is:

$$n_1 = 200, \ \ \hat{p}_1 = \frac{130}{200} = 0.65 \qquad n_2 = 300, \ \ \hat{p}_2 = \frac{150}{300} = 0.5$$

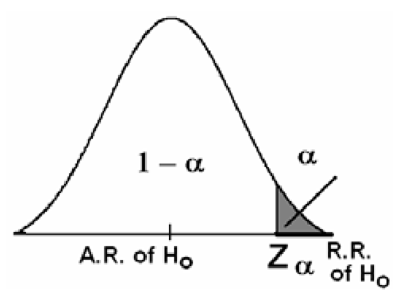
$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{130 + 150}{200 + 300} = 0.56$$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.65 - 0.5)}{\sqrt{(0.56)(0.44)\left(\frac{1}{200} + \frac{1}{300}\right)}} = 3.31$$

(A) -3.31 *(B)5.96* *(C)1.15* *(D) 3.31*

(2) The decision is:

$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$



(A) Reject H_0 *(B) Do not reject (Accept) H_0*
(C) Accept both H_0 and H_A *(D) Reject both H_0 and H_A*

(3) We can conclude that from confidence interval that

(A) The diabetes proportions may be equal for both proportion.
(B) The diabetes proportions may not be equal for both proportion.

Question 2:

In a study of diabetes, the following results were obtained from samples of males and females between the ages of 20 and 75. Male sample size is 300 of whom 129 are diabetes patients, and female sample size is 200 of whom 50 are diabetes patients. If P_M, P_F are the diabetes proportions in both populations and $\hat{p}_M, \hat{p}_F$ are the sample proportions, then:
A researcher claims that the Proportion of diabetes patients is found to be more in males than in female (H_0: $P_M - P_F \leq 0$ vs $H_A: P_M - P_F > 0$) . Do you agree with his claim, take α = 0.10

(1) The pooled proportion is:

$$\hat{p} = \frac{x_m + x_f}{n_m + n_f} = \frac{129 + 50}{300 + 200} = 0.358$$

(A) 0.43 *(B) 0.18* *(C) 0.358* *(D) 0.68*

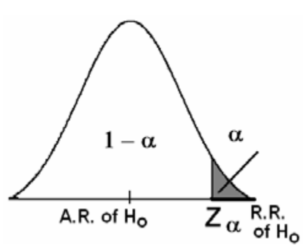
(2) The value of the test statistic is:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.43 - 0.25)}{\sqrt{(0.358)(1 - 0.358)\left(\frac{1}{300} + \frac{1}{200}\right)}} = 0.411$$

(A) -4.74 *(B) 4.74* *(C) 4.11* *(D) - 4.11*

(3) The decision is:

$$Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.05}{2}} = Z_{0.975} = 1.96$$



(A) Agree with the claim *(B) do not agree with the claim* *(C) Can't say*

* $n = 25 \rightarrow$ normal
* $\bar{X} = 4.8$
* $S = 2 \rightarrow \sigma$ is unknown
* $\alpha = 0.05$

Exercise #10

Q1: A study was made of a random sample of 25 records of patients seen at a chronic disease hospital on an outpatient basis, the mean number of outpatient visits per patient was 4.8 with standard deviation was 2. Can it be concluded from these data that the population mean is greater than four visits per patient. Let the probability of committing a type I error be 0.05.

1-what is the assumption?

σ unknown, normal, n small ($n<30$)

2-Hypothesis is?

$H_A: \mu > 4$

$H_0: \mu \leq 4$

3-Test statistic =

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{4.8 - 4}{\frac{2}{\sqrt{25}}} = 2$$

4-Reject H₀ if

من القانون $t > t_{1-\alpha}$ $\rightarrow$ $2 > 1.711$ ✓ تحقق الشرط

5-conclusion is:

$\boxed{\text{a)reject } H_0}$ b)accept H_0

* الرفض بالرسم:

وقعت في منطقة الرفض

* إيجاد $t_{1-\alpha}$ " من الجدول "

* $df = n - 1 = 25 - 1 = 24$

* $t_{1-\alpha} = t_{1-0.05} = t_{0.95}$

df	$t_{0.95}$
24	1.711

---

* $n = 49$
* $\bar{X} = 21$
* $S = 11$

Q2: In a sample of 49 adolescents who served as the subjects in an immunologic study, one variable of interest was the diameter of a skin test reaction to an antigen. The sample mean and standard deviation were 21 and 11 mm erythematic, respectively. Can it be concluded from these data that the population mean is less than 30? let α=0.05

1-what is the assumption?

σ unknown, non-normal, n large ($n > 30$)

2-Hypothesis is?

$H_A: \mu < 30$

$H_0: \mu \geq 30$

3-Test statistic=

$$Z = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{21 - 30}{\frac{11}{\sqrt{49}}} = -5.727$$

4-Reject H₀ if

القانون $Z < -Z_{1-\alpha}$

$-5.727 < -1.645$ ✓ تحقق الشرط

$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$

5-conclusion is:

$\boxed{\text{a)reject } H_0}$ b)accept H_0

---

* الرفض بالرسم لا طريقة اخرى للرفض " :

$\boxed{Z = -5.73}$ القانون

* تقع في منطقة الرفض

$\boxed{-Z_{1-\alpha} = -1.645}$

Scanned by CamScanner

* n = 100   * $\sigma = 6.5$
* $\bar{X} = 27$ * $\alpha = 0.05$

Q3: A survey of 100 similar-sized hospitals revealed a mean daily census in the pediatrics service of 27. The population distributed normally with standard deviation of 6.5 .Do these data provide sufficient evidence to indicate that the population mean is not equal 25?let α=0.05

1-what is the assumption?

σ known, normal , n large (n>30)

2-Hypothesis is?

$H_A: \mu \neq 25$

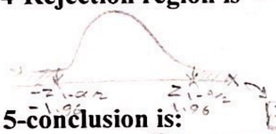
$H_0: \mu = 25$

3-Test statistic=

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{27 - 25}{\frac{6.5}{\sqrt{100}}} = 3.077$$

4-Rejection region is $R.R = (-\infty, -1.96) \cup (1.96, \infty)$

$Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.05}{2}} = Z_{1-0.025} = Z_{0.975} = 1.96$



5-conclusion is: $\boxed{Z = 3.077} \in R.R$

(R.R) وقعت في منطقة الرفض

a)reject H_0 b)accept H_0

6- P-value =

two sided (=)(≠)

$2 \times P(Z > |Z_c|) = 2 \times P(Z > 3.077) = 2 \times 0.00104 = 0.00208$

H.W 1:

* $\sigma = 16$ → known
* n = 64 → n large
* $\bar{X} = 133$
* $\alpha = 0.05$

A research team is willing to assume that systolic blood pressures in a certain population of males are approximately normally distributed with a standard deviation of 16. A simple random sample of 64 males from the population had a mean systolic blood pressure reading of 133 . At the 0.05 level of significance, do these data provide sufficient evidence for us to conclude that the population mean is greater than 130.

1-what is the assumption? **(Answer: Normal , σ known , n large)**

2-Hypothesis is? **(Answer: $H_0: \mu \leq 130$, $H_A: \mu > 130$)**

* إيجاد قيمة $Z_{1-\alpha}$:

$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95}$

$\therefore Z = 1.645$

3-Test statistic= **(Answer: Z = 1.5)**

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{133 - 130}{\frac{16}{\sqrt{64}}} = 1.5$$

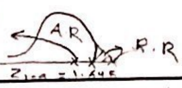
4-Reject H_0 if **(Answer: $Z > Z_{1-\alpha}$)**

5-conclusion is:

a)reject H_0 b)accept H_0

* الرفض بالرسم :

$\boxed{Z = 1.5}$ وقعت في منطقة القبول A.R ⇐ accept H_0



* الرفض بالقانون :

$Z > Z_{1-\alpha}$

$1.5 > 1.645$ ✗

لم يتحقق شرط الرفض

⇓

accept H_0

Q4: The objective of a study by Sairam et al. (A-8) was to identify the role of various disease states and additional risk factors in the development of thrombosis. One focus of the study was to determine if there were differing levels of the anticardiolipin antibody IgG in subjects with and without thrombosis.

Group	Mean IgG Level (ml/unit)	Sample Size	Population Standard deviation
Thrombosis	59.01	53	44.89
No thrombosis	46.61	54	34.85

We wish to know if we may conclude, on the basis of these results, that, in general, persons with thrombosis have, on the average, higher IgG levels than persons without thrombosis.
let α=0.01

$H_A : \mu_1 > \mu_2$

& تكون مساواة

1-what is the assumption?

(σ_1, σ_2) known , non-normal , (n_1, n_2) large

2-Hypothesis is?

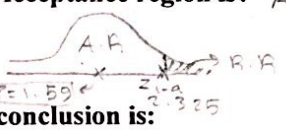
$H_A : \mu_1 > \mu_2$

$H_0 : \mu_1 \leq \mu_2$

3-Test statistic=

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{59.01 - 46.61}{\sqrt{\frac{44.89^2}{53} + \frac{34.85^2}{54}}} = 1.59$$

4-Acceptance region is? $A.R = (-\infty, 2.325)$ $\quad Z_{1-\alpha} = Z_{1-0.01} = Z_{0.99} = 2.325$



$\boxed{Z = 1.59}$ وتقع في منطقة القبول

$Z \in A.R$

5-conclusion is:

a)reject H_0 $\quad$ $\boxed{\text{b)accept } H_0}$

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Q5: A test designed to measure mothers' attitudes toward their labor and delivery experiences was given to two groups of new mothers. Sample 1 (attenders) had attended prenatal classes held at the local health department. Sample 2 (nonattenders) did not attend the classes. The sample sizes and means and standard deviations of the test scores were as follows:

sample	n	$\bar{x}$	S
1	15	4.75	1.0
2	22	3.00	1.5

Assume equal variances. Do these data provide sufficient evidence to indicate that attenders, on the average, score less than non attenders? Let α =0.05. Assume normal population

$H_A: \mu_1 < \mu_2$

1-what is the assumption?

(σ_1, σ_2) unknown but equal , normal, (n_1, n_2) small

2-Hypothesis is?

$H_A: \mu_1 < \mu_2$

$H_0: \mu_1 \geq \mu_2$

3- find pooled variance

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{(15-1)(1)^2 + (22-1)(1.5)^2}{15+22-2} = 1.75$$

4-Test statistic=

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}} = \frac{4.75-3}{\sqrt{\frac{1.75}{15} + \frac{1.75}{22}}} = 3.95$$

5-Reject H_0 if

$T < -t_{1-a}$

$t_{1-a} = t_{1-0.05} = t_{0.95} = 1.6896$

$\boxed{df = n_1+n_2-2 = 35}$

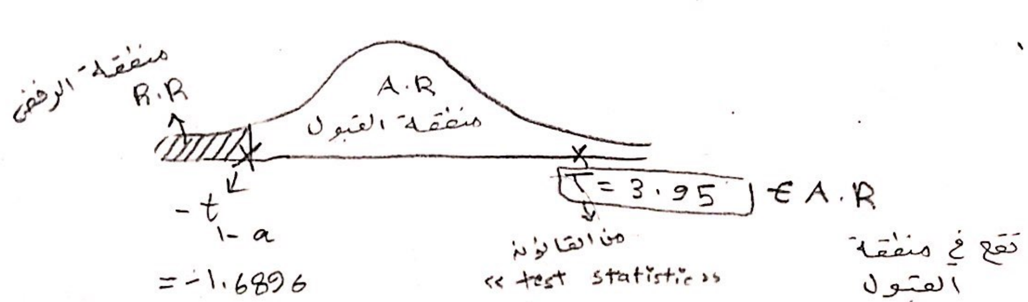
$3.95 < -1.6896$ X لم تتحقق شرط الرفض

6-conclusion is:

a)reject H_0 $\boxed{\text{b)accept } H_0}$

---

* طريقة أخرى للرفض : الرفض بالرسم :-



Q6:

Woo and McKenna (A-18) investigated the effect of broadband ultraviolet B (UVB) therapy and topical calcipotriol cream used together on areas of psoriasis. One of the outcome variables is the Psoriasis Area and Severity Index (PASI). The following table gives the PASI scores for 20 subjects measured at baseline and after eight treatments. Do these data provide sufficient evidence, at the .01 level of significance, to indicate that the combination therapy reduces PASI scores?

subject	Baseline X_i	After 8 treatments Y_i	$D_i = X_i - Y_i$
1	5.9	5.2	0.7
2	7.6	12.2	-4.6
3	12.8	4.6	8.2
4	16.5	4.0	12.5
5	6.1	0.4	5.7
6	14.4	3.8	10.6
7	6.6	1.2	5.4
8	5.4	3.1	2.3
9	9.6	3.5	6.1
10	11.6	4.9	6.7
11	11.1	11.1	0
12	15.6	8.4	7.2
13	6.9	5.8	1.1
14	15.2	5.0	10.2
15	21.0	6.4	14.6
16	5.9	0.0	5.9
17	10.0	2.7	7.3
18	12.2	5.1	7.1
19	20.2	4.8	15.4
20	6.2	4.2	2

1-what is the assumption?

$(\bar{X})$ (mean) $\bar{D} = 6.22$ باستخدام الآلة

(S_x) (standard) $S_D = 5.04$

2-Hypothesis is?

$H_0 : \mu_D \le 0$

$H_A : \mu_D > 0$

3-Test statistic=

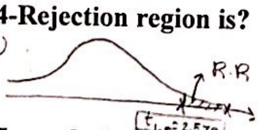
$$T = \frac{\bar{D}}{\frac{S_D}{\sqrt{n}}} = \frac{6.22}{\frac{5.04}{\sqrt{20}}} = 5.519$$

4-Rejection region is?

R.R = (2.539, ∞)

$t_{1-\alpha} = t_{1-0.01} = t_{0.99} = 2.539$

$\boxed{df = n-1 = 19}$



5-conclusion is:

since $t = 5.519 \in R.R$

$\boxed{\text{a) reject H0}}$ b)accept H0

* طريقة استخدام الآلة

① تنظيف : Shift → 9 → 1: Setup → = → AC

② Mode → 2 (stat) → 1 (1-Var) → D_i ندخل ثم → AC

③ حساب $\bar{D}$: Shift → 1 → 4 (Var) → 2 ($\bar{x}$) → =

④ حساب S_D : Shift → 1 → 4 (Var) → 4 (Sx) → =

$* \; n = 295$

$* \; x = 90$ $\qquad * \; \hat{p} = \frac{x}{n} = \frac{90}{295} = 0.31$

$* \; \alpha = 0.05$

Q7: Jacquemyn et al. (A-21) conducted a survey among gynecologists-obstetricians in the Flanders region and obtained 295 responses. Of those responding, 90 indicated that they had performed at least one cesarean section on demand every year. Does this study provide sufficient evidence for us to conclude that less than 35 percent of the gynecologists-obstetricians in the Flanders region perform at least one cesarean section on demand each year? Let α =0.05.

1-Hypothesis is?

$H_A: P < 0.35$

$H_0: P \geq 0.35$

2-Test statistic=

$$Z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0 q_0}{n}}} = \frac{0.31 - 0.35}{\sqrt{\frac{0.35(1-0.35)}{295}}} = -1.44$$

$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$

3-Rejection region is

$R.R = (-\infty, -1.645)$

وقعت

A.R ؟

4-conclusion is:

a)reject H_0 **b)accept H_0**

P.value $<$ α ?

6- P-value = من القانون

$P(Z > -Z_c) = P(Z > 1.44) = 0.07493 \not< 0.05$

لم يتحقق الشرط ⟸ Accept H_0

H.W4

In an article in the journal Health and Place, Hui and Bell (A-22) found that among 2428 boys ages 7 to 12 years, 461 were overweight or obese. On the basis of this study, can we conclude that more than 15 percent of the boys ages 7 to 12 in the sampled population are obese or overweight? Let α=0.05

1-Hypothesis is?   **(Answer : H_0: $P \leq 0.15$, H_A: $P > 0.15$)**

2-Test statistic=   **(Answer : Z = 4.91)**

3-Acceptance region is

(Answer : ($-\infty$, 1.645))

4-conclusion is:

a)reject H_0 **b)accept H_0**

$* \; n_1 = 1222$ $\qquad$ $* \; n_2 = 282$

$* \; x_1 = 72$ $\qquad$ $* \; x_2 = 30$

Q8:

$\boxed{* \; \hat{P}_1 = \frac{x_1}{n_1} = \frac{72}{1222} = 0.059}$ $\qquad$ $\boxed{* \; \hat{P}_2 = \frac{x_2}{n_2} = \frac{30}{282} = 0.106}$

Ho et al. (A-25) used telephone interviews of randomly selected respondents in Hong Kong to obtain information regarding individuals' perceptions of health and smoking history. Among 1222 current male smokers, 72 reported that they had "poor" or "very poor" health, while 30 among 282 former male smokers reported that they had "poor" or "very poor" health. Is this sufficient evidence to allow one to conclude that among Hong Kong men there is a difference → $P_1 - P_2 \neq 0$ between current and former smokers with respect to the proportion who perceive themselves as having "poor" and "very poor" health? Let $\boxed{\alpha=0.01.}$

يوم خرمة

$\bar{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{72+30}{1222+282} = 0.068$

$\bar{q} = 1 - \bar{P} = 1 - 0.068 = 0.932$

$Z_{1-\alpha} = Z_{1-0.01} = Z_{0.99} = 2.325$

1-Hypothesis is?

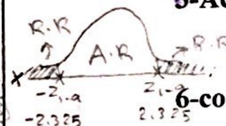
$H_A: P_1 - P_2 \neq 0 \; , \; H_A: P_1 \neq P_2$

$H_0: P_1 - P_2 = 0 \; , \; H_0: P_1 = P_2$

2-Test statistic=

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\frac{\bar{P}\bar{q}}{n_1} + \frac{\bar{P}\bar{q}}{n_2}}} = \frac{0.059 - 0.106}{\sqrt{\frac{0.068(1-0.068)}{1222} + \frac{0.068(1-0.068)}{282}}} = -2.83$$

3-Acceptance region is?



$A.R = (-2.325, 2.325)$

$Z = -2.83$

وقعت في R.R

$Z \in R.R$

6-conclusion is:

a)reject H_0 $\qquad$ b)accept H_0

H.W5:

In a study of obesity the following results were obtained from samples of males and females between the ages of 20 and 75:

	n	Number overweight
Males	150	21
Females	200	48

Can we conclude from these data that in the sampled populations there is a difference in the proportions who are overweight? Let α= 0.05.

1-Hypothesis is?

$H_0: P_1 = P_2$, $\qquad$ $H_A: P_1 \neq P_2$

2-Test statistic=

$Z = -2.328$

3-Acceptance region is?

(- 1.645 , 1.645)

6-conclusion is:

a)reject H_0 $\qquad$ b)accept H_0

CHAPTER 6: Using Sample Data to Make Estimations About Population Parameters

فترة الثقة

Confidence Interval (C.I)

جدول z

1) C.I of the Mean (μ): σ^2 is known

جدول t

2) C.I of the Mean (μ): σ^2 is unknown

جدول z

3) C.I of the Difference between two Means ($\mu_1 - \mu_2$) σ^2_1 and σ^2_2 are known

جدول t

4) C.I of the Difference between two Means ($\mu_1 - \mu_2$) σ^2_1 and σ^2_2 are unknown

جدول z

5) C.I of a Proportion

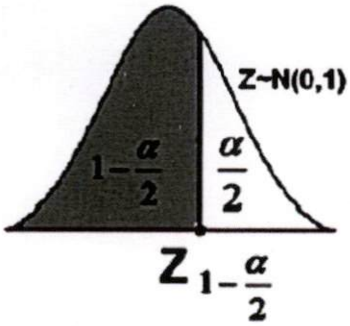
جدول z

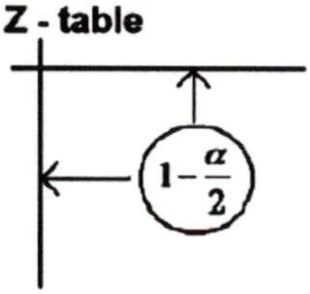
6) C.I of the Difference between Two Proportions

أي درس

1. C.I □ ± △ · ○
2. Point estimate = □
3. Standard error = ○
4. Max. Error (error will not exceed = e) = △ · ○
5. Upper = +
6. Lower = −
7. Length(width) of c.i = u − L

(Finding Reliability Coefficient)





$$Z_{1-\alpha/2}$$

90%	95%
مكملة الثقة $\longrightarrow \alpha = 10\% = 0.1$ $Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.1}{2}} = Z_{0.9500}$ $= \frac{1.64+1.65}{2}$ $= 1.645$	
$Z_{1-\alpha/2} = 1.645$	$Z_{1-\alpha/2} = 1.96$
96%	**99%**
$Z_{1-\alpha/2} = 2.055$	$Z_{1-\alpha/2} = 2.575$

6.2 Confidence Interval for a Population Mean (μ) :

1) C.I of the Mean (μ): σ^2 is known

السؤال

حفظ

Mean = average = $\bar{x}$

Sample = n

Population standard deviation = σ

$\square$ % $\xrightarrow{\text{نستنتج}}$ $z_{1-\frac{\alpha}{2}} = \bigcirc$

C.I = $\boxed{\bar{x}} \pm z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

P.S = $\bar{x}$

S T . E = $\frac{\sigma}{\sqrt{n}}$

MAX . E = $z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

U = $\bar{x} + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

L = $\bar{x} - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

L . OF C.I = $u - l$

$$\overline{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$\left(\overline{X} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \,,\, \overline{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$$

1. We are $(1-\alpha)100\%$ confident that the true value of μ belongs to the interval $(\overline{X} - Z_{1-\frac{\alpha}{2}} \frac{}{\sqrt{n}} \,,\, \overline{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}})$.

2. Upper limit of the confidence interval = $\overline{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

3. Lower limit of the confidence interval = $\overline{X} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

4. $Z_{1-\frac{\alpha}{2}}$ = Reliability Coefficient

5. $Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$ = margin of error = precision of the estimate

6. In general the interval estimate (confidence interval) may be expressed as follows:

$$\overline{X} \pm Z_{1-\frac{\alpha}{2}} \sigma_{\overline{X}}$$

estimator ± (reliability coefficient) × (standard Error)

estimator ± margin of error

Example: (The case where σ^2 is known)

Diabetic ketoacidosis is a potential fatal complication of diabetes mellitus throughout the world and is characterized in part by very high blood glucose levels. In a study on 123 patients living in Saudi Arabia of age 15 or more who were admitted for diabetic ketoacidosis, the mean blood glucose level was 26.2 mmol/l. Suppose that the blood glucose levels for such patients have a normal distribution with a standard deviation of 3.3 mmol/l.

(1) Find a point estimate for the mean blood glucose level of such diabetic ketoacidosis patients.

(2) Find a 90% confidence interval for the mean blood glucose level of such diabetic ketoacidosis patients.

[1] $C.I = \boxed{\bar{x}} \pm Z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

$= 26.2 \pm (1.645)\left(\frac{3.3}{\sqrt{123}}\right)$

$= (26.11, 27.09)$

$26.11 \le \mu \le 27.09$

$\bar{x} = 26.2$

$\sigma = 3.3$

$n = 123$

$\boxed{90\%}$

$Z_{1-\frac{\alpha}{2}} = 1.645$

R.E

[2] P.S $= \bar{x} = 26.2$

[3] ST.e $= \frac{\sigma}{\sqrt{n}} = \frac{3.3}{\sqrt{123}} = 0.297$

[4] Max.e $= Z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} = 0.49$

[5] U = + = 27.09

[6] L = − = 26.11

[7] L.of C.I = U − L = 27.09 − 26.11 = 0.98

6.3 The t Distribution: (Confidence Interval Using t)

2) C.I of the Mean (μ): σ^2 is Unknown

السؤال

Mean = average = $\bar{x}$

Sample = n

~~Population~~ standard deviation = S

جدول T

☐ % $\xrightarrow{\text{نستنتج}}$ $\alpha =$ $T_{(1-\frac{\alpha}{2},\ df)}$

جدول T

df $1-\frac{\alpha}{2}$

C.I = $\boxed{\bar{x}} \pm T_{1-\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}$

P.S = $\bar{x}$

S T . E = $\frac{S}{\sqrt{n}}$

MAX . E = $T_{1-\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}$

U = $+$ $=$

L = $-$ $=$

L . OF C.I = $u - L$

$$\overline{X} \pm t_{\cancel{1-}\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

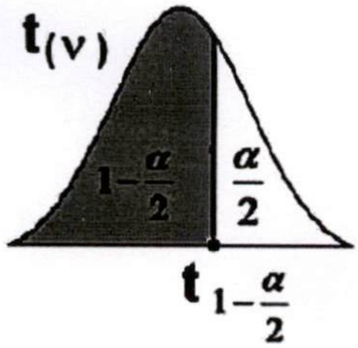
$$\left(\overline{X} - t_{\cancel{1-}\frac{\alpha}{2}} \frac{S}{\sqrt{n}}, \overline{X} + t_{\cancel{1-}\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \right)$$

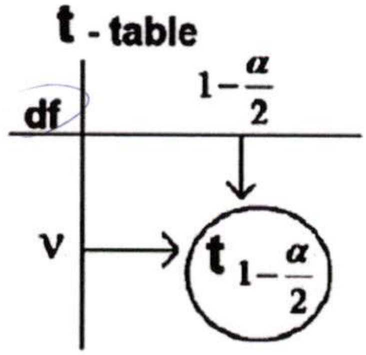
1. We are $(1-\alpha)100\%$ confident that the true value of μ belongs to the interval $\left(\overline{X} - t_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}, \overline{X} + t_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \right)$.
2. $\hat{\sigma}_{\overline{X}} = \frac{S}{\sqrt{n}}$ (estimate of the standard error of $\overline{X}$)
3. $t_{\cancel{1-}\frac{\alpha}{2}}$ = Reliability Coefficient
4. In this case, we replace σ by S and Z by t.
5. In general the interval estimate (confidence interval) may be expressed as follows:

Estimator ± (Reliability Coefficient) × (Estimate of the Standard Error)

$$\overline{X} \pm t_{1-\frac{\alpha}{2}} \hat{\sigma}_{\overline{X}}$$

(Finding Reliability Coefficient)





Example:

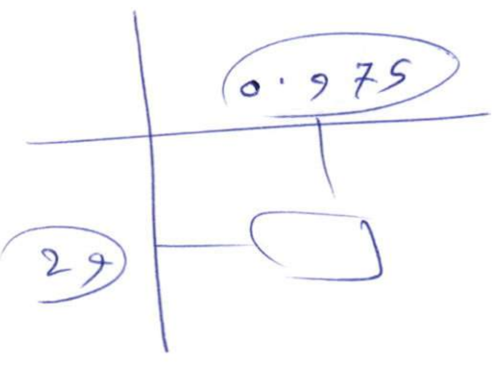
Suppose that $t \sim t(30)$. Find $t_{1-\frac{\alpha}{2}}$ for $\alpha = 0.05$.

n

$1-\frac{\alpha}{2} = 1-\frac{0.05}{2} = 0.975$

$df = v = n-1 = 30-1 = 29$

$T_{1-\frac{\alpha}{2}} = 2.045$



Example: (The case where σ^2 is unknown)

A study was conducted to study the age characteristics of Saudi women having breast lump. A sample of 121 Saudi women gave a mean of 37 years with a standard deviation of 10 years. Assume that the ages of Saudi women having breast lumps are normally distributed.

(a) Find a point estimate for the mean age of Saudi women having breast lumps.

(b) Construct a 99% confidence interval for the mean age of Saudi women having breast lumps

1) (b) $C.I = \boxed{\bar{x}} \pm T_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

$= 37 \pm 2.6174 \cdot \frac{10}{\sqrt{121}}$

$= (34.62, 39.38)$

$n = 121$

$\bar{x} = 37$

$S = 10$

99%

$\alpha = 1\% = 0.01$

$1 - \frac{\alpha}{2} = 1 - \frac{0.01}{2} = 0.995$

$df = v = n - 1 = 120$

	0.995
120	2.6174

$T_{1-\frac{\alpha}{2}}$

R.C

(a) 2) $p.s = \boxed{\bar{x}} = 37$

3) $ST.e = \frac{S}{\sqrt{n}} = \frac{10}{\sqrt{121}} =$

4) $max.e = \triangle \cdot \bigcirc =$

5) $U = + = 39.38$

6) $L = - = 34.62$

7) $L.of\ C.I = U - L =$

6.4 Confidence Interval for the Difference between Two Population Means ($\mu_1-\mu_2$):

(i) First Case: σ_1^2 and σ_2^2 are known:

3) C.I of the Difference between two Means ($\mu_1-\mu_2$) σ^2_1 and σ^2_2 are known

السؤال

	A	B
Mean = average	$\bar{x}_1 =$	$\bar{x}_2 =$
Sample	$n_1 =$	$n_2 =$
Population standard deviation	$\sigma_1 =$	$\sigma_2 =$

☐ % $\xrightarrow{\text{نستنتج}}$ $Z_{1-\frac{\alpha}{2}}$:

1) C.I = $(\bar{x}_1 - \bar{x}_2) \pm Z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

2) P.S =

3) S T . E =

4) MAX . E =

5) U =

6) L =

7) L . OF C.I =

$$(\overline{X}_1-\overline{X}_2)\pm Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

1. Mean of $\overline{X}_1-\overline{X}_2$ is: $\mu_{\overline{X}_1-\overline{X}_2}=\mu_1-\mu_2$

2. Variance of $\overline{X}_1-\overline{X}_2$ is: $\sigma^2_{\overline{X}_1-\overline{X}_2}=\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}$

3. Standard error of $\overline{X}_1-\overline{X}_2$ is: $\sigma_{\overline{X}_1-\overline{X}_2}=\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$

Example: (1st Case: σ_1^2 and σ_2^2 are known)

An experiment was conducted to compare time length (duration time) of two types of surgeries (A) and (B). 75 surgeries of type (A) and 50 surgeries of type (B) were performed. The average time length for (A) was 42 minutes and the average for (B) was 36 minutes.

(1) Find a point estimate for $\mu_A - \mu_B$, where μ_A and μ_B are population means of the time length of surgeries of type (A) and (B), respectively.

(2) Find a 96% confidence interval for $\mu_A - \mu_B$. Assume that the population standard deviations are 8 and 6 for type (A) and (B), respectively.

C.I

A	B
$n_A = 75$	$n_B = 50$
$\bar{x}_A = 42$	$\bar{x}_B = 36$
$\sigma_A = 8$	$\sigma_B = 6$

96%

$Z_{1-\frac{\alpha}{2}} = 2.055$

$$\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}} = \sqrt{\frac{8^2}{75} + \frac{6^2}{50}}$$

$$= 1.25$$

1) C.I $\boxed{(\bar{x}_A - \bar{x}_B)} \pm Z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}$

$(42 - 36) \pm (2.055)(1.25)$

$= (3.43 , 8.5687)$

---

2) P.S = $\square$

3) ST.e = $\bigcirc$

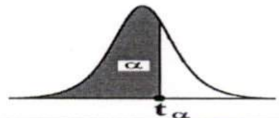
4) Max.e = $\triangle \cdot \bigcirc$

5) u = 8.56

6) l = 3.43

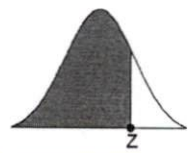
7) L.of.C.I = u − l = $\bigcirc$

Critical Values of the t-distribution (t_α)



ν=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.3062	1.6896	2.0301	2.4377	2.7238
40	1.3030	1.6840	2.0210	2.4230	2.7040
45	1.3006	1.6794	2.0141	2.4121	2.6896
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.2922	1.6641	1.9901	2.3739	2.6387
90	1.2910	1.6620	1.9867	2.3685	2.6316
100	1.2901	1.6602	1.9840	2.3642	2.6259
120	1.2886	1.6577	1.9799	2.3578	2.6174
140	1.2876	1.6558	1.9771	2.3533	2.6114
160	1.2869	1.6544	1.9749	2.3499	2.6069
180	1.2863	1.6534	1.9732	2.3472	2.6034
200	1.2858	1.6525	1.9719	2.3451	2.6006
∞	1.282	1.645	1.960	2.326	2.576

Areas Under The Standard Normal Curve



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

CHAPTER 7: Using Sample Statistics To Test Hypotheses About Population Parameters:

Tests of Hypotheses

اختبارات الفروض

1) T.H of the Mean (μ): σ^2 is known

جدول z

2) T.H of the Mean (μ): σ^2 is unknown

جدول t

3) T.H of the Difference between two Means ($\mu_1 - \mu_2$) σ^2_1 and σ^2_2 are known

جدول z

4) T.H of the Difference between two Means ($\mu_1 - \mu_2$) σ^2_1 and σ^2_2 are unknown

جدول t

5) T.H of a Proportion

جدول z

6) T.H of the Difference between Two Proportions

جدول z

أي درس

1)Hypothesis

الفروض

فرض العدم null ← H_0 : = | = | =

الفرض البديل Alternative → H_1 : ≠ | > | <

~~more~~ ~~less~~ | more | Less

Test statistic

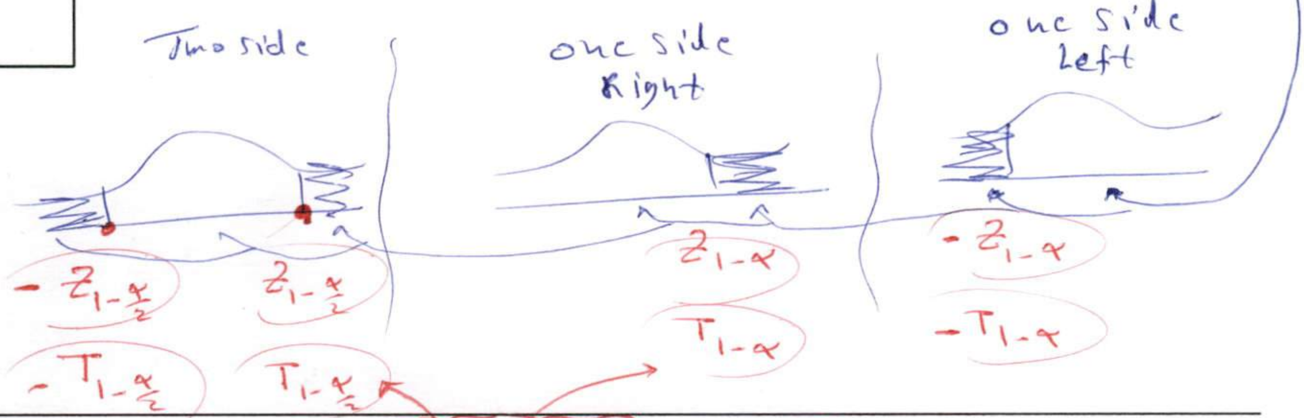
الاختبار الإحصائي

Z =

T =

قانون خاص بكل درس

الرسم



Critical point النقطة الحرجة

Decision

القرار

قبول
☐ Accept H_0 or reject H_1

رفض
■ Reject H_0 or accept H_1

accept = not reject
reject = not accept

7.2 Hypothesis Testing: A Single Population Mean (μ):

1) T.H of the Mean (μ): σ^2 is known

السؤال

Sample n =

Average $\overline{x}$ =

Population standard deviation σ =

Greater, less, equal μ_o =

Use α level of significance ⟹ $Z_{1-\frac{\alpha}{2}}$ or $Z_{1-\alpha}$ جدول z

---

1) Hypothesis

H_0 :	$\mu = \mu_0$	$\mu = \mu_0$	$\mu = \mu_0$
H_1 :	$\mu \neq \mu_0$	$\mu > \mu_0$	$\mu < \mu_0$

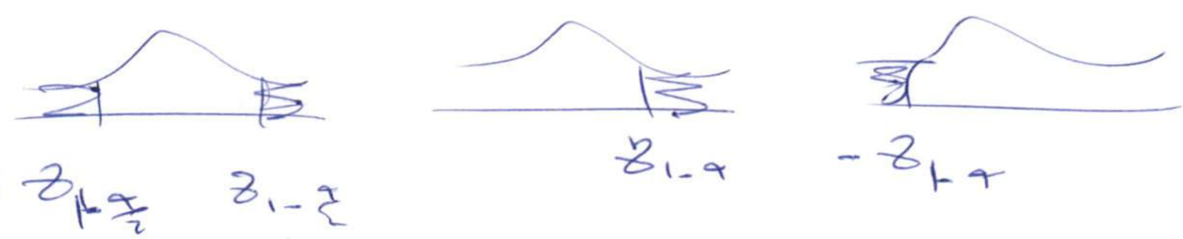
---

2) Test statistic

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

---

3) الرسم



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4) Decision

☐ Accept H_0

▨ reject H_0

Example: (first case: variance σ^2 is known)

A random sample of 100 recorded deaths in the United States during the past year showed an average of 71.8 years. Assuming a population standard deviation of 8.9 year, does this seem to indicate that the mean life span today is greater than 70 years? Use a 0.05 level of significance.

① Hyp

$H_0: \mu = \mu_0 \Rightarrow \mu = 70$ or $\mu - 70 = 0$

$H_1: \mu > \mu_0 \Rightarrow \mu > 70$ or $\mu - 70 > 0$

$n = 100$

$\bar{x} = 71.8$

$\sigma = 8.9$

Greater than

$\mu_0 = 70$

$\alpha = 0.05$

② T-S

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{71.8 - 70}{\frac{8.9}{\sqrt{100}}} = 2.022$$

③

$-\infty$ $\quad Z_{1-\alpha} \quad$ ∞

Critical point $\rightarrow$ 1.645

$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.9500} = \frac{1.64 + 1.65}{2} = 1.645$

④ DC reject H_0

السبب

Since $\qquad Z > Z_{1-\alpha}$

منطقة قبول الفرض العدم

Accepte region of H_0 $\quad (-\infty, 1.645)$

منطقة رفض الفرض العدم

reject region of H_0 $\quad (1.645, \infty)$

2)T.H of the Mean (μ): σ^2 is unknown

السؤال

Sample n =

Average $\overline{x}$ =

~~Population~~ standard deviation S =

t جدول

Greater, less, equal μ_0 =

$1-\frac{\alpha}{2}$

$1-\alpha$

df

Use α level of significance → $t_{1-\alpha}$, $df = v = n-1$

or $t_{1-\frac{\alpha}{2}}$

---

1) Hypothesis

$H_0: \mu = \mu_0$	$\mu = \mu_0$	$\mu = \mu_0$
$H_1: \mu \neq \mu_0$	$\mu > \mu_0$	$\mu < \mu_0$

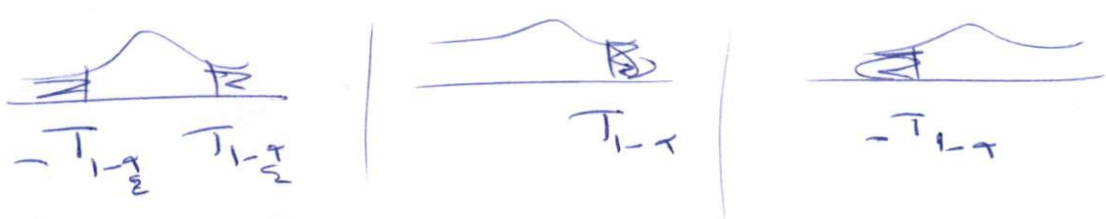
---

2) Test statistic

$$T = \frac{\overline{x} - \mu_0}{\frac{S}{\sqrt{n}}}$$

---

3) الرسم



---

4) Decision

Accept H_0

reject H_0

Example: (second case: variance σ^2 is unknown)

The manager of a private clinic claims that the mean time of the patient-doctor visit in his clinic is 8 minutes. Test the hypothesis that $\mu=8$ minutes against the alternative that $\mu\neq8$ minutes if a random sample of 50 patient-doctor visits yielded a mean time of 7.8 minutes with a standard deviation of 0.5 minutes. It is assumed that the distribution of the time of this type of visits is normal. Use a 0.01 level of significance.

① Hyp $H_0: \mu = 8$

$H_1: \mu \neq 8$

② T.S $T = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{7.8 - 8}{\frac{0.5}{\sqrt{50}}} = -2.83$

$\mu_0 = 8$

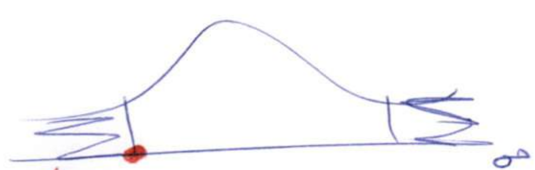
$n = 50$

$\bar{x} = 7.8$

$S = 0.5$

$\alpha = 0.01$

③



$-\infty$ $\quad -T_{1-\frac{\alpha}{2}} \quad T_{1-\frac{\alpha}{2}}$ $\quad \infty$

$-2.6778 \qquad 2.6778$

$T_{1-\frac{\alpha}{2}} = T_{1-\frac{0.01}{2}} = T_{0.995}$

$df = n - 1 = 50 - 1 = 49$

	0.995
49	2.6778

④ D.C reject H_0

Since

$T < T_{1-\frac{\alpha}{2}}$

Accepte region of H_0 $(-2.6778, 2.6778)$

Reject region of H_0

$(-\infty, -2.6778) \cup (2.6778, \infty)$

7.3 Hypothesis Testing: The Difference Between Two Population Means: (Independent Populations)

3)T.H of the Difference between two Means ($\mu1-\mu2$) σ^2_1 and σ^2_2 are known

السؤال

	A	B
Sample	$n_1 =$	$n_2 =$
Average	$\overline{x}_1 =$	$\overline{x}_2 =$
Population standard deviation	$\sigma_1 =$	$\sigma_2 =$

موجود σ

d= $= D$ إذا فرق

Use α level of significance $\longrightarrow$ $z_{1-\alpha}$, $z_{1-\frac{\alpha}{2}}$

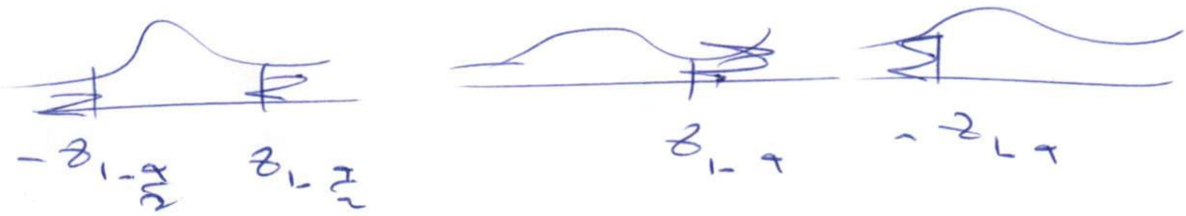
1) Hypothesis

H_0 :	$\mu_1 - \mu_2 = d$ $\mu_1 = \mu_2$	$\mu_1 - \mu_2 = d$ $\mu_1 = \mu_2$	$\mu_1 - \mu_2 = d$ $\mu_1 = \mu_2$
H_1 :	$\mu_1 - \mu_2 \neq d$ $\mu_1 \neq \mu_2$	$\mu_1 - \mu_2 > d$ $\mu_1 > \mu_2$	$\mu_1 - \mu_2 < d$ $\mu_1 < \mu_2$

2) Test statistic

$$Z = \frac{(\overline{x}_1 - \overline{x}_2) - d}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

3) الرسم



4) Decision

Accepte H_0

reject H_0

Example: (σ_1^2 and σ_2^2 are known)

Researchers wish to know if the data they have collected provide sufficient evidence to indicate the difference in mean serum uric acid levels between individuals with Down's syndrome and normal individuals. The data consist of serum uric acid on 12 individuals with Down's syndrome and 15 normal individuals. The sample means are $\bar{X}_1 = 4.5$ mg/100ml and $\bar{X}_2 = 3.4$ mg/100ml. Assume the populations are normal with variances σ_1^2=1 and σ_2^2=1.5. Use significance level α=0.05.

Sol

1	2
$n_1 = 12$	$n_2 = 15$
$\bar{x}_1 = 4.5$	$\bar{x}_2 = 3.4$
$\sigma_1^2 = 1$	$\sigma_2^2 = 1.5$

$\alpha = 0.05$

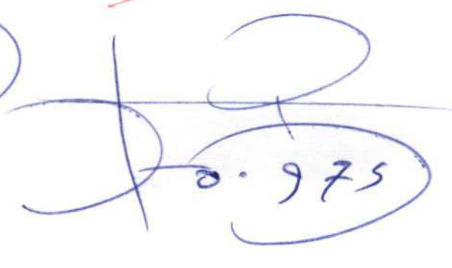
$Z_{1-\frac{\alpha}{2}}$

$Z_{1-\frac{0.05}{2}}$

$Z_{0.975} = 1.96$

$d = 0$

لم يذكر



① Hyp $H_0: \mu_1 = \mu_2$ ~~Greater~~

$H_1: \mu_1 \neq \mu_2$ ~~Less~~

---

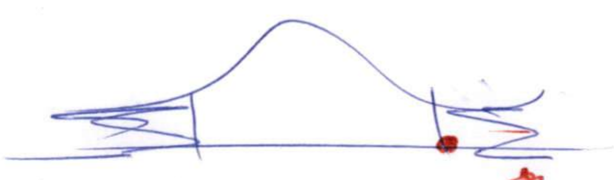
② T.S

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \cancel{d}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{4.5 - 3.4}{\sqrt{\frac{1}{12} + \frac{1.5}{15}}} = 2.569$$

---

③ الرسم



$-z_{1-\frac{\alpha}{2}}$ $z_{1-\frac{\alpha}{2}}$

-1.96 1.96

---

④ DC reject H_0

since $z > z_{1-\frac{\alpha}{2}}$

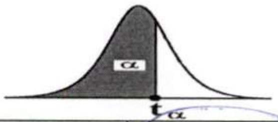
ACCepte region of H_0

$(-1.96, 1.96)$

reject region of H_0

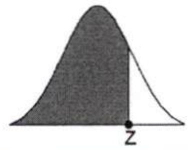
$(-\infty, -1.96) \cup (1.96, \infty)$

Critical Values of the t-distribution (t_α)



ν=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.3062	1.6896	2.0301	2.4377	2.7238
40	1.3030	1.6840	2.0210	2.4230	2.7040
45	1.3006	1.6794	2.0141	2.4121	2.6896
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.2922	1.6641	1.9901	2.3739	2.6387
90	1.2910	1.6620	1.9867	2.3685	2.6316
100	1.2901	1.6602	1.9840	2.3642	2.6259
120	1.2886	1.6577	1.9799	2.3578	2.6174
140	1.2876	1.6558	1.9771	2.3533	2.6114
160	1.2869	1.6544	1.9749	2.3499	2.6069
180	1.2863	1.6534	1.9732	2.3472	2.6034
200	1.2858	1.6525	1.9719	2.3451	2.6006
∞	1.282	1.645	1.960	2.326	2.576

Areas Under The Standard Normal Curve



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

4) T.H of the Difference between two Means(μ1−μ2) σ_1^2 and σ_2^2 are unknown

السؤال

	A	B
Sample	$n_1 =$	$n_2 =$
Average	$\overline{x}_1 =$	$\overline{x}_2 =$
~~Population~~ standard deviation	$s_1 =$	$s_2 =$

d =

نستنتج **Pooled estimate** $s_p^2 = \dfrac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

Use α level of significance

Pooled variance

$T_{1-\frac{\alpha}{2}}$ $df = v = n_1+n_2-2$

1) Hypothesis

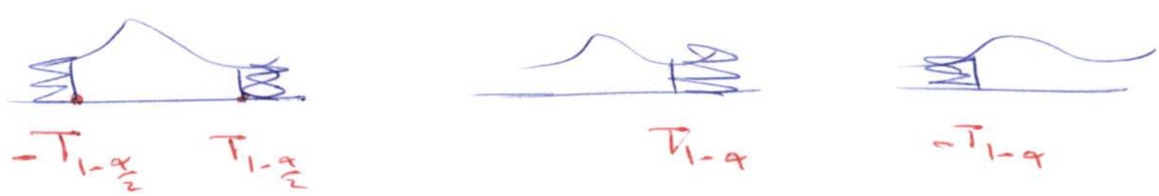
null $H_0:$ $\mu_1 = \mu_2$ ($\mu_1-\mu_2=d$) | $\mu_1 = \mu_2$ ($\mu_1-\mu_2=d$) | $\mu_1 = \mu_2$ ($\mu_1-\mu_2=d$)

Alternative $H_1:$ $\mu_1 \neq \mu_2$ ($\mu_1-\mu_2\neq d$) ~~more Less~~ | $\mu_1 > \mu_2$ ($\mu_1-\mu_2>d$) more | $\mu_1 < \mu_2$ ($\mu_1-\mu_2<d$) less

2) Test statistic

$$T = \frac{(\overline{x}_1-\overline{x}_2)-d}{\sqrt{\dfrac{s_p^2}{n_1}+\dfrac{s_p^2}{n_2}}}$$

3) الرسم



4) Decision

Accepte H_0

reject H_0

Example: ($\sigma_1^2=\sigma_2^2=\sigma^2$ is unknown)

An experiment was performed to compare the abrasive wear of two different materials used in making artificial teeth. 12 pieces of material 1 were tested by exposing each piece to a machine measuring wear. 10 pieces of material 2 were similarly tested. In each case, the depth of wear was observed. The samples of material 1 gave an average wear of 85 units with a sample standard deviation of 4, while the samples of materials 2 gave an average wear of 81 and a sample standard deviation of 5. Can we conclude at the 0.05 level of significance that the mean abrasive wear of material 1 is greater than that of material 2? Assume normal populations with equal variances.

+ العينة الصغيرة 12,10 الجدول T

1

$n_1 = 12$

$\bar{x}_1 = 85$

$s_1 = 4$

2

$n_2 = 10$

$\bar{x}_2 = 81$

$s_2 = 5$

$\alpha = 0.05$

Greater than

لم يذكر $d = 0$

$$\Longrightarrow S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = 20.05$$

$$\sqrt{\frac{S_p^2}{n_1}+\frac{S_p^2}{n_2}} = \sqrt{\frac{20.05}{12}+\frac{20.05}{10}} = 1.91$$

Greater

1) HYP $H_0 : \boxed{\mu_1 = \mu_2}$

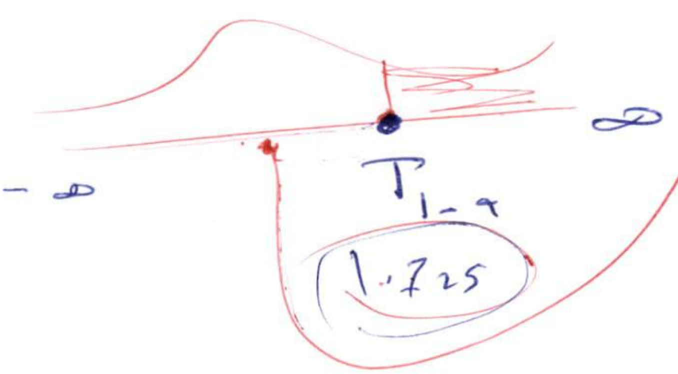
$H_1 : \boxed{\mu_1 > \mu_2}$

2) T.S

$$T = \frac{(\bar{x}_1 - \bar{x}_2) - \delta^{\circ}}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$$

$$T = \frac{12 - 10}{1.94} = 1.031$$

3) الرسم



$\alpha = 0.05$

$T_{1-\alpha} = T_{0.95}$

$df = n_1 + n_2 - 2 = 20$

df	0.95
20	1.725

4) DC

Accept H_0

→ Since $T < T_{1-\alpha}$

→ Accept region of H_0 $(-\infty, 1.725)$

→ reject region of H_0 $(1.725, \infty)$

5) T.H of a Proportion

السؤال

n = لكبير

x = صغير

نستنتج → $\hat{p} = \frac{x}{n}$ → $\cancel{q = 1 - \hat{p}}$

$p_0 = \boxed{}$ % ← نسبة في السؤال

نستنتج → $q_0 = 1 - p_0$

يوجد Z

Use α level of significance

---

1) Hypothesis

null $H_0: p = p_0$ $\quad p \le p_0$ $\quad p \ge p_0$

Alternative $H_1: p \ne p_0$ $\quad p > p_0$ $\quad p < p_0$

~~less more~~ more Less

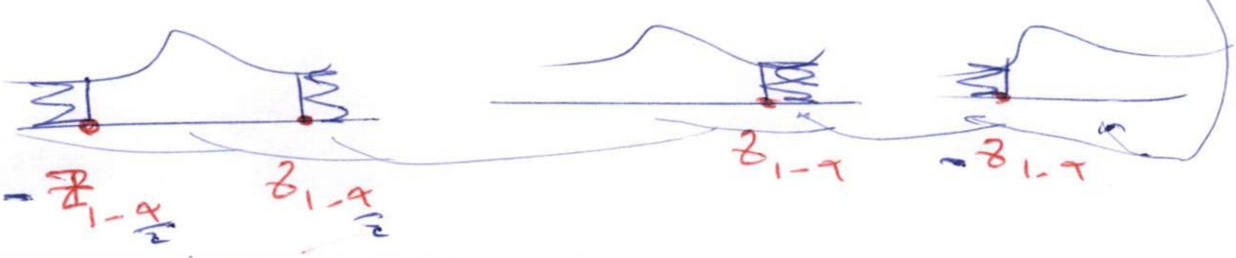
---

2) Test statistic

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

---

3) الرسم



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4) Decision

$\boxed{}$ Accepte H_0

$\boxed{}$ reject H_0

Example:

A researcher was interested in the proportion of females in the population of all patients visiting a certain clinic. The researcher claims that 70% of all patients in this population are females. Would you agree with this claim if a random survey shows that 24 out of 45 patients are females? Use a 0.10 level of significance.

① $H_0 : P = P_0$ $\quad P = 0.7$

$H_1 : P \neq P_0$ $\quad P \neq 0.7$

$P_0 = 70\% = 0.7$

تسيير $\longrightarrow q_0 = 1 - P_0 = 0.3$

② T.S

$x = 24$

$n = 45$

$$Z = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0 q_0}{n}}}$$

$\longrightarrow \hat{P} = \frac{x}{n} = \frac{24}{45} = 0.533$

$\longrightarrow \cancel{\hat{q} = 1 - \hat{P}}$

$$= \frac{0.533 - 0.7}{\sqrt{\frac{(0.7)(0.3)}{45}}} = -2.44$$

③ ارسم

$\alpha = 0.10$

$-\infty \quad -Z_{1-\frac{\alpha}{2}} \quad Z_{1-\frac{\alpha}{2}} \quad \infty$

$-1.645 \quad 1.645$

$Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.10}{2}}$

$= Z_{0.95} = \frac{1.64 + 1.65}{2}$

$= 1.645$

4) D e reject H_0

—— Since $z < -z_{1-\frac{\alpha}{2}}$

—— $\boxed{\text{accept}}$ region of H_0 $(-1.645, 1.645)$

—— reject region of H_0 $(-\infty, -1.645) \cup (1.645, \infty)$

6) T.H of the Difference between Two Proportions

السؤال

	A	B
	$n_1 =$ الكبير	$n_2 =$ الكبير
	$x_1 =$ الصغير	$x_2 =$ الصغير
نستنتج →	$\hat{p}_1 = \frac{x_1}{n_1}$	$\hat{p}_2 = \frac{x_2}{n_2}$

نستنتج → **Pooled estimate** $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$ $\quad \hat{q} = 1 - \hat{p}$

Use α level of significance

جدول z

---

1) Hypothesis

$H_0 : P_1 = P_2 \quad | \quad P_1 = P_2 \quad | \quad P_1 = P_2$

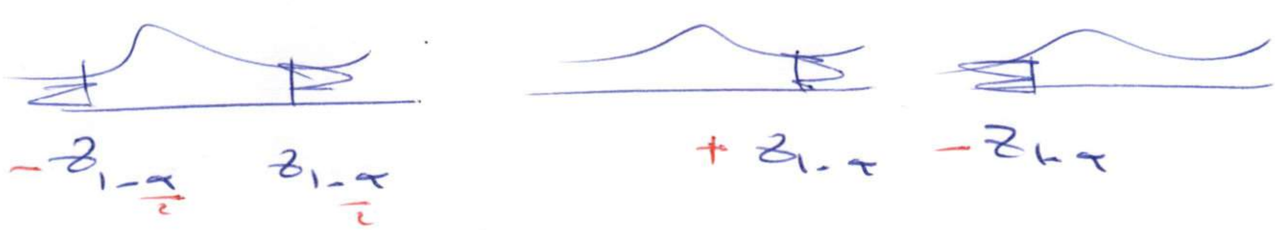
$H_1 : P_1 \neq P_2 \quad | \quad P_1 > P_2 \quad | \quad P_1 < P_2$

2) Test statistic

$$z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}}$$

$$\cancel{\frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}}}}$$

3) الرسم



4) Decision

Accepte H_0

reject H_0

Example:

In a study about the obesity (overweight), a researcher was interested in comparing the proportion of obesity between males and females. The researcher has obtained a random sample of 150 males and another independent random sample of 200 females. The following results were obtained from this study.

	n	Number of obese people
Males	150	21
Females	200	48

Can we conclude from these data that there is a difference between the proportion of obese males and proportion of obese females? Use $\alpha = 0.05$.

M | f

$n_1 = 150$ | $n_2 = 200$

$x_1 = 21$ | $x_2 = 48$

$\hat{p}_1 = \frac{21}{150} = 0.14$ | $\hat{p}_2 = \frac{48}{200} = 0.24$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{21 + 48}{150 + 200} = 0.197$$

$$\hat{q} = 1 - \hat{p} = 0.8$$

$$\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}} = \sqrt{\frac{(0.197)(0.8)}{150} + \frac{(0.197)(0.8)}{200}} = 0.043$$

1) Hyp

$H_0: P_1 = P_2$

$H_1: P_1 \neq P_2$

tess
more

---

2) T.S

$$z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\frac{P q}{n_1} + \frac{P q}{n_2}}}$$

$$= \frac{0.14 - 0.24}{0.043} = -2.328$$

---

3) الرسم

$-z_{1-\frac{\alpha}{2}}$ $\qquad$ $z_{1-\frac{\alpha}{2}}$

-1.96 $\qquad$ 1.96

$\alpha = 0.05$

$$z_{1-\frac{0.05}{2}} = z_{0.975}$$

	0.06
1.9	0.975

4) we reject H_0

since $z < -z_{1-\frac{\alpha}{2}}$

Acceptance region of H_0 $(-1.96, 1.96)$

rejection region of H_0

$(-\infty, -1.96) \cup (1.96, \infty)$

1

descriptive

Statisticis: field of stady concerned with.

① collecting data. "جمع البيانات"

② Organize and summarize data. "ترتيب و تلخيص البيانات"
[by tablets and Charts]
باستخدام الجداول والأعمدة

③ Analysis. "تحليل البيانات"

④ Conclusions or decision. "إصدار القرارات والاستنتاجات"
[inferential statics]
احصاء استقرائي

Data: Raw material of statistics.

معدود ← Quantitative
[numbers: age, weights.]

Qualitative ← غير معدود
[Letter: وصفي]
مثلاً: جنس ، اللون

Sources of Data ← مصادر جمع البيانات

[1] Routinely Kept Records
سجلات محفوظة بشكل روتيني
مثال:
* U.N records.
سجلات الأمم المتحدة
* school records.
سجلات مدرسية

[2] Surveys
دراسة مسحية أو استقصائية
مثال:
* Interviews.
مقابلات شخصية
* A questionnaire
استبيان

[3] Experiments
التجارب

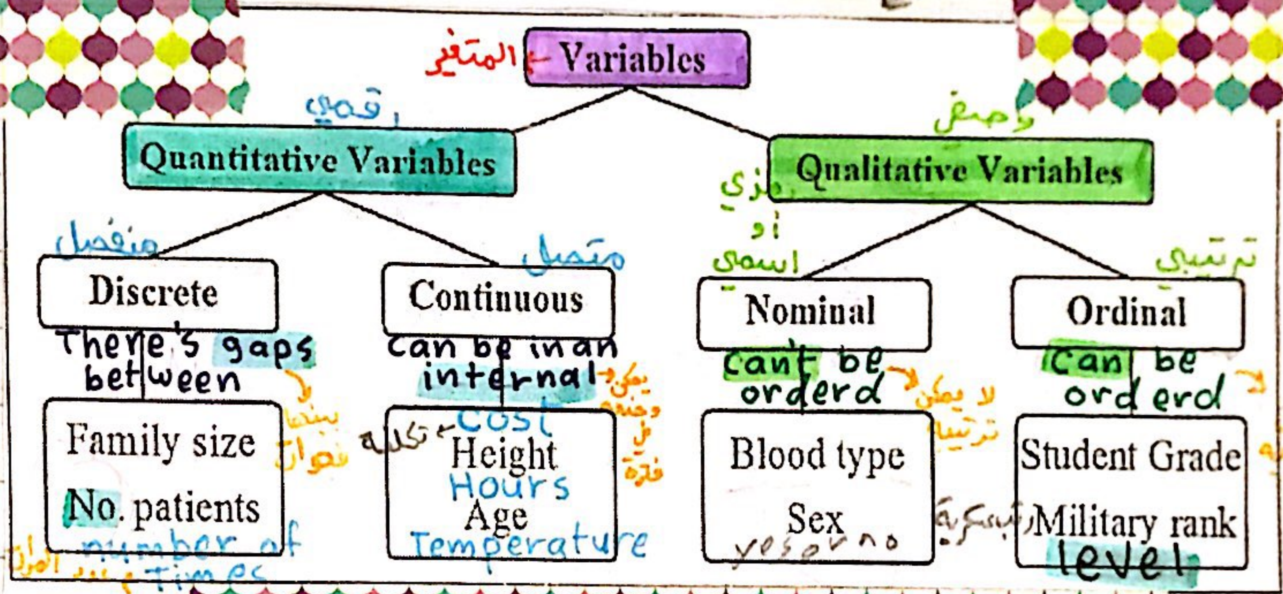
[4] External sources
مصادر خارجية
مثال:
* Books.
* Data Bank.
* Published reports.
التقارير المنشورة

---

[1] Population "N"
largest Collection of entities.
أكبر مجموعة من الموجود

[2] Sample "n"
Part of population.
جزء من المجتمع

the characteristic to be measured on the elements is called



How to chose Sample?

1 Simple random sample

عينة بسيطة عشوائية

2 Stratified Sample

عينة طبقية

لـ يستخدم

في مجتمع

مقسم

لطبقات

Scanned by CamScanner

2

* Mid-point = $\dfrac{\text{UPPER LIMIT} + \text{LOWER LIMIT}}{2}$

* d = LOWER LIMIT2 − UPPER LIMIT1

* TRUE

• TRUE UPPER LIMIT = UPPER LIMIT + d/2

• TRUE LOWER LIMIT = LOWER LIMIT − d/2

0.20 * RELATIVE = FREQUENCY/n
Proportion

% * PERCENTAGE = RELATIVE × 100
Percent

* WIDTH
- W = TRUE UPPER LIMIT$_1$ − TRUE LOWER LIMIT$_1$ → بس في TRUE
- W = Mid-point$_2$ − Mid-point$_1$ → يطبق على ال TRUE والعادي
- W = LOWER LIMIT$_2$ − LOWER LIMIT$_1$ لا نستعمل

★ أي طريقة أفضل لحساب البيانات من مقاييس النزعة المركزية؟

1) Mode - المنوال: عند تكرار القيم # قيمة وحيدة #

2) Mean - المتوسط: لا يوجد تكرار ولا قيمة متطرفة.

[illegible]

3) Median - الوسيط: لا يوجد تكرار ويوجد قيمة متطرفة.

[illegible]

measures of central Tendency	MEAN unit	MEDIAN unit	MODE unit
Advantages ✓	• Simplicity easily • Uniqueness only one • The mean takes into account all values of the data	• Simplicity • Uniqueness • The median is not affected by extreme values.	• Simplicity • The mode is not affected by extreme values • The mode may be found for both quantitative and qualitative.
DisAdvantages ✗	قيمة متطرفة • Extreme values have an influence on the mean. • The mean can only be found for quantitative variables	• The median doesn't take into account all values of the sample • The median can only be found for quantitative variables, in some cases can be found for ordinal	• The mode is not a "good" measure of location, because it depends on a few values of the data. • The mode doesn't take into account all values of the sample. • There might be no mode for a data set. • There might be more mode for a data set.

Population
- $X_1, X_2, \dots, X_N$
- any measure here it called "parameter"

→ that obtained

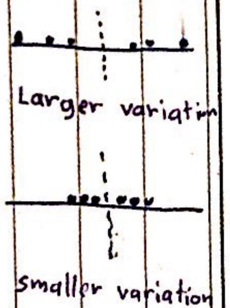
↳ Unknown ✗

Sample
- $x_1, x_2, \dots, x_n$
- any measure here it called "staistic"

↳ Known ✓

Measures of central tendency (of location)	MEAN متوسط	MEDIAN وسيط	MODE منوال
	• Population "parameter" $\mu = \frac{X_1 + X_2 + \dots + X_N}{N}$ $\mu = \frac{\sum_{i=1}^{N} X_i}{N}$ • Sample "staistic" $\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$ $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$	• That value which divides the "ordered array" into tow equal parts • Odd number * There is only one value in the middle Ex: ~~10~~, ~~21~~, 38 ✓, ~~53~~, ~~54~~ • Even number * There are two values in the middle Ex: ~~16~~, ~~35~~, 40, 46, ~~66~~, ~~63~~ ↳ $\frac{40+46}{2}$ $= 43$ ✓	• That value with highest frequency • one mode Ex: 27, $\boxed{28, 28}$, 60 • more than one mode Ex: 2, $\boxed{3, 3}$, 4, 5, $\boxed{6, 6}$ • No mode Ex: 2, 2, 3, 3, 4, 4 Ex: 1, 2, 3, 4, 5

Measures of Dispersion (Variation)	unit ✓ RANGE المدى	$unit^2$ ✓ VARIANCE → تباين	unit ✓ STANDARD DEVIATION الانحراف المعياري	free of unit × C.V معامل الاختلاف coefficient of variation
The variation or dispersion in a set of values refers to how spread out the values is from each other.	• The difference between the largest value and the smallest value.	• The variance is a measure that uses the mean as a point of reference.	• The square root of the variance.	• باختصار: الـ C.V نقدر نقارن قيم ووحدات مختلفة ببعض ودائماً القيمة الأصغر هي الأفضل لأن تشتتها اقل.
	• $R = Max - Min$	* it small when the observations are close to the mean.	• Population σ $\sigma = \sqrt{\sigma^2}$	• $C.V = \frac{S}{\bar{x}} \times 100$ ← متوسط العينة
	Ex: 26, 27, 30, 33, 39	* it large when the observations are spread out from the mean.	• sample S $S = \sqrt{S^2}$	جوابه %
	$R = 39 - 26$ $= 13$ unit	* it zero when all observations have the same value.	* We wish to express the concept of dispersion in terms of the original unit.	* We need a measure of the relative variation that will not depend on either the units or on how large the values are.
	نفس الوحدة المعطاة في السؤال g/cm/m/Kg	• The population variance σ^2: $\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$ (μ → population mean, N → population size) parameter it is unknown. $\sigma^2 \geq 0$ دائماً قيمته ايجابية او تساوي صفر مستحيل سالب	لأ قصده نرجع القيم لمقياس كويس لأنه موحد مربع مثل التباين عشان كان ذا الجذر عشان نتخلص من التربيع.	
		• The sample variance S^2: $S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$ ($\bar{x}$ → sample mean, n → sample size) statistic it is known. $S^2 \geq 0$ $S^2 = 0$ → اذا كانت كل القيم متساوية		



6

3

① Probability — measure (مقياس) used to measure the chance of occurrence of event (which is between 0 and 1)
الاحتمال

$P(E) = 0 \leq P(E) = \frac{n(E)}{n(\Omega)} \leq 1$ → $P(\emptyset)=0$, $P(\Omega)=1$

② Sample space (Ω) — set of all possible outcomes of experiment
فضاء العينة
(where $n(\Omega)$ is the number of outcomes -elements- (الكلية) in Ω)

③ Experiment — Some procedure or process that we do.
التجربة
Equally likely outcomes: If the outcomes have the same chance of occurrence. ← فرص الظهور متساوية

④ Event (E) — Any subset of Ω (where $n(E)$ is the number of outcomes in E
الحدث
$\emptyset \subseteq \Omega$ (impossible event) // $\Omega \subseteq \Omega$ (sure event)

Operations on events

Union $\cup$ or	Intersection $\cap$ and	not Complement $\bar{A}$ A^c A'
نأخذ بالاعتبار العناصر الموجودة في a أو b أو كليهما بدون تكرار.	نأخذ بالاعتبار العناصر المشتركة فقط	كل العناصر الموجودة في Ω وغير موجودة في A
The exhaustive events is: الحوادث الي تأخذ كل فضاء العينة Ω وتصير $\boxed{P(E)=1}$ $\boxed{event=\Omega}$	The disjoint or mutually exclusive events is: الحوادث الي ما فيها عناصر مشتركة أبداً وتصير $\boxed{P(E)=0}$ $\boxed{event=\emptyset}$	Rule: $P(A^c) = 1 - P(A)$ $*A \cup A' = \Omega$ [exhaustive] $*A \cap A' = \emptyset$ [disjoint]
Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$		
$P(A \cup B) = \frac{n(A \cup B)}{n(\Omega)}$ يعني $P(A \cup B) = \frac{n(A) + n(B) - n(A \cap B)}{n(\Omega)}$	$P(A \cap B) = \frac{n(A \cap B)}{n(\Omega)}$	

	B	$\bar{B}$	TOTAL
A	0.2	0.3	0.5
$\bar{A}$	0.4	0.1	0.5
TOTAL	0.6	0.4	1.00

الجواب السؤال

$P(\bar{A}) = 0.5$

$P(\bar{B}) = 0.4$

$P(A \cap \bar{B}) = 0.3$

$P(\bar{A} \cap B) = 0.4$

$P(\bar{A} \cap \bar{B}) = 0.1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.6 - 0.2 = 0.9$

$P(A \cup \bar{B}) = P(A) + P(\bar{B}) - P(A \cap \bar{B}) = 0.5 + 0.4 - 0.3 = 0.6$

$P(\bar{A} \cup B) =$ *exercise*

$P(\bar{A} \cup \bar{B}) =$ *exercise*

مثال لل exhaustive

Are A, B exhaustive?

لـ نشوف الاتحاد

$P(A \cup B) \overset{?}{=} 1 \leftarrow$ الشرط

الحل المعتمد: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.5 + 0.6 - 0.2$

$P(A \cup B) = 0.9$

$P(A \cup B) \overset{?}{=} 1$

$0.9 \neq 1$

(not exhaustive)

مثال للdisjoint

Are A, B disjoint?

لـ نشوف التقاطع

الشرط حقنا تكون $(A \cap B) = \emptyset$: جواب

or

$\overset{?}{=}$

$P(A \cap B) \overset{?}{=} 0$

من الجدول

يطلع لي تقاطع

A و B

جاهز إلي هو

$\boxed{0.2}$

و أعوض مباشر

$0.2 \overset{?}{=} 0$

$0.2 \neq 0$ (not disjoint)

8

الحوادث المستقلة

*Independent Events

تكون الحادثة مستقلة اذا تحققت أحد هذه الشروط :

① $P(A|B) = P(A)$

② $P(B|A) = P(B)$

③ $P(B \cap A) = P(A) \times P(B)$

* نختار واحدة منهم حسب المعطيات .

مثال على الحوادث المستقلة

Ex: $P(B|A) = 0.5$

$P(A) = 0.5$

$P(B) = 0.3$

*Are A, B independent?

$P(B|A) \stackrel{?}{=} P(B)$

$0.5 \neq 0.3$

A, B are not independent.

احتمال مشروط

*Conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(B) \neq 0$$

A: مجهول   B: معلوم

كلمات دالة :

- given ~~that~~
- Knowing that
- Found that

mid 2

test — نتيجة التحليل
true — حقيقة وجود المرض

3.5

FP + $P(T|\bar{D}) = \dfrac{P(T \cap \bar{D})}{P(\bar{D})}$ This result happens when a test indicates a positive status when the true status is negative.

Fn − $P(\bar{T}|D) = \dfrac{P(\bar{T} \cap D)}{P(D)}$ This result happens when a test indicates a negative status when the true status is positive.

sen $P(T|D) = \dfrac{P(T \cap D)}{P(D)}$ The sensitivity of a test is the Probability of a positive test result given the presence of the disease.

sp $P(\bar{T}|\bar{D}) = \dfrac{P(\bar{T} \cap \bar{D})}{P(\bar{D})}$ The specificity of a test is the Probability of a negative test result given the absence of the disease.

Predictive $P(D)$ → Rate of the disease. (معدل)

sen عكس ← [1]

The predictive value positive

The Probability that a subject has the disease, given that the subject has a positive screening.

$$P(D|T) = \frac{\text{sen} \times P(D)}{\text{نفس البسط} + (1 - sp) \times P(\bar{D})}$$

$P(D)$ → معطاة

$P(\bar{D})$ مكملة لـ $P(D)$

عكس SP ← [2]

The predictive value negative

The probability that a subject doesn't have the disease, given that the subject has a negative screening.

$$P(\bar{D}|\bar{T}) = \frac{sp \times P(\bar{D})}{\text{نفس البسط} + (1 - \text{Sen}) \times P(D)}$$

$P(D)$ معطاة

تشابتر 4 الجزء 1

Random variables

- Discreate ← Ex: Number of any thing [no.of]
- Continuous

Probability Distributions

★ شروطه

1. $0 \leq P(X=x) \leq 1$

كل احتمال مابين الـ0 والـ1

2. $\sum_x P(X=x) = 1$

مجموع الاحتمال = 1

★ طرق الحل

بالنظر للعمود الأخضر

① $P(X\leq 2)$ بالجمع

$= P(X=2)+P(X=1)+P(X=0)$

$= 0.07 + 0.20 + 0.67$

$= 0.94$

② $P(X\leq 2)$ بالمكملة

$= 1 - P(X>2)$ ليش؟

$= 1 - P(X=3)$

$= 1 - 0.06$

$= 0.94$

نفس الجواب

المكملة دائماً تاخذ العكس تماماً، اذا هي أكبر تاخذ أصغر واذا فيه مساواة نشيلها وهكذا

★ $P(X=-1) = 0$

ليش؟ احتمال القيمة الغير موجودة بـ(0).

x	freq. of x $n(X=x)$	R. freq. $P(X=x) = \frac{n(X=x)}{n(\Omega)}$	C. freq. $P(X\leq x)$
0	10,000	10,000/14,900 = 0.67	0.67
1	3,000	0.20	0.87
2	1,000	0.07	0.94
3	900	0.06	1
	Total = 14,900	Total = 1	

Cumulative Distributions

★ $P(X\leq x) = \sum_{a\leq x} P(X=a)$

للاطلاع فقط طريقة ايجاده في الجدول

★ طرق الحل

بالنظر للجدول البرتقالي

$P(X\leq 2)$

$= 0.94$

نلقاها جاهزة في C.f

مجموعه عكس الـR.f انتبهوا تجمعوا

لا توجد بالحاسبة لكن للاطلاع

★ mean = expected قانونه $\mu = \sum_x x\,P(X=x)$

★ variance $\sigma^2 = \sum_x (x-\mu)^2 P(X=x)$

Scanned by CamScanner

تشابتر ٤ جزء ٣   Discreate distribution → Bar Chart تستخدم شكل

Binomial distribution

* تعريفه → The experiment has a sequence of "n" [Bernoulli trials] → is an experiment with only tow possible outcomes.

* p → The probability of success معطى دائماً في السؤال
"Constant for each trial"

q → The probability of failure. مكملة الـ p
$1 - p = q$

* توزيعه → $X \sim \text{Binomial}(n, p)$

n: Sample size
p: % أو جزء من ١٠

Parameter

وهي إذا أعطانا في السؤال تعتبر دلالة على أننا المفروض نستخدم [Binomial]

* قانونه → $P(X=x) = {}_nC_x \, p^x \, q^{n-x}$ ← أي قيمة يطلبها موجودة ضمن الـ X قيمتها $\boxed{0}$

where: X = The number of successes in the "n" trials.

سؤال: The possible values of "X" are?
$x = 0, 1, 2, 3, \dots, n$

* المتوسط، التباين، الانحراف المعياري →
The mean $\mu = np$
The variance $\sigma^2 = npq$
The standard deviation $\sigma = \sqrt{\sigma^2}$

Poisson distribution

* توزيعه → $X \sim \text{Poisson}(\lambda)$

Parameter

إذا نحطّ لنا في السؤال فهي دلالة على أننا المفروض نستخدم توزيع [Poisson]
يعطينا مثلاً رقم يعبر عن المتوسط ويعبر λ وبعدها فترة زمنية، سنة، شهر، أسبوع...

* وقانونه $P(X=x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ ← أي قيمة يطلبها موجودة = $\boxed{0}$

سؤال: The possible values of "X" are?
$x = 0, 1, 2, 3, \dots, \infty$

مهم: لما يطلب مني احتمال "أكبر من"
$P(X \geq x) = 1 - P(X < x)$
أجيب المكملة لأن الـ "Poisson" قيمه! كما لا نهاية "∞"

* المتوسط، التباين، الانحراف المعياري →
The mean (μ) $= \lambda$
The variance (σ^2) $= \lambda$
The standard deviation (σ) $= \sqrt{\sigma^2} = \sqrt{\lambda}$

ارجعوا للتمارين ✓ مثلاً السابقة تنتبهوا لو تحطون كل شيء ما تعرفونه $\boxed{0}$

تتشابه بتمرين جزء ي والأخير — Continuous distributions — Bell-shaped — تستخدم شكل

Normal distribution X

* متوزعه → $X \sim Normal(\mu, \sigma^2)$ — μ: location, σ^2: the shape — Parameter
* شرط → $-\infty < X < \infty$ قيمه محصورة بين
* معلومة خطيرة → The highest point of The curve of $f(x)$ at the [mean = mode = median].

standard Normal distribution Z

* متوزعه → $Z \sim N(0, 1)$ — μ, σ^2 — ثابت يعني قياسي
* شرط → $-\infty < Z < \infty$ قيمه محصورة
* معلومة خطيرة → جداول ال Z ماتحسب إلا احتمال الأقل من عشان كذا إذا طلب احتمال أكبر من نجيب المكملة وبس.

معادلة تحويل من X ⇐ Z

$$Z_{standard} = \frac{X - \mu}{\sigma} \sim Normal(0, 1)$$

* معلومة خطيرة #2 → $P(Z < 0) = P(Z \geq 0) = 0.5$ دائماً

ما تفرق المساواة في ال "Continuous"

تشابتر 5 sampling distributions

$\boxed{\text{توزيع}} \sim N(\text{mean}, \text{variance})$

single $\hat{p}$ "proprtion"

mean

$P = \frac{N(A)}{N}$ عدد النجاح / عدد كلي

P ← احتمال النجاح

variance

$\frac{pq}{n}$

distribution

$\hat{p} \sim N(P, \frac{pq}{n})$

mean ← P

variance ← $\frac{pq}{n}$

Two: $\hat{p}_1 - \hat{p}_2$

$P_1 \boxed{-} P_2$

P_1 ← احتمال نجاح ① 2

P_2 ← احتمال نجاح ② 2

$\frac{P_1 q_1}{n_1} \boxed{+} \frac{P_2 q_2}{n_2}$

لما أبي أطلع احتماله بيتغير التوزيع من $Z \leftarrow \hat{p}$ عن طريق استخدام معادلة التحويل

$Z = \frac{\hat{p} - \text{mean}}{\text{standard error}}$

standard error ← نفسه deviation

"$Z \sim N(0,1)$"

وأوجده من جداول Z

single $\bar{X}$ "mean"

mean

μ

variance

- Known ← population normal distribution: $\frac{\sigma^2}{n}$
- unKnown ← sample size: $\frac{S^2}{n}$

distribution

- Known: $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

  نفس الكلام بس يتغير الرمز من $\hat{p}$ إلى $\bar{X}$

- unKnown: $\bar{X} \sim N(\mu, \frac{S^2}{n})$

  لما أبي أطلع احتماله باستخدم جداول ال t وتوزيع t

  $T = \frac{\bar{X} - \text{mean}}{\text{standard error}}$

  standard error = $\sqrt{\text{variance}}$

  "$T \sim t_{(n-1)}$" $v = df$

  Note: فيها جزء نظري عن "t" ...

Two: $\bar{X}_1 - \bar{X}_2$

$\mu_1 \boxed{-} \mu_2$

$\frac{\sigma_1^2}{n_1} \boxed{+} \frac{\sigma_2^2}{n_2}$

$\frac{S_1^2}{n_1} \boxed{+} \frac{S_2^2}{n_2}$

★ ملاحظات : • في القناة أشياء مهمة عن كيف تعرفون إذا "variance" معلومة أو مجهولة .

• لازم تتعلمون على جدول "t"

• افهموا الملخصات والزبدة وانطلقوا حلوا التجميعات وتدربوا كثير "لازم تأخذ يدكم عالحل وتعرفون تفرقون بين التوزيعات"!

• أخيراً ، أتمنى تسامحوني إذا فيه خطأ أو نقصان وآسفة عالخط + احتاج دعاء :).

Estimation تشابتر σ

نجيبه من الالة أحيان $\sqrt{\frac{S^2}{n}}$ $\bar{X}$ $\sqrt{\frac{\sigma^2}{n}}$ $\sqrt{\frac{pq}{n}}$

نجيب هذا الاول
pooled اسمه
variance

$$Sp^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

$\bar{X}_1-\bar{X}_2$

$\sigma_1^2 = \sigma_2^2$

$\bar{X}_1-\bar{X}_2$ $\hat{P}_1-\hat{P}_2$

$\bar{X}$ $\hat{P}$

هذا اللي نبي نوصل له
يحل محل standard error

$$Sp = \sqrt{\frac{Sp^2}{n_1} + \frac{Sp^2}{n_2}}$$

σ unknown σ Known

$\bar{D}$

$\hat{P}_1-\hat{P}_2$

$\hat{P}$

$\bar{X}$

$\bar{X}_1-\bar{X}_2$

← Point estimate $\pm$ Reability of coefficient (standard error)

كل شي Z عدا $\star$

ضرب بعض
يعطونا
margin of error
(precission)

$Z_{1-\frac{\alpha}{2}}$

① σ Known
سواء كانت
normal $n<30$ (small)
او
non-normal $n>30$ (Large)

② σ unknown
+
non-normal
$n>30$ (Large)

$\star$ $t_{1-\frac{\alpha}{2}}$ → $df=n-1$

σ unknown
+
normal
$n<30$ (small)

لو كتاب دانيل يقول ممكن نحلها ب t بعد
حطيها بالك احتياط !!

تشابتر 7

test statistic

Z

1. σ Known → $\bar{X}$ → $Z = \dfrac{\bar{x} - \mu_0}{\sqrt{\dfrac{\sigma^2}{n}}}$

2. σ Known → $(\bar{x}_1 - \bar{x}_2)$ → $Z = \dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\dfrac{\sigma^2}{n_1} + \dfrac{\sigma^2}{n_2}}}$
(independent)

3. $\hat{P}$ → $Z = \dfrac{\hat{P} - P_0}{\sqrt{\dfrac{P_0 q_0}{n}}}$ → H_0
$\frac{x}{n}$

4. $(\hat{P}_1 - \hat{P}_2)$ → $Z = \dfrac{(\hat{P}_1 - \hat{P}_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}}$ → $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$
Poded proprtion

5. σ UN-Known
non-normal
$n > 30$ → $\bar{X}$ → $Z = \dfrac{\bar{x} - \mu_0}{\sqrt{\dfrac{s^2}{n}}}$

T

1. σ Un-Known
normal
$n < 30$ → $\bar{X}$ → $T = \dfrac{\bar{x} - \mu_0}{\sqrt{\dfrac{s^2}{n}}}$

2. σ un-Known → $(\bar{x}_1 - \bar{x}_2)$ → $T = \dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\dfrac{sp^2}{n_1} + \dfrac{sp^2}{n_2}}}$
(independent)

3. $(\bar{x}_1 - \bar{x}_2)$ → MD → $T = \dfrac{\bar{D}}{\dfrac{SD}{\sqrt{n}}}$
$\bar{D}$ → mean
SD → standard
(dependent)
لنفس المجتمع قبل و بعد

$\bar{D} = \bar{x} - \bar{y}$

σ → $\bar{D} \pm t_{1-\frac{\alpha}{2}} \left(\dfrac{SD}{\sqrt{n}}\right)$

الملخص للخطوات الأساسية في أختبار الفرضيات فقط

أ/ خلود باسالم

ch 7

Test Hypothesis

Hypothese Tests about Unknown parameters:

- μ — page (128)
- $\mu_1 - \mu_2$
  - indep. — page(135)
  - dep. paired T. test — page (141)
- p — page (146)
- $p_1 - p_2$ — page (151)

أختبار الفرضيات يمر بأربع مراحل

① **Hypothese:**

H_0: $=$, $\leq$, $\geq$

(مكملة لـ H_A) عكس H_A + علامة المساواة دائماً هنا

null hypothesis

H_1 / H_A: $\neq$, $>$, $<$

معطاه في السؤال

فرضية الباحث

Alternative Hypothesis

research Hypothesis

② **Test statistic (T.S.):**

قانون الاحصاءه

أما

- Z
- T

$\alpha = P(\text{Type I error}) = P(\text{Rejecting } H_0 \mid H_0 \text{ true})$

α = Level significance

③ Rejection region of H_0 (Critical region)

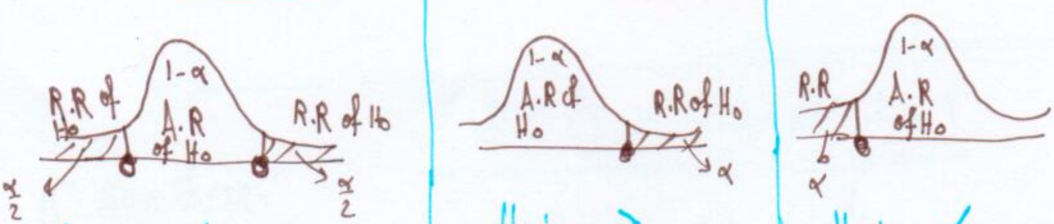
قيم حرجة

*** Reliability Cofficient (Critical Values) :** القيم اللي أستخرجها من الجداول وهي تفصل منطقة رفض H_0 عن منطقة قبول H_0

Two-Sided	right one sided	one sided (left)
$\pm Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
$\pm t_{1-\frac{\alpha}{2}}$	$t_{1-\alpha}$	$-t_{1-\alpha}$

* الرسومات :

الرسومات معتمده على H_A



$H_A: \neq$ | $H_A: >$ | $H_A: <$

④ Decision :

القرار

قوانين الرفض H_0

أقارن قيمة (T.S.) بالقيم الحرجة

⇓

إذا تحقق الشرط نرفض H_0

reject H_0

الرسم

① نرسم ونحدد القيم الحرجة على الرسم

② أشوف قيمة (T.S.) وين تقع في منطقة رفض H_0 (R.R) أو منطقة قبول H_0 (A.R)

P-Value $\boxed{\text{تستخدم في حالة Z}}$

الجدول موجود في صفحة (131)

H_A	$\neq$	$>$	$<$
P-Value	$2P(Z>\lvert Z_c\rvert)$	$P(Z>Z_c)$	$P(Z<-Z_c)$

- if P-value $\leq \alpha$ $\Rightarrow$ reject H_0
- if P-value $> \alpha$ $\Rightarrow$ Accept H_0

الملخص لخطوات أجراء الأختبار فقط

أ/ خلود باسالم

H.W3

One of the purposes of an investigation by Porcellini et al. (A-19) was to investigate the effect on CD4 T cell count of administration of intermittent interleukin (IL-2) in addition to highly active antiretroviral therapy (HAART). The following table shows the CD4 T cell count at baseline and then again after 12 months of HAART therapy with IL-2. Do the data show, at the .05 level, a significant change in CD4 T cell count?

$n=7$

Subject	1	2	3	4	5	6	7
X — CD4 T cell count at entry	173	58	103	181	105	301	169
Y — CD4 T cell count at end of follow-up	257	108	315	362	141	549	369
$D=X-Y$	-84	-50	-212	-181	-36	-248	-200

~~1-what is the assumption?~~

$\bar{D} = -144.43$

$S_D = 85.677$

2-Hypothesis is?

$H_0: \mu_x - \mu_y = 0$ vs $H_A: \mu_x - \mu_y \neq 0$

or $H_0: \mu_D = 0$ vs $H_A: \mu_D \neq 0$

3-Test statistic=

$$T = \frac{\bar{D}}{S_D/\sqrt{n}} = \frac{-144.43}{85.677/\sqrt{7}} = -4.46$$

$\alpha = 0.05$

$df = n-1 = 7-1 = 6$

4-Rejection region is?

$-2.447 = -t_{1-\frac{\alpha}{2}}$ $t_{1-\frac{\alpha}{2}} = t_{0.975} = 2.447$

الجواب :

$(-\infty, -2.447) \cup (2.447, \infty)$

5-conclusion is:

a)reject H0 b)accept H0

Parameters: measure computed from population

a. [μ σ^2 σ p

Chapter 6

هذا الملخص لا يغني عن الملزمة
أ/ خلود باسالم

Statistical Inferences

Estimation ⇒ تقدير بقيمة حقيقية للمعلمة المجهولة
estimating the actual value of unknown parameters

Hypothesis test ⇒ ch7

Point Estimate

- single value used to estimate the corresponding population parameter...

	Population parameters	point estimation
mean	μ	$\bar{x}$
Variance	σ^2	S^2
standard devation	σ	S
proportion	p	$\hat{p}$
Difference between tow mean	$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$
Difference between Tow proportion	$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$

Confidence ~~Interval~~ = Interval Estimate

- consists of two numerical values defining a rang of values that most likely includes the parameter...

$$(L , U)$$

$$L < \text{parameter} < U$$

L ⇒ Lower Limit

U ⇒ upper Limit

- $(1-\alpha)$ ⇒ [• Confidence cofficient... • Confidence level...

الطريقة العامة لكتابة الفترة التقديرية:

estimator ± $\underbrace{\text{(reliability cofficent)(standard Error)}}$

⇓

estimator ± margin Error

- Confidence Interval of μ
- Interval Estimate of μ

- normal + σ^2 known
- non-normal + σ^2 known + $n \geq 30$

$\Downarrow$

$$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$\bar{X}$ $\Downarrow$ estimator

$Z_{1-\frac{\alpha}{2}}$ $\Downarrow$ reliability coefficient

$\frac{\sigma}{\sqrt{n}}$ → Standard Error of $\bar{X}$

$\Downarrow$

- margin of eror

=

- Precision of the estimate

Upper Limit: $\bar{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

Lower Limit: $\bar{X} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

- normal + σ Unknown + $n < 30$

$\Downarrow$

$$\bar{X} \pm t_{1-\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}$$

df = n-1

$\bar{X}$ $\Downarrow$ estimator

$t_{1-\frac{\alpha}{2}}$ $\Downarrow$ reliability Coefficient

$\frac{S}{\sqrt{n}}$ → Standard Error of $\bar{X}$

$\Downarrow$

- margin of eror

=

- Precision of estimate

Upper Limit: $\bar{X} + t_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

Lower Limit: $\bar{X} - t_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

$df = v = n-1$

- Confidence Interval for difference between two mean $\mu_1 - \mu_2$
- Interval Estimate of $\mu_1 - \mu_2$

- σ_1^2 and σ_2^2 known

$$\underbrace{(\bar{X}_1 - \bar{X}_2)}_{\text{estimator}} \pm \underbrace{\underbrace{Z_{1-\frac{\alpha}{2}}}_{\text{reliability coefficient}} \underbrace{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}_{\text{Standard Error of } (\bar{x}_1 - \bar{x}_2)}}_{\text{margin of error / Precision of estimate}}$$

- margin of error
- Precision of estimate

Upper Limit: $(\bar{X}_1 - \bar{X}_2) + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Lower Limit: $(\bar{X}_1 - \bar{X}_2) - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

- normal + $\sigma_1 = \sigma_2 = \sigma$ Un known + $n_1 < 30$, $n_2 < 30$

$$\underbrace{(\bar{X}_1 - \bar{X}_2)}_{\text{estimator}} \pm \underbrace{\underbrace{t_{1-\frac{\alpha}{2}}}_{\text{reliability Coefficient}} \underbrace{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}_{\text{S.E}}}_{\text{margin of error / Precision of estimate}}$$

- margin of error
- Precision of estimate

$$S_p^2 = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}$$

$df = V = n_1 + n_2 - 2$

Upper Limit: $(\bar{X}_1 - \bar{X}_2) + t_{1-\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$

Lower Limit: $(\bar{X}_1 - X_2) - t_{1-\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$

- Confidence Interval for difference between two proportion $P_1 - P_2$

- Interval Estimation $P_1 - P_2$

$\Downarrow$

$$\left[\begin{array}{llll} n_1 \geq 30, & n_1 P_1 > 5, & n_1 q_1 > 5, & q_1 = 1 - P_1 \\ n_2 \geq 30, & n_2 P_2 > 5, & n_2 q_2 > 5, & q_2 = 1 - P_2 \end{array}\right] \quad \hat{P}_1 = \frac{X_1}{n_1} \quad \hat{P}_2 = \frac{X_2}{n_2}$$

$\Downarrow$

$$\underbrace{(\hat{P}_1 - \hat{P}_2)}_{\text{estimator}} \pm \underbrace{\underbrace{Z_{1-\frac{\alpha}{2}}}_{\text{reliability Cofficient}} \underbrace{\sqrt{\frac{\hat{P}_1 \hat{q}_1}{n_1} + \frac{\hat{P}_2 \hat{q}_2}{n_2}}}_{\text{standard Error of } \hat{P}_1 - \hat{P}_2}}_{=\text{margin of Error}}$$

- margin of Error
- precision of estimate

Upper Limit : $(\hat{P}_1 - \hat{P}_2) + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}_1 \hat{q}_1}{n_1} + \frac{\hat{P}_2 \hat{q}_2}{n_2}}$

Lower Limil : $(\hat{P}_1 - \hat{P}_2) - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}_1 \hat{q}_2}{n_1} + \frac{\hat{P}_2 \hat{q}_2}{n_2}}$

- Confidence Interval (p)
- Interval Estimation (p)

$$\Downarrow$$

$$\left[\, n \geq 30 \;,\; np > 5 \;,\; n(1-p) = nq > 5 \,\right]$$

$$\Downarrow$$

$$\underbrace{\hat{p}}_{\text{Estimator}} \pm \underbrace{\underbrace{Z_{1-\frac{\alpha}{2}}}_{\text{reliability Coefficient}} \underbrace{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}_{\text{standard Error of } \hat{p}}}_{\text{Margin of Error}}$$

- Margin of Error
- precision of estimate

Upper Limit: $\hat{p} + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\boxed{\hat{q} = 1 - \hat{p}}$

Lower Limit: $\hat{p} - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

هذا الملخص لا يغني عن الملزمة الرئيسية للمقرر

أ/ خلود باسالم

(5)

Estimation

Point estimate

Population	Sample
μ	$\bar{X}$
$\mu_1 - \mu_2$	$\bar{X}_1 - \bar{X}_2$
P	$\hat{P}$
$P_1 - P_2$	$\hat{P}_1 - \hat{P}_2$

(C.I) Interval Estimate

$(\bar{X})$

1) σ^2 known, norml, non-normal (n large, $(n \geq 30)$

$$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

2) σ^2 unknown, normal, (n small) $\rightarrow$ $(n < 30)$

$$\bar{x} \pm t_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \qquad (df = n-1)$$

3) σ^2 unknown, normal, (n large) $(n \geq 30)$

$$\bar{x} \pm Z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$(\bar{X}_1 - \bar{X}_2)$

σ_1^2, σ_2^2 known

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

σ_1^2, σ_2^2 unknown but equal $(\sigma_1^2 = \sigma_2^2)$

$$sp^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{1-\frac{\alpha}{2}} \sqrt{\frac{sp^2}{n_1} + \frac{sp^2}{n_2}}$$

$(df = n_1 + n_2 - 2)$

$(\hat{p})$

$$\hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{q}\hat{p}}{n}}$$

$(\hat{P}_1 - \hat{P}_2)$

$$(\hat{P}_1 - \hat{P}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}}$$

* خطوات حل C.I :

1 Find α → alpha

2 Calculate $1-\frac{\alpha}{2}$

3 Find $Z_{1-\frac{\alpha}{2}}$ or $t_{1-\frac{\alpha}{2}}$ من الجداول

4 التعويض بالقانون

* القانون العام :

$$\text{Point estimate} \pm \underbrace{\underbrace{(\text{Reability coffictent})}_{Z_{1-\frac{\alpha}{2}} \;\; t_{1-\frac{\alpha}{2}}} (S.E)}_{\text{margin of error (precission)}}$$

« Hypotheses testing »

① كتابة Hypothesis

- H_0: الفرضية التي تحتوي على مساواة
  - «one sided»: $\leq$, $\geq$
  - «two sided»: $=$
- H_A: الفرضية التي لا تحتوي مساواة
  - «one sided»: $<$, $>$
  - «two sided»: $\neq$

② test statistis

- «Z»
  - μ
    - σ known → $Z = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$
    - σ unknown, non-normal «n large» → $Z = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}}$
  - $\mu_1 - \mu_2$: σ_1, σ_2 known, independent → $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
  - P → $Z = \dfrac{\hat{P} - P_0}{\sqrt{\frac{P_0 q_0}{n}}}$
  - $P_1 - P_2$ → $Z = \dfrac{\hat{P}_1 - \hat{P}_2}{\sqrt{\frac{\bar{P}\bar{q}}{n_1} + \frac{\bar{P}\bar{q}}{n_2}}}$
- «t»
  - $\mu_1 - \mu_2$ → dependent $= \mu_D$ → $T = \dfrac{\bar{D}}{\frac{S_D}{\sqrt{n}}}$
  - $\mu_1 - \mu_2$ → σ_1, σ_2 unknown «independent» but equal → $T = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{sp^2}{n_1} + \frac{sp^2}{n_2}}}$
  - μ → σ unknown, normal, n (small) → $T = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}}$

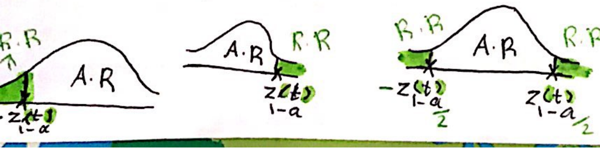
* $\bar{P} = \dfrac{X_1 + X_2}{n_1 + n_2}$

* $sp^2 = \dfrac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2}{n_1 + n_2 - 2}$

③ Rejection Rejion «R.R»

H_A تحدد بإشارة

- one sided
  - $<$ أقل من
  - $>$ أكبر من
- two sided
  - $\neq$



④ Decision «اتخاذ القرار»

1- Rejection Rejion «الرسم»
- ① نحدد قيمة Z أو t
- ② المحسوبة في الخطوة
- ③ على الرسم في الخطوة
  - A.R accept H_0
  - R.R reject H_0

2- بالقانون * اشارة H_A *
- $>$ أكبر من: if Z or t (من القانون «خطوة 2») $> Z_{1-a}$ → Reject H_0
- $<$ أقل من: if Z or t $< -Z_{1-a}$ → Reject H_0
- if $\neq$: $Z < -Z_{1-a/2}$ أو $Z > Z_{1-a/2}$ → Reject H_0 / Reject H_0

* تابع :

4 Decision << اتخاذ القرار >>

3- P-Value

* أولاً : نوجد قيمة P-value (احتمال)

* باستخدام إشارة H_A *

$>$ أكبر من

[دائماً أكبر من ثابتة]

نوجد احتمال

$P(Z > Z) = ?$

من القانون خطوة 2

$<$ أعلى من

$P(Z > -Z) = ?$

من خطوة 2

$\neq$

$2 \times P(Z > Z)$

من الخطوة 2

* ثانياً : نقارن قيمة P-value (من الخطوة السابقة) بـ α

if

[دائماً أقل من أو يساوي]

P-Value $\leq \alpha$

Reject H_0

Estimation $\underline{6}$ تشابتر

نجيبه من الآلة أحيان → $\sqrt{\frac{S^2}{n}}$ $\bar{x}$

$\sqrt{\frac{\sigma^2}{n}}$ $\sqrt{\frac{pq}{n}}$

نجيب هذا أول
اسمه pooled variance →

$$Sp^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

$\bar{x}_1 - \bar{x}_2$ ← $\sigma = \sigma$

$\bar{x}_1 - \bar{x}_2$, $\bar{x}$ ← σ Known

$\hat{p}_1 - \hat{p}_2$, $\hat{p}$

هذا الي نبي نوصل له
يحل محل standard error →

$$Sp = \sqrt{\frac{Sp^2}{n_1} + \frac{Sp^2}{n_2}}$$

σ unknown

$\hat{p}_1 - \hat{p}_2$
$\hat{p}$
$\bar{x}$
$\bar{x}_1 - \bar{x}_2$

← Point estimate $\pm$ Reability of coefficient (standard error)

كل شي Z عدا ★

ضرب بعض يعطونا margin of error (Precission)

$Z_{1-\frac{\alpha}{2}}$

① σ Known
سواء كانت
normal $n<30$ (small)
أو
non-normal $n>30$ (Large)

② σ unknown
+
non-normal
$n>30$ (Large)

★ $t_{1-\frac{\alpha}{2}}$ → $df = n-1$

σ unknown
+
normal
$n<30$ (small)

لو كتاب دانيل يقول ممكن نحلها ب t بعد
حطيها ببالك احتياط Z!

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parameters.

لأن هذه الفترة upper & lower

تقدير بنقطة 1 – **Point Estimate:** A point estimate is single value used to estimate the corresponding population parameter. فترة ثقة

تقدير بفترة 2 – **Interval Estimate** (or **Confidence Interval**): An interval estimate consists of two numerical values defining a range of values that most likely includes the parameter being estimated with a specified degree of confidence. درجة ثقة

6.1: The Point Estimates of the Population Parameters:

يتم تقدير قيم Population المجهولة بالقيم التي تقابلها في الجدول.

	Population Parameters (مجهولة)	Point estimator (معلومة)
Mean	μ	$\overline{X}$
Variance	σ^2	S^2
Standard Deviation	σ	S
Proportion	P	$\hat{p}$
The Difference between Two Means	$\mu_1 - \mu_2$	$\overline{X_1} - \overline{X_2}$
The Difference between Two Proportion	$P_1 - P_2$	$\widehat{P_1} - \widehat{P_2}$

مهم نظري + حل الأسئلة

the true value of μ with a probability of $1-\alpha$.

« degree of confidence »

* $1-\alpha$ = is called the confidence coefficient (level)
* L = lower limit of the confidence interval
* U = upper limit of the confidence interval

الرمز مهم (ثيتا) (Theta)

7.1 Introduction:

Consider a population with some unknown parameter θ. We are interested in testing (confirming or denying) some conjectures about θ. For example, we might be interested in testing the conjecture that $\theta > \theta_o$, where θ_o is a given value.

definition (تعريف) مهم تذكر

- A hypothesis is a statement about one or more populations. فرضية / ادعاء
- A research hypothesis is the conjecture or supposition that motivates the research.
- A statistical hypothesis is a conjecture (or a statement) concerning the population which can be evaluated by appropriate statistical technique.
- For example, if θ is an unknown parameter of the population, we might be interested in testing the conjecture sating that $\theta \geq \theta_o$ against $\theta < \theta_o$ (for some specific value θ_o). الفرضيتين لكونهما ضد بعض
- We usually test the null hypothesis (H_o) against the alternative (or the research) hypothesis (H_1 or H_A) by choosing one of the following situations:

الفرضية البديلة — الفرضية الأساسية — الرموز مهمة

هناك 3 حالات لكتابة الفرضيات

(i)	$H_o: \theta = \theta_o$	against	$H_A: \theta \neq \theta_o$
(ii)	$H_o: \theta \geq \theta_o$	against	$H_A: \theta < \theta_o$
(iii)	$H_o: \theta \leq \theta_o$	against	$H_A: \theta > \theta_o$

H_o و H_A نختار إدراجها حسب معطيات السؤال

- Equality sign must appear in the null hypothesis.
- H_o is the null hypothesis and H_A is the alternative hypothesis. (H_o and H_A are complement of each other)
- The null hypothesis (H_o) is also called "the hypothesis of no difference". H_o عكس H_A — الفرضية الأساسية
- The alternative hypothesis (H_A) is also called the research hypothesis.

- There are 4 possible situations in testing a statistical hypothesis:

		Condition of Null Hypothesis H_o (Nature/reality)	
		H_o is true	H_o is false
Possible Action (Decision)	Accepting H_o	Correct Decision	Type II error (β)
	Rejecting H_o	Type I error (α)	Correct Decision

- There are two types of Errors:
  - Type I error = Rejecting H_o when H_o is true
    P(Type I error) = P(Rejecting Ho | Ho is true) = α
  - Type II error = Accepting Ho when Ho is false
    P(Type II error) = P(Accepting Ho | Ho is false) = β

- The level of significance of the test is the probability of rejecting true H_o:
  α = P(Rejecting H_o | H_o is true) = P(Type I error)

- There are 2 types of alternative hypothesis:
  - One-sided alternative hypothesis:
    - $H_O: \theta \geq \theta_o$ against $H_A: \theta < \theta_o$
    - $H_O: \theta \leq \theta_o$ against $H_A: \theta > \theta_o$
  - Two-sided alternative hypothesis:
    - $H_O: \theta = \theta_o$ against $H_A: \theta \neq \theta_o$

- We will use the terms "accepting" and "not rejecting" interchangeably. Also, we will use the terms "acceptance" and "nonrejection" interchangeably.
- We will use the terms "accept" and "fail to reject" interchangeably

The Procedure of Testing H_o (against H_A):

The test procedure for rejecting H_o (accepting H_A) or accepting H_o (rejecting H_A) involves the following steps:

Definition
مهم
نفكر !

3 Note: Using P- Value as a decision tool:

P-value is the smallest value of α for which we can reject the null hypothesis H_o.

Calculating P-value: