Determine the volume of a cylinder using Vernier Caliper

In 1631, a French instrument maker, Pierre Vernier, devised a way to improve the precision of length measurements. The **Vernier Caliper**, commonly called a Vernier, consists of a main engraved scale and a movable jaw with an engraved Vernier scale (figure 1). The span of the lower jaw is used to measure length and is particularly convenient for measuring the diameter of a cylindrical object. The span of the upper jaw is used to measure distances between two surfaces, such as the inside diameter of a hollow cylindrical object.

- Vernier caliper is used to measure internal dimensions, external dimensions, and depth of an object

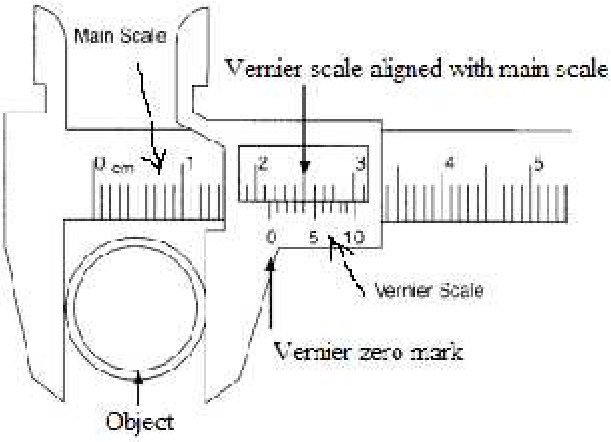

Figure 1. A Vernier Caliper

Zeroing

Before making a measurement, one should check the zero of the Vernier Caliper with the jaws completely closed. It is possible that through misuse the caliper is no longer zeroed and thus gives erroneous readings (systematic error). If this is the case, a zero correction should be made for each reading.

- In zeroing, if the Vernier zero lies to the right of the main-scale zero, measurements will be too large and the error is taken to be *positive*. In this case, the zero correction is made by subtracting the zero reading from the measurement reading.
- Similarly, if the error is *negative*, or the Vernier zero lies to the left of the main-scale zero, measurements will be too small, and the zero correction must be added to the measurement readings.

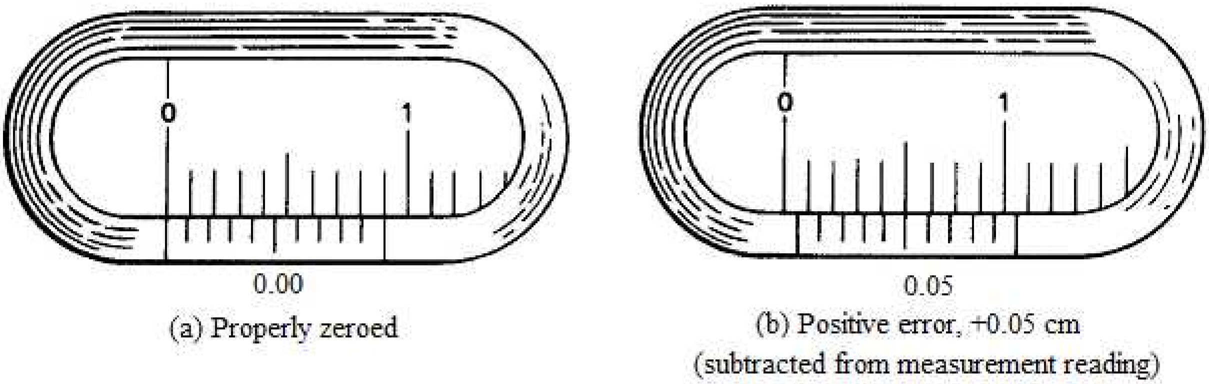

Figure 2 Zeroing and error. The zero of the Vernier Caliper is checked with the jaws closed. (a) Zero error. (b) Positive error, +0.05 cm.

How to read a Vernier Caliper

- The main scale is calibrated in centimeters with a millimeter least count, and the movable Vernier scale has 10 divisions that cover 9 divisions on the main scale. The function of the Vernier scale is to assist in the accurate reading of the fractional part of the scale division, thus increasing the precision.

The leftmost mark on the Vernier scale is the zero mark (lower scale for metric reading and upper scale for inches). The zero mark is often unlabeled.

- A measurement is made by closing the jaws on the object to be measured and reading where the zero mark on the Vernier scale falls on the main scale (figure 3). In figure 3, the first two significant figures are read directly from the main scale. The Vernier zero mark is past the 2-mm line after the 1-cm major division mark, so there is a reading of 1.2 cm (12 mm).
- The next significant figure is the fractional part of the smallest subdivision on the main scale. This is obtained by referring to the Vernier scale markings below the main scale.

If a Vernier mark coincides with a mark on the main scale, then the Vernier mark number is the fractional part of the main-scale division. In the figure, this is the third mark to the right of the Vernier zero, so the third significant figure is 3 (0.03 cm (0.3 mm)). Finally, since the 0.03-cm reading is known exactly, a zero is added as the doubtful figure for a reading of 1.230 cm or 12.30 mm.

- However, a mark on the Vernier scale may not always line up exactly with one on the main scale, and we say there is a change of "phase" between two successive Vernier markings. In that situation take the middle of the range. Thus a 5 would be put in the thousandth-of-a-centimeter digit (hundredth of a millimeter digit).

Example (calculating the least count, and reading the Vernier scale on a Vernier caliper.**)**

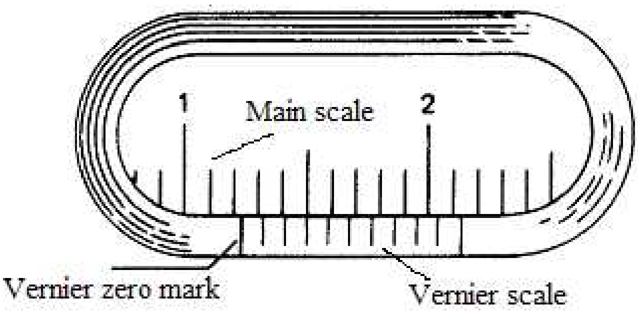

Figure 3. An example of calculating the least count, and reading the Vernier scale on a Vernier caliper.

$$Least\ count\,(L.C) = \frac{1mm}{10} = 0.1mm$$

Main scale reading = a = 12 mm

Vernier coincidence (aligned mark) = n = 3

Vernier scale reading = b = n x least count = 3x 0.1 = 0.3 mm

Total reading = a + b = 12.30 mm = 1.230 cm

Volume of a cylinder

$V = \circledR r^2 l$, where l is the length and r is the radius of the cylinder

$$Least\,count\,(L.C) = \frac{smallest\,main\,scale\,division}{total\,number\,of\,\,vernier\,scale\,division} =$$

Length of the cylinder:

Main scale reading a (cm)	Vernier coincidence n	Vernier scale reading b = n x least count (cm)	Total reading l = (a + b) (cm)

Length of the cylinder = l = (cm)

Diameter of the cylinder:

Main scale reading a (cm)	Vernier coincidence n	Vernier scale reading b = n x least count (cm)	Total reading d = (a + b) (cm)

Diameter of the cylinder = d = (cm)
Radius of the cylinder = r = (cm)

$$V = \circledR r^2 l$$

Volume in $V =$ cm^3

Volume in $V =$ m^3

Use: $\pm_r = \pm_l = \pm 0.05\,mm$ (reading error)

Standard deviation (error propagation)

$$\pm_V = V\left[\left(\frac{n}{x}\right)^2 \cdot \pm_x{}^2 + \left(\frac{m}{y}\right)^2 \cdot \pm_y{}^2\right]^{1/2} = V\left[\left(\frac{2}{r}\right)^2 \cdot \pm_r{}^2 + \left(\frac{1}{l}\right)^2 \cdot \pm_l{}^2\right]^{1/2}$$

$\therefore Volume = V \pm \pm_V =$