KINGDOOM OF SAUDI ARABIA		المملكة العربية السعودية
King Saud University	جامعة الملك سعود King Saud University	جامعة الملك سعود
Deanship of Common First Year	السنة الأولى المشتركة	عمادة السنة الأولى المشتركة
Basic Sciences Department		قسم العلوم الأساسية

Second Homework for
Introduction to Probability and Statistics (101 Stat)

(3 marks)

1) Give an example for each of the following:

a) A random experiment with finite space of elementary events.

b) A random experiment with infinite countable space of elementary events.

c) A random experiment with continuous space of elementary events.

(5 marks)

2) Classify each of the following as discrete or continuous.

a) The space of elementary events of the experiment of throwing a stone randomly in a well.

b) The space of elementary events of the experiment of tossing a coin **1000** times.

c) The random variable that recording the minimum appearance numbers by the experiment of rolling two dice for three times.

d) The random variable that recording the number of tails by the experiment of tossing a coin infinite times.

e) The random variable that measures the depth of the hole caused by a falling meteorites on the Earth's surface.

(8 marks)

3) Any of the following function is a probability mass function (explaining why) ?

a) $P(X=k)=\dfrac{1}{k} \quad ; k=1,2,3,4,5 .$

b) $P(X=q)=1/q \quad ; k=-2,1,2,3,4,5 .$

c) $P(X=j)=\dfrac{1}{9} \quad ; j=1,2,3,4,5,6,7,8,9 .$

d) $P(X=x)=-x \quad ; x\in[0,1] .$

(6 marks)

4) Any of the following function is a probability density function (explaining why) ?

a) $f(x)=\begin{cases} \dfrac{8}{3}x & for \quad x\in[-0.5,1] \\ 0 & \text{otherwise} \end{cases}$

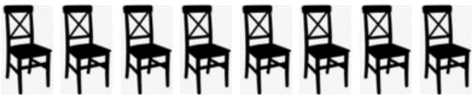
b) $f(x)=\begin{cases} 2x & for \quad x\in[0,1] \\ 0 & \text{otherwise} \end{cases}$

c) $f(x)=\begin{cases} x & for \quad x\in[0,1] \\ 0 & \text{otherwise} \end{cases}$

(18 marks)

5) Answer all following questions:

a) There are eight men who want to sit on eight chairs. How many ways can these men sitting on these chairs if:



i) The chairs have a straight side **?** **(2 marks)**

ii) The chairs have circle form **?** **(2 marks)**

b) We have **12** students in a semester, and we want to form a committee of **5** students. How many ways can we select these committees if:

i) We select student after another **?** **(2 marks)**

ii) We select all **5** students at the same time **?** **(2 marks)**

c) How many possible different hands of **7** cards can be selected (at the same time) from a standard deck of **52** cards **?** **(2 marks)**

d) If an automobile license plate must consist **2** Arabic letters followed by **4** single-digit numbers or **3** English letters followed by **5** single-digit numbers, how many different license plates are possible **?** **(2 marks)**

e) In a school, there are **16** teachers and **6** administrative staffs. Then, if a committee of **6** teachers and **3** administrative staffs is to be chosen. How many different possibilities are there **?** **(6 marks)**

(30 marks)

6) Let $[\Omega, \mathscr{A}, P]$ **be a probability space, and** A, B **and** $C \in \mathscr{A}$ **with:** $P(A \setminus B) = P(B \setminus A) = P(C \setminus A) = \mathbf{0.25}$ **and** $P(A \cap B) = P(A \cap C) = P(B \cap C) = \mathbf{0.25}$ **and** $P(A \cap B \cap C) = \mathbf{0.125}$ **. Then:**

a) Calculate the probabilities $P(A)$, $P(B)$, $P(C)$, $P(A \setminus C)$, $P(C \setminus B)$, $P(B \setminus C)$, $P(A|B)$ and $P(C \mid A \cap B)$. **(16 marks)**

b) Calculate the probabilities $P(\bar{A} \cup \bar{B} \cup \bar{C})$ and $P(A \cup B \cup C)$. **(4 marks)**

c) Are the events A, B and C statically independent **?** **(10 marks)**

(20 marks)

7) In a residential neighborhood there are four schools S_1, S_2, S_3 and S_4. If the number of students in this neighborhood is distributed equally to these schools. But the percentage of those who excellent in these schools are 6%, 5%, 2% and 7% respectively. If a student from this neighborhood selected at random, then:

a) Calculate the probability that the selected student is excellent. **(15 marks)**

b) If we find that, the selected student is not excellent, what is the probability that this student from the school S_3 **?** **(5 marks)**

(18 marks)

8) We select 4 balls randomly of a box contains 7 black, 4 green and 2 yellow balls. If all balls have the same chance at selecting. Then:

a) If we select the balls at the same time, and we consider A the event that all balls are yellow, then calculate $P(A)$. **(5 marks)**

b) If we select the balls one after another, and we consider B the event that the selected balls have the same colors, then calculate $P(B)$. **(5 marks)**

c) If we select the balls at the same time, What is the probability that the selected balls have thee colors **?** **(5 marks)**

c) What is the probability that the selected balls have four colors **?** **(3 marks)**

(25 marks)

9) Cars arrive successive at a gas station independently. If you know that the possibility that the car wants to refuel is 0.95, then:

a) What is the possibility that a car entered the station and do not want refueling **?** **(3 marks)**

b) If two cars enter the station, what is the probability that two cars will be refueling **?** **(4 marks)**

c) What is the probability that three cars will be not refueling **?** **(4 marks)**

d) What is the probability that of the next **10** cars, at least one want refueling **?** **(4 marks)**

d) Let ***X*** be a random variable recording the number of cars which has refueled in particular day. Now, if **10** cars arrive in this day. What is the probability that **7** cars had refueled **?** **(5 marks)**

e) If the time required to supply a car with fuel is exponential distributed with an average of **5** minutes, what is the probability that a car will come and be refueled within two minutes at most **?** **(5 marks)**

(40 marks)

10) Let *X* be a discrete random variable representing the maximum value of the two numbers on throwing two identical balanced dice for one time only. Then:

a) Find the possible values of the random variable ***X*** for the following cases: **(3 marks)**

b) Determine the probability mass function $P\left(X=\bullet\right)$. **(3 marks)**

c) Draw the graphical representation of the probability mass function $P\left(X=\bullet\right)$. **(3 marks)**

d) Determine the distribution function F_X . **(3 marks)**

e) Sketch the functions in part (**a**). **(3 marks)**

f) Calculate the mean and variance for the random variable ***X***. **(5 marks)**

g) Calculate the standard deviation of ***X***. **(8 marks)**

h) Calculate the standard deviation of the random variable $Y := 2X+5$. **(12 marks)**

(35 marks)

11) A probability density function of a continuous random variable *X* is given by:

$$f_X(x)=\begin{cases} ax+b & for\ -1<x<1 \\ 0 & for\ otherwise \end{cases}$$

And verifying the equation $P\left(X>0\right)=0.25$**. Then:**

a) Use the properties of f_X and the above probability to determine the values of ***a*** and ***b***. **(8 marks)**

b) Calculate $P\left(-1.2<X<0.5\right)$ **(7 marks)**

c) Derive the distribution function F_X . **(10 marks)**

f) Calculate the mean and standard deviation for the random variable ***X***. **(10 marks)**

(16 marks)

12) Assume that heights of men are normal distributed with mean equals to 175 cm and standard deviation equals to 15 cm. Then:

a) What is the probability that a man has height less than **155** **?** **(5 marks)**

b) What is the probability that a man has height between **177** and **188** **?** **(6 marks)**

c) What is the probability that a man has height greater than **195** **?** **(5 marks)**

(10 marks)

13) Let *X* be a discrete random variable with the following probability mass function:

$$P\left(X = k\right) = \frac{c}{5k} \quad ; k = 1, 2, 3, 4, 5$$

Then:

a) Determine the value of the constant ***c***. **(5 marks)**

b) Construct the tabular representation for the given random variable ***X***. **(5 marks)**

(4 marks)

14) Determine the value of ***k*** in the following probability distribution of a random variable X.

x	**0**	**1**	**2**	**3**	**4**	**5**
$P(X = k)$	**0.01**	**0.29**	***k***	**0.20**	**0.35**	**0.02**

End of Questions