إعداد المدرس: فراس قلعه جي

حل اختبار نواس الفتل

السؤال الأول:

1) يتضح من الشكل أن: $\theta_{max} = 0.8\,rad$

$\frac{1}{2} T_0 = 2 \Rightarrow T_0 = 4s \Rightarrow$

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}\, rad.s^{-1}$

نحسب $\bar{\varphi}$ من شروط البدء:

$$\left.\begin{array}{l} t=0 \\ \theta=\theta_{max} \end{array}\right\} \Rightarrow \bar{\theta} = \theta_{max}\cos(\omega_0 t + \bar{\varphi})$$

$\theta_{max} = \theta_{max}\cos\bar{\varphi} \Rightarrow$

$\cos\bar{\varphi} = 1 \Rightarrow \bar{\varphi} = 0\, rad$

$\Rightarrow \bar{\theta} = 0.8 \cos \frac{\pi}{2} t$

2) $T_0 = 2\pi\sqrt{\frac{I_0}{k}}$

$T_0' = 2\pi\sqrt{\frac{4I_0}{k}} = 2 \times 2\pi\sqrt{\frac{I_0}{k}}$

$T_0' = 2T_0 = 2 \times 2 = 4s$

3) $T_0 = 2\pi\sqrt{\frac{I_0}{k}} \qquad k = k'\frac{(2r)^4}{l}$

نلاحظ عند زيادة طول السلك إلى الضعف

$k^* = k'\frac{(2r)^4}{2l} = \frac{k}{2} \Rightarrow$

$T_0' = 2\pi\sqrt{\frac{I_0}{k^*}} = 2\pi\sqrt{\frac{I_0}{\frac{k}{2}}}$

$T_0' = 2\pi\sqrt{\frac{2I_0}{k}} = \sqrt{2} \times 2\pi\sqrt{\frac{I_0}{k}}$

$T_0' = \sqrt{2}\, T_0 = \sqrt{2} \times 0.5 = \frac{\sqrt{2}}{2}\, s$

4) $\Gamma_{\Delta} = -k\theta$

5) $E_k = E - E_p = \frac{1}{2}k\theta_{max}^2 - \frac{1}{2}k\theta^2$

$= \frac{1}{2}k\theta_{max}^2 - \frac{1}{2}k\left(\frac{\theta_{max}}{\sqrt{2}}\right)^2$

$= \frac{1}{2}k\theta_{max}^2 - \frac{1}{2}k\frac{\theta_{max}^2}{2}$

$= \frac{1}{2}k\theta_{max}^2 - \frac{1}{4}k\theta_{max}^2$

$= \frac{1}{4}k\theta_{max}^2 = \frac{1}{2} \times \frac{1}{2}k\theta_{max}^2$

$= \frac{1}{2}E$

من منتصف

السؤال الثاني: صفحة 22 من الكتاب

+ من منتصف صفحة 23 من الكتاب + صفحة 24

السؤال الثالث:

$E = E_p + E_k$

$\frac{1}{2}k\theta_{max}^2 = \frac{1}{2}k\theta^2 + \frac{1}{2}I_0\omega^2$

نشتق طرفي المعادلة:

$0 = \frac{1}{2}k\,2\theta\theta' + \frac{1}{2}I_0\,2\omega\omega'$

$0 = k\theta\omega + I_0\omega\alpha$

$0 = \omega(k\theta + I_0\alpha)$

بما أن $\omega \neq 0$

$k\theta + I_0\alpha = 0 \Rightarrow$

$k\theta + I_0(\theta)''_t = 0$

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$$(\theta)''_t = -\frac{k}{I_0}\bar{\theta} \quad ...(1)$$

وهي معادلة تفاضلية من المرتبة الثانية تقبل حلاً جيبياً من الشكل:

$$\bar{\theta} = \theta_{max}\cos(\omega_0 t + \bar{\varphi}) \quad ...(2)$$

للتأكد من أن المعادلة (2) حل للمعادلة (1) نشتق المعادلة (2) مرتين بالنسبة للزمن

$$(\theta)'_t = -\omega_0\theta_{max}\sin(\omega_0 t + \bar{\varphi})$$

$$(\theta)''_t = -\omega_0^2\theta_{max}\cos(\omega_0 t + \bar{\varphi})$$

$$(\theta)''_t = -\omega_0^2\bar{\theta} \quad ...(3)$$

بمساواة (1) و(3) نجد أن:

$$\omega_0 = \sqrt{\frac{k}{I_0}} > 0$$

لأن k، I_0 موجبة، ومنه

$$\omega_0 = \frac{2\pi}{T_0} = \sqrt{\frac{k}{I_0}} \Rightarrow T_0 = 2\pi\sqrt{\frac{I_0}{k}}$$

وحركة نواس الفتل حركة دورانية جيبية توافقية بسيطة.

السؤال الرابع:

$$K_1 = k'\frac{(2r)^4}{\ell_1} \qquad K_2 = k'\frac{(2r)^4}{\ell_2}$$

$$\Rightarrow \ell_1 = k'\frac{(2r)^4}{K_1} \qquad \Rightarrow \ell_2 = k'\frac{(2r)^4}{K_2}$$

ولكن

$$\ell_1 = 4\ell_2$$

$$\frac{\cancel{k'(2r)^4}}{K_1} = 4\,\frac{\cancel{k'(2r)^4}}{K_2}$$

2/

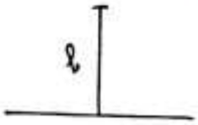
$$\frac{1}{K_1} = \frac{4}{K_2} \Rightarrow K_2 = 4K_1$$

ولكن

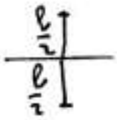
$$T_{0_2} = 2\pi\sqrt{\frac{I_0}{K_2}} = 2\pi\sqrt{\frac{I_0}{4K_1}}$$

$$T_{0_2} = \frac{1}{2}\times 2\pi\sqrt{\frac{I_0}{K_1}} = \frac{1}{2}T_{0_1}$$

السؤال الخامس: (1



$$K = k'\frac{(2r)^4}{\ell} \qquad T_0 = 2\pi\sqrt{\frac{I_0}{K}}$$



$$K^* = K_1 + K_2 = k'\frac{(2r)^4}{\frac{\ell}{2}} + k'\frac{(2r)^4}{\frac{\ell}{2}}$$

$$K^* = 2k'\frac{(2r)^4}{\ell} + 2k'\frac{(2r)^4}{\ell}$$

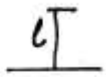
$$K^* = 2K + 2K = 4K$$

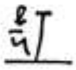
$$\Rightarrow T_0' = 2\pi\sqrt{\frac{I_0}{K^*}} = 2\pi\sqrt{\frac{I_0}{4K}}$$

$$T_0' = \frac{1}{2}\times 2\pi\sqrt{\frac{I_0}{K}} = \frac{1}{2}T_0$$

(2

$$K = k'\frac{(2r)^4}{\ell} \qquad T_0 = 2\pi\sqrt{\frac{I_0}{K}}$$





$$K^* = k'\frac{(2r)^4}{\frac{\ell}{4}} = 4k'\frac{(2r)^4}{\ell} = 4K$$

$$T_0' = 2\pi\sqrt{\frac{I_0}{K^*}} = 2\pi\sqrt{\frac{I_0}{4K}} = \frac{1}{2}\times 2\pi\sqrt{\frac{I_0}{K}}$$

$$T_0' = \frac{1}{2}T_0$$

3/

السؤال السادس:

المسألة الأولى:

(1

$$\bar{\theta} = \theta_{max} \cos(\omega_0 t + \bar{\varphi})$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{1} = 2\pi \ rad.s^{-1}$$

$$\left.\begin{matrix} t = 0 \\ \omega = 0 \end{matrix}\right\} \Rightarrow \theta = \theta_{max} = \frac{\pi}{2} \ rad$$

نحسب $\bar{\varphi}$ من شروط البدء:

$$\left.\begin{matrix} t = 0 \\ \theta = \theta_{max} \end{matrix}\right\} \Rightarrow \begin{matrix} \bar{\theta} = \theta_{max} \cos(\omega_0 t + \bar{\varphi}) \\ \theta_{max} = \theta_{max} \cos\bar{\varphi} \Rightarrow \end{matrix}$$

$$\cos\bar{\varphi} = 1 \Rightarrow \bar{\varphi} = 0 \ rad$$

$$\Rightarrow \bar{\theta} = \frac{\pi}{2} \cos(2\pi t)$$

(2

$$\omega = (\theta)'_t = -\omega_0 \theta_{max} \sin(\omega_0 t + \bar{\varphi})$$

$$\omega = -2\pi \times \frac{\pi}{2} \sin(2\pi t)$$

$$\omega = -10 \sin 2\pi t$$

لحظة المرور الأول بوضع التوازن

$$t = \frac{T_0}{4} = \frac{1}{4} s \Rightarrow$$

$$\omega = -10 \sin \frac{2\pi}{4} = -10 \sin \frac{\pi}{2}$$

$$\omega = -10 \ rad.s^{-1}$$

(3

$$\alpha = -\omega_0^2 \theta = -(2\pi)^2 \times -\frac{\pi}{4}$$

$$\alpha = +40 \times \frac{\pi}{4} = 10\pi \ rad.s^{-2}$$

(4 بدون وجود كتلتين

$$T_0 = 2\pi \sqrt{\frac{I_{0\,\text{ساق}}}{K}}$$

بوجود كتلتين

$$T_0' = 2\pi \sqrt{\frac{I_{0\,\text{ساق}} + I_{0/m_1} + I_{0/m_2}}{K}}$$

$$T_0' = 2\pi \sqrt{\frac{I_{0\,\text{ساق}} + m_1 r_1^2 + m_2 r_2^2}{K}}$$

$$T_0' = 2\pi \sqrt{\frac{I_{0\,\text{ساق}} + 2m_1 r_1^2}{K}}$$

$$\frac{T_0'}{T_0} = \frac{\cancel{2\pi} \sqrt{\frac{I_{0\,\text{ساق}} + 2m_1 r_1^2}{\cancel{K}}}}{\cancel{2\pi} \sqrt{\frac{I_{0\,\text{ساق}}}{\cancel{K}}}}$$

$$\frac{T_0'}{T_0} = \sqrt{\frac{I_{0\,\text{ساق}} + 2m_1 r_1^2}{I_{0\,\text{ساق}}}}$$

$$T_0' = T_0 \sqrt{1 + \frac{2m_1 r_1^2}{I_{0\,\text{ساق}}}} \qquad r = \frac{l}{2}$$

$$T_0' = T_0 \sqrt{1 + \frac{2 \times 20 \times 10^{-3} \times (\frac{1}{2})^2}{2 \times 10^{-3}}}$$

$$T_0' = 1 \sqrt{1 + 5}$$

$$T_0' = \sqrt{6} \ s$$

حساب K:

$$K = \omega_0^2 I_0 = \left(\frac{2\pi}{T_0}\right)^2 \times I_0$$

$$K = 40 \times 2 \times 10^{-3} = 8 \times 10^{-2} \ m.N.rad^{-1}$$

5)

$$E = \frac{1}{2}k\theta_{max}^2 = \frac{1}{2}\times 8\times 10^{-2}\left(\frac{\pi}{2}\right)^2$$

$$E = 0.1\ J$$

المسألة الثانية:

1)

$$\bar{\theta} = \theta_{max}\cos(\omega_0 t + \bar{\varphi})$$

$$\omega_0 = \sqrt{\frac{k}{I_0}} = \sqrt{\frac{8\times 10^{-2}}{2\times 10^{-3}}} = 2\pi\ rad.s^{-1}$$

$$\left.\begin{matrix} t=0 \\ \omega=0 \end{matrix}\right\} \Rightarrow \theta = \theta_{max} = \frac{\pi}{2}\ rad$$

نحسب $\bar{\varphi}$ من شروط البدء:

$$\left.\begin{matrix} t=0 \\ \theta=\theta_{max} \end{matrix}\right\} \Rightarrow \bar{\theta} = \theta_{max}\cos(\omega_0 t + \bar{\varphi})$$

$$\theta_{max} = \theta_{max}\cos\bar{\varphi} \Rightarrow$$

$$\cos\bar{\varphi} = 1 \Rightarrow \bar{\varphi} = 0\ rad$$

$$\Rightarrow \bar{\theta} = \frac{\pi}{2}\cos 2\pi t$$

2)

$$E_p = \frac{1}{2}k\theta^2 = \frac{1}{2}\times 8\times 10^{-2}\left(\frac{\pi}{6}\right)^2$$

$$= 4\times 10^{-2}\times\frac{10}{36} = \frac{1}{90}\ J$$

$$E = \frac{1}{2}k\theta_{max}^2 = \frac{1}{2}\times 8\times 10^{-2}\times\left(\frac{\pi}{2}\right)^2$$

$$E = \frac{1}{10}\ J$$

$$\Rightarrow E_K = E - E_p = \frac{1}{10} - \frac{1}{90}$$

$$E_K = \frac{8}{90}\ J$$

3)

$$I_{o/c} = \frac{1}{2}mr^2 \Rightarrow$$

$$m = \frac{2I_0}{r^2} = \frac{2\times 2\times 10^{-3}}{(0.2)^2}$$

$$m = 0.1\ kg$$

المسألة الثالثة:

1)

$$k = \omega_0^2 I_0 \quad \text{---}\ (1)$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi\ rad.s^{-1}$$

$$I_0 = I_{\Delta/m_1} + I_{\Delta/m_2}$$

حيث

$$= m_1 r_1^2 + m_2 r_2^2 = 2m_1 r_1^2 \qquad r_1 = \frac{\ell}{2}$$

$$= 2\times 100\times 10^{-3}\times(20\times 10^{-2})^2$$

$$= 0.2\times 400\times 10^{-4} = 8\times 10^{-3}\ kg.m^2$$

نعوض بـ (1):

$$k = (\pi)^2\times 8\times 10^{-3} = 8\times 10^{-2}\ m.N.rad^{-1}$$

2)

$$\bar{\theta} = \theta_{max}\cos(\omega_0 t + \bar{\varphi})$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi\ rad.s^{-1}$$

$$\left.\begin{matrix} t=0 \\ \omega=0 \end{matrix}\right\} \Rightarrow \theta = \theta_{max} = \frac{\pi}{3}\ rad$$

نحسب $\bar{\varphi}$ من شروط البدء:

$$\left.\begin{matrix} t=0 \\ \theta=\theta_{max} \end{matrix}\right\} \Rightarrow \bar{\theta} = \theta_{max}\cos(\omega_0 t + \bar{\varphi})$$

$$\theta_{max} = \theta_{max}\cos\bar{\varphi}$$

$$\cos\bar{\varphi}=1 \Rightarrow \bar{\varphi}=0\,rad$$

$$\Rightarrow \bar{\theta}=\frac{\pi}{3}\cos(\pi t)$$

3)

$$T_0=2\pi\sqrt{\frac{I_\Delta}{k}} \qquad k=k'\frac{(2r)^4}{\ell}$$

$$T_0'=2\pi\sqrt{\frac{I_\Delta}{k^*}}$$

$$k^*=k'\frac{(2r)^4}{\frac{\ell}{2}}=2k'\frac{(2r)^4}{\ell}=2k$$

$$\Rightarrow T_0'=2\pi\sqrt{\frac{I_\Delta}{2k}}=\frac{1}{\sqrt{2}}\times 2\pi\sqrt{\frac{I_\Delta}{k}}$$

$$T_0'=\frac{T_0}{\sqrt{2}}=\frac{2}{\sqrt{2}}\,s$$

المسألة الرابعة: 1)

$$T_0=2\pi\sqrt{\frac{I_\Delta}{k}}$$

$$I_{\Delta_{كلي}}=I_{\Delta_{قرص}}+I_{\Delta/m_1}+I_{\Delta/m_2}$$

$$=\frac{1}{2}M_1R^2+2m_1R^2$$

$$=\frac{1}{2}(0.2)(0.05)^2+2(20\times10^{-3})(0.05)^2$$

$$=0.01\times25\times10^{-4}+4\times10^{-2}\times25\times10^{-4}$$

$$=25\times10^{-6}+100\times10^{-6}$$

$$=125\times10^{-6}\ Kg\,m^2$$

5/

$$T_0=2\pi\sqrt{\frac{125\times10^{-6}}{5\times10^{-4}}}$$

$$T_0=\pi\ s$$

2)

$$T_0'=T_0-1.14=3.14-1.14$$

$$T_0'=2\,s$$

$$T_0'=2\pi\sqrt{\frac{I_{\Delta_{قرص}}+2m_1r'^2}{k}}$$

$$2=2\pi\sqrt{\frac{25\times10^{-6}+2\times20\times10^{-3}\,r'^2}{5\times10^{-4}}}$$

نربع الطرفين

$$4=40\,\frac{25\times10^{-6}+4\times10^{-2}\,r'^2}{5\times10^{-4}}$$

$$\frac{4\times5\times10^{-4}}{40}=25\times10^{-6}+4\times10^{-2}\,r'^2$$

$$5\times10^{-5}-2.5\times10^{-5}=4\times10^{-2}\,r'^2$$

$$2.5\times10^{-5}=4\times10^{-2}\,r'^2$$

$$r'^2=\frac{2.5\times10^{-5}}{4\times10^{-2}}=625\times10^{-6}$$

$$r'=25\times10^{-3}\,m$$

بالتالي البعد بين الكتلتين: $2r'=0.05\ m$

المسألة الخامسة:

1)

$$\bar{\theta} = \theta_{max}\cos(\omega_0 t + \bar{\varphi})$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}\ rad.s^{-1}$$

$$\left.\begin{array}{l} t=0 \\ \omega=0 \end{array}\right] \Rightarrow \theta = \theta_{max} = \frac{\pi}{3}\ rad$$

حسب شروط البدء:

$$\left.\begin{array}{l} t=0 \\ \theta=\theta_{max} \end{array}\right\} \Rightarrow \bar{\theta} = \theta_{max}\cos(\omega_0 t + \bar{\varphi})$$

$$\theta_{max} = \theta_{max}\cos\bar{\varphi}$$

$$\cos\bar{\varphi} = 1 \Rightarrow \bar{\varphi} = 0\ rad$$

$$\Rightarrow \bar{\theta} = \frac{\pi}{3}\cos\left(\frac{\pi}{2}t\right)$$

2)

$$T_0 = 2\pi\sqrt{\frac{I_{0\ جملة}}{k}}$$

$$T_0 = 2\pi\sqrt{\frac{I_{0/m_1} + I_{0/m_2}}{k}}$$

$$T_0 = 2\pi\sqrt{\frac{2m_1 r_1^2}{k}} \qquad r = \frac{l}{2}$$

$$T_0 = 2\pi\sqrt{\frac{2m_1\frac{l^2}{4}}{k}}$$

نربع الطرفين

$$T_0^2 = 40\,\frac{m_1\frac{l^2}{2}}{k} \Rightarrow$$

$$\frac{l^2}{2} = \frac{T_0^2 \times k}{40m} \Rightarrow l^2 = \frac{T_0^2 \times k}{20m}$$

$$l = T_0\sqrt{\frac{K}{20m}} = 4\sqrt{\frac{8\times10^{-4}}{20\times20\times10^{-3}}}$$

$$l = 4\sqrt{\frac{8}{4000}} = 4\sqrt{\frac{1}{500}}$$

$$l = \frac{4}{10\sqrt{5}} = \frac{2}{5\sqrt{5}}\ m$$

3)

$$\omega = -\omega_0\theta_{max}\sin(\omega_0 t + \bar{\varphi})$$

$$\omega = -\frac{\pi}{2}\times\frac{\pi}{3}\sin\left(\frac{\pi}{2}t\right)$$

$$\omega = -\frac{10}{6}\sin\frac{\pi}{2}t = -\frac{5}{3}\sin\frac{\pi}{2}t$$

لحظة المرور الثاني بوضع التوازن

$$t = \frac{3T_0}{4} = \frac{3\times4}{4} = 3s$$

$$\Rightarrow \omega = -\frac{5}{3}\sin\frac{3\pi}{2}$$

$$\omega = +\frac{5}{3}\ rad.s^{-1}$$