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0.5 Function and Their Graphs

Example 1 : For $f(x) = x^2 - 2x$, find and simplify :

(a) $f(4)$

(b) $f(4+h)$

(c) $f(4+h) - f(4)$

(d) $[f(4+h) - f(4)]/h$

Solution

(a) $f(4) = 4^2 - 2(4) = 16 - 8 = \mathbf{8}$

(b) $f(4+h) = (4+h)^2 - 2(4+h) = 16 + 8h + h^2 - 8 - 2h = \boldsymbol{h^2 + 6h + 8}$

(c) $f(4+h) - f(4) = h^2 + 6h + 8 - 8 = \boldsymbol{h^2 + 6h}$

(d) $\frac{[f(4+h)-f(4)]}{h} = \frac{\boldsymbol{h^2+6h}}{h} = \frac{h(h+6)}{h} = \boldsymbol{h+6}$

- **Natural Domain of Functions :**

المجال	الدالة
R (all real numbers)	كثيرة الحدود
R - {اصفار المقام}	الكسرية
ماتحت الجذر ≥ 0	الجذرية
ماتحت الجذر > 0	كسرية مقامها جذر

Example 2 : Find the natural domains for :

(a) $f(x) = \frac{1}{x-3}$

Solution

$$R - \{3\}$$

(b) g(t) = $\sqrt{9-t^2}$

Solution

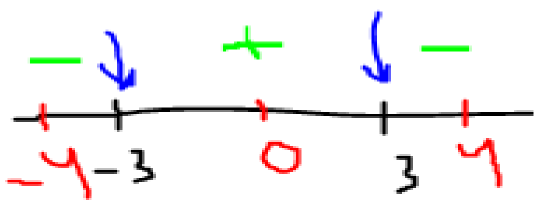
$$9 - t^2 \geq 0$$

a= -1 , b= 0 , c=9

$$x_{1,2} = \frac{\pm\sqrt{36}}{-2}$$

$$x_1 = \frac{6}{-2} = -\mathbf{3}$$

$$x_2 = \frac{-6}{-2} = \mathbf{3}$$



$\therefore$ ***The solutoin set = Domain*** : $[-3, 3]$

(c) h(w) = $1/\sqrt{9-w^2}$

Solution

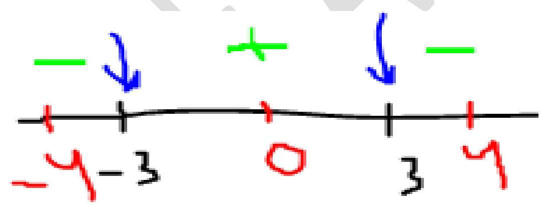
$$9 - w^2 > 0$$

a= -1 , b= 0 , c=9

$$x_{1,2} = \frac{\pm\sqrt{36}}{-2}$$

$$x_1 = \frac{6}{-2} = -\mathbf{3}$$

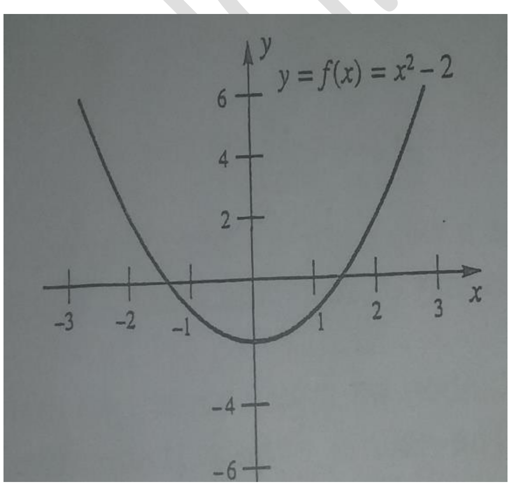
$$x_2 = \frac{-6}{-2} = \mathbf{3}$$



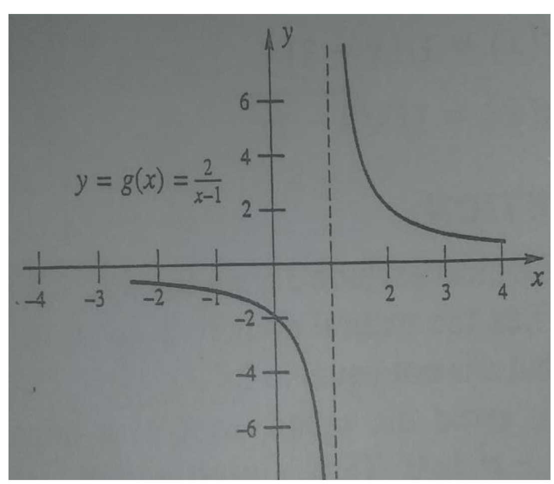
$$\therefore \textit{The solutoin set} = \textit{Domain} : (-3, 3)$$

Example 4 : Sketch the graphs of

(a) $f(x) = x^2 - 2$



(b) $g(x) = 2/(x-1)$



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- **Even and Odd Function:**

خطوات التأكد من ان الدالة زوجية او فردية او غير ذلك :

1- $f(-x) = f(x)$ $\therefore even$

2- $f(-x) = -f(x)$ $\therefore odd$

3- $f(-x) \neq f(x) \neq -f(x)$ $\therefore neither$

➢ **Examples :**

(a) $f(x) = x^2 - 2$

Solution

$$f(-x) = (-x)^2 - 2 = \boldsymbol{x^2 - 2} = f(x) \qquad \therefore even$$

(b) $f(x) = 3x^6 - 2x^4 + 11x^2 - 5$

Solution

$$f(-x) = 3(-x^6) - 2(-x^4) + 11(-x^2) - 5$$

$$= \boldsymbol{3x^6 - 2x^4 + 11x^2 - 5} = f(x) \qquad \therefore even$$

(c) $f(x) = \frac{x^2}{1+x^4}$

Solution

$$f(-x) = \frac{(-x)^2}{1+(-x)^4} = \frac{\boldsymbol{x^2}}{\boldsymbol{1+x^4}} = f(x) \qquad \therefore even$$

(d) $f(x) = \frac{x^3-2x}{3x}$

Solution

$$f(x) = \frac{(-x)^3 - 2(-x)}{3(-x)} = \frac{-x^3 + 2x}{-3x} = \frac{-(x^3 - 2x)}{-3x}$$

$$= \frac{x^3 - 2x}{3x} = f(x) \qquad \therefore \textit{even}$$

(e) $f(x) = x^3 - 2x$

Solution

$$f(-x) = (-x^3) - 2(-x) = -x^3 + 2x \neq f(x) \qquad \therefore \textit{not even}$$
$$f(-x) = -x^3 + 2x = -(x^3 - 2x) = -f(x) \qquad \therefore \textit{odd}$$

(f) $f(x) = \frac{2}{x-1}$

Solution

$$f(-x) = \frac{2}{-x-1} \neq f(x) \neq -f(x) \qquad \therefore \textit{neither (not even and not odd)}$$

Example 5 : Is $f(x) = \frac{x^3+3x}{x^4-3x^2+4}$ even, odd or neither ?

Solution

$$f(-x) = \frac{(-x)^3 + 3(-x)}{(-x^4) - 3(-x)^2 + 4} = \frac{-x^3 - 3x}{x^4 - 3x^2 + 4} = \frac{-(x^3 + 3x)}{x^4 - 3x^2 + 4}$$
$$= -\frac{x^3+3x}{x^4-3x^2+4} = -f(x) \qquad \therefore \textit{odd}$$