مدونة المناهج السعودية

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الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

Complex Numbers

DEFINITION 1 **Complex Number**

A **complex number** is a number of the form

$$a + bi \qquad \textbf{Standard Form}$$

where a and b are real numbers and i is called the **imaginary unit.**

Some examples of complex numbers are

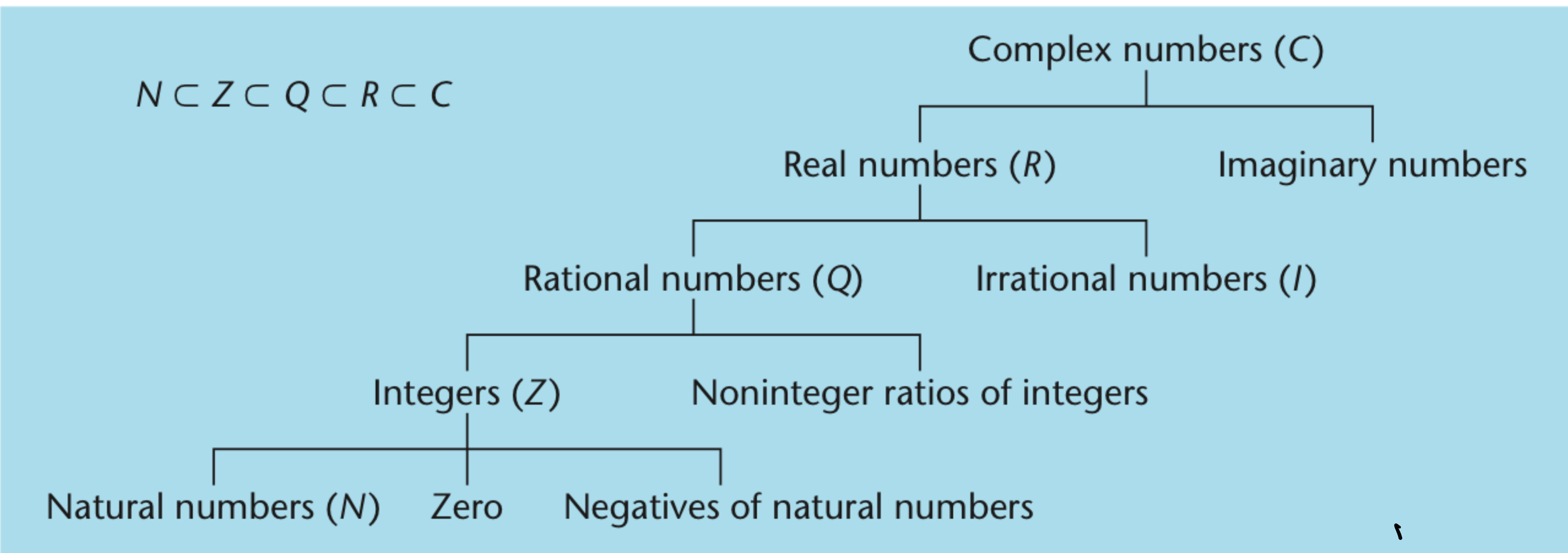
$$3 - 2i \qquad \tfrac{1}{2} + 5i \qquad 2 - \tfrac{1}{3}i$$
$$0 + 3i \qquad 5 + 0i \qquad 0 + 0i$$

The notation $3 - 2i$ is shorthand for $3 + (-2)i$.

DEFINITION 2 **Special Terms**

i		**Imaginary Unit**
$a + bi$	a and b real numbers	**Complex Number**
$a + bi$	$b \neq 0$	**Imaginary Number**
$0 + bi = bi$	$b \neq 0$	**Pure Imaginary Number**
bi		**Imaginary Part of $a + bi$**
$a + 0i = a$		**Real Number**
a		**Real Part of $a + bi$**
$0 = 0 + 0i$		**Zero**
$a - bi$		**Conjugate of $a + bi$**

The relationship of the complex number system to the other number systems:



Example1:

Identify the real part, the imaginary part, and the conjugate of each of the following numbers:

(A) $3 - 2i$ (B) $2 + 5i$ (C) $7i$ (D) 6

Real Part	**Imaginary part**	**Conjugate**	ملاحظات
3	$-2i$	$3+2i$	يأخذ الجزء التخيلي بإشارته
2	$5i$	$2-5i$	
0	$7i$	$-7i$	العدد تخيلي إذن يكون له مرافق
6	0	6	لأن العدد حقيقي و المرافق يكون في الجزء التخيلي

Operations with Complex Number

DEFINITION 3 Equality and Basic Operations

1. **Equality:** $a + bi = c + di$ if and only if $a = c$ and $b = d$
2. **Addition:** $(a + bi) + (c + di) = (a + c) + (b + d)i$
3. **Multiplication:** $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Example 2: **Carry out each operation and express the answer in stand form**

(A) $(2-3i) + (6+2i)$ (B) $(-5+4i) + (0+0i)$

(C) $(7-3i) - (6+2i)$ (D) $(-2+7i) + (2-7i)$

Solution

(A) $(2-3i) + (6+2i) = 2-3i+6+2i$

$= (2+6) + (-3+2)i$

$= 8 - i$

(B) $(-5+4i)+(0+0i) = -5+4i+0+0i$

$= -5+4i$

(C) $(7-3i)-(6+2i) = 7-3i-6-2i$

$= (7-6)+(-3-2)i$

$= 1-5i$

(D) $(-2+7i)+(2-7i) = -2+7i+2-7i = 0$

Example 3: **Carry out each operation and express the answer in standard form**

(A) $(2-3i)(6+2i)$ (B) $1(3-5i)$

(C) $i(1+i)$ (D) $(3+4i)(3-4i)$.

Solution:

(A) $(2-3i)(6+2i) = 12+4i-18i-6i^2$

$= 12-14i-6(-1)$

$= 12-14i+6$

$= 18-14i$

(B) $1(3-5i) = 3-5i$

فقط تعديل للشكل

(C) $i(1+i) = i+i^2 = i-1 = -1+i$

(D) $(3+4i)(3-4i) = 9 \quad 12i - 12i - 16i^2$

$= 9 - 16(-1)$

› **THEOREM 1** Product of a Complex Number and Its Conjugate

$$(a + bi)(a - bi) = a^2 + b^2$$ A real number

عدد مرافقه

معنى النظريه أنة عند الضرب في العدد ومرافقه نستطيع مباشره ان نربع a و b ونجمعهم دون الحاجة لتطبيق خطوات الضرب

For example : $(3+4i)(3-4i) = 3^2 + 4^2 = 9+16 = 25$ real!

Remarks

For any complex number $a + bi$,

$$1(a + bi) = (a + bi)1 = a + bi$$

$\frac{1}{a + bi}$ is the **reciprocal** or multiplicative inverse المعكوس الضربي of $a + bi$ $a + bi \neq 0$

Example4: Reciprocals and Quotients

Write each expression in standard form:

(A) $\frac{1}{2 + 3i}$ (B) $\frac{7 - 3i}{1 + i}$

كي نكتب المعكوس في الصورة القياسية للأعداد المركبة لابد أن نضرب البسط والمقام في مرافق المقام

Solution:

$$\frac{1}{2 + 3i} = \frac{1}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} \boxed{= \frac{2 - 3i}{4 - 9i^2} = \frac{2 - 3i}{4 + 9}}$$

$$= \frac{2 - 3i}{13} = \frac{2}{13} - \frac{3}{13}i$$

CHECK

$$(2 + 3i)\left(\frac{2}{13} - \frac{3}{13}i\right) = \frac{4}{13} - \frac{6}{13}i + \frac{6}{13}i - \frac{9}{13}i^2$$

$$= \frac{4}{13} + \frac{9}{13} = 1$$

(B) $\dfrac{7-3i}{1+i} = \dfrac{7-3i}{1+i}\cdot\dfrac{1-i}{1-i} = \boxed{\dfrac{7-7i-3i+3i^2}{1-i^2}}$

$$= \frac{4-10i}{2} = 2-5i$$

CHECK $(1+i)(2-5i) = 2-5i+2i-5i^2 = 7-3i$

Natural number powers of i take on particularly simple forms:

i | $i^5 = i^4\cdot i = (1)i = i$
$i^2 = -1$ | $i^6 = i^4\cdot i^2 = 1(-1) = -1$
$i^3 = i^2\cdot i = (-1)i = -i$ | $i^7 = i^4\cdot i^3 = 1(-i) = -i$
$i^4 = i^2\cdot i^2 = (-1)(-1) = 1$ | $i^8 = i^4\cdot i^4 = 1\cdot 1 = 1$

نلاحظ بان بعد الاس ٤
تتكرر النتائج وهذا يعني
أن قوى i تكون دوريه
بعد الأس ٤

Example 5: Evaluate each the following:

طريقة الحل : نقسم الاس
على ٤ ونأخذ الباقي

(A) $i^{17} = i^{1} = i$ beacaus $17 = 4\times4+1$

$i^{24} = i^{0} = 1$ $24 = 4\times6+0$

$i^{38} = i^{2} = -1$ $38 = 4\times9+2$

$i^{47} = i^{3} = -i$ $47 = 4\times11+3$

Relating Complex Numbers and Radicals

> **DEFINITION 4** Principal Square Root of a Negative Real Number

The **principal square root of a negative real number,** denoted by $\sqrt{-a}$, where a is positive, is defined by

$$\sqrt{-a} = i\sqrt{a} \qquad \sqrt{-3} = i\sqrt{3} \qquad \sqrt{-9} = i\sqrt{9} = 3i$$

The other square root of $-a$, $a > 0$, is $-\sqrt{-a} = -i\sqrt{a}$.

Complex Numbers and Radicals

Write in standard form:

(A) $\sqrt{-4}$ (B) $4 + \sqrt{-5}$ (C) $\dfrac{-3 - \sqrt{-5}}{2}$ (D) $\dfrac{1}{1 - \sqrt{-9}}$

SOLUTIONS

(A) $\sqrt{-4} = i\sqrt{4} = 2i$ (B) $4 + \sqrt{-5} = 4 + i\sqrt{5}$

(C) $\dfrac{-3 - \sqrt{-5}}{2} = \dfrac{-3 - i\sqrt{5}}{2} = -\dfrac{3}{2} - \dfrac{\sqrt{5}}{2}i$

(D) $\dfrac{1}{1 - \sqrt{-9}} = \dfrac{1}{1 - 3i} = \dfrac{1 \cdot (1 + 3i)}{(1 - 3i) \cdot (1 + 3i)}$

$$= \frac{1 + 3i}{1 - 9i^2} = \frac{1 + 3i}{10} = \frac{1}{10} + \frac{3}{10}i$$

› Solving Equations Involving Complex Numbers

Equations Involving Complex Numbers

(A) Solve for real numbers x and y:

$(3x + 2) + (2y - 4)i = -4 + 6i$

(B) Solve for complex number z:

$(3 + 2i)z - 3 + 6i = 8 - 4i$

SOLUTIONS

(A) Equate the real and imaginary parts of each side of the equation to form two equations:

Real Parts	**Imaginary Parts**
$3x + 2 = -4$	$2y - 4 = 6$
$3x = -6$	$2y = 10$
$x = -2$	$y = 5$

(B) $(3 + 2i)z - 3 + 6i = 8 - 4i$ — Add $3 - 6i$ to both sides.

$(3 + 2i)z = 11 - 10i$ — Divide both sides by $3 + 2i$.

$z = \dfrac{11 - 10i}{3 + 2i}$ — Multiply numerator and denominator by $3 - 2i$.

$= \dfrac{(11 - 10i)(3 - 2i)}{(3 + 2i)(3 - 2i)}$ — Simplify.

$= \dfrac{13 - 52i}{13}$

$= 1 - 4i$

ملاحظة:
الأسئلة (هاي
لايت اصفر)
متعلقة بدرس
الأعداد المركبه

Try to solve it

لمزيد من الملومات:

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