Testing for the two populations Proportions

If we have two independent samples of size and with proportions and respectively. Thus, we will use the following steps:

1-data needed: x_1 , $\hat{p}_1 = \frac{a_1}{n_1}$ and x_2 , $\hat{p}_2 = \frac{a_2}{n_2}$

2- the hypothesis: $H_0: P_1 = P_2 \longrightarrow P_1 - P_2 = 0$

$$H_1: \begin{cases} P_1 < P_2 \longrightarrow P_1 - P_2 < 0 \\ P_1 > P_2 \longrightarrow P_1 - P_2 > 0 \\ P_1 \neq P_2 \longrightarrow P_1 - P_2 \neq 0 \end{cases}$$

3- the statistic:

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where} \quad \hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_1}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$$

4- Determining the rejection of H_0, that is:

i)if $H_1: P_1 > P_2 \longrightarrow P_1 - P_2 > 0$,reject H_0 if $Z > Z_{1-\alpha}$

ii) if $H_1: P_1 < P_2 \longrightarrow P_1 - P_2$,reject H_0 if $Z < Z_{\alpha}$

iii) if $H_1: P_1 \neq P_2 \longrightarrow P_1 - P_2 \neq 0$, reject H_0 if $Z > Z_{1-\frac{\alpha}{z}}$ or $Z < Z_{\frac{\alpha}{z}}$

Ex (9):

Two machine A and B, a random sample of size 300 units from machine A with defective proportion 8% and another sample of size 200 units from machine B with defective proportion 4%. The manager think that the defective proportion from machine A is differ from the defective proportion from machine B, is he right?. use $\alpha = 0.05$

Solu.

1-data needed: $n_1 = 300$, $x_1 = 0.08$ and $n_2 = 200$, $x_2 = 0.04, \alpha = 0.05$

2- the hypothesis: $H_0 : P_1 = P_2 \longrightarrow P_1 - P_2 = 0$

$H_1 : P_1 \neq P_2 \longrightarrow P_1 - P_2 \neq 0$

3- the statistic:

$$Z = \frac{0.08 - 0.04}{\sqrt{0.064(1-0.064)\left(\frac{1}{300} + \frac{1}{200}\right)}} = 0.895$$

where $\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_1}{n_1 + n_2} = \frac{300(0.08)+200(0.04)}{500} = 0.064$

4- reject H_0 if $Z < Z_{\frac{\alpha}{2}} = Z_{0.025} = -1.96$ or $Z \rangle Z_{1-\frac{\alpha}{z}} = 1.96$

Thus , we accept H_0 and reject H_1 that says there is a difference between the defective proportions from machines A and B.