King Saud University
Department of Mathematics
Second Semester 1435-1436 H

MATH 244 (Linear Algebra)
First Midterm (Make-up) Exam
Duration: 105 Minutes

Student's Name	Student's ID	Group No.	Lecturer's Name

Question No.	I	II	III	IV	V	Total
Mark						

[**I**] Determine whether the following is **True** or **False**. [9 *Points*]

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(1) If A is an invertible matrix, then AA^T is invertible. ( )

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(2) If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -2 \\ 0 & 2 & 1 \end{bmatrix}$, then $\det B = \det A$. ( )

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(3) The following equations form a linear system

$$\begin{aligned} x + 3y^2 &= 1 \\ \sin x + y &= 0 \end{aligned}$$

( )

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(4) If the characteristic polynomial of a matrix A is $P(\lambda) = \lambda^2 + 1$, then A is invertible. ( )

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(5) Any set containing three vectors from $\mathbb{R}^3$ is a basis for $\mathbb{R}^3$. ( )

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(6) If $A = \begin{bmatrix} 2 & 0 \\ 3 & -1 \end{bmatrix}$, then the eigenvalues of A^4 are 16 and 4.

( )

OVER

(7) $W = \{(x, y) \in \mathbb{R}^2, x^2 = y^2\}$ is a subspace of $\mathbb{R}^2$ ( )

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(8) The vector $u = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is a unit vector. ( )

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(9) The inverse of an invertible upper triangular matrix is upper triangular. ( )

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(10) If $(v)_S = (1, -1)$ and $S = \{(5, 3), (2, 1)\}$, then $v = (3, 2)$. ( )

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(11) If $S = \{(1, 4), (2, 1)\}$ and $T = \{(1, 4), (2, 1), (3, 5)\}$, then Span($S$) =span($T$). ( )

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(12) If $m_1 \neq m_2$ in the system $\begin{cases} -m_1x_1 + x_2 = b_1 \\ -m_2x_1 + x_2 = b_2 \end{cases}$, then the system has a unique solution. ( )

OVER

[II] Choose the correct answer. [5 *Points*]

(1) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation, such that $T\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $T\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $T\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

Find $T\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$.

(a) $T\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$ **(b)** $T\begin{bmatrix} 5 \\ 7 \\ 7 \end{bmatrix}$ **(c)** $T\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$ **(d)** None of the previous

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(2) If W =span$\{(1,1,1),(2,2,2)\}$, then $\dim W$ is

(a) 0 **(b)** 2 **(c)** 1 **(d)** None of the previous

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(3) The values of c, if any, for which the matrix $A = \begin{bmatrix} c & -c & c \\ 1 & c & 1 \\ 0 & 0 & c \end{bmatrix}$ is invertible are

(a) $c \neq 0, 1$ **(b)** $c \neq 0, -1$ **(c)** $c = 0, -1$ **(d)** None of the previous

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(4) If A and B are 4×4 matrices, then $\det(3A + 3AB) =$

(a) $3^4(\det A + \det A \cdot \det B)$ **(b)** $3^4 \det A \cdot (1 + \det B)$ **(c)** $3^4 \det A \cdot \det(I + B)$ **(d)** None of the previous

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(5) If $T_1(x,y) = (x - y, x + y)$ and $T_2(x,y) = (4x, 3x + 2y)$, then the standard matrix for $T_2 \circ T_1$ is

(a) $\begin{bmatrix} 1 & -2 \\ 7 & 2 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & -4 \\ 5 & -1 \end{bmatrix}$ **(c)** $\begin{bmatrix} 4 & 5 \\ -4 & -1 \end{bmatrix}$ **(d)** None of the previous

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(6) If T is the rotation about the origin, through an angle $\theta = 60°$, then

(a) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **(b)** $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **(c)** $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$ **(d)** None of the previous

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(7) For $a = 2$, the system $\begin{cases} 2x + a^2 y = 1 \\ X + 2y = 1 \end{cases}$ has

(a) no solution **(b)** one solution **(c)** infinitely many solutions **(d)** None of the previous

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(8) The number of parameters in the general solution of $A\mathbf{x} = \mathbf{0}$, if $A = [a_{ij}]_{5\times 7}$ is of rank 3 is

(a) 3 **(b)** 4 **(c)** 2 **(d)** None of the previous

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(9) Given $v_1 = (-1, 2, 1)$ and $v_2 = (0, 4, -5)$, then $2v_1 \cdot v_2$ is

(a) $(0, 16, -10)$ **(b)** -4 **(c)** 6 **(d)** None of the previous

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OVER

[III]

(a) Show that $\lambda = 2$ is an eigenvalue of $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

(b) Compute the eigenvalues of $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

OVER

[IV] Let $S = \{v_1, v_2, v_3\}$, where $v_1 = (1, -3, 1, 1)$, $v_2 = (2, -1, 1, 1)$, $v_3 = (4, -7, 3, 3)$.

(i) Prove that S is not a basis for $\mathbb{R}^4$;

(ii) Find a basis B for $\text{span}\{v_1, v_2, v_3\}$ that contains only vectors from S;

(iii) Express the vector of S which is not in B as a linear combination of vectors from B.

[V]

(a) Find the standard matrix for the composed transformation in $\mathbb{R}^3$ given by a reflection about the xy-plane, followed by a reflection about the xz-plane, followed by an orthogonal projection on the yz-plane.

(b) Determine whether the matrix operator $T : \mathbb{R}^3 \to \mathbb{R}^3$, defined by

$$\begin{cases} w_1 = & x_1 & -3x_2 & +4x_3 \\ w_2 = & -x_1 & +x_2 & +x_3 \\ w_3 = & & -2x_2 & +5x_3 \end{cases}$$

is one-to-one. If so, find the standard matrix for the inverse operator and find $T^{-1}(w_1, w_2, w_3)$.

(3) Compute $\text{rank}([T])$.

Good Luck