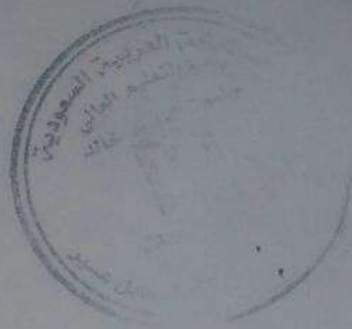


(ii) Let $E = \{e_1, e_2, e_3\} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ be the usual basis of Euclidean space \mathbb{R}^3 . Show that E is orthonormal set of \mathbb{R}^3 . ✓

(iii) Consider the matrix $A = \begin{bmatrix} 4 & 3 \\ 0 & -2 \end{bmatrix}$ find the eigenvalues and eigenvectors of A . ✓

(15 Marks)

GOOD LUCK



اسم ورمز المقرر: ٣٤٣ رياض الجبر الخطي (٢)

المستوى الخامس
القسم: الرياضيات
الزمن: ساعتان



المملكة العربية السعودية
وزارة التعليم العالي
جامعة الملك خالد
كلية العلوم والآداب بحائل

الاختبار النهائي للفصل الدراسي الأول للعام الجامعي ١٤٣٥ / ١٤٣٦ هـ

Answer the following questions:

Q1:

(i) Consider the set \mathbb{R}^n . Show that \mathbb{R}^n is a vector space under the sum of vectors and scalar multiplication of vectors defined as follows:

$$u+v = (a_1, a_2, \dots, a_n) + (b_1, b_2, \dots, b_n) = (a_1+b_1, a_2+b_2, \dots, a_n+b_n)$$

$$ku = k(a_1, a_2, \dots, a_n) = (ka_1, ka_2, \dots, ka_n), k \text{ scalar.}$$

(ii) Let $V = M_{n,n}$ the vector space of all matrices of the size $n \times n$. Let W be a set of all symmetric matrices subset of V . Show that W is a subspace of V .

(iii) Consider the vectors $e_1=(1,1,1), e_2=(1,1,0), e_3=(1,0,0)$ show that these vectors are basis on \mathbb{R}^3 and find the dimension of \mathbb{R}^3 .

(iv) Consider the two vectors v_1, v_2 one of them is multiple of the other, verify that the two vectors are linear dependent or independent.

$$v_1 = [3 \ 0 \ 1] \in \mathbb{R}^3$$
$$v_2 = [3 \ 0 \ 9] \in \mathbb{R}^3$$
$$v_1 = 3v_2$$

(20 Marks)

Q2:

(i) Let A be any real $m \times n$ matrix and the transformation

$$F_A: \mathbb{R}^n \rightarrow \mathbb{R}^m \text{ defined by } F_A(u) = Au$$

Show that F_A is a linear transformation.

(ii) Let $F: V \rightarrow U$ and $G: U \rightarrow W$ be linear transformation. Show that $G \circ F$ is also linear transformation.

(iii) Let F be the linear operator on \mathbb{R}^2 defined by $F(x,y) = (2x-y, 2x+y)$. Show that F is invertible and find the inverse F^{-1} .

(15 Marks)

Q3:

(i) Consider the inner product space V . Show that

$$\langle u, v+w \rangle = \langle u, v \rangle + \langle u, w \rangle$$

Continue