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1.5 Infinite Limits

Example 2 : Prove that $\lim_{x\to\infty} \frac{x}{1+x^2} = 0$

Solution

$$\lim_{x\to\infty} \frac{x}{1+x^2} = \lim_{x\to\infty} \frac{\frac{x}{x^2}}{\frac{1}{x^2}+\frac{x^2}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{x^2}+1} = \frac{\frac{1}{\infty}}{\frac{1}{\infty^2}+1}$$

$$= \frac{\frac{1}{\infty}}{\frac{1}{\infty}+1} = \frac{0}{0+1} = \frac{0}{1} = \mathbf{0}$$

Example 3 : Find $\lim_{x\to\infty} \frac{2x^3}{1+x^3}$

Solution

$$\lim_{x\to\infty} \frac{2x^3}{1+x^3} = \lim_{x\to\infty} \frac{\frac{2x^3}{x^3}}{\frac{1}{x^3}+\frac{x^3}{x^3}} = \lim_{x\to\infty} \frac{2}{\frac{1}{x^3}+1} = \frac{2}{\frac{1}{\infty}+1} = \frac{2}{0+1} = \frac{2}{1} = \mathbf{2}$$

Example 5 : Find :-

(a) $\lim_{x\to1^-} \frac{1}{(x-1)^2}$

(b) $\lim_{x\to1^+} \frac{1}{(x-1)^2}$

Solution

(a) $\lim_{x\to1^-} \frac{1}{(x-1)^2} = \frac{1}{(1-1)^2} = \frac{1}{(-0)^2} = \frac{1}{0} = \infty$

(b) $\lim_{x\to1^+} \frac{1}{(x-1)^2} = \frac{1}{(1-1)^2} = \frac{1}{(+0)^2} = \frac{1}{0} = \infty$

Example 6 : Find $\lim_{x\to2^+} \frac{x+1}{x^2-5x+6}$

Solution

$$\lim_{x\to2^+} \frac{x+1}{x^2-5x+6} = \lim_{x\to2^+} \frac{x+1}{(x-3)(x-2)} = \frac{2+1}{(2-3)(2-2)} = \frac{3}{(-1)(+0)}$$
$$= \frac{3}{-0} = -\frac{3}{0} = -\infty$$

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- **Asymptotes خطوط التقارب :**

- لإيجاد خط التقارب الراسي نوجد النهاية عند صفر المقام من الناحية اليمنى واليسرى.
- لإيجاد خط التقارب الافقي نوجد النهاية عند $\pm\infty$

Example 7 : Find the vertical and horizontal asymptotes of the graph of $y = f(x)$ if $f(x) = \frac{2x}{x-1}$

Solution

Vertical asymptote:

$$\lim_{x\to 1^+} \frac{2x}{x-1} = \frac{2}{+0} = \infty \qquad and \qquad \lim_{x\to 1^-} \frac{2x}{x-1} = \frac{2}{-0} = -\infty$$

$$\therefore f(x) \text{ has vertical asympotete at } x = 1$$

Horizontal asymptote:

$$\lim_{x\to \infty} \frac{2x}{x-1} = \frac{2}{1} = 2 \qquad and \qquad \lim_{x\to -\infty} \frac{2x}{x-1} = \frac{2}{1} = 2$$

$$\therefore f(x) \text{ has horizontal asympotete at } y = 2$$