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1.3 Limit Theorems

Example 1 : Find $\lim_{x\to 3} 2x^4$

Solution

$$\lim_{x\to 3} 2x^4 = 2(3^4) = 2(81) = \mathbf{162}$$

Example 2 : Find $\lim_{x\to 4}(3x^2 - 2x)$

Solution

$$\lim_{x\to 4}(3x^2 - 2x) = 3(16) - 2(4) = 48 - 8 = \mathbf{40}$$

Example 3 : Find $\lim_{x\to 4} \frac{\sqrt{x^2+9}}{x}$

Solution

$$\lim_{x\to 4} \frac{\sqrt{x^2+9}}{x} = \frac{\sqrt{16+9}}{4} = \frac{\sqrt{25}}{4} = \mathbf{\frac{5}{4}}$$

Example 4: If $\lim_{x\to 3} f(x) = 4$ *and* $\lim_{x\to 3} g(x) = 8$,

find $\lim_{x\to 3}\left[f^2(x)\cdot\sqrt[3]{g(x)}\right]$

Solution

$$\lim_{x\to 3} f(x) = 4 \quad \rightarrow \quad \therefore \boldsymbol{f(x) = 4}$$
$$\lim_{x\to 3} g(x) = 8 \quad \rightarrow \quad \therefore \boldsymbol{g(x) = 8}$$

$$\lim_{x\to 3}\left[f^2(x)\cdot\sqrt[3]{g(x)}\right] = \lim_{x\to 3}\left[4^2\cdot\sqrt[3]{8}\right] = \lim_{x\to 3}[16\cdot 2] = \lim_{x\to 3}[32] = \mathbf{32}$$

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Example 5 : Find $\lim_{x\to 2}\frac{7x^5-10x^4-13x+6}{3x^2-6x-8}$

Solution

$$\lim_{x\to 2}\frac{7x^5-10x^4-13x+6}{3x^2-6x-8}=\frac{7(2)^5-10(2)^4-13(2)+6}{3(2)^2-6(2)-8}=-\frac{11}{2}$$

Example 6 : Find $\lim_{x\to 1}\frac{x^3+3x+7}{x^2-2x+1}$

Solution

$$\lim_{x\to 1}\frac{x^3+3x+7}{x^2-2x+1}=\frac{1+3+7}{1-2+1}=\frac{11}{0}=\infty$$

Example 7 : Find $\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}$

Solution

$$\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}=\frac{1-1}{\sqrt{1}-1}=\frac{0}{0}$$

اذًا نبحث عن حل اخر (عن طريق الضرب في مرافق المقام) :

$$\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}=\lim_{x\to 1}\frac{(x-1)(\sqrt{x}+1)}{x-1}=\lim_{x\to 1}(\sqrt{x}+1)=2$$

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Example 8 : Find $\lim_{x\to 2} \frac{x^2+3x-10}{x^2+x-6}$

Solution

$$\lim_{x\to 2} \frac{x^2+3x-10}{x^2+x-6} = \frac{4+6-10}{4+2-6} = \frac{0}{0}$$

اذًا نبحث عن حل اخر (عن طريق التحليل) :

$$\lim_{x\to 2} \frac{(x-2)(x+5)}{(x-2)(x+3)} = \lim_{x\to 2} \frac{(x+5)}{(x+3)} = \frac{2+5}{2+3} = \frac{7}{5}$$