QUESTION FIVE: Verify that the function $f(x) = x^2 - 4x + 3$ satisfies the three hypotheses of **Rolle's Theorem** on the interval $[1,3]$. Then find all possible values of c that satisfy the conclusion of the theorem.

(3 Marks)

QUESTION SIX: Evaluate the following limits using **L'Hopital's rule**:

(2) $\lim_{x \to 0} \dfrac{e^x - 1}{\sin x}$

(2) $\lim_{x \to \infty} x \tan\left(\dfrac{1}{x}\right)$

(5 Marks: 2 + 3)

QUESTION SEVEN: Given $f(x) = x^3 - 12x$. Find:

(1) The critical numbers of f.

(2) The intervals on which f is increasing.

(3) The relative extrema of f.

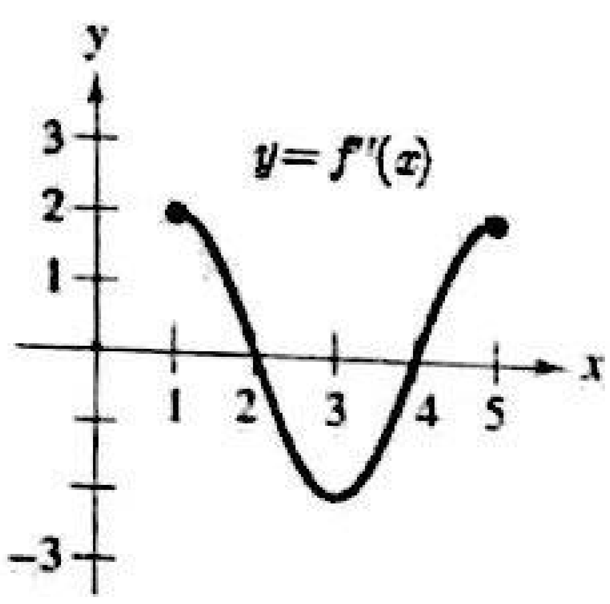
(4) The intervals on which f is concave down.

(5) The x-coordinates of inflection points of f.

1 2 3 4 5

(7 Marks: 2 + 1 + 1 + 2 + 1)

QUESTION EIGHT: Use the graph of $y = f''(x)$ below to find the following:



(1) The intervals on which f is concave up.

(2) The x-coordinates of inflection points of f.

(3 Marks: 1 + 2)

QUESTION NINE: Find two nonnegative real numbers whose sum is 12 with the property that the product of them is largest.

(3 Marks)

GOOD LUCK

KING SAUD UNIVERSITY

PREPARATORY YEAR DEANSHIP

BASIC SCIENCE DEPARTMENT

MATH 150 | FINAL EXAM / WINTER 2014-2015 | DATE: 04/01/2015

INSTRUCTOR: .... SECTION: .... ST. NAME: ....

TIME ALLOWED: 3 Hours | ST. ID: ....

⋆ *This exam consists of **9** essay questions pointed in two pages for a total of **50** marks.*

QUESTION ONE: Find the limit if exists.

(A) $\lim\limits_{x\to 1} \dfrac{2x^2+5}{x^3+2}$ (B) $\lim\limits_{x\to 3} \dfrac{x^2-9}{x^2-4x+3}$ (C) $\lim\limits_{x\to 0} \dfrac{\tan(9x)}{\sin(6x)}$ (D) $\lim\limits_{x\to 3^-} \dfrac{x+2}{x-3}$

(E) $\lim\limits_{x\to 0^+} x^2 \sin\left(\dfrac{1}{x}\right)$ (Use the Squeeze Theorem).

(8 Marks: 1 + 2 + 1 + 2 + 2)

QUESTION TWO:

(A) Find the horizontal asymptotes for the function $f(x) = \dfrac{x-4x^2}{2x^2-1}$.

(B) Find the value of the constant c such that the function

$$f(x) = \begin{cases} cx^2, & \text{if } x < 2 \\ 5cx-2, & \text{if } x \geq 2 \end{cases}$$

is continuous on $(-\infty, \infty)$.

(5 Marks: 2 + 3)

QUESTION THREE: Find the first derivative $\dfrac{dy}{dx}$ for the following functions:

(1) $y = x^7 - 4x^{-3}$

(2) $y = \sqrt{x^3 - x + 5}$

(3) $y = \cot^2(7x)$

(4) $y = e^{x-\cos x}$

(5) $y = \log_2(x^3 + 3x - 4)$

(6) $y^2 + xy = x^3 + 2$

(11 Marks: 1 + 1 + 2 + 2 + 2 + 3)

QUESTION FOUR:

(A) Let $y = t^2 - t$ and $t = x^2$. Find $\dfrac{dy}{dx}$.

(B) Find the equation of the tangent line to the curve of $f(x) = x^3 - 2x + 1$ at the point $x = 1$.

(5 Marks: 2 + 3)

ملاحظة: أكتب خطوات الحل بالتفصيل لجميع الأسئلة داخل دفتر الإجابة
علما بأن عدد الأسئلة (5) ، وعدد الصفحات (2).

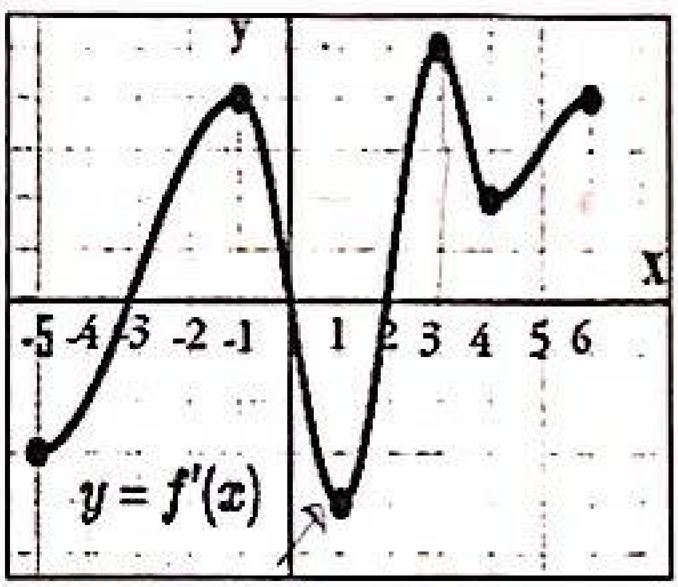
QUESTION 1: (12 Marks)

Evluate each of the following limits, (if exist):

1. $\lim_{x\to 3} x^2 - 2x + 1$
2. $\lim_{x\to 2} \dfrac{2x^2 - 8}{x^2 + x - 6}$
3. $\lim_{x\to 0} \dfrac{1-\cos^2 x}{3x^2}$
4. $\lim_{x\to 2} \dfrac{-1}{(x-2)^2}$
5. $\lim_{x\to 0} (x^2 \cos(\dfrac{1}{x}))$
6. $\lim_{x\to \infty} (x \sin(\dfrac{\pi}{x}))$

QUESTION 2: (8 Marks)

Use the graph of $y = f'(x)$ to Find the following:



a) The interval(s) on which $f(x)$ is increasing and decreasing.

b) The critical numbers of $f(x)$.

c) The x - coordinates on which f has a relative maximum and a relative minimum.

d) The interval(s) on which $f(x)$ is concave up/ concave down.

QUESTION 3:

(12 Marks)

Find $\frac{dy}{dx}$ for the following :

1. $y = x^4 + x^{-3} + 2$
2. $y = \cos(3x^2)$
3. $y = \log_3(x^2 + 5)$
4. $y = \pi^{x \tan x}$
5. $x^2 - y^2 = e^{3y}$
6. $y = x^x$

QUESTION 4:

(10 Marks)

Consider the function $f(x) = \frac{x-1}{x-3}$ to find the following:

a) Discuss the continuity of $f(x)$.

b) The horizontal asymptote(s) of $f(x)$, (if any).

c) The interval(s) on which $f(x)$ is decreasing, (if any).

d) The interval(s) on which $f(x)$ is concave up / concave down, (if any).

e) Sketch the graph of $f(x)$.

QUESTION 5:

(8 Marks)

a) Use the limit definition of derivatives to find $f'(x)$ For $f(x) = 5x^2$

b) Let $y = 1 + 3u^2$ and $u = \sqrt[3]{x}$. Find $\frac{dy}{dx}$

c) Verify that the function $f(x) = \sin(2x)$ satisfies the hypotheses of Rolle's Theorem over the interval $[0, \pi]$. Then find all possible values of c that satisfy the conclusion of the theorem.

d) Find two nonnegative real numbers whose sum is 15 with the property that the product between one of them and the square of the second is largest.

Good Luck

QUESTION 3: **(12 Marks)**

Find $\frac{dy}{dx}$ for the following :

1. $y = x^4 + x^{-3} + 2$
2. $y = \cos(3x^2)$
3. $y = \log_3(x^2 + 5)$
4. $y = \pi^{x \tan x}$
5. $x^2 - y^2 = e^{3y}$
6. $y = x^x$

QUESTION 4: **(10 Marks)**

Consider the function $f(x) = \dfrac{x-1}{x-3}$ to find the following:

a) Discuss the continuity of $f(x)$.

b) The horizontal asymptote(s) of $f(x)$, (if any).

c) The interval(s) on which $f(x)$ is decreasing, (if any).

d) The interval(s) on which $f(x)$ is concave up / concave down, (if any).

e) Sketch the graph of $f(x)$.

UESTION 5: **(8 Marks)**

a) Use the limit definition of derivatives to find $f'(x)$ For $f(x) = 5x^2$

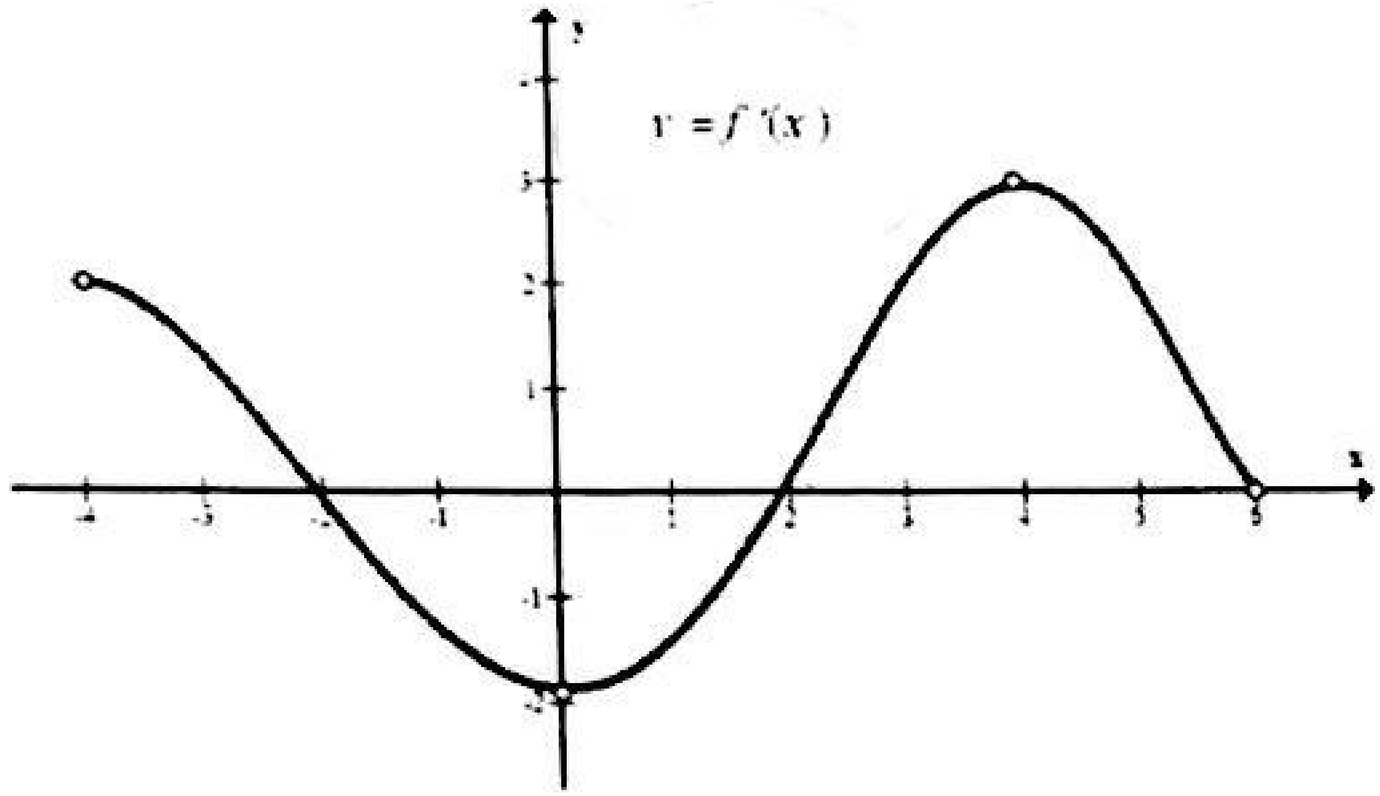
b) Let $y = 1 + 3u^2$ *and* $u = \sqrt[3]{x}$. Find $\frac{dy}{dx}$

c) Verify that the function $f(x) = \sin(2x)$ satisfies the hypotheses of Rolle's Theorem over the interval $[0, \pi]$. Then find all possible values of c that satisfy the conclusion of the theorem.

d) Find two nonnegative real numbers whose sum is 15 with the property that the product between one of them and the square of the second is largest.

Good Luck

QUESTION FIVE : Use the graph of $f'(x)$ to find the following



[1] The intervals on which f is increasing and the intervals on which f is decreasing

[2] The critical numbers of $f(x)$

[3] The x – coordinates on which f has relative extrema

[4] The interval on which f is concave up / concave down

solution:

[1] * The interval of increasing is $(-4,-2) \cup (2,6)$

* The interval of deacreasing is $(-2,2)$

[2] The critical numbers are $x=-2$ and $x=2$

[3] * f has a relative maximum at $x=-2$

* f has a relative minimum at $x=2$

[4] * f is concave up on $(0,4)$

* f is concave down on $(-4,0) \cup (4,6)$