Fundamentals of Physics

8^{th} Edition

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Chapter 1

Measurement

1-2 Measuring Physical quantities

Base quantities

Length
Mass
Time
Temperature

Derived quantities

Quantities are defined in term of base quantities

Area
Volume
Density

1-3 Units

A unique name we assign to measures of that quantity

Systems of units

SI

Length	m
Time	s
Mass	Kg

FR

Length	cm
Time	s
Mass	g

BR

Length	feet
Time	s
Mass	slug

To express the very large and very small quantity
We use Scientific notation

Scientific notation

Power of ten

For example:

1. 3560000000 m = 3.56×10^{9} m

2. 0.000000492 s = 4.92×10^{-7} s

Exponent of ten

For example:

3.56 E 9 m

4.92 E -7 s

When we dealing with very large or very small measurements we use Prefixes Listed

Prefixes of units

Factor	10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{-1}	1	10^{1}	10^{2}	10^{3}	10^{6}	10^{9}	10^{12}
Prefix	pico	nano	micro	milli	centi	deci		deka	hecto	kilo	mega	giga	tera
symbol	p	n	μ	m	c	d		da	h	k	M	g	T

For example,

1) a microsecond is 10^{-6} s

2) 1.27 x 10^{9} watts = 1.27 gw

3) 1.2 x 10^{6} m = 1.2 Mm

1-4 Changing Units

We often need to change the units in the physical quantity by a method called chain link conversion

In this method we multiply the original measurement by A conversion Factor

** **conversion Factor is (a ratio of units that is equal to unity)**

1 min = 60 s

$$\frac{1\ \text{min}}{60\ \text{s}} = 1 \qquad \frac{60\ \text{s}}{1\ \text{min}} = 1$$

1) For example, the conversion factor to convert 6 m to mm is

1 m → 10^{3} mm

6 m → ?

$$6\ \text{m} \times \frac{10^{3}\ \text{mm}}{1\ \text{m}}$$

→ Conversion factor

معامل التحويل

2) Convert 2 mins to seconds

1-5 Length

The unit of Length –the meter- is defined as the length of the path traveled by light in vacuum during a time interval 1/229 792 458 of a second

1-6 Time

The unit of Time –the second- is defined in terms of the oscillations of light emitted by an atomic source (cesium-133) .

1-7 Mass

The unit of mass –one Kilogram - is defined in terms of a platinum-iridium cylinder kept near Paris.

Atomic mass units $1u = 1.660\ 538\ 86 \times 10^{-27}$ kg

Density

The density ρ of a material is the mass per unit volume

$$\rho = \frac{m}{v} = \frac{\text{mass}}{\text{Volume}} \qquad \frac{Kg}{m^3}$$

Chapter 2

Motion Along a Straight Line

In this chapter we will study the motion of objects and the basic physics of motion .

Examples of motion :

1-Earth's orbit around the Sun

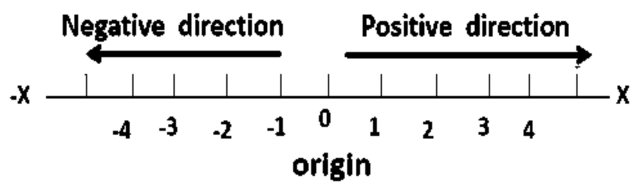
2-Earth's rotation on its axis

Every thing in the world moves, even the stationary objects move with Earth's rotation.

The motion on a straight line may be vertical, horizontal, or slanted, but it must be straight .

2-3 Position and Displacement

To locate an object means to find its position relative to a reference point origin (or Zero point) of an axis, such as the X axis. FIG. 2-1



The displacement

A change from initial position to final position

$\Delta X = X_2 - X_1$

final position — initial position

- **unit of ΔX is meter**

- **ΔX is a vector quantity**

Example page 15

(a) A particle moves from $\vec{x_1} = (5\,\text{m})$ to $\vec{x_2} = (12\,\text{m})$

So Δx =

(b) A particle moves from $\vec{x_1} = (12\,\text{m})$ to $\vec{x_2} = (5\,\text{m})$

So Δx =

(c) Find the distance in (a) and (b)

(d) a particle moves from y_1= 2m to y_2 = 8m, so Δy =

Features of a Displacement

1- Its magnitude is the distance, such as the number of meters, between the original and final positions.

2- Its direction from an original position to a final position can be represented by a plus sign (+) or a minus sign (-) if a motion is along a single axis.

- Distance is a scalar quantity **(Absolute value)**

(number of meters)

CHECKPOINT 1 Here are three pairs of initial and final positions, respectively, along an x axis. Which pairs give a negative displacement: (a) -3 m, $+5$ m; (b) -3 m, -7 m; (c) 7 m, -3 m?

2-4 Average Velocity and Average Speed

***Average Velocity is: The ratio of the displacement to time interval (Δt)**

$$* \; V_{avg} = \frac{displacement}{\Delta t} = \frac{\Delta X}{\Delta t} = \frac{X_2 - X_1}{t_2 - t_1}$$

*** Unit of the V_{avg} is m/s , Km/s**

*** V_{avg} Is vector quantity**

*** V_{avg} Is the slope of the Straight line**

Example motion of armadillo

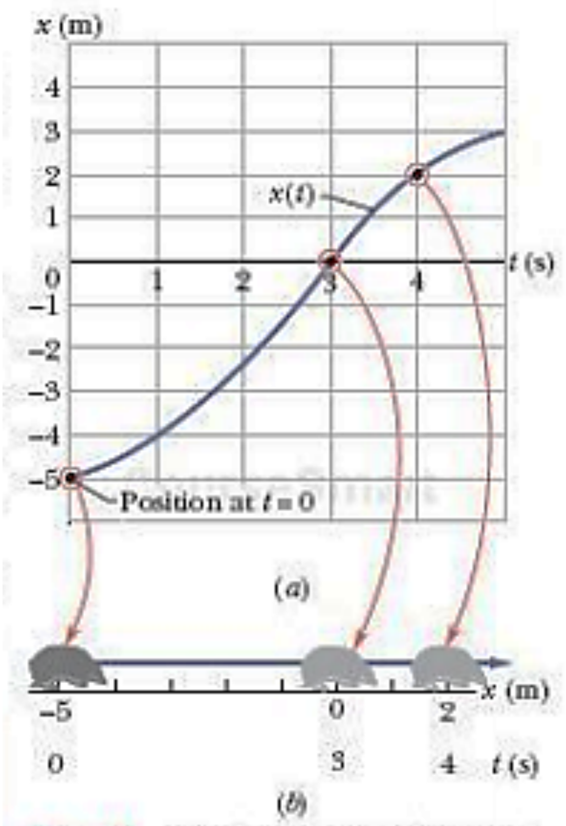


Fig 2-2

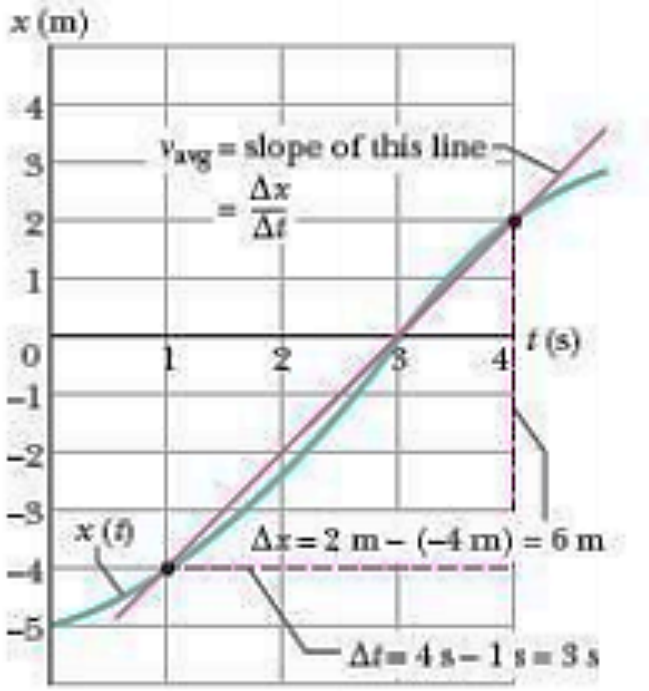


Fig 2-3

Fig 2-3 shows how to find V_{avg} for the time interval t_1=1s to t_2=4 s

Position is x_1= -4m and x_2= 2m

The average velocity is 6m/3s = 2 m/second

Average Speed

Average Speed: **is a ratio of the total distance that occurs during a particular time interval Δ t to that interval.**

$$S_{avg} = \frac{total\ distance}{\Delta t} = \frac{numbers\ of\ meters}{\Delta t}$$

Unit of S_{avg} is m/s , Km/s

S_{avg} **Is Scalar quantity**

Sample Problem 2-1

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

$$\Delta x = x_2 - x_1 = 10.4\ \text{km} - 0 = 10.4\ \text{km}.$$

In the positive direction of the x axis

(b) What is the time interval Δt from the beginning of your drive to your arrival at the station?

Calculations: We first write

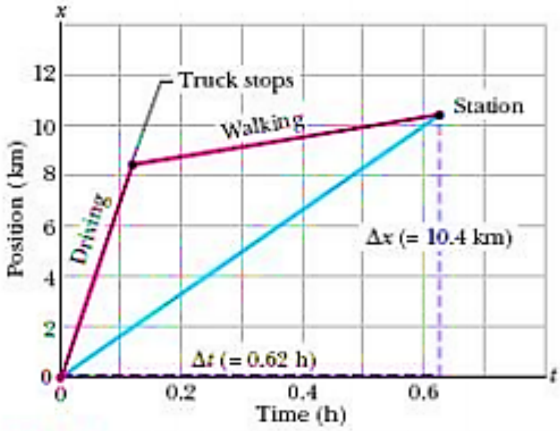
$$v_{avg,dr} = \frac{\Delta x_{dr}}{\Delta t_{dr}}.$$

Rearranging and substituting data then give us

$$\Delta t_{dr} = \frac{\Delta x_{dr}}{v_{avg,dr}} = \frac{8.4\ \text{km}}{70\ \text{km/h}} = 0.12\ \text{h}.$$

So, $\Delta t = \Delta t_{dr} + \Delta t_{wlk}$
$= 0.12\ \text{h} + 0.50\ \text{h} = 0.62\ \text{h}.$ (Answer)

(c) What is your average velocity v_{avg} from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.



Calculation: Here we find

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{10.4 \text{ km}}{0.62 \text{ h}}$$
$$= 16.8 \text{ km/h} \approx 17 \text{ km/h}. \qquad \text{(Answer)}$$

(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

Calculation: The total distance is 8.4 km + 2.0 km + 2.0 km = 12.4 km. The total time interval is 0.12 h + 0.50 h + 0.75 h = 1.37 h. Thus, Eq. 2-3 gives us

$$s_{avg} = \frac{12.4 \text{ km}}{1.37 \text{ h}} = 9.1 \text{ km/h}. \qquad \text{(Answer)}$$

2-5 Instantaneous Velocity & Speed

Velocity at any instant

$$V_{ins} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Unit of the V_{ins} is m/s

V_{ins} **Is vector quantity**

Speed is the magnitude of Velocity

The Speedometer in a car measures speed, not velocity

A velocity of +5 m/s or -5m/s

Then the speed is 5m/s

CHECKPOINT 2 The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $t > 0$): (1) $x = 3t - 2$; (2) $x = -4t^2 - 2$; (3) $x = 2/t^2$; and (4) $x = -2$. (a) In which situation is the velocity v of the particle constant? (b) In which is v in the negative x direction?

Check point 2

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SP 2-3 Page 19

Sample Problem 2-3

The position of a particle moving on an x axis is given by

$$x = 7.8 + 9.2t - 2.1t^3, \qquad (2\text{-}5)$$

with x in meters and t in seconds. What is its velocity at $t = 3.5$ s? Is the velocity constant, or is it continuously changing?

$$v = \frac{dx}{dt} = \frac{d}{dt}(7.8 + 9.2t - 2.1t^3),$$

which becomes

$$v = 0 + 9.2 - (3)(2.1)t^2 = 9.2 - 6.3t^2. \qquad (2\text{-}6)$$

At $t = 3.5$ s,

$$v = 9.2 - (6.3)(3.5)^2 = -68 \text{ m/s.} \qquad \text{(Answer)}$$

At $t = 3.5$ s, the particle is moving in the negative direction of x (note the minus sign) with a speed of 68 m/s. Since the quantity t appears in Eq. 2-6, the velocity v depends on t and so is continuously changing.

2-6 Acceleration

Acceleration is the ratio of the velocity Δv to time interval (Δt)

$$a_{avg} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t}$$

Where V_1 is Velocity at t_1 ,V_2 is Velocity at t_2

Unit of a_{avg} is m/S^2 (length /Time^2)

a_{avg} is Vector quantity .

Instantaneous acceleration

$$a_{ins} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt}$$

$$a_{ins} = \frac{dV}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

a_{ins} is the second derivative of position (x) with respect to time .

The unit of a_{ins} is m/s^2 OR Length/time^2 .

a_{ins} is vector quantity (magnitude and direction).

An acceleration's sign

If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

Checkpoint 3

Sample problem 2-4

Page 20-21(a,b)

CHECKPOINT 3 A wombat moves along an x axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

Answer: (a) Plus

(b) minus

(c) minus

(d) plus

Sample Problem 2-4 Build your skill

A particle's position on the x axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with x in meters and t in seconds.

(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

Calculations: Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad \text{(Answer)}$$

with v in meters per second. Differentiating the velocity function then gives us

$$a = +6t, \qquad \text{(Answer)}$$

with a in meters per second squared.

(b) Is there ever a time when $v = 0$?

Calculation: Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

which has the solution

$$t = \pm 3 \text{ s}. \qquad \text{(Answer)}$$

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

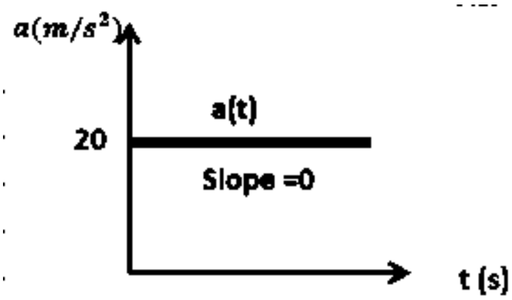
2-7Constant Acceleration: a special case

What is constant acceleration ?

Constant acceleration is the movement of particles with constant velocity in equal time.

Example:

T(s)	V(m/s)	a=V/t (m/s^2)
1	20	20
2	40	20
3	60	20
4	80	20
5	100	20



$$a_{avg} = \frac{V_2 - V_1}{t_2 - t_1}$$

When the acceleration is constant, the average acceleration and instantaneous acceleration are equal

$$V_2 \rightarrow V \qquad t_2 \rightarrow t \qquad X_2 \rightarrow X$$

$$V_1 \rightarrow V_0 \qquad t_1 \rightarrow 0 \qquad X_1 \rightarrow X_0$$

$$a = a_{avg} = \frac{V - V_0}{t - 0}$$

TABLE 2-1

Equations for Motion with Constant Acceleration[a]

Page 23

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Check point 4

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Sample problem 2-5

Page 23

CHECKPOINT 4 The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

Sample Problem 2-5

The head of a woodpecker is moving forward at a speed of 7.49 m/s when the beak makes first contact with a tree limb. The beak stops after penetrating the limb by 1.87 mm. Assuming the acceleration to be constant, find the acceleration magnitude in terms of g.

2-9 Free-Fall Acceleration

If you toss an object either up or down without an air effect, you would find that the object accelerates downward at a certain constant rate .

That rate is called the free-fall acceleration and its magnitude is g = 9.8 . The acceleration is independent of the object's characteristics, such as mass, density, or shape.

The free –fall acceleration near Earth's surface is
$a= -g = -9.8 m/s^2$

and the magnitude of the acceleration is $g= 9.8\ m/s^2$

-As these objects fall, they acceleration downward at the same rate $g = 9.8\ m/s^2$

-Their speed increases at the same rate

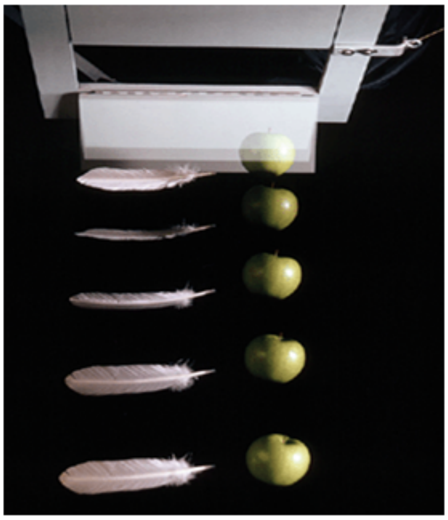
-They fall together

-The value of g varies slightly with the latitude and with the elevation .

Examples of free –fall acceleration:

- Feather

- An apple



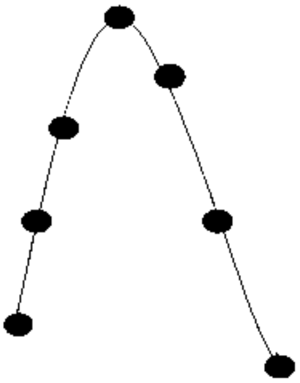
The direction of motion are now along a vertical y axis instead of the x axis

Direction:

Upward → positive (y)

Downward → negative (y)

Upward
V (+)
V_o (+)
Y (+)
a (-)



downward
V (-)
V_o (-)
Y (-)
a (-)

free-fall acceleration is always negative and thus downward

The equation of motion in table 2-1 for constant acceleration also apply to free-fall near Earth's surface

$a = -g$, $x_o = y_o$, $x = y$

$$V = V_0 - gt \qquad\qquad y - y_0 = V_0 - \frac{1}{2} g\, t^2$$

$$V^2 = V_0^2 - 2g\,(y - y_0) \qquad\qquad y - y_0 = \frac{1}{2}(V_0 + V)t$$

$$y - y_0 = V\,t + \frac{1}{2} g\, t^2$$

Checkpoint 5
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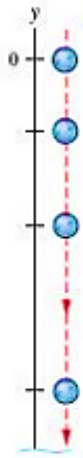
CHECKPOINT 5 (a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

Sample Problem 2-7

On September 26, 1993, Dave Munday went over the Canadian edge of Niagara Falls in a steel ball equipped with an air hole and then fell 48 m to the water (and rocks). Assume his initial velocity was zero, and neglect the effect of the air on the ball during the fall.

(a) How long did Munday fall to reach the water surface?



t (s)	y (m)	v (m/s)	a (m/s^2)
0	0	0	−9.8
1	−4.9	−9.8	−9.8
2	−19.6	−19.6	−9.8
3	−44.1	−29.4	−9.8
	−48.0		−9.8

(b) Munday could count off the three seconds of free fall but could not see how far he had fallen with each count. Determine his position at each full second.

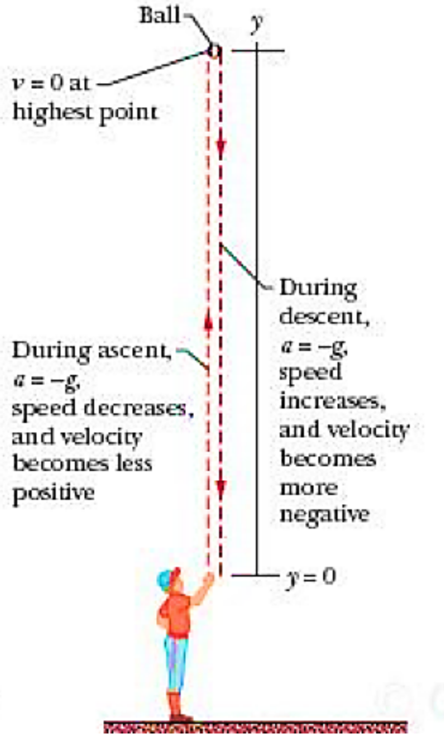
(c) What was Munday's velocity as he reached the water surface?

(d) What was Munday's velocity at each count of one full second? Was he aware of his increasing speed?

Sample Problem 2-8

In Fig. 2-12, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?



(b) What is the ball's maximum height above its release point?

(c) How long does the ball take to reach a point 5.0 m above its release point?

Chapter 3
Vectors

3-2 Vectors and scalars

1- Vector quantities → magnitude and direction. Example: displacement, velocity

2- Scalar quantities → only magnitude (no direction) Example: temperature, pressure

The simplest vector quantity is displacement vector

which we represent by an arrow → from A to B

The length of the arrow = magnitude of the vector quantity

The head of the arrow shows the direction for the vector quantity

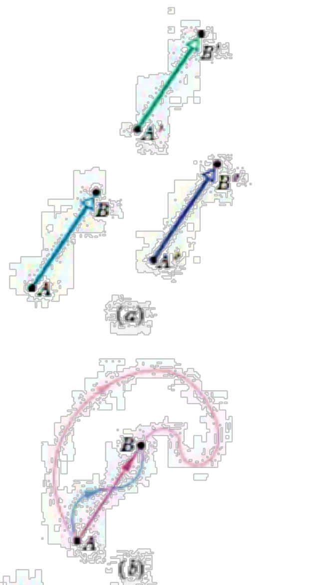
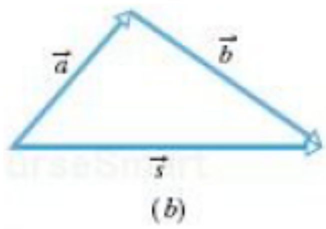


FIG. 3-1 (a) All three arrows have the same magnitude and direction and thus represent the same displacement. (b) All three paths connecting the two points correspond to the same displacement vector.

3-3 Adding Vectors Geometrically

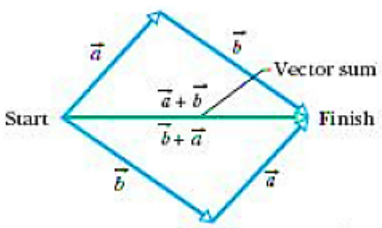


*If a particle moves from **a** to **b** we will get two displacement vectors.

The net displacement from **a** to **b** is called the **vector sum (s)**

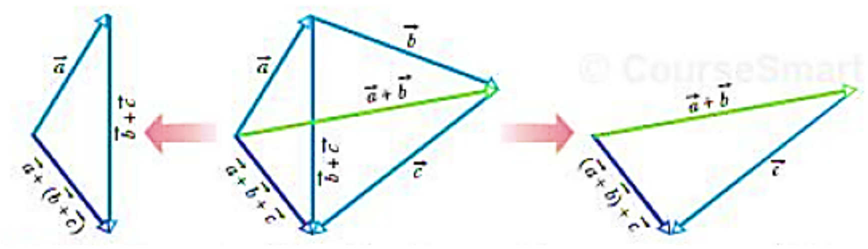
Vector equation $\vec{s} = \vec{a} + \vec{b}$,

***Commutative Law**



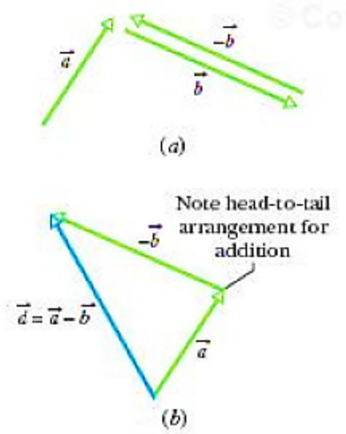
$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \quad \text{(commutative law).}$$

***Associative Law**



$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \quad \text{(associative law).}$$

***Vector Subtraction**



$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \quad \text{(vector subtraction);}$$

CHECKPOINT 1 The magnitudes of displacements $\vec{a}$ and $\vec{b}$ are 3 m and 4 m, respectively, and $\vec{c} = \vec{a} + \vec{b}$. Considering various orientations of $\vec{a}$ and $\vec{b}$, what is (a) the maximum possible magnitude for $\vec{c}$ and (b) the minimum possible magnitude?

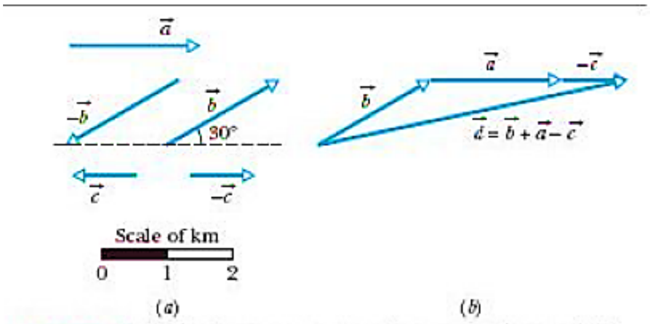
Answer:

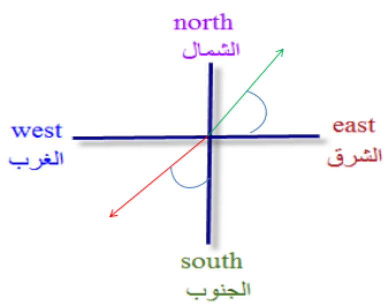
(a) 7 m (a and b are in same direction)

(b) 1 m (a and b are in opposite direction

Sample Problem 3-1

In an orienteering class, you have the goal of moving as far (straight-line distance) from base camp as possible by making three straight-line moves. You may use the following displacements in any order: (a) $\vec{a}$, 2.0 km due east (directly toward the east); (b) $\vec{b}$, 2.0 km 30° north of east (at an angle of 30° toward the north from due east); (c) $\vec{c}$, 1.0 km due west. Alternatively, you may substitute either $-\vec{b}$ for $\vec{b}$ or $-\vec{c}$ for $\vec{c}$. What is the greatest distance you can be from base camp at the end of the third displacement?





North of east = toward the north from due east

West of south= = toward the west from due south

3-4 Components of Vectors

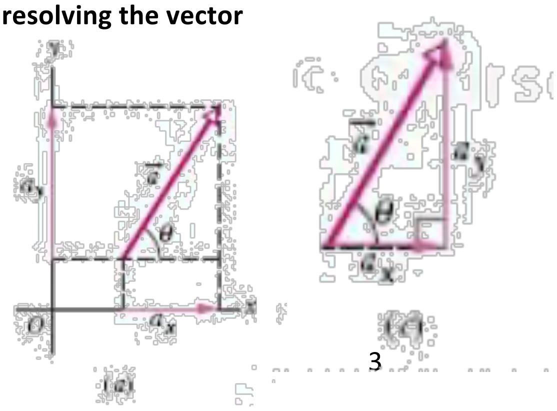
A component of a vector is the projection of a vector on an axis

The process of finding the components of a vector is calling resolving the vector

We can find the components of $\vec{a}$ ifrom the right tri-angle there:

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta,$$

where θ is the angle that the vector $\vec{a}$ makes with the positive direction of the x axis, and a is the magnitude of $\vec{a}$.

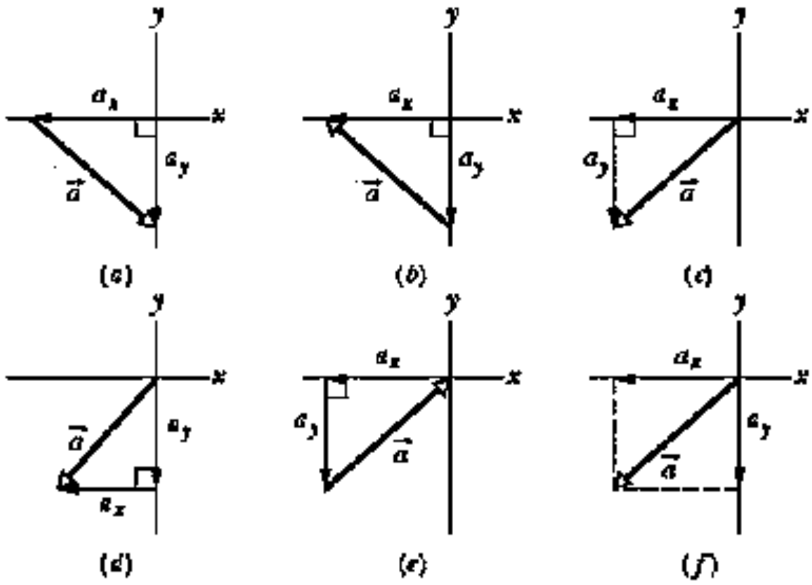


To find the magnitude, we use: $a = \sqrt{a_x^2 + a_y^2}$

To find direction, we use: $\tan\theta = \dfrac{a_y}{a_x}$

CHECKPOINT 2

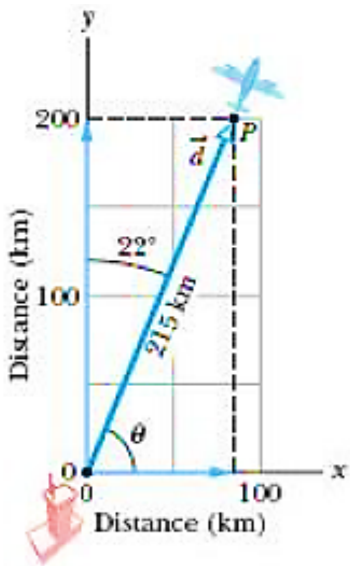
In the figure, which of the indicated methods for combining the x and y components of vector $\vec{a}$ are proper to determine that vector?



Answer: **c, d, f (components must be head-to-tail; a must extend form tail of one component to head of the other)**

Sample Problem 3-2

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. How far east and north is the airplane from the airport when sighted?



Answer:

To find the components of $\vec{d}$,
$\theta = 68°$ ($= 90° - 22°$):

$$d_x = d\cos\theta = (215\text{ km})(\cos 68°) = 81\text{ km} \quad \text{(Answer)}$$

$$d_y = d\sin\theta = (215\text{ km})(\sin 68°) = 199\text{ km} \approx 2.0\times 10^2\text{ km.} \quad \text{(Answer)}$$

Thus, the airplane is 81 km east and 2.0×10^2 km north of the airport.

3-5 Unit Vectors

* A **unit vector** is a vector that has a magnitude of exactly 1 and points in a particular direction.

* The unit vectors in the positive directions of the x, y, and z axes are labeled $\hat{i}$, $\hat{j}$, and $\hat{k}$,

* , where the hat ˆ is used instead of an overhead arrow as for other vectors (Fig. 3-14)

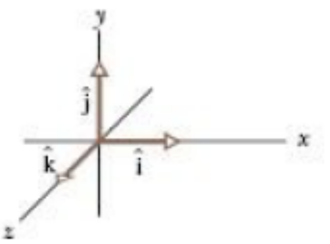
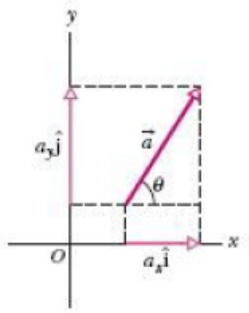


FIG. 3-14 Unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ define the directions of a right-handed coordinate system.

for example, we can express $\vec{a}$ and $\vec{b}$



(a)

The quantities $a_x\hat{i}$ and $a_y\hat{j}$ are vectors, called the **vector components** of $\vec{a}$.

The quantities a_x and a_y are scalars, called the scalar components of $\vec{a}$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

Example

$$\vec{d} = -(2.6 \text{ km})\hat{i} + (0.025 \text{ km})\hat{j} + (3.9 \text{ km})\hat{k}. \longrightarrow \textbf{vector components of } \vec{d}$$

Scalar components of **d**

3-6 Adding vectors by components

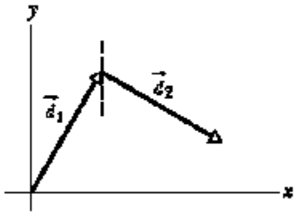
$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\vec{a} + \bar{b} = (a_x + b_x)\hat{\imath} + (a_y + b_y)\hat{\jmath} + (a_z + b_z)\hat{k}$$

****Find a – b**

✓**CHECKPOINT 3** (a) In the figure here, what are the signs of the x components of $\vec{d}_1$ and $\vec{d}_2$? (b) What are the signs of the y components of $\vec{d}_1$ and $\vec{d}_2$? (c) What are the signs of the x and y components of $\vec{d}_1 + \vec{d}_2$?



Answers

(a) +,+

(b) +, -

(c) +,+ Draw vector from tail of d_1 to head of d_2

Sample Problem 3-4

Figure 3-16*a* shows the following three vectors:

$$\vec{a} = (4.2\text{ m})\hat{\imath} - (1.5\text{ m})\hat{\jmath},$$

$$\vec{b} = (-1.6\text{ m})\hat{\imath} + (2.9\text{ m})\hat{\jmath},$$

and $$\vec{c} = (-3.7\text{ m})\hat{\jmath}.$$

What is their vector sum $\vec{r}$ which is also show

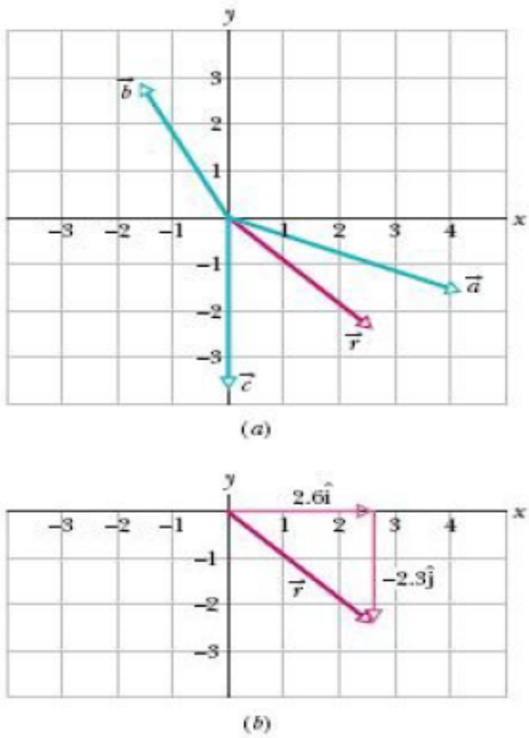


FIG. 3-16 Vector $\vec{r}$ is the vector sum of the other three vectors.

3-8 Multiplying Vectors

There are three ways in which vectors can be multiplied

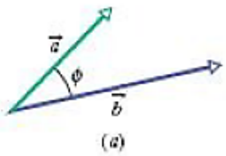
1) Multiplying vector by scalar: If we multiply a vector **a** by scalar s we get a new vector

If s is positive, **a** will become positive, EX:

If s is negative, **a** will become negative, EX:

2) Multiplying vector by vector; we get: → The vector product / → The scalar product

a) The Scalar product (or dot product)



The **scalar product** of the vectors $\vec{a}$ and $\vec{b}$ in Fig. 3-20*a* is written as $\vec{a}\cdot\vec{b}$ and defined to be

$$\vec{a}\cdot\vec{b} = ab\cos\phi,$$

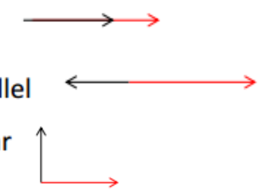
where a is the magnitude of $\vec{a}$, b is the magnitude of $\vec{b}$, and ϕ is the angle between $\vec{a}$ and $\vec{b}$

If the angle ϕ between two vectors is 0°, the component of one vector along the other is maximum, and so also is the dot product of the vectors. If, instead, ϕ is 90°, the component of one vector along the other is zero, and so is the dot product.

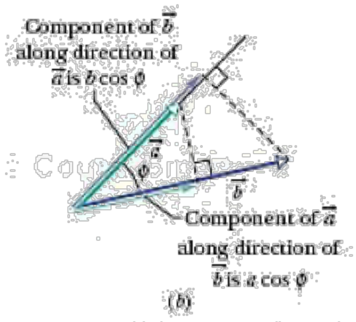
If $\theta = 0 \Rightarrow \vec{a}\,.\vec{b} = a\,b$ ➡ vectors are parallel

$\theta = 180 \Rightarrow \vec{a}\,.\vec{b} = -\,a\,b$ ➡ vectors are anti parallel

$\theta = 90 \Rightarrow \vec{a}\,.\vec{b} = 0$ ➡ vectors are perpendicular



*** The scalar product is commutative** $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$.



*** When two vectors are in unit vector notation,**

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

we write the dot product as follows:

$$\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z$$

***Multiplying Unit Vectors**

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$
$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

CHECKPOINT 4 Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if $\vec{C}\cdot\vec{D}$ equals (a) zero, (b) 12 units, and (c) −12 units?

Answer:

Sample Problem 3-7

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$?

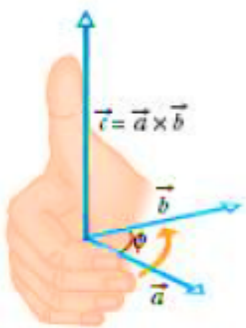
b) The Vector Product (or cross product)

The **vector product** of $\vec{a}$ and $\vec{b}$, written $\vec{a}\times\vec{b}$, produces a third vector $\vec{c}$ whose magnitude is

$$c = ab\sin\phi,$$

where ϕ is the *smaller* of the two angles between $\vec{a}$ and $\vec{b}$.

The direction of **c** is perpendicular to both **a** and **b**



(a)

Right hand rule

If $\vec{a}$ and $\vec{b}$ are parallel or antiparallel, $\vec{a}\times\vec{b} = 0$. The magnitude of $\vec{a}\times\vec{b}$, which can be written as $|\vec{a}\times\vec{b}|$, is maximum when $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

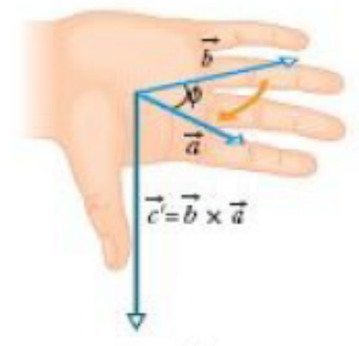
If $\theta = 0 \Rightarrow \vec{a} \times \vec{b} = 0$ ➡ vectors are parallel

$\theta = 180 \Rightarrow \vec{a} \times \vec{b} = 0$ ➡ vectors are anti parallel

$\theta = 90 \Rightarrow |\vec{a} \times \vec{b}| = a\,b$ ➡ vectors are perpendicular

***The order of the vector in this case is important**

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b}).$$



(b)

Multiplying Unit Vectors

$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k} \quad \hat{k} \times \hat{j} = -\hat{i} \quad \hat{i} \times \hat{k} = -\hat{j}$$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

*If the two vectors are given in unit vector notation, we multiply by matrix

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j}\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k}\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$= (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j}$$

$$+ (a_x b_y - b_x a_y)\hat{k}$$

CHECKPOINT 5 Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if the magnitude of the vector product $\vec{C} \times \vec{D}$ is (a) zero and (b) 12 units?

Sample Problem 3-9

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

Chapter 4

Motion in Two and Three Dimensions

In this chapter, we will study the motion in two and three dimensions. Examples of two dimention motion include projectile motion and uniform circular motion

4-2 | Position and Displacement

Position Vector The location of a particle relative to the origin of a coordinate system is given by a *position vector* $\vec{r}$, which in unit-vector notation is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}. \tag{4-1}$$

Here $x\hat{i}$, $y\hat{j}$, and $z\hat{k}$ are the vector components of position vector $\vec{r}$, and x, y, and z are its scalar components

Fig. 4-1 shows a particle with position vector

$$\vec{r} = (-3\text{ m})\hat{i} + (2\text{ m})\hat{j} + (5\text{ m})\hat{k}$$

and rectangular coordinates $(-3\text{ m}, 2\text{ m}, 5\text{ m})$. Along the x axis the particle is 3 m from the origin, in the $-\hat{i}$ direction. Along the y axis it is 2 m from the origin, in the $+\hat{j}$ direction. Along the z axis it is 5 m from the origin, in the $+\hat{k}$ direction.

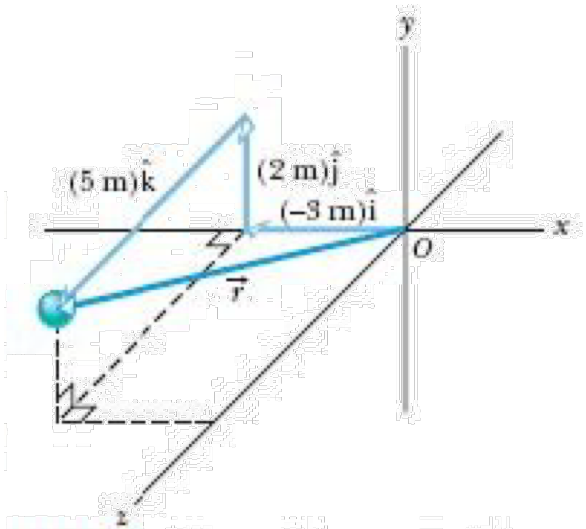


FIG. 4-1 The position vector $\vec{r}$ for a particle is the vector sum of its vector components.

If the position vector changes—say, from $\vec{r}_1$ to $\vec{r}_2$ during a certain time interval—then the particle's **displacement** $\Delta\vec{r}$ during that time interval is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1. \tag{4-2}$$

Using the unit-vector notation of Eq. 4-1, we can rewrite this displacement as

$$\Delta\vec{r} = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

or as

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}, \tag{4-3}$$

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}. \tag{4-4}$$

Sample Problem 4-1

In Fig. 4-2, the position vector for a particle initially is

$$\vec{r}_1 = (-3.0\text{ m})\hat{i} + (2.0\text{ m})\hat{j} + (5.0\text{ m})\hat{k}$$

and then later is

$$\vec{r}_2 = (9.0\text{ m})\hat{i} + (2.0\text{ m})\hat{j} + (8.0\text{ m})\hat{k}.$$

What is the particle's displacement $\Delta\vec{r}$ from $\vec{r}_1$ to $\vec{r}_2$?

Calculation: The subtraction gives us

$$\begin{aligned}\Delta\vec{r} &= \vec{r}_2 - \vec{r}_1\\ &= [9.0 - (-3.0)]\hat{i} + [2.0 - 2.0]\hat{j} + [8.0 - 5.0]\hat{k}\\ &= (12\text{ m})\hat{i} + (3.0\text{ m})\hat{k}. \qquad \text{(Answer)}\end{aligned}$$

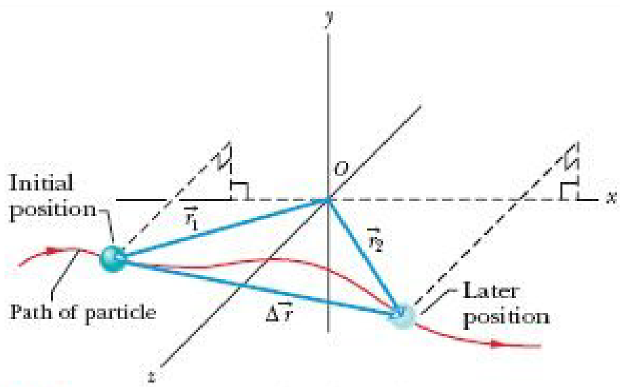


FIG. 4-2 The displacement $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ extends from the head of the initial position vector $\vec{r}_1$ to the head of the later position vector $\vec{r}_2$.

Sample Problem 4-2

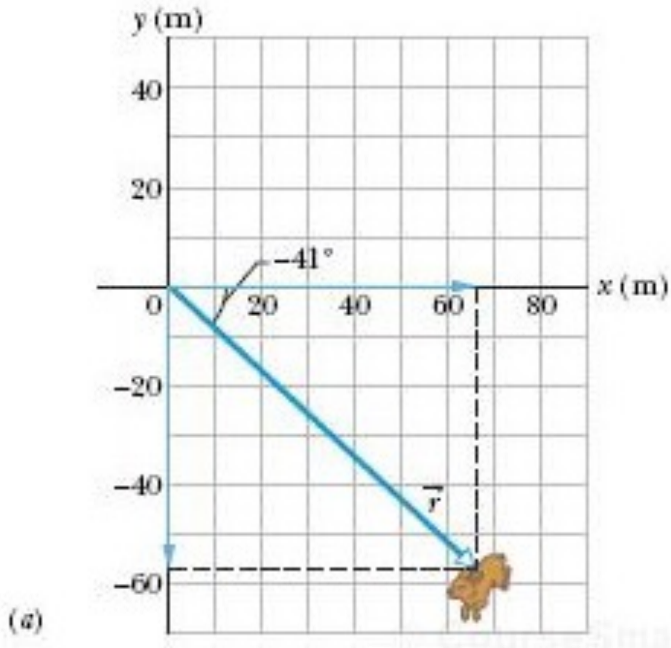
A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \tag{4-5}$$

and

$$y = 0.22t^2 - 9.1t + 30. \tag{4-6}$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?



Calculations: We can write

$$\vec{r}(t) = x(t)\hat{\mathrm{i}} + y(t)\hat{\mathrm{j}}. \qquad (4\text{-}7)$$

(We write $\vec{r}(t)$ rather than $\vec{r}$ because the components are functions of t, and thus $\vec{r}$ is also.)

At $t = 15$ s, the scalar components are

$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66 \text{ m}$$

and $$y = (0.22)(15)^2 - (9.1)(15) + 30 = -57 \text{ m},$$

so $$\vec{r} = (66 \text{ m})\hat{\mathrm{i}} - (57 \text{ m})\hat{\mathrm{j}}, \qquad \text{(Answer)}$$

which is drawn in Fig. 4-3*a*. To get the magnitude and angle of $\vec{r}$, we use Eq. 3-6:

$$r = \sqrt{x^2 + y^2} = \sqrt{(66 \text{ m})^2 + (-57 \text{ m})^2}$$
$$= 87 \text{ m}, \qquad \text{(Answer)}$$

and $$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-57 \text{ m}}{66 \text{ m}}\right) = -41°. \qquad \text{(Answer)}$$

4-3 | Average Velocity and Instantaneous Velocity

If a particle moves through a displacement $\Delta\vec{r}$ in a time interval Δt, then its **average velocity** $\vec{v}_{avg}$ is

$$\text{average velocity} = \frac{\text{displacement}}{\text{time interval}},$$

or $$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}. \qquad (4\text{-}8)$$

$$\vec{v}_{avg} = \frac{\Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\mathrm{i}} + \frac{\Delta y}{\Delta t}\hat{\mathrm{j}} + \frac{\Delta z}{\Delta t}\hat{\mathrm{k}}. \qquad (4\text{-}9)$$

For example, if the particle in Sample Problem 4-1 moves from its initial position to its later position in 2.0 s, then its average velocity during that move is

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(12 \text{ m})\hat{\mathrm{i}} + (3.0 \text{ m})\hat{\mathrm{k}}}{2.0 \text{ s}} = (6.0 \text{ m/s})\hat{\mathrm{i}} + (1.5 \text{ m/s})\hat{\mathrm{k}}.$$

When we speak of the **velocity** of a particle, we usually mean the particle's **instantaneous velocity** $\vec{v}$ at some instant. This $\vec{v}$ is the value that $\vec{v}_{avg}$ approaches in the limit as we shrink the time interval Δt to 0 about that instant. Using the language of calculus, we may write $\vec{v}$ as the derivative

$$\vec{v} = \frac{d\vec{r}}{dt}. \qquad (4\text{-}10)$$

$$\vec{v} = \frac{d}{dt}(x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}) = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}.$$

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}}, \qquad (4\text{-}11)$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}. \qquad (4\text{-}12)$$

To find the instantaneous velocity of the particle at, say, instant t_1 (when the particle is at position 1), we shrink interval Δt to 0 about t_1. Three things happen as we do so. (1) Position vector $\vec{r}_2$ in Fig. 4-4 moves toward $\vec{r}_1$ so that $\Delta\vec{r}$ shrinks toward zero. (2) The direction of $\Delta\vec{r}/\Delta t$ (and thus of $\vec{v}_{avg}$) approaches the direction of the line tangent to the particle's path at position 1. (3) The average velocity $\vec{v}_{avg}$ approaches the instantaneous velocity $\vec{v}$ at t_1.

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In the limit as $\Delta t \to 0$, we have $\vec{v}_{avg} \to \vec{v}$ and, most important here, $\vec{v}_{avg}$ takes on the direction of the tangent line. Thus, $\vec{v}$ has that direction as well:

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

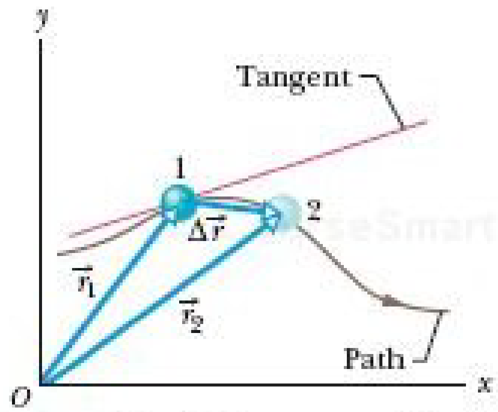


FIG. 4-4 The displacement $\Delta\vec{r}$ of a particle during a time interval Δt, from position 1 with position vector $\vec{r}_1$ at time t_1 to position 2 with position vector $\vec{r}_2$ at time t_2. The tangent to the particle's path at position 1 is shown.

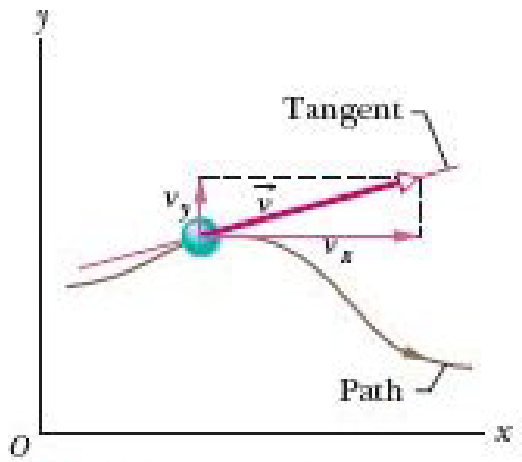


FIG. 4-5 The velocity $\vec{v}$ of a particle, along with the scalar components of $\vec{v}$.

Sample Problem 4-3

For the rabbit in Sample Problem 4-2 find the velocity $\vec{v}$ at time $t = 15$ s.

Calculations: Applying the v_x part of Eq. 4-12 to Eq. 4-5, we find the x component of $\vec{v}$ to be

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28) = -0.62t + 7.2. \tag{4-13}$$

At $t = 15$ s, this gives $v_x = -2.1$ m/s. Similarly, applying the v_y part of Eq. 4-12 to Eq. 4-6, we find

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30) = 0.44t - 9.1. \tag{4-14}$$

At $t = 15$ s, this gives $v_y = -2.5$ m/s. Equation 4-11 then yields

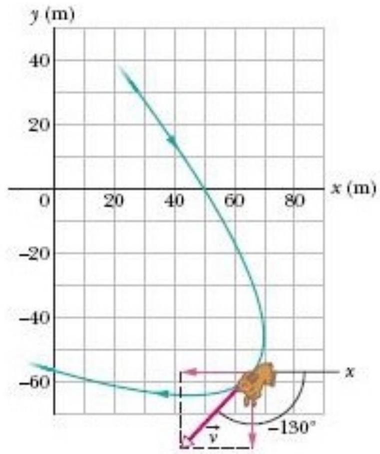


FIG. 4-6 The rabbit's velocity $\vec{v}$ at $t = 15$ s.

$$\vec{v} = (-2.1 \text{ m/s})\hat{i} + (-2.5 \text{ m/s})\hat{j}, \quad \text{(Answer)}$$

which is shown in Fig. 4-6, tangent to the rabbit's path and in the direction the rabbit is running at $t = 15$ s.

★ To get the magnitude and angle of $\vec{v}$.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.1 \text{ m/s})^2 + (-2.5 \text{ m/s})^2} = 3.3 \text{ m/s} \quad \text{(Answer)}$$

and

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-2.5 \text{ m/s}}{-2.1 \text{ m/s}}\right) = \tan^{-1} 1.19 = -130°. \quad \text{(Answer)}$$

4-4 | Average Acceleration and Instantaneous Acceleration

When a particle's velocity changes from $\vec{v}_1$ to $\vec{v}_2$ in a time interval Δt, its **average acceleration** $\vec{a}_{avg}$ during Δt is

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time interval}},$$

or

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}. \tag{4-15}$$

If we shrink Δt to zero about some instant, then in the limit $\vec{a}_{avg}$ approaches the **instantaneous acceleration** (or **acceleration**) $\vec{a}$ at that instant; that is,

$$\vec{a} = \frac{d\vec{v}}{dt}. \tag{4-16}$$

If the velocity changes in *either* magnitude *or* direction (or both), the particle must have an acceleration.

We can write Eq. 4-16 in unit-vector form by substituting Eq. 4-11 for $\vec{v}$ to obtain

$$\vec{a} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$$
$$= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}.$$

We can rewrite this as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}, \tag{4-17}$$

where the scalar components of $\vec{a}$ are

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad \text{and} \quad a_z = \frac{dv_z}{dt}. \tag{4-18}$$

CHECKPOINT 2 Here are four descriptions of the position (in meters) of a puck as it moves in an xy plane:

(1) $x = -3t^2 + 4t - 2$ and $y = 6t^2 - 4t$ (3) $\vec{r} = 2t^2\hat{i} - (4t + 3)\hat{j}$

(2) $x = -3t^3 - 4t$ and $y = -5t^2 + 6$ (4) $\vec{r} = (4t^3 - 2t)\hat{i} + 3\hat{j}$

Are the x and y acceleration components constant? Is acceleration $\vec{a}$ constant?

Answer: (1) and (3) a_x and a_y are constant and thus a is constant

(2) and (4) a_y is constant but a_x is not, thus a is not

Sample Problem 4-4

For the rabbit in Sample Problems 4-2 and 4-3, find the acceleration $\vec{a}$ at time $t = 15$ s.

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62 \text{ m/s}^2.$$

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44 \text{ m/s}^2.$$

$$\vec{a} = (-0.62 \text{ m/s}^2)\hat{i} + (0.44 \text{ m/s}^2)\hat{j}, \quad \text{(Answer)}$$

Magnitude:

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.62 \text{ m/s}^2)^2 + (0.44 \text{ m/s}^2)^2}$$
$$= 0.76 \text{ m/s}^2. \qquad \text{(Answer)}$$

For the angle we have

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\left(\frac{0.44 \text{ m/s}^2}{-0.62 \text{ m/s}^2}\right) = -35°.$$

$$-35° + 180° = 145°. \qquad \text{(Answer)}$$

Sample Problem 4-5

A particle with velocity $\vec{v}_0 = -2.0\hat{i} + 4.0\hat{j}$ (in meters per second) at $t = 0$ undergoes a constant acceleration $\vec{a}$ of magnitude $a = 3.0$ m/s² at an angle $\theta = 130°$ from the positive direction of the x axis. What is the particle's velocity $\vec{v}$ at $t = 5.0$ s?

Calculations: We find the velocity components v_x and v_y from the equations

$$v_x = v_{0x} + a_x t \quad \text{and} \quad v_y = v_{0y} + a_y t.$$

In these equations, v_{0x} ($= -2.0$ m/s) and v_{0y} ($= 4.0$ m/s)

$$a_x = a\cos\theta = (3.0 \text{ m/s}^2)(\cos 130°) = -1.93 \text{ m/s}^2,$$
$$a_y = a\sin\theta = (3.0 \text{ m/s}^2)(\sin 130°) = +2.30 \text{ m/s}^2.$$

When these values are inserted into the equations for v_x and v_y, we find that, at time $t = 5.0$ s,

$$v_x = -2.0 \text{ m/s} + (-1.93 \text{ m/s}^2)(5.0 \text{ s}) = -11.65 \text{ m/s},$$
$$v_y = 4.0 \text{ m/s} + (2.30 \text{ m/s}^2)(5.0 \text{ s}) = 15.50 \text{ m/s}.$$

Thus, at $t = 5.0$ s, we have, after rounding,

$$\vec{v} = (-12 \text{ m/s})\hat{i} + (16 \text{ m/s})\hat{j}. \qquad \text{(Answer)}$$

Magnitude:

Angle:

4-5 | Projectile Motion

Projectile Motion *Projectile motion* is the motion of a particle that is launched with an initial velocity $\vec{v}_0$. During its flight, the particle's horizontal acceleration is zero and its vertical acceleration is the free-fall acceleration $-g$.

The projectile is launched with an initial velocity $\vec{v}_0$ that can be written as

$$\vec{v}_0 = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}}. \tag{4-19}$$

The components v_{0x} and v_{0y} can then be found if we know the angle θ_0 between $\vec{v}_0$ and the positive x direction:

$$v_{0x} = v_0 \cos\theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin\theta_0. \tag{4-20}$$

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

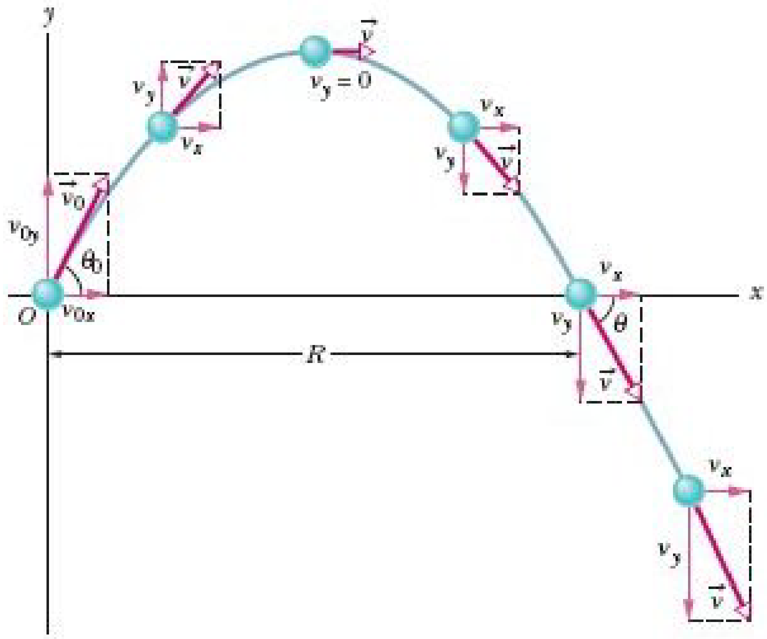


FIG. 4-10 The path of a projectile that is launched at $x_0 = 0$ and $y_0 = 0$, with an initial velocity $\vec{v}_0$. The initial velocity and the velocities at various points along its path are shown, along with their components. Note that the horizontal velocity component remains constant but the vertical velocity component changes continuously. The *range* R is the horizontal distance the projectile has traveled *when it returns to its launch height.*

CHECKPOINT 3 At a certain instant, a fly ball has velocity $\vec{v} = 25\hat{i} - 4.9\hat{j}$ (the x axis is horizontal, the y axis is upward, and $\vec{v}$ is in meters per second). Has the ball passed its highest point?

Answer: no

4-6 | Projectile Motion Analyzed

Projectile motion

$$\underbrace{\text{Horizontal Motion} \qquad\qquad \text{Vertical Motion}}$$

Horizontal Motion

$\boxed{a_x = 0}$

$\boxed{v_{0x} = v_0 \cos\theta_0}$

↓

$\boxed{v_x = v_{0x}}$

$$x - x_0 = v_{0x}t.$$

$$x - x_0 = (v_0 \cos\theta_0)t. \qquad \boxed{4\text{-}21}$$

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

Vertical Motion

$\boxed{a_y = -g}$

$\boxed{v_{0y} = v_0 \sin\theta_0}$

$\boxed{v_y = v_{0y} - g\,t}$

↓

$$v_y = v_0 \sin\theta_0 - g\,t \qquad \boxed{4\text{-}23}$$

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

$$= (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2, \qquad \boxed{4\text{-}22}$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0). \qquad \boxed{4\text{-}24}$$

Time of flight:

★ $T = \dfrac{2v_o \sin\theta}{g}$

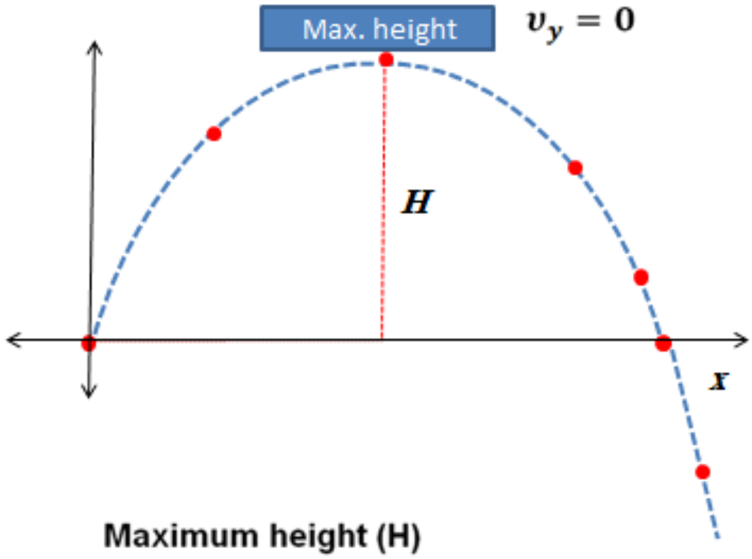
From Eq 4-22 (y=0)

★ $t = \sqrt{\dfrac{2h}{g}}$

From Eq 4-22 (ɵ=0)

As is illustrated in Fig. 4-10 and Eq. 4-23, the vertical velocity component behaves just as for a ball thrown vertically upward. It is directed upward initially, and its magnitude steadily decreases to zero, *which marks the maximum height of the path.* The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

READ

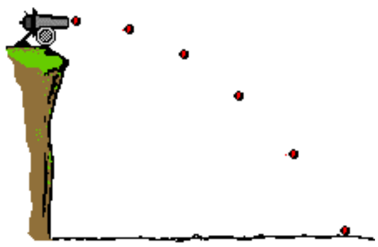


Maximum height (H)

$$\boxed{H = \frac{(v_0 \sin \theta_0)^2}{2g}}$$

From Eq 4-24 (v_y = 0)

PROBLEMS



•21 A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s. (a) How long does the projectile remain in the air?

Answer:

(a) From Eq. 4-22 (with $\theta_0 = 0$), $h = (-gt^2)/2$,, $h = -45.0$m

the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(45.0\text{ m})}{9.80\text{ m/s}^2}} = 3.03\text{ s}.$$

(b) At what horizontal distance from the firing point does it strike the ground?

Answer:

(b) From (Eq. 4-21)

$$\Delta x = v_0 t = (250\text{ m/s})(3.03\text{ s}) = 758\text{ m}.$$

(c) What is the magnitude of the vertical component of its velocity as it strikes the ground?

Answer:

(c) from Eq.(4-23)

$$|v_y| = gt = (9.80\text{ m/s}^2)(3.03\text{ s}) = 29.7\text{ m/s}.$$

••38 You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-38). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall?

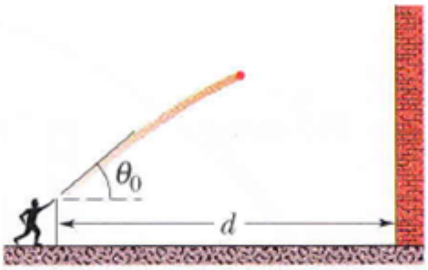


FIG. 4-38 Problem 38.

Answer:

(a) from Eq. 4-21

$$t = \frac{\Delta x}{v_x} = \frac{22.0\text{ m}}{(25.0\text{ m/s})\cos 40.0°} = 1.15\text{ s}.$$

The vertical distance (from Eq. 4-22)

$$\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = (25.0\text{ m/s})\sin 40.0°(1.15\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(1.15\text{ s})^2 = 12.0\text{ m}.$$

What are the (b) horizontal and
(c) vertical components of its velocity as it hits the wall?

Answer:

(b) $v_x = v_0 \cos 40.0° = 19.2$ m/s.

(c) from (Eq. 4-23)

$$v_y = v_0 \sin\theta_0 - gt = (25.0\text{ m/s})\sin 40.0° - (9.80\text{ m/s}^2)(1.15\text{ s}) = 4.80\text{ m/s}.$$

(d) When it hits, has it passed the highest point on its trajectory?

Answer:

(d) As $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

The Equation of the Path

We can find the equation of the projectile's path (its **trajectory**) by eliminating time t between Eqs. 4-21 and 4-22. Solving Eq. 4-21 for t and substituting into Eq. 4-22, we obtain, after a little rearrangement,

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2} \quad \text{(trajectory).} \qquad (4\text{-}25)$$

$y = ax + bx^2$, in which a and b are constants.

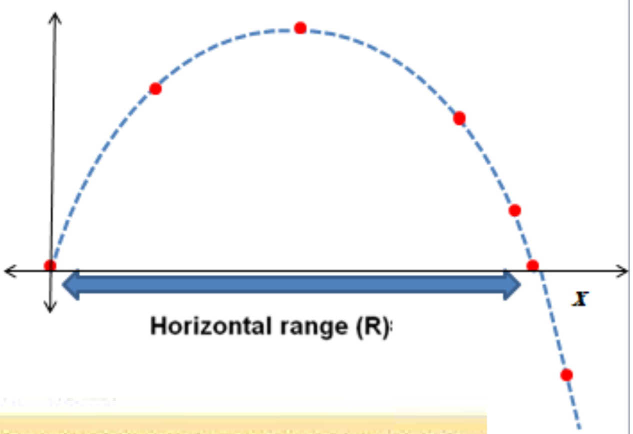
This is the equation of a parabola, so the path is *parabolic*.

horizontal range R, which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

$$R = \frac{v_0^2}{g}\sin 2\theta_0$$

Maximum range

$$\theta_0 = 45^0 \rightarrow R_{max} = \frac{v_0^2}{g}$$



The horizontal range R is maximum for a launch angle of 45°.

CHECKPOINT 4 A fly ball is hit to the outfield. During its flight (ignore the effects of the air), what happens to its (a) horizontal and (b) vertical components of velocity? What are the (c) horizontal and (d) vertical components of its acceleration during ascent, during descent, and at the topmost point of its flight?

Answer: (a) v_x constant

(b) v_y initially positive, decreases to zero and then becomes more negative

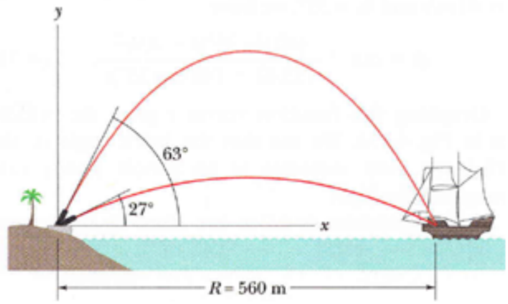
(c) $a_x = 0$

(d) $a_y = -g$

Sample Problem 4-7

Figure 4-16 shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed $v_0 = 82$ m/s.

(a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship?



(b) What is the maximum range of the cannonballs?

4-7 | Uniform Circular Motion If a particle travels along a circle or circular arc of radius r at constant speed v, it is said to be in *uniform circular motion* and has an acceleration $\vec{a}$ of constant magnitude

$$a = \frac{v^2}{r}. \quad (4\text{-}34)$$

The direction of $\vec{a}$ is toward the center of the circle or circular arc, and $\vec{a}$ is said to be *centripetal*. The time for the particle to complete a circle is

$$T = \frac{2\pi r}{v}. \quad (4\text{-}35)$$

T is called the *period of revolution*, or simply the *period*, of the motion.

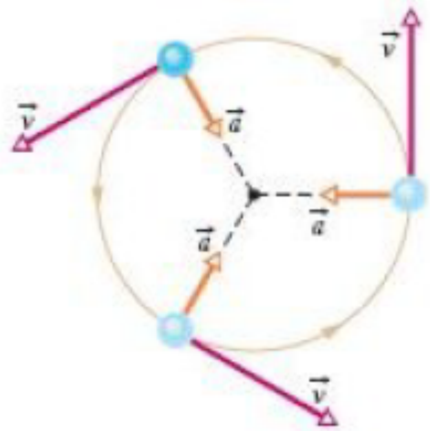


FIG. 4-19 Velocity and acceleration vectors for uniform circular motion.

The velocity is tangent to the circle in the direction of motion

CHECKPOINT 5 An object moves at constant speed along a circular path in a horizontal xy plane, with the center at the origin. When the object is at $x = -2$ m, its velocity is $-(4 \text{ m/s})\hat{j}$. Give the object's (a) velocity and (b) acceleration at $y = 2$ m.

Sample Problem 4-10

What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\hat{i} + 500\hat{j})$ m/s and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\hat{i} - 500\hat{j})$ m/s?

Answer:

$$a = \frac{2\pi v}{T}.$$

$$v = \sqrt{(400 \text{ m/s})^2 + (500 \text{ m/s})^2} = 640.31 \text{ m/s}.$$

Half circle time: 24 s

Thus full circle T = 48 s

$$a = \frac{2\pi(640.31 \text{ m/s})}{48.0 \text{ s}} = 83.81 \text{ m/s}^2 \approx 8.6g.$$

Chapter 5

Force and Motion

5 -1

We have seen the acceleration is change in velocity and the cause in acceleration is the force (push or pull)

5 -2 Newton Mechanics

* The relation between a force and acceleration it causes by Isaac Newton

*The study of that relation as Newton presented it ,is called Newtonian mechanics.

* If the speeds of the interacting bodies are very large , Newtonian mechanics does not apply , and we must replace Newtonian mechanics with another mechanics as Einstein's theory of relativity

or with quantum mechanics as object in size very small .

*Newtonian mechanics is very important special case for the motion of objects between Einstein's theory and quantum mechanics.

5-3 Newton's First law

➢The first law of Newton's that a body will keep moving with constant velocity if no force acts on it, and the body cannot accelerate

Newton's First Law: If no force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.

➢ In other words , if the body is at rest , It stays at rest , if it is moving, it continues to move with the same velocity (same magnitude and same direction).

Newton's First Law: If no *net* force acts on a body ($\vec{F}_{net} = 0$), the body's velocity cannot change; that is, the body cannot accelerate.

5-4 Force

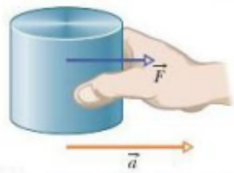


FIG. 5-1 A force $\vec{F}$ on the standard kilogram gives that body an acceleration $\vec{a}$.

 # We know that a force can cause the acceleration of body

 # We shall define the unit of force in terms of the acceleration that a force gives to a standard reference body, we take to be the standard (kilogram).

 # If we put the standard body on a horizontal frictionless table and pull the body to the right. The acceleration of body is $1 m/s^2$ where the magnitude of force acting an standard body equal (1N)

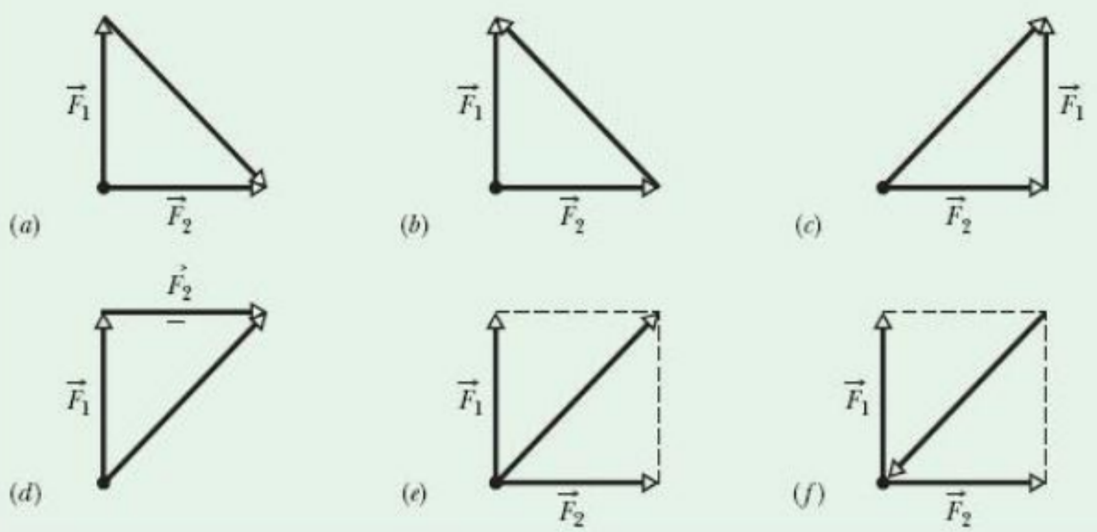
 # In general if the our standard body of 1kg mass has an acceleration of magnitude a, and the force F acting on it we find that :The magnitude of the force (N)is equal the magnitude of the acceleration(m/s^2).

 # The acceleration is a vector quantity then the force a vector quantity.

If we acts on a body with two or more forces we find the **net force** by adding the forces vectorially. and the direction of the net force has the same effect on the body as all the individual forces together .This fact is called the principle of super-position for forces.

The net force or force have components forces along coordinate axis and then have components acceleration. $(F_x,a_x),(F_y,a_y),(F_z,a_z)$.

CHECKPOINT 1 Which of the figure's six arrangements correctly show the vector addition of forces $\vec{F}_1$ and $\vec{F}_2$ to yield the third vector, which is meant to represent their net force $\vec{F}_{net}$?



5-5 Mass

* Mass is depends on the properties of bodies.

* Mass is a scalar quantity.

* We can say that the mass of body is the characteristic that relates a force on the body to the resulting acceleration.

* The ratio of the masses of two bodies is equal to the inverse of the ratio of their accelerations when the same force is applied to both. For body X and the standard body, this tells us that

$$\frac{m_X}{m_0} = \frac{a_0}{a_X}.$$

Solving for m_X yields

$$m_X = m_0 \frac{a_0}{a_X} = (1.0\ \text{kg}) \frac{1.0\ \text{m/s}^2}{0.25\ \text{m/s}^2} = 4.0\ \text{kg}.$$

where; mass m_0 is defined to be 1.0 kg & we find that this body X accelerates at 0.25 m/s^2.
Suppose that the standard body accelerates at 1.0 m/s^2.

5.6 Newton's Second Law

Newton's Second Law: The net force on a body is equal to the product of the body's mass and its acceleration.

$$\vec{F}_{net} = m\vec{a} \qquad \text{(Newton's second law).} \qquad (5\text{-}1)$$

The net force $\vec{F}_{net}$ must be the vector sum of all the forces that act on that body

Like other vector equations, Eq. 5-1 is equivalent to three component equations, one for each axis of an xyz coordinate system:

$$F_{\text{net},x} = ma_x, \quad F_{\text{net},y} = ma_y, \quad \text{and} \quad F_{\text{net},z} = ma_z. \qquad (5\text{-}2)$$

Each of these equations relates the net force component along an axis to the acceleration along that same axis.

The acceleration component along a given axis is caused *only by* the sum of the force components along that *same* axis, and not by force components along any other axis.

TABLE 5-1

Units in Newton's Second Law (Eqs. 5-1 and 5-2)

System	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
CGS[a]	dyne	gram (g)	cm/s^2
British[b]	pound (lb)	slug	ft/s^2

[a] $1\ \text{dyne} = 1\ \text{g}\cdot\text{cm/s}^2$.
[b] $1\ \text{lb} = 1\ \text{slug}\cdot\text{ft/s}^2$.

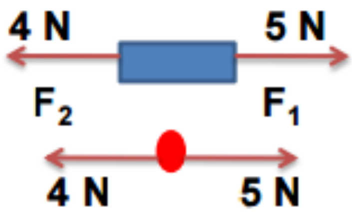
From eq.(5-1) we find that: $\boxed{\vec{F}_{net} = m\,\vec{a}}$ **(5-1)**

(1) *If the net force on a body is zero ,the body's acceleration a=0

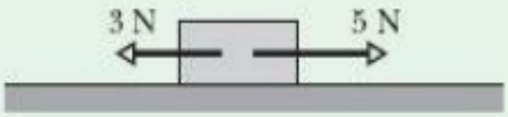
(2) *If the body's is at rest , it stays at rest , if it is moving it continues to move at constant velocity .

(3)*In such cases (1,2) we find that if any forces on the body balance one another , we say that the forces and the body are to be in equilibrium state the forces also said to cancel one another .

To solve problem with Newton's Second law we often draw a free – body diagram which is usually represent with a dot (●) , and each force on the body is drawn as a vector arrow with its tail on the body



CHECKPOINT 2 The figure here shows two horizontal forces acting on a block on a frictionless floor. If a third horizontal force $\vec{F}_3$ also acts on the block, what are the magnitude and direction of $\vec{F}_3$ when the block is (a) stationary and (b) moving to the left with a constant speed of 5 m/s? page 92



Answer:

(a) and (b) 2N, Leftward (acceleration is zero in each situation)

Sample Problem 5-1 page 93

Figures 5-3*a* to *c* show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck's mass is $m = 0.20$ kg. Forces $\vec{F}_1$ and $\vec{F}_2$ are directed along the axis and have magnitudes $F_1 = 4.0$ N and $F_2 = 2.0$ N. Force $\vec{F}_3$ is directed at angle $\theta = 30°$ and has magnitude $F_3 = 1.0$ N. In each situation, what is the acceleration of the puck?

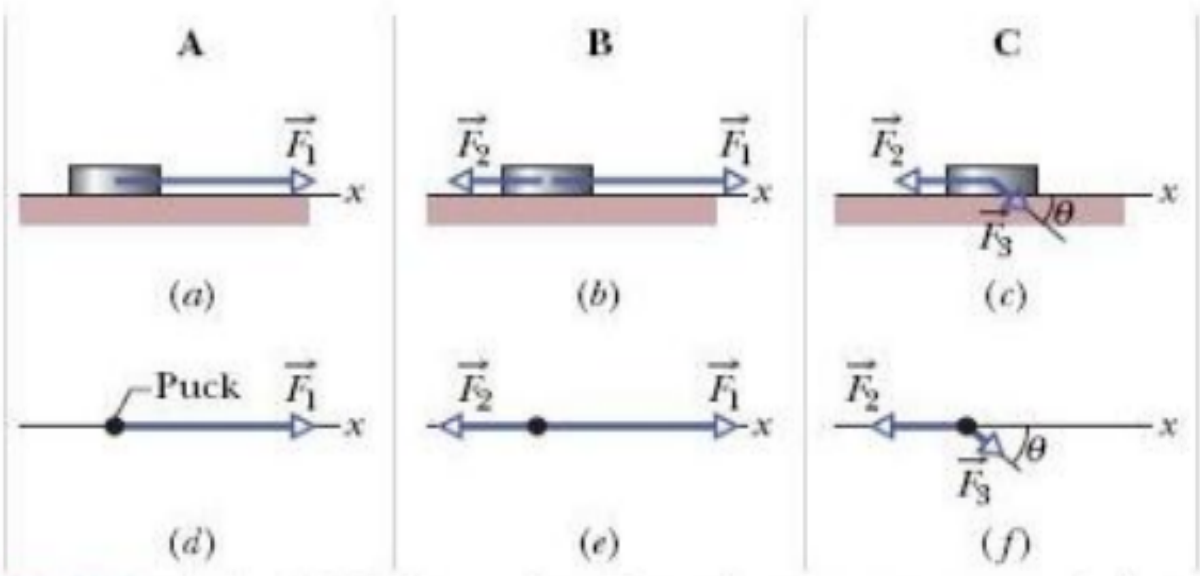


FIG. 5-3 (*a*)–(*c*) In three situations, forces act on a puck that moves along an *x* axis. (*d*)–(*f*) Free-body diagrams.

Situation A:

$$F_{\text{net},x} = ma_x.$$

$$F_1 = ma_x,$$

$$a_x = \frac{F_1}{m} = \frac{4.0\text{ N}}{0.20\text{ kg}} = 20\text{ m/s}^2.$$

Situation B:

$$F_{\text{net},x} = ma_x.$$

$$F_1 - F_2 = ma_x,$$

$$a_x = \frac{F_1 - F_2}{m} = \frac{4.0\text{ N} - 2.0\text{ N}}{0.20\text{ kg}} = 10\text{ m/s}^2.$$

Situation C:

$$F_{\text{net},x} = ma_x.$$

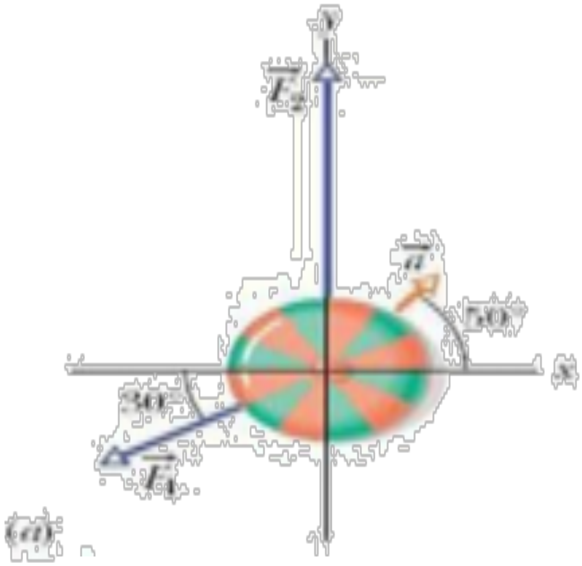
$$F_{3,x} - F_2 = ma_x.$$

$$a_x = \frac{F_{3,x} - F_2}{m} = \frac{F_3\cos\theta - F_2}{m}$$

$$= \frac{(1.0\text{ N})(\cos 30^\circ) - 2.0\text{ N}}{0.20\text{ kg}} = -5.7\text{ m/s}^2.$$

Sample Problem 5-2 page 93

In the overhead view of Fig. 5-4*a*, a 2.0 kg cookie tin is accelerated at 3.0 m/s^2 in the direction shown by $\vec{a}$, over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: $\vec{F}_1$ of magnitude 10 N and $\vec{F}_2$ of magnitude 20 N. What is the third force $\vec{F}_3$ in unit-vector notation and in magnitude-angle notation?



$$(\vec{F}_{net} = m\vec{a})$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a},$$

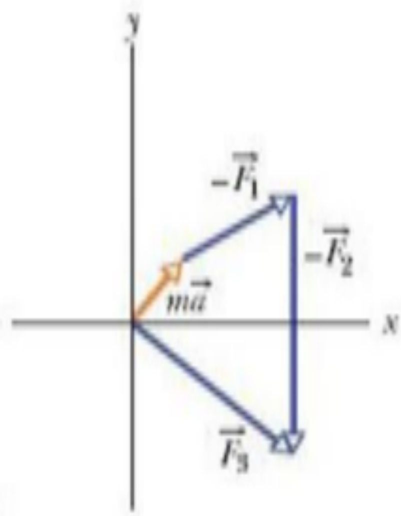
$$\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2.$$

***x* components:** Along the x axis we have

$$F_{3,x} = ma_x - F_{1,x} - F_{2,x}$$
$$= m(a\cos 50°) - F_1\cos(-150°) - F_2\cos 90°.$$

Then, substituting known data, we find

$$F_{3,x} = (2.0\text{ kg})(3.0\text{ m/s}^2)\cos 50° - (10\text{ N})\cos(-150°)$$
$$- (20\text{ N})\cos 90°$$
$$= 12.5\text{ N}.$$



(*b*)

***y* components:** Similarly, along the y axis we find

$$F_{3,y} = ma_y - F_{1,y} - F_{2,y}$$
$$= m(a\sin 50°) - F_1\sin(-150°) - F_2\sin 90°$$
$$= (2.0\text{ kg})(3.0\text{ m/s}^2)\sin 50° - (10\text{ N})\sin(-150°)$$
$$- (20\text{ N})\sin 90°$$
$$= -10.4\text{ N}.$$

Vector: In unit-vector notation, we can write

$$\vec{F}_3 = F_{3,x}\hat{i} + F_{3,y}\hat{j} = (12.5\text{ N})\hat{i} - (10.4\text{ N})\hat{j}$$

$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2} = 16\text{ N}$$

$$\theta = \tan^{-1}\frac{F_{3,y}}{F_{3,x}} = -40°.$$

5-7 Some Particular Forces

1. The Gravitational Force

(1) A gravitational force $\vec{F}_g$ on a body is a certain type of pull that is directed toward a second body.

(2) F_g **is force between two objects**

(3) If the second body is Earth ,thus it is a force that pulls on a body directly toward the center of earth .

(4) The direction of F_g is directly down toward the ground .

*** Free Fall Acceleration**

$$\vec{F} = m\,\vec{a}$$

$$\vec{F_g} = m\,g$$

The magnitude of the gravitational is equal to the product mg

We can write Newton's Second law for the gravitational force in these vector forms

$$\boxed{\vec{F}_g = -F_g\hat{j} = -mg\hat{j} = m\vec{g}.}$$

Where $\hat{j}$ is the unit vector , g is the free fall acceleration

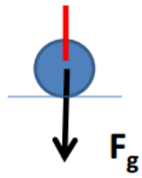
2-Weight

We can write Newton's second law for vertical y axis, with the positive direction upward as

$$\boxed{F_{net,y} = ma_y.}$$

$$\boxed{W - F_g = m(0)}$$

$$\boxed{W = F_g}$$

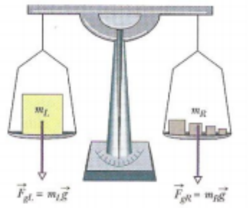


The weight W of a body is equal to the magnitude F_g of the gravitational force on the body.

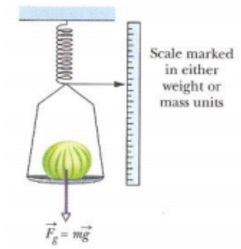
$$W = mg$$ Weight

To weigh a body (or measure its weight) we have two methods

An equal – arm balance



A spring scale



The weight of a body must be measured when the body is not accelerating vertically relative to the ground .

For example : you can measure your weight on a scale in your bathroom or on a fast train . But you can't do that at elevator .

Caution **A body's weight is not its mass.**

For example : The body has mass m , then the weight is different from the earth and moon because the acceleration on the moon is only 1.6 m/s^2

	On Earth	On the moon
ball	Mass = 0.3 Kg g = 9.8 m/s^2 W= (0.3) (9.8) = 2.9 N	Mass = 0.3 Kg a = 1.6 m/s^2 W= (0.3) (1.6) = 0.49 N

3- The normal force F_N

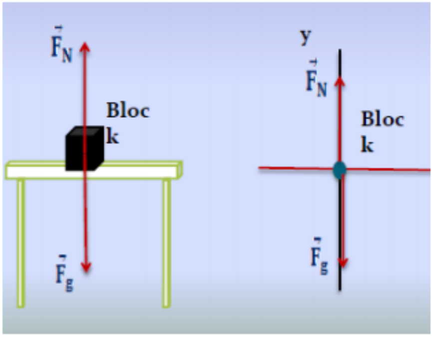
When a body presses against a surface , the surface (even a seemingly rigid one) deforms and pushes on the body with a normal force F_N that is perpendicular to the surface .

$$\boxed{(F_{net,y} = ma_y)}$$

$$\boxed{F_N - F_g = ma_y.}$$

$$\boxed{F_N - mg = ma_y.}$$

$$\boxed{F_N = mg + ma_y = m(g + a_y)}$$



If the table and block are not accelerating $a_y = 0$

$$\boxed{F_N = mg.}$$

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CHECKPOINT 3 In Fig. 5-7, is the magnitude of the normal force $\vec{F}_N$ greater than, less than, or equal to mg if the block and table are in an elevator moving upward (a) at constant speed and (b) at increasing speed?

$$\boxed{F_N = mg + ma_y = m(g + a_y)}$$

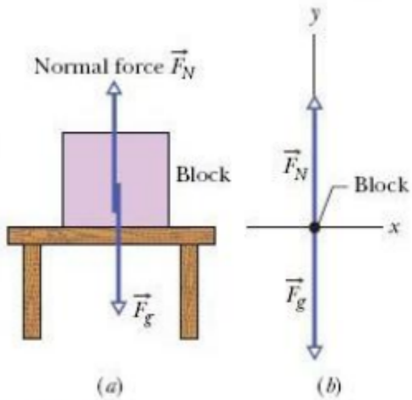
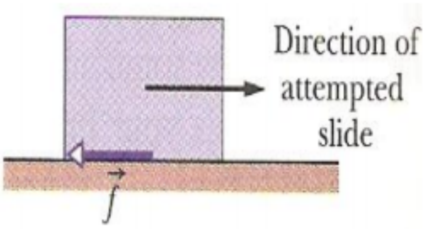


FIG. 5-7 (*a*) A block resting on a table experiences a normal force $\vec{F}_N$ perpendicular to the tabletop. (*b*) The free-body diagram for the block.

4- Friction

IF we slide a body on a surface the motion is resisted by a bonding between the body and the surface .

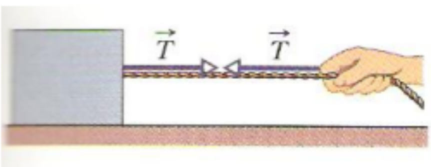
- $\vec{f}_f$ **Is directed a long the surface , but in opposite the direction of motion .**

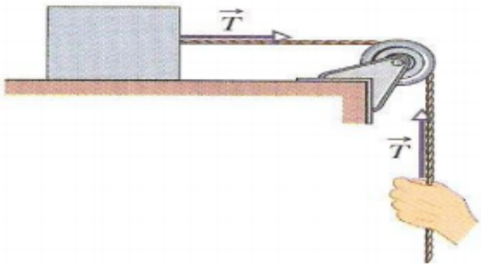


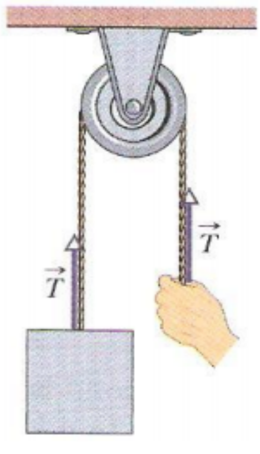
5- Tension

IF a cord or rope or other such object is attached to a body , the cord pull's on the body with T

T is directed away from the body and a long the cord .







CHECKPOINT 4 The suspended body in Fig. 5-9*c* weighs 75 N. Is *T* equal to, greater than, or less than 75 N when the body is moving upward (a) at constant speed, (b) at increasing speed, and (c) at decreasing speed?

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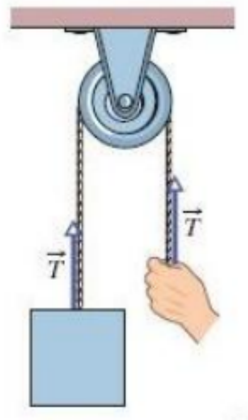
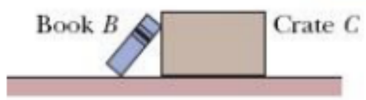


(c)

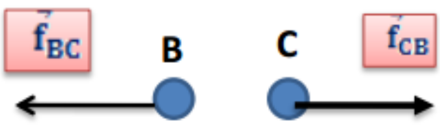
5-8 Newton's Third Law

Newton's Third Law: When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

The action and reaction forces are in opposite directions

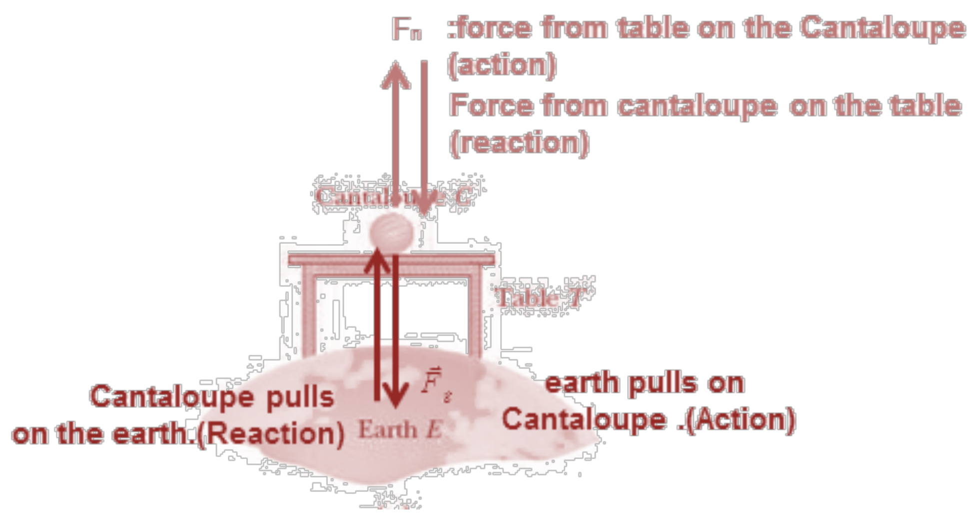


$$F_{BC} = F_{CB} \qquad \text{(equal magnitudes)}$$



$$\vec{f}_{BC} = -\vec{f}_{CB} \qquad \text{(equal magnitudes and opposite directions),}$$

Another Example:



5-9 Applying Newton's Law

Sample Problem 5-4 Build your skill page 100

Figure 5-13 shows a block S (the *sliding block*) with mass $M = 3.3$ kg. The block is free to move along a horizontal frictionless surface and connected, by a cord that wraps over a frictionless pulley, to a second block H (the *hanging block*), with mass $m = 2.1$ kg. The cord and pulley have negligible masses compared to the blocks (they are "massless"). The hanging block H falls as the sliding block S accelerates to the right. Find (a) the acceleration of block S, (b) the acceleration of block H, and (c) the tension in the cord.

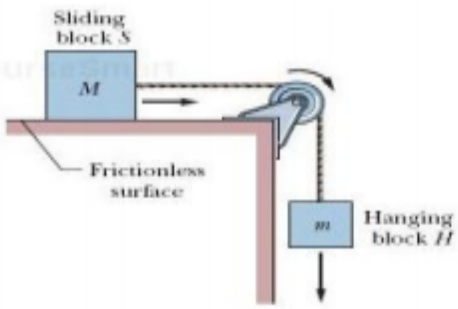


FIG. 5-13 A block S of mass M is connected to a block H of mass m by a cord that wraps over a pulley.

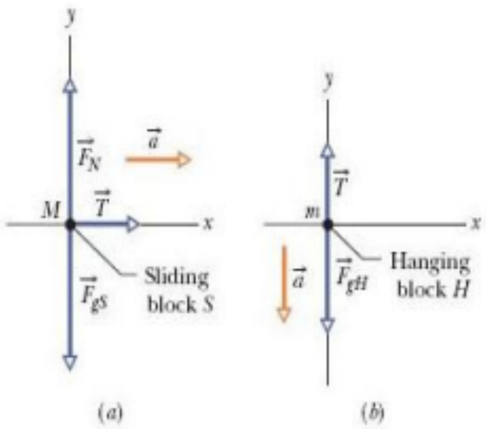


FIG. 5-15 (*a*) A free-body diagram for block S of Fig. 5-13. (*b*) A free-body diagram for block H of Fig. 5-13.

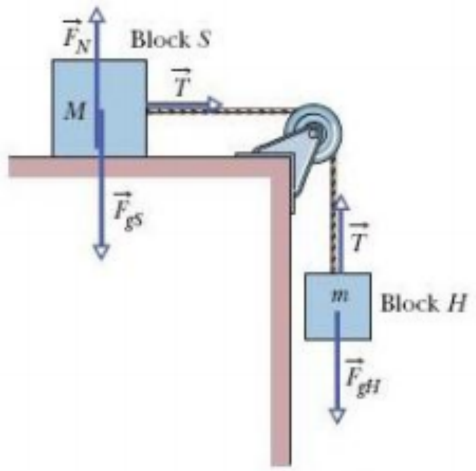


FIG. 5-14 The forces acting on the two blocks of Fig. 5-13.

Sample Problem 5-5

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In Fig. 5-16*a*, a cord pulls on a box of sea biscuits up along a frictionless plane inclined at $\theta = 30°$. The box has mass $m = 5.00$ kg, and the force from the cord has magnitude $T = 25.0$ N. What is the box's acceleration component a along the inclined plane?

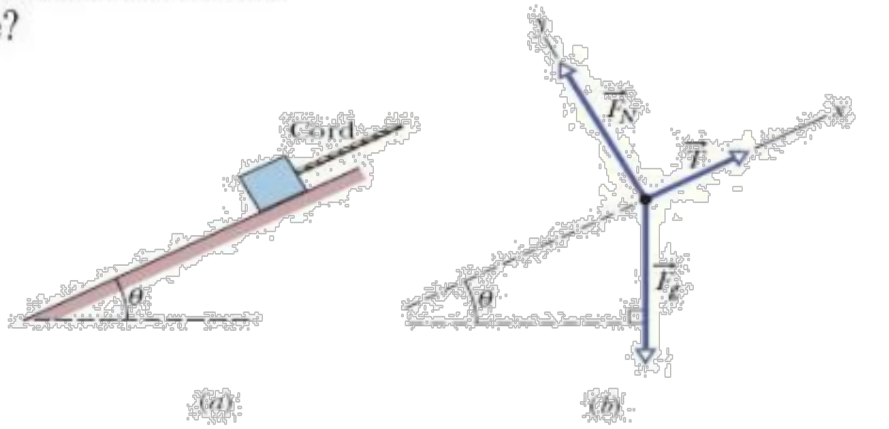
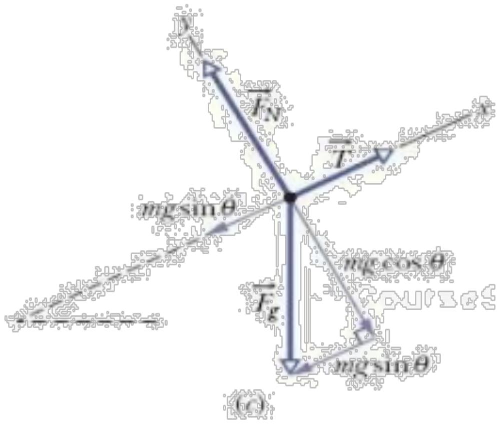


FIG. 5-16 (*a*) A box is pulled up a plane by a cord. (*b*) The three forces acting on the box: the cord's force $\vec{T}$, the gravitational force $\vec{F}_g$, and the normal force $\vec{F}_N$. (*c*) The components of $\vec{F}_g$ along the plane and pependicular to it.

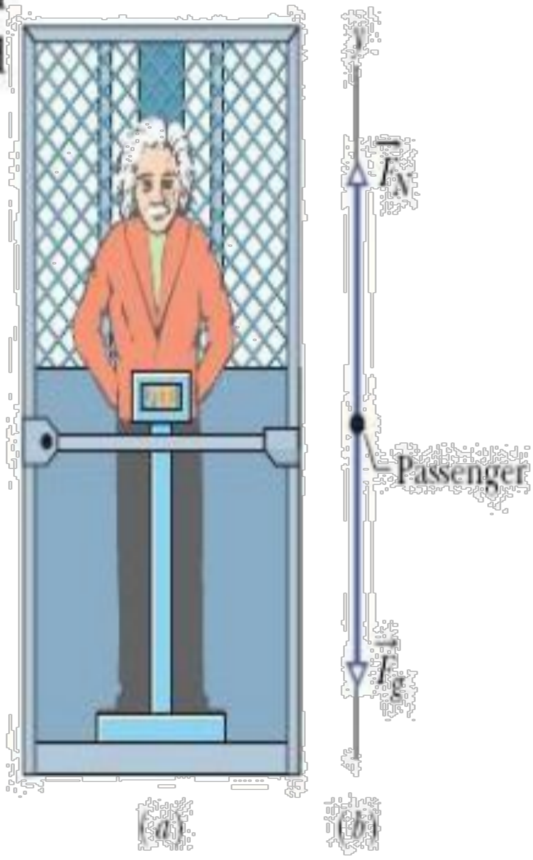


$$m = 5Kg$$
$$T = 25N$$
$$\theta = 30^0$$
$$F_{net} = ma$$
$$T - F_{g,x} = ma$$
$$T - mg\sin\theta = ma$$
$$a = \frac{T - mg\sin\theta}{m} \rightarrow \frac{25-(5)(9.8)\sin 30}{5} = .1\, m/s^2$$

Sample Problem 5-8 Build your skill page 104

In Fig. 5-19*a*, a passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

FIG. 5-19 (*a*) A passenger stands on a platform scale that indicates either his weight or his apparent weight. (*b*) The free-body diagram for the passenger, showing the normal force $\vec{F}_N$ on him from the scale and the gravitational force $\vec{F}_g$.



(a) Find a general solution for the scale reading, whatever the vertical motion of the cab.

$$m = 72.2\,Kg$$

$$F_{net} = ma \rightarrow F_N - F_g = ma \rightarrow F_N = mg + ma$$

$$F_N = m(a+g)$$

(b) What does the scale read if the cab is stationary or moving upward at a constant 0.50 m/s?

$$m = 72.2$$

$$v = .5\,m/s$$

$$a = 0$$

$$F_N = m(a+g) \rightarrow F_N = m(0+g) \rightarrow F_N = mg$$

$$F_N = (72.2)(9.8)$$

$$F_N = 708\,N$$

(c) What does the scale read if the cab accelerates upward at 3.20 m/s² and downward at 3.20 m/s²?

$a = 3.2\, m/s^2$ **Upward**

$F_N = m(a+g) \rightarrow F_N = 72.2(3.2+9.8)$

$F_N = 939\, N$

$a = -3.2\, m/s^2$ **Downward**

$F_N = m(a+g) \rightarrow F_N = 72.2(-3.2+9.8)$

$F_N = 477\, N$

(d) During the upward acceleration in part (c), what is the magnitude F_{net} of the net force on the passenger,

$F_{net} = F_N - F_g$

$F_{net} = 939 - (72.2)(9.8)$

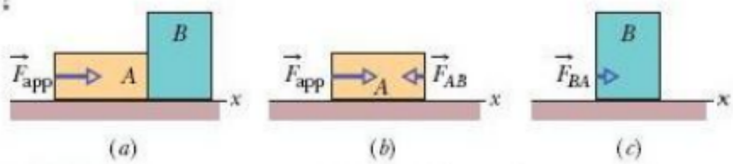
$F_{net} = 231\, N$

Sample Problem 5-9 Build your skill Page 104-105

In Fig. 5-20a, a constant horizontal force $\vec{F}_{app}$ of magnitude 20 N is applied to block A of mass $m_A = 4.0$ kg,

(a) What is the acceleration of the blocks?

(b) What is the (horizontal) force $\vec{F}_{BA}$ on block B from block A (Fig. 5-20c)?



Chapter 6

Force and Motion – II

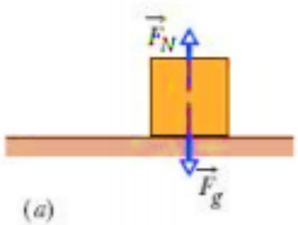
6-1

In this chapter, we will study two types of forces: friction force and centripetal force.

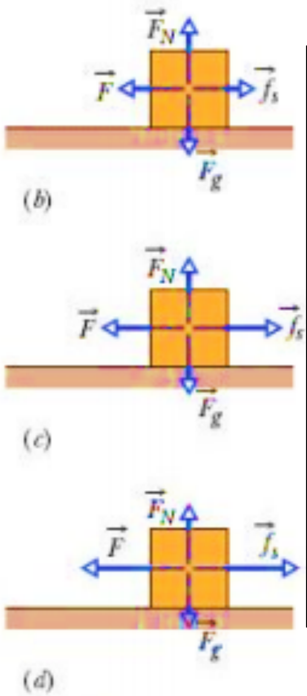
6-2 Friction **There are two types of friction force:**

1- static friction force

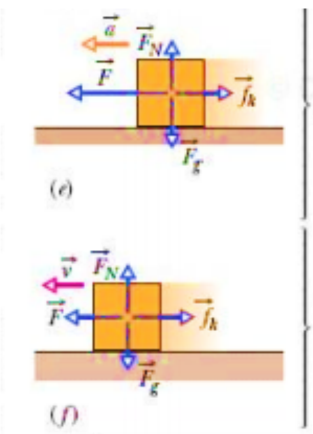
2- kinetic friction forces



(a)A block lies on a table, so friction force = zero



The block in (b), (c), (d) is not moving, then static friction force = applied force in magnitude but opposite direction



(e) The block is accelerating in direction of F. The block slides along the surface. The friction force is called kinetic friction force

(f) The block moves at constant velocity

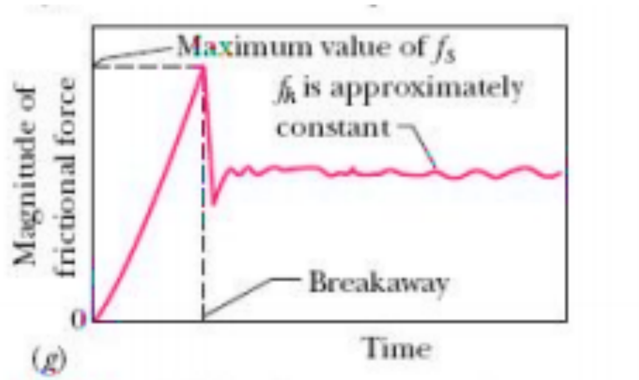


FIG. 6-1 (*a*) The forces on a stationary block. (*b–d*) An external force $\vec{F}$, applied to the block, is balanced by a static frictional force $\vec{f}_s$. As F is increased, f_s also increases, until f_s reaches a certain maximum value. (*e*) The block then "breaks away," accelerating suddenly in the direction of $\vec{F}$. (*f*) If the block is now to move with constant velocity, F must be reduced from the maximum value it had just before the block broke away. (*g*) Some experimental results for the sequence (*a*) through (*f*).

6-3 Properties of Friction

Property 1. If the body does not move, then the static frictional force $\vec{f}_s$ and the component of $\vec{F}$ that is parallel to the surface balance each other. They are equal in magnitude, and $\vec{f}_s$ is directed opposite that component of $\vec{F}$.

Property 2. The magnitude of $\vec{f}_s$ has a maximum value $f_{s,max}$ that is given by

$$f_{s,\max} = \mu_s F_N,$$

where μ_s is the coefficient of static friction and F_N is the magnitude of the normal force on the body from the surface. If the magnitude of the component of $\vec{F}$ that is parallel to the surface exceeds $f_{s,max}$, then the body begins to slide along the surface.

Property 3. If the body begins to slide along the surface, the magnitude of the frictional force rapidly decreases to a value f_k given by

$$f_k = \mu_k F_N, \tag{6-2}$$

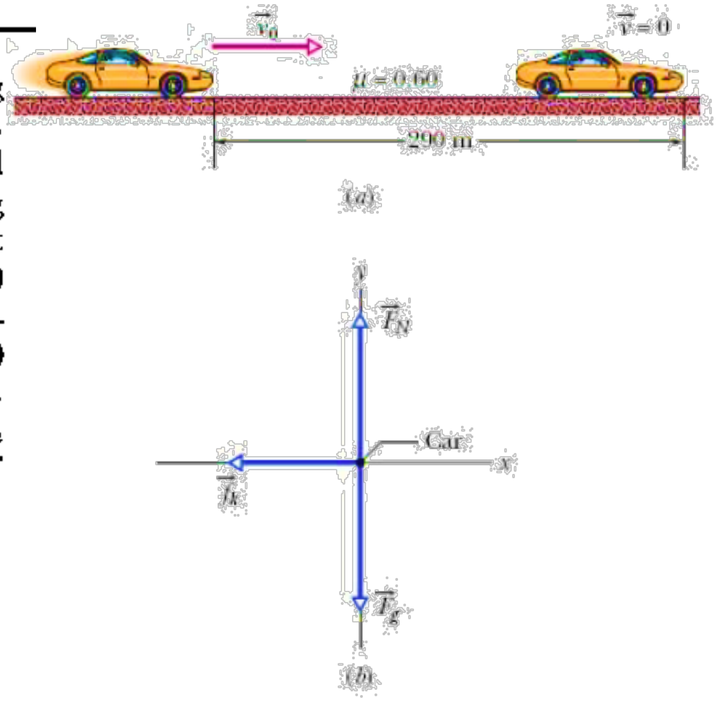
where μ_k is the coefficient of kinetic friction. Thereafter, during the sliding, a kinetic frictional force $\vec{f}_k$ with magnitude given by Eq. 6-2 opposes the motion.

The coefficients μ_s and μ_k are dimensionless and must be determined experimentally. Their values depend on certain properties of both the body and the surface

CHECKPOINT 1 A block lies on a floor. (a) What is the magnitude of the frictional force on it from the floor? (b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the frictional force on it? (c) If the maximum value $f_{s,max}$ of the static frictional force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N? (d) If it is 12 N? (e) What is the magnitude of the frictional force in part (c)?

Answer: **(a) zero (b) 5 N (c) no (d) yes (e) 8 N**

Sample Problem 6-1



If a car's wheels are "locked" (kept from rolling) during emergency braking, the car slides along the road. Ripped-off bits of tire and small melted sections of road form the "skid marks" that reveal that cold-welding occurred during the slide. The record for the longest skid marks on a public road was reportedly set in 1960 by a Jaguar on the M1 highway in England (Fig. 6-3a)—the marks were 290 m long! Assuming that $\mu_k = 0.60$ and the car's acceleration was constant during the braking, how fast was the car going when the wheels became locked?

Answer: To find the car's initial speed, we can use the constant acceleration equation because the acceleration is constant

$$v^2 = v_0^2 + 2a(x - x_0).$$

$$-f_k = ma.$$

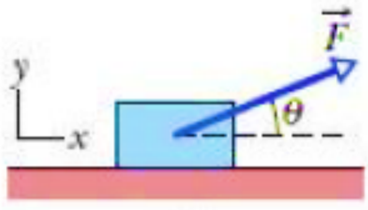
$$a = -\frac{f_k}{m} = -\frac{\mu_k mg}{m} = -\mu_k g,$$

= (-0.6) (9.8) = -5.88 m/s^2

Acceleration is in negative direction of x axis

Sample Problem 6-2

In Fig. 6-4a, a block of mass $m = 3.0$ kg slides along a floor while a force $\vec{F}$ of magnitude 12.0 N is applied to it at an upward angle θ. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.40$.



(a)

What is the acceleration of the block?

6-5 Uniform Circular Motion

From Section 4-7, recall that when a body moves in a circle (or a circular arc) at constant speed v, it is said to be in uniform circular motion. Also recall that the body has a centripetal acceleration (directed toward the center of the circle) of constant magnitude given by

$$a = \frac{v^2}{R} \qquad \text{(centripetal acceleration)}, \qquad (6\text{-}17)$$

where R is the radius of the circle.

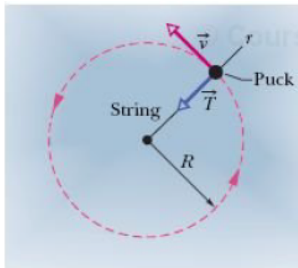
We have three exampl es

1- Rounding a curve in a car: in this situation, centripetal force is frictional force on the tires from the road

2- Orbiting Earth: in this situation,

cetripetal force is gravity force

3- Hockey Puck: in thi situation, centripetal force on the puck is tension



A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

From Newton's second law and Eq. 6-17 ($a = v^2/R$), we can write the magnitude F of a centripetal force (or a net centripetal force) as

$$F = m\frac{v^2}{R} \qquad \text{(magnitude of centripetal force)}. \qquad (6\text{-}18)$$

Because the speed v here is constant, the magnitudes of the acceleration and the force are also constant.

CHECKPOINT 2 When you ride in a Ferris wheel at constant speed, what are the directions of your acceleration $\vec{a}$ and the normal force $\vec{F}_N$ on you (from the always upright seat) as you pass through (a) the highest point and (b) the lowest point of the ride?

Answer: (a) a downward, F_N upward (b) a and F_N upward

Sample Problem 6-6

Igor is a cosmonaut on the International Space Station, in a circular orbit around Earth, at an altitude h of 520 km and with a constant speed v of 7.6 km/s. Igor's mass m is 79 kg.

(a) What is his acceleration?

Answer:

R_E is Earth's radius (6.37×10^6 m,

$$a = \frac{v^2}{R} = \frac{v^2}{R_E + h}$$

$$= \frac{(7.6 \times 10^3 \text{ m/s})^2}{6.37 \times 10^6 \text{ m} + 0.52 \times 10^6 \text{ m}}$$

$$= 8.38 \text{ m/s}^2 \approx 8.4 \text{ m/s}^2. \qquad \text{(Answer)}$$

(b) What force does Earth exert on Igor?

$$F_g = ma = (79 \text{ kg})(8.38 \text{ m/s}^2)$$

$$= 662 \text{ N} \approx 660 \text{ N}. \qquad \text{(Answer)}$$

Chapter 7

Kinetic Energy and Work

7-2 | What Is Energy?

energy is a scalar quantity associated with the state (or condition) of one or more objects.

Energy is a number that we associate with a system of one or more objects.

If a force changes one of the objects by, say, making it move, then the energy number changes.

Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same (energy is *conserved*).

7-3 | Kinetic Energy

Kinetic energy K is energy associated with the *state of motion* of an object.

For an object of mass m whose speed v is well below the speed of light

$$K = \tfrac{1}{2}mv^2 \qquad \text{(kinetic energy).}$$

For example, a 3.0 kg duck flying past us at 2.0 m/s

K = 6 J

The SI unit of kinetic energy is the **joule** (J).

$$\boxed{1\text{ joule} = 1\text{ J} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2.}$$

Sample Problem 7-1

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4-km-long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed (Fig. 7-1) in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed 1.2×10^6 N and its acceleration was a constant 0.26 m/s^2, what was the total kinetic energy of the two locomotives just before the collision?

$$v^2 = v_0^2 + 2a(x - x_0).$$

With $v_0 = 0$ and $x - x_0 = 3.2 \times 10^3$ m (half the initial separation), this yields

$$v^2 = 0 + 2(0.26 \text{ m/s}^2)(3.2 \times 10^3 \text{ m}),$$

or

$$v = 40.8 \text{ m/s}$$

(about 150 km/h).

We can find the mass of each locomotive by dividing its given weight by g:

$$m = \frac{1.2 \times 10^6 \text{ N}}{9.8 \text{ m/s}^2} = 1.22 \times 10^5 \text{ kg}.$$

we find the total kinetic energy of the two locomotives just before the collision as

$$K = 2(\tfrac{1}{2}mv^2) = (1.22 \times 10^5 \text{ kg})(40.8 \text{ m/s})^2$$
$$= 2.0 \times 10^8 \text{ J.} \qquad \text{(Answer)}$$

7-4 | Work

If you accelerate an object to a greater speed by applying a force to the object, you increase the kinetic energy K $(= \frac{1}{2}mv^2)$ of the object. Similarly, if you decelerate the object to a lesser speed by applying a force, you decrease the kinetic energy of the object. We account for these changes in kinetic energy by saying that your force has transferred energy *to* the object from yourself or *from* the object to yourself. In such a transfer of energy via a force, **work** W is said to be *done on the object by the force*. More formally, we define work as follows:

> Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

★ Work has the same units as energy and is a scalar quantity.

★ we shall use the symbol W only for work and shall represent a weight with its equivalent mg.

7-5 | Work and Kinetic Energy

I) Work done by constant force:

Let us find an expression for work by considering a bead that can slide along a frictionless wire that is stretched along a horizontal x axis (Fig. 7-2). A constant force $\vec{F}$, directed at an angle ϕ to the wire, accelerates the bead along the wire. We can relate the force and the acceleration with Newton's second law, written for components along the x axis:

$$F_x = ma_x, \qquad (7\text{-}3)$$

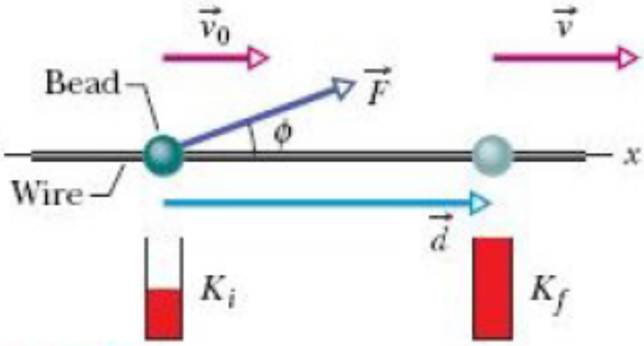


FIG. 7-2

$$v^2 = v_0^2 + 2a_x d. \qquad (7\text{-}4)$$

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = F_x d. \qquad (7\text{-}5)$$

$$W = F_x d. \qquad (7\text{-}6)$$

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

$$W = Fd\cos\phi \qquad \text{(work done by a constant force).} \qquad (7\text{-}7)$$

where ϕ is the angle between the directions of the displacement $\vec{d}$ and the force $\vec{F}$.

$$W = \vec{F}\cdot\vec{d} \qquad \text{(work done by a constant force),} \qquad (7\text{-}8)$$

Cautions:

First, the force must be a *constant force;*

Second, the object must be *particle-like.*

Signs for work.

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

$$when\ \phi = 90^o \Rightarrow \cos\phi = 0 \Rightarrow W = 0$$

$$when\ \phi < 90^o \Rightarrow \cos\phi = +ve \Rightarrow W = +ve$$

$$when\ \phi > 90^\circ (up\ to\ 180^\circ) \Rightarrow \cos\phi = -ve \Rightarrow W = -ve$$

Units for work.

Work has the SI unit of the joule.

$$1\ \mathrm{J} = 1\ \mathrm{kg \cdot m^2/s^2} = 1\ \mathrm{N \cdot m}$$

Net work done by several forces.

We can calculate the net work in two ways.

1) Find the work done by each force and then sum those works

$$W_1 = F_1 d$$

$$W_2 = F_2 d$$

$$W_3 = F_3 d$$

$$W_{net} = W_1 + W_2 + W_3 + \cdots$$

2) Find the net force $\vec{F}_{net}$ then

$$W_{net} = (F_{net}) d \cos\phi,$$

$where\ \phi\ is\ the\ angle\ between\ \vec{F}_{net}\ and\ \vec{d}$

II) Work-Kinect Energy Theorem

Equation 7-5 relates the change in kinetic energy of the bead (from an initial $K_i = \frac{1}{2}mv_0^2$ to a later $K_f = \frac{1}{2}mv^2$) to the work W $(= F_x d)$ done on the bead. For such particle-like objects, we can generalize that equation. Let ΔK be the change in the kinetic energy of the object, and let W be the net work done on it. Then

$$\Delta K = K_f - K_i = W, \tag{7-10}$$

which says that

$$\begin{pmatrix}\text{change in the kinetic}\\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on}\\ \text{the particle}\end{pmatrix}.$$

We can also write

$$K_f = K_i + W, \tag{7-11}$$

which says that

$$\begin{pmatrix}\text{kinetic energy after}\\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy}\\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net}\\ \text{work done}\end{pmatrix}.$$

For example, if the kinetic energy of a particle is initially 5 J and there is a net transfer of 2 J to the particle (positive net work), the final kinetic energy is 7 J. If, instead, there is a net transfer of 2 J from the particle (negative net work), the final kinetic energy is 3 J.

CHECKPOINT 1 A particle moves along an x axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from -3 m/s to -2 m/s and (b) from -2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

(a) decrease

(b) same

(c) Work done in (a) is negative and (b) is zero

Sample Problem 7-2

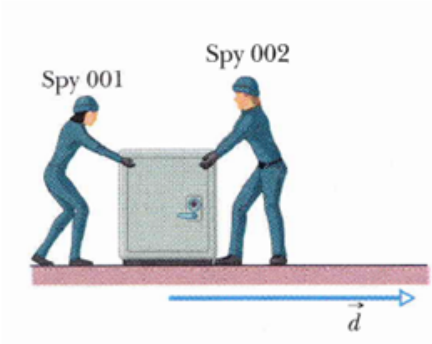


Figure 7-4*a* shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m, straight toward their truck. The push $\vec{F}_1$ of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?

Calculations: From Eq. 7-7 and the free-body diagram for the safe in Fig. 7-4*b*, the work done by $\vec{F}_1$ is

$$W_1 = F_1 d \cos \phi_1 = (12.0\text{ N})(8.50\text{ m})(\cos 30.0°) = 88.33\text{ J},$$

and the work done by $\vec{F}_2$ is

$$W_2 = F_2 d \cos \phi_2 = (10.0\text{ N})(8.50\text{ m})(\cos 40.0°) = 65.11\text{ J}.$$

Thus, the net work W is

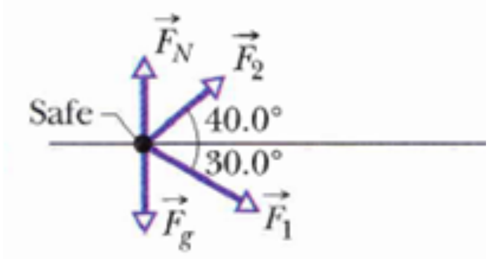
$$W = W_1 + W_2 = 88.33\text{ J} + 65.11\text{ J} = 153.4\text{ J} \approx 153\text{ J}. \quad \text{(Answer)}$$

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

Calculations: Thus, with mg as the magnitude of the gravitational force, we write

$$W_g = mgd \cos 90° = mgd(0) = 0 \quad \text{(Answer)}$$

and $$W_N = F_N d \cos 90° = F_N d(0) = 0. \quad \text{(Answer)}$$



(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

Calculations: We relate the speed to the work done by combining Eqs. 7-10 and 7-1:

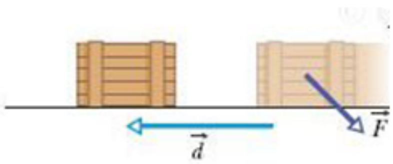
$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\text{ J})}{225\text{ kg}}}$$
$$= 1.17\text{ m/s.} \qquad \text{(Answer)}$$

Sample Problem 7-3

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0\text{ m})\hat{\text{i}}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0\text{ N})\hat{\text{i}} + (-6.0\text{ N})\hat{\text{j}}$. The situation and coordinate axes are shown in Fig. 7-5.



(a) How much work does this force do on the crate during the displacement?

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

(a) $W = \vec{F}\cdot\vec{d} = -6.0\text{ J.}$

(b) $K_f = K_i + W = 10\text{ J} + (-6.0\text{ J}) = 4.0\text{ J.} \quad \text{(Answer)}$

7-6 | Work Done by the Gravitational Force

$$\boxed{W = Fd\cos\phi}$$

$$W_g = F_g\, d\, \cos\phi$$

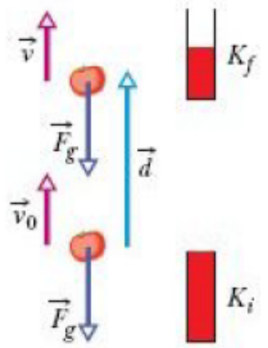
$$W_g = mgd\cos\phi \qquad \text{(work done by gravitational force)}$$

For a rising object, force $\vec{F}_g$ is directed opposite the displacement $\vec{d}$,

$$W_g = mgd\cos 180° = mgd(-1) = -mgd.$$

For falling object, force $\vec{F}_g$ is directed along the displacement $\vec{d}$

$$W_g = mgd\cos 0° = mgd(+1) = +mgd$$



Sample Problem 7-4

One of the lifts of Paul Anderson (Fig. 7-8) in the 1950s remains a record: Anderson stooped beneath a reinforced wood platform, placed his hands on a short stool to brace himself, and then pushed upward on the platform with his back, lifting the platform straight up by 1.0 cm. The platform held automobile parts and a safe filled with lead, with a total weight of 27 900 N (6270 lb).

(a) As Anderson lifted the load, how much work was done on it by the gravitational force $\vec{F}_g$?

Answer:

$$W_g = mgd\cos\phi = (27\,900\text{ N})(0.010\text{ m})(\cos 180^\circ)$$
$$= -280\text{ J}.$$

7-7 | Work Done by a Spring Force

The Spring Force

Figure 7-11*a* shows a spring in its **relaxed state**—that is, neither compressed nor extended. One end is fixed, and a particle-like object—a block, say—is attached to the other, free end. If we stretch the spring by pulling the block to the right as in Fig. 7-11*b*, the spring pulls on the block toward the left. (Because a spring force acts to restore the relaxed state, it is sometimes said to be a *restoring force.*) If we compress the spring by pushing the block to the left as in Fig. 7-11*c*, the spring now pushes on the block toward the right.

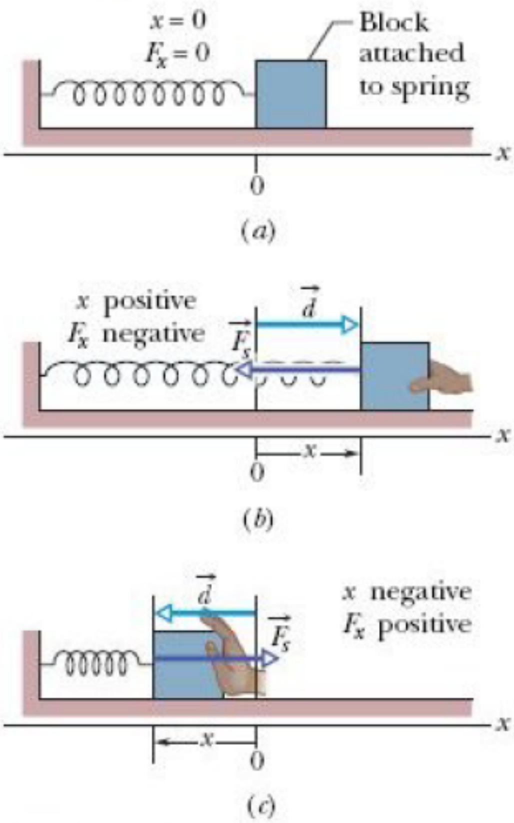


FIG. 7-11 (*a*) A spring in its relaxed state. The origin of an x axis has been placed at the end of the spring that is attached to a block. (*b*) The block is displaced by $\vec{d}$, and the spring is stretched by a positive amount x. Note the restoring force $\vec{F}_s$ exerted by the spring. (*c*) The spring is compressed by a negative amount x. Again, note the restoring force.

. The *spring force* is given by

$$\vec{F}_s = -k\vec{d} \qquad \text{(Hooke's law)}.$$

☆ The constant k is called the **spring constant** (or **force constant**)

☆. The SI unit for k is the newton per meter.

☆ The minus sign indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end.

$$F_x = -kx \qquad \text{(Hooke's law)},$$

The Work Done by a Spring Force

$$W_s = \int_{x_i}^{x_f} -F_x\,dx.$$

$$W_s = \int_{x_i}^{x_f} -kx\,dx = -k\int_{x_i}^{x_f} x\,dx$$

$$= (-\tfrac{1}{2}k)[x^2]_{x_i}^{x_f} = (-\tfrac{1}{2}k)(x_f^2 - x_i^2). \qquad (7\text{-}24)$$

Multiplied out, this yields

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force)}. \qquad (7\text{-}25)$$

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

$$If\ x_i > x_f \Rightarrow W_s = +ve$$

$$If\ x_f > x_i \Rightarrow W_s = -ve$$

If $x_i = 0$ and if we call the final position x, then Eq. 7-25 becomes

$$W_s = -\tfrac{1}{2}kx^2 \qquad \text{(work by a spring force)}. \qquad (7\text{-}26)$$

CHECKPOINT 2 For three situations, the initial and final positions, respectively, along the x axis for the block in Fig. 7-11 are (a) -3 cm, 2 cm; (b) 2 cm, 3 cm; and (c) -2 cm, 2 cm. In each situation, is the work done by the spring force on the block positive, negative, or zero?

Sample Problem 7-7

A package of spicy Cajun pralines lies on a frictionless floor, attached to the free end of a spring in the arrangement of Fig. 7-11*a*. A rightward applied force of magnitude $F_a = 4.9$ N would be needed to hold the package at $x_1 = 12$ mm.

(a) How much work does the spring force do on the package if the package is pulled rightward from $x_0 = 0$ to $x_2 = 17$ mm?

Answer:

$$k = -\frac{F_x}{x_1} = -\frac{-4.9\text{ N}}{12 \times 10^{-3}\text{ m}} = 408\text{ N/m}.$$

Now, with the package at $x_2 = 17$ mm, Eq. 7-26 yields

$$\begin{aligned} W_s &= -\tfrac{1}{2}kx_2^2 = -\tfrac{1}{2}(408\text{ N/m})(17 \times 10^{-3}\text{ m})^2 \\ &= -0.059\text{ J}. \end{aligned}$$

the spring force F_x would have to be -4.9 N

(b) Next, the package is moved leftward to $x_3 = -12$ mm. How much work does the spring force do on

the package during this displacement? Explain the sign of this work.

Calculation: Now $x_i = +17$ mm and $x_f = -12$ mm, and Eq. 7-25 yields

$$\begin{aligned} W_s &= \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 = \tfrac{1}{2}k(x_i^2 - x_f^2) \\ &= \tfrac{1}{2}(408\text{ N/m})[(17 \times 10^{-3}\text{ m})^2 - (-12 \times 10^{-3}\text{ m})^2] \\ &= 0.030\text{ J} = 30\text{ mJ}. \qquad \text{(Answer)} \end{aligned}$$

This work done on the block by the spring force is positive because the spring force does more positive work as the block moves from $x_i = +17$ mm to the spring's relaxed position than it does negative work as the block moves from the spring's relaxed position to $x_f = -12$ mm.

7-9 | Power

The time rate at which work is done by a force is said to be the **power** due to the force. If a force does an amount of work W in an amount of time Δt, the **average power**

$$P_{\text{avg}} = \frac{W}{\Delta t} \qquad \text{(average power).} \tag{7-42}$$

The **instantaneous power** P is the instantaneous time rate of doing work, which we can write as

$$P = \frac{dW}{dt} \qquad \text{(instantaneous power).} \tag{7-43}$$

The SI unit of power is the joule per second.

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s}$$

$$1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W}. \tag{7-45}$$

$$\begin{aligned} 1 \text{ kilowatt-hour} &= 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s}) \\ &= 3.60 \times 10^6 \text{ J} = 3.60 \text{ MJ}. \end{aligned} \tag{7-46}$$

$$P = \frac{dW}{dt} = \frac{F \cos \phi \, dx}{dt} = F \cos \phi \left(\frac{dx}{dt}\right),$$

or

$$P = Fv \cos \phi. \tag{7-47}$$

Reorganizing the right side of Eq. 7-47 as the dot product $\vec{F} \cdot \vec{v}$, we may also write the equation as

$$P = \vec{F} \cdot \vec{v} \qquad \text{(instantaneous power).} \tag{7-48}$$

CHECKPOINT 3 A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Sample Problem 7-11

Figure 7-16 shows constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

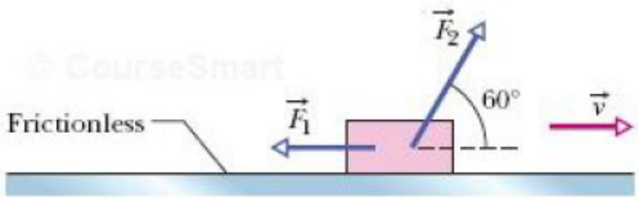


FIG. 7-16 Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a box that slides rightward across a frictionless floor. The velocity of the box is $\vec{v}$.

Calculation: We use Eq. 7-47 for each force. For force $\vec{F}_1$, at angle $\phi_1 = 180°$ to velocity $\vec{v}$, we have

$$P_1 = F_1 v \cos\phi_1 = (2.0\text{ N})(3.0\text{ m/s})\cos 180° = -6.0\text{ W}. \qquad \text{(Answer)}$$

This negative result tells us that force $\vec{F}_1$ is transferring energy *from* the box at the rate of 6.0 J/s.

For force $\vec{F}_2$, at angle $\phi_2 = 60°$ to velocity $\vec{v}$, we have

$$P_2 = F_2 v \cos\phi_2 = (4.0\text{ N})(3.0\text{ m/s})\cos 60° = 6.0\text{ W}. \qquad \text{(Answer)}$$

This positive result tells us that force $\vec{F}_2$ is transferring energy *to* the box at the rate of 6.0 J/s.

The net power is the sum of the individual powers:

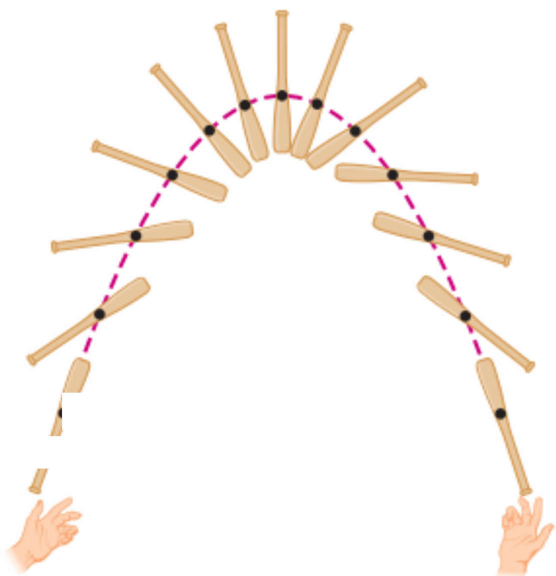
$$P_{net} = P_1 + P_2 = -6.0\text{ W} + 6.0\text{ W} = 0, \qquad \text{(Answer)}$$

Chapter 9
Center of Mass and Linear Momentum

9-2 The Center of Mass

We define the **center of mass** (com) of a system of particles (such as a person) in order to predict the possible motion of the system.

The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.



(*b*)

Fig. 9-1 (*a*) A ball tossed into the air follows a parabolic path. (*b*) The center of mass (black dot) of a baseball bat flipped into the air follows a parabolic path, but all other points of the bat follow more complicated curved paths. *(a: Richard Megna/Fundamental Photographs)*

Systems of Particles

Figure 9-2*a* shows two particles of masses m_1 and m_2 separated by distance d. We have arbitrarily chosen the origin of an x axis to coincide with the particle of mass m_1. We *define* the position of the center of mass (com) of this two-particle system to be

$$x_{\text{com}} = \frac{m_2}{m_1 + m_2} d. \tag{9-1}$$

Suppose, as an example, that $m_2 = 0$. Then there is only one particle, of mass m_1, and the center of mass must lie at the position of that particle; Eq. 9-1 dutifully reduces to $x_{\text{com}} = 0$. If $m_1 = 0$, there is again only one particle (of mass m_2), and we have, as we expect, $x_{\text{com}} = d$. If $m_1 = m_2$, the center of mass should be halfway between the two particles; Eq. 9-1 reduces to $x_{\text{com}} = \frac{1}{2}d$, again as we expect. Finally, Eq. 9-1 tells us that if neither m_1 nor m_2 is zero, x_{com} can have only values that lie between zero and d; that is, the center of mass must lie somewhere between the two particles.

Figure 9-2*b* shows a more generalized situation, in which the coordinate system has been shifted leftward. The position of the center of mass is now defined as

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}. \tag{9-2}$$

Note that if we put $x_1 = 0$, then x_2 becomes d and Eq. 9-2 reduces to Eq. 9-1, as it must. Note also that in spite of the shift of the coordinate system, the center of mass is still the same distance from each particle.

We can rewrite Eq. 9-2 as

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{M}, \tag{9-3}$$

in which M is the total mass of the system. (Here, $M = m_1 + m_2$.) We can extend this equation to a more general situation in which n particles are strung out along the x axis. Then the total mass is $M = m_1 + m_2 + \cdots + m_n$, and the location of the center of mass is

$$\begin{aligned} x_{\text{com}} &= \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{M} \\ &= \frac{1}{M}\sum_{i=1}^{n} m_ix_i. \end{aligned} \tag{9-4}$$

The subscript i is an index that takes on all integer values from 1 to n.

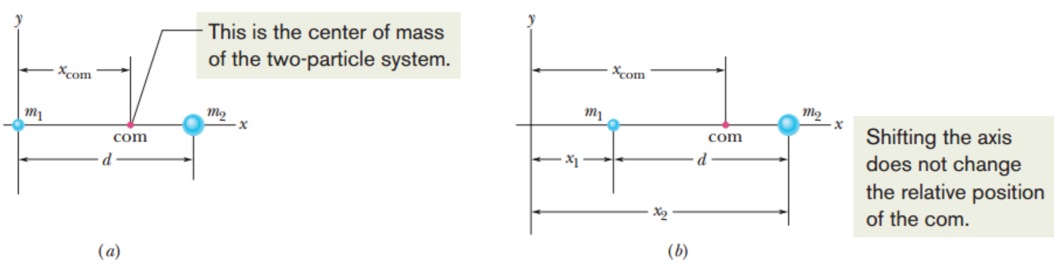


Fig. 9-2 (*a*) Two particles of masses m_1 and m_2 are separated by distance d. The dot labeled com shows the position of the center of mass, calculated from Eq. 9-1. (*b*) The same as (*a*) except that the origin is located farther from the particles. The position of the center of mass is calculated from Eq. 9-2. The location of the center of mass with respect to the particles is the same in both cases.

If the particles are distributed in three dimensions, the center of mass must be identified by three coordinates. By extension of Eq. 9-4, they are

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \qquad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \qquad z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i. \tag{9-5}$$

We can also define the center of mass with the language of vectors. First recall that the position of a particle at coordinates x_i, y_i, and z_i is given by a position vector:

$$\vec{r}_i = x_i\hat{\mathrm{i}} + y_i\hat{\mathrm{j}} + z_i\hat{\mathrm{k}}. \tag{9-6}$$

Here the index identifies the particle, and $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$, and $\hat{\mathrm{k}}$ are unit vectors pointing, respectively, in the positive direction of the x, y, and z axes. Similarly, the position of the center of mass of a system of particles is given by a position vector:

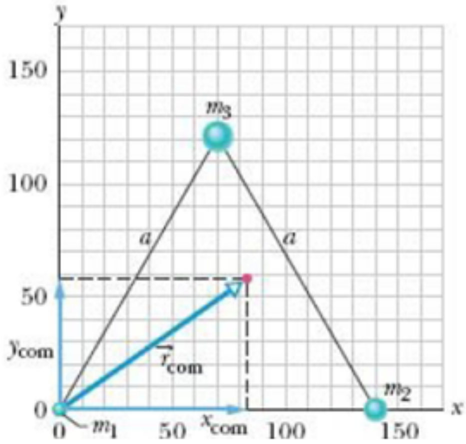
$$\vec{r}_{com} = x_{com}\hat{\mathrm{i}} + y_{com}\hat{\mathrm{j}} + z_{com}\hat{\mathrm{k}}. \tag{9-7}$$

The three scalar equations of Eq. 9-5 can now be replaced by a single vector equation,

$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i, \tag{9-8}$$

Sample Problem 9-1

Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg form an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this system?



Particle	Mass (kg)	x (cm)	y (cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	120

The total mass M of the system is 7.1 kg.

From Eq. 9-5, the coordinates of the center of mass are

$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M}$$
$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ cm})}{7.1\text{ kg}}$$
$$= 83\text{ cm} \qquad \text{(Answer)}$$

and
$$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$
$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(0) + (3.4\text{ kg})(120\text{ cm})}{7.1\text{ kg}}$$
$$= 58\text{ cm}. \qquad \text{(Answer)}$$

the center of mass is located by the position vector $\vec{r}_{com}$, which has components x_{com} and y_{com}.

9-3 | Newton's Second Law for a System of Particles

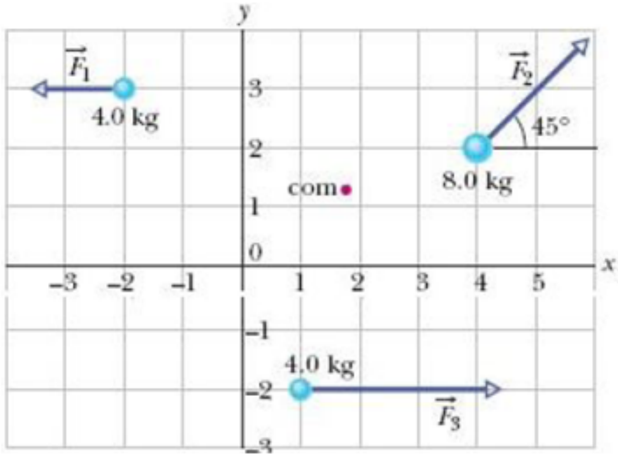
$$\vec{F}_{net} = M\vec{a}_{com}$$

1. $\vec{F}_{net}$ is the net force of *all external forces* that act on the system.
2. M is the *total mass* of the system. We assume that no mass enters or leaves the system as it moves, so that M remains constant. The system is said to be **closed.**
3. $\vec{a}_{com}$ is the acceleration of the *center of mass* of the system.

$$F_{net,x} = Ma_{com,x} \qquad F_{net,y} = Ma_{com,y} \qquad F_{net,z} = Ma_{com,z}$$

Sample Problem 9-3

The three particles in Fig. 9-7*a* are initially at rest. Each experiences an *external* force due to bodies outside the three-particle system. The directions are indicated, and the magnitudes are $F_1 = 6.0$ N, $F_2 = 12$ N, and $F_3 = 14$ N. What is the acceleration of the center of mass of the system, and in what direction does it move?



9-4 Linear Momentum

In this section, we discuss only a single particle.

The **linear momentum** of a particle is a vector quantity $\vec{p}$ that is defined as

$$\vec{p} = m\vec{v} \qquad \text{(linear momentum of a particle),} \qquad (9\text{-}22)$$

in which m is the mass of the particle and v is its velocity.

Eq. 9-22 tells us that p and v have the same direction. From Eq. 9-22,

the SI unit for momentum is the kilogram•meter per second (kg • m/s).

Newton expressed his second law of motion in terms of momentum:

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

In equation form this becomes

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}. \tag{9-23}$$

In words, Eq. 9-23 says that the net external force $\vec{F}_{\text{net}}$ on a particle changes the particle's linear momentum $\vec{p}$. Conversely, the linear momentum can be changed only by a net external force. If there is no net external force, $\vec{p}$ *cannot* change. As we shall see in Section 9-7, this last fact can be an extremely powerful tool in solving problems.

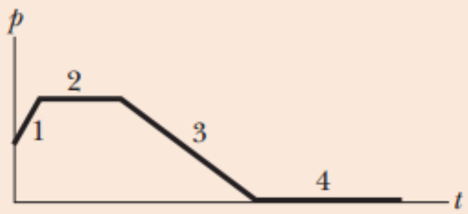
Manipulating Eq. 9-23 by substituting for $\vec{p}$ from Eq. 9-22 gives, for constant mass m,

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

Thus, the relations $\vec{F}_{\text{net}} = d\vec{p}/dt$ and $\vec{F}_{\text{net}} = m\vec{a}$ are equivalent expressions of Newton's second law of motion for a particle.

CHECKPOINT 3

The figure gives the magnitude p of the linear momentum versus time t for a particle moving along an axis. A force directed along the axis acts on the particle. (a) Rank the four regions indicated according to the magnitude of the force, greatest first. (b) In which region is the particle slowing?



Answer:

Consider slopes and equation 9-23

(a) 1,3, and then 2 and 4 tie (zero force);

(b) 3

9-5 The Linear Momentum of a System of Particles

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_n$$
$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n. \quad (9\text{-}24)$$

If we compare this equation with Eq. 9-17, we see that

$$\vec{P} = M\vec{v}_{\text{com}} \quad \text{(linear momentum, system of particles)}, \quad (9\text{-}25)$$

which is another way to define the linear momentum of a system of particles:

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

If we take the time derivative of Eq. 9-25, we find

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{v}_{\text{com}}}{dt} = M\vec{a}_{\text{com}}. \quad (9\text{-}26)$$

Comparing Eqs. 9-14 and 9-26 allows us to write Newton's second law for a system of particles in the equivalent form

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} \quad \text{(system of particles)}, \quad (9\text{-}27)$$

where $\vec{F}_{\text{net}}$ is the net external force acting on the system. This equation is the generalization of the single-particle equation $\vec{F}_{\text{net}} = d\vec{p}/dt$ to a system of many particles. In words, the equation says that the net external force $\vec{F}_{\text{net}}$ on a system of particles changes the linear momentum $\vec{P}$ of the system. Conversely, the linear momentum can be changed only by a net external force. If there is no net external force, $\vec{P}$ *cannot* change.

9-7 Conservation of Linear Momentum

Suppose that the net external force $\vec{F}_{net}$ (and thus the net impulse $\vec{J}$) acting on a system of particles is zero (the system is isolated) and that no particles leave or enter the system (the system is closed). Putting $\vec{F}_{net} = 0$ in Eq. 9-27 then yields $d\vec{P}/dt = 0$, or

$$\vec{P} = \text{constant} \qquad \text{(closed, isolated system).} \tag{9-42}$$

In words,

If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

This result is called the **law of conservation of linear momentum.** It can also be written as

$$\vec{P}_i = \vec{P}_f \qquad \text{(closed, isolated system).} \tag{9-43}$$

In words, this equation says that, for a closed, isolated system,

$$\begin{pmatrix}\text{total linear momentum}\\ \text{at some initial time } t_i\end{pmatrix} = \begin{pmatrix}\text{total linear momentum}\\ \text{at some later time } t_f\end{pmatrix}.$$

Caution: Momentum should not be confused with energy. In the sample problems of this section, momentum is conserved but energy is definitely not.

Equations 9-42 and 9-43 are vector equations and, as such, each is equivalent to three equations corresponding to the conservation of linear momentum in three mutually perpendicular directions as in, say, an *xyz* coordinate system. Depending on the forces acting on a system, linear momentum might be conserved in one or two directions but not in all directions. However,

If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

CHECKPOINT 6

An initially stationary device lying on a frictionless floor explodes into two pieces, which then slide across the floor. One piece slides in the positive direction of an x axis. (a) What is the sum of the momenta of the two pieces after the explosion? (b) Can the second piece move at an angle to the x axis? (c) What is the direction of the momentum of the second piece?

Answer:

No net external force,

(a) 0

(b) no

(c) -x

Sample Problem 9-6

One-dimensional explosion: A ballot box with mass $m = 6.0$ kg slides with speed $v = 4.0$ m/s across a frictionless floor in the positive direction of an x axis. The box explodes into two pieces. One piece, with mass $m_1 = 2.0$ kg, moves in the positive direction of the x axis at $v_1 = 8.0$ m/s. What is the velocity of the second piece, with mass m_2?

Section Exercises

sec. 9-2 The Center of Mass

•1 A 2.00 kg particle has the xy coordinates (-1.20 m, 0.500 m), and a 4.00 kg particle has the xy coordinates (0.600 m, -0.750 m). Both lie on a horizontal plane. At what (a) x and (b) y coordinates must you place a 3.00 kg particle such that the center of mass of the three-particle system has the coordinates (-0.500 m, -0.700 m)?

1. We use Eq. 9-5 to solve for (x_3, y_3).

(a) The *x* coordinates of the system's center of mass is:

Solving the equation yields $x_3 = -1.50$ m.

(b) The *y* coordinates of the system's center of mass is:

$$y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(0.500\text{ m}) + (4.00\text{ kg})(-0.750\text{ m}) + (3.00\text{ kg})\,y_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.700\text{ m}.$$

Solving the equation yields $y_3 = -1.43$ m.

sec. 9-5 The Linear Momentum of a System of Particles

•18 A 0.70 kg ball moving horizontally at 5.0 m/s strikes a vertical wall and rebounds with speed 2.0 m/s. What is the magnitude of the change in its linear momentum?

18. The magnitude of the ball's momentum change is

Chapter 7

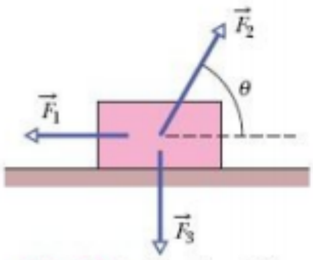


FIG. 7-28 Problem 13.

••13 Figure 7-28 shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F_1 = 5.00$ N, $F_2 = 9.00$ N, and $F_3 = 3.00$ N, and the indicated angle is $\theta = 60.0°$. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease? GO

•43 A 100 kg block is pulled at a constant speed of 5.0 m/s across a horizontal floor by an applied force of 122 N directed 37° above the horizontal. What is the rate at which the force does work on the block? SSM ILW

13. (a) The forces are constant, so the work done by any one of them is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d \cos\phi_1 = (5.00\,\text{N})(3.00\,\text{m})\cos 0° = 15.0 \text{ J}.$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d \cos\phi_2 = (9.00\,\text{N})(3.00\,\text{m})\cos 120° = -13.5 \text{ J}.$$

Force $\vec{F}_3$ is perpendicular to the displacement, so

$$W_3 = F_3 d \cos\phi_3 = 0 \text{ since } \cos 90° = 0.$$

The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0 \text{ J} - 13.5 \text{ J} + 0 = +1.50 \text{ J}.$$

(b) If no other forces do work on the box, its kinetic energy increases by 1.50 J during the displacement.

43. The power associated with force $\vec{F}$ is given by $P = \vec{F} \cdot \vec{v}$, where $\vec{v}$ is the velocity of the object on which the force acts. Thus,

$$P = \vec{F} \cdot \vec{v} = Fv\cos\phi = (122 \text{ N})(5.0 \text{ m/s})\cos 37° = 4.9 \times 10^2 \text{ W}.$$