CH. 1.1
1.2

Notes

4

- التركيز على المفاهيم الأساسية.
- شرح أبواب المنهج حسب الخطة.
- أمثلة توضيحية وتدريبات.
- نماذج اختبارات.

السعدي

رياضيات ١١٠
Math.110

جمال السعدي
استاذ الرياضيات والإحصاء للمرحلة الجامعية
0566664790

Functions

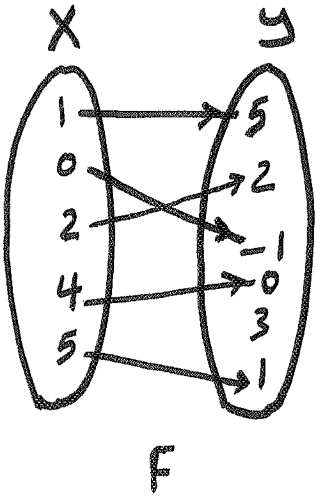
The relation from x to y is function

$$F : x \longrightarrow y$$

X is independent variable متغير مستقل

y is dependent variable متغير تابع

$$y = f(x)$$



- المجال Domain: $X = \{1, 0, 2, 4, 5\}$
- Codomain المجال المقابل : $y = \{5, 2, -1, 0, 3, 1\}$
- Range المدى $= \{5, 2, -1, 0, 1\}$

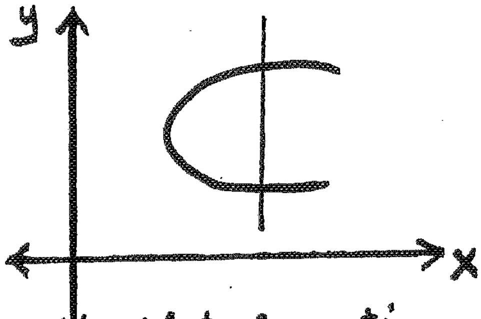
* المدى هو مجموعة صور عناصر المجموعة X

* المدى هو المجموعة F(x)

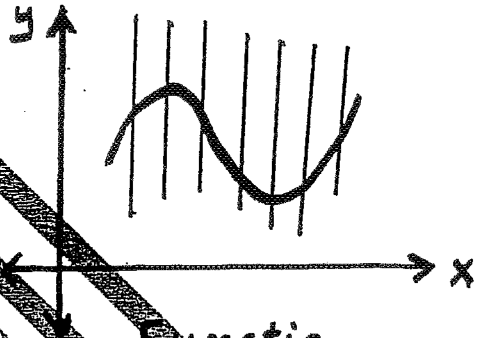
* Range $\subseteq$ codomain

Vertical line test . اختبار الخط الرأسي

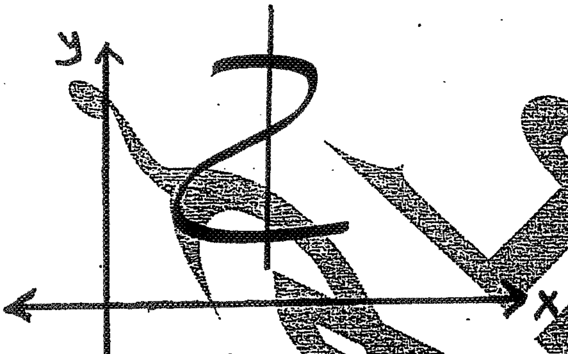
* اذا قطع خط رأسي المنحنى في أكثر من نقطة فإن هذا المنحنى لا يمثل داله.
* اذا قطع أي خط رأسي المنحنى في نقطة واحده فإن هذا المنحنى يمثل داله.



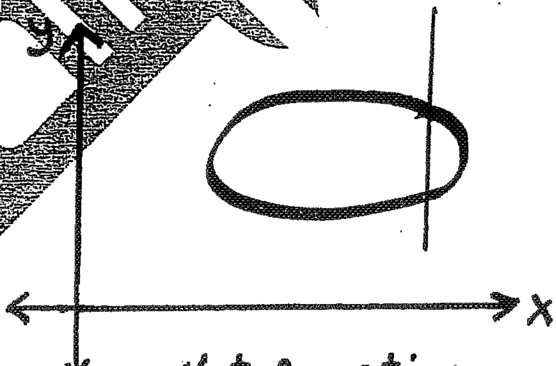
Not function



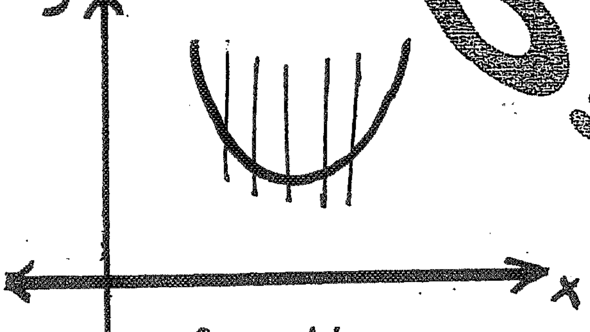
Function



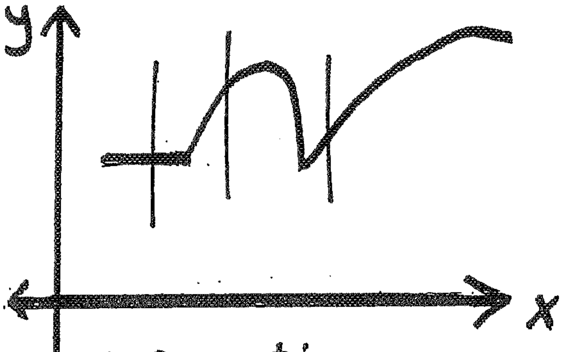
Not function



Not function



Function



Function

Domain المجال

- **Polynomial** كثيره الحدود

$R = (-\infty , \infty)$ ⟵ مجالها

$F(x) = 2x^3 - x^2 + x + 1$

$\downarrow DF = R = (-\infty , \infty)$

$F(x) = 2x + 1$
$F(x) = -2$
$F(x) = x^2$

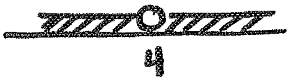
$\left.\begin{array}{l} F(x) = 2x+1 \\ F(x) = -2 \\ F(x) = x^2 \end{array}\right\} DF = R = (-\infty , \infty)$

- **Rational** الداله الكسري

$R - \{$ أصفار المقام $\}$ ⟵ مجالها

$F(x) = \frac{x+3}{x-4}$

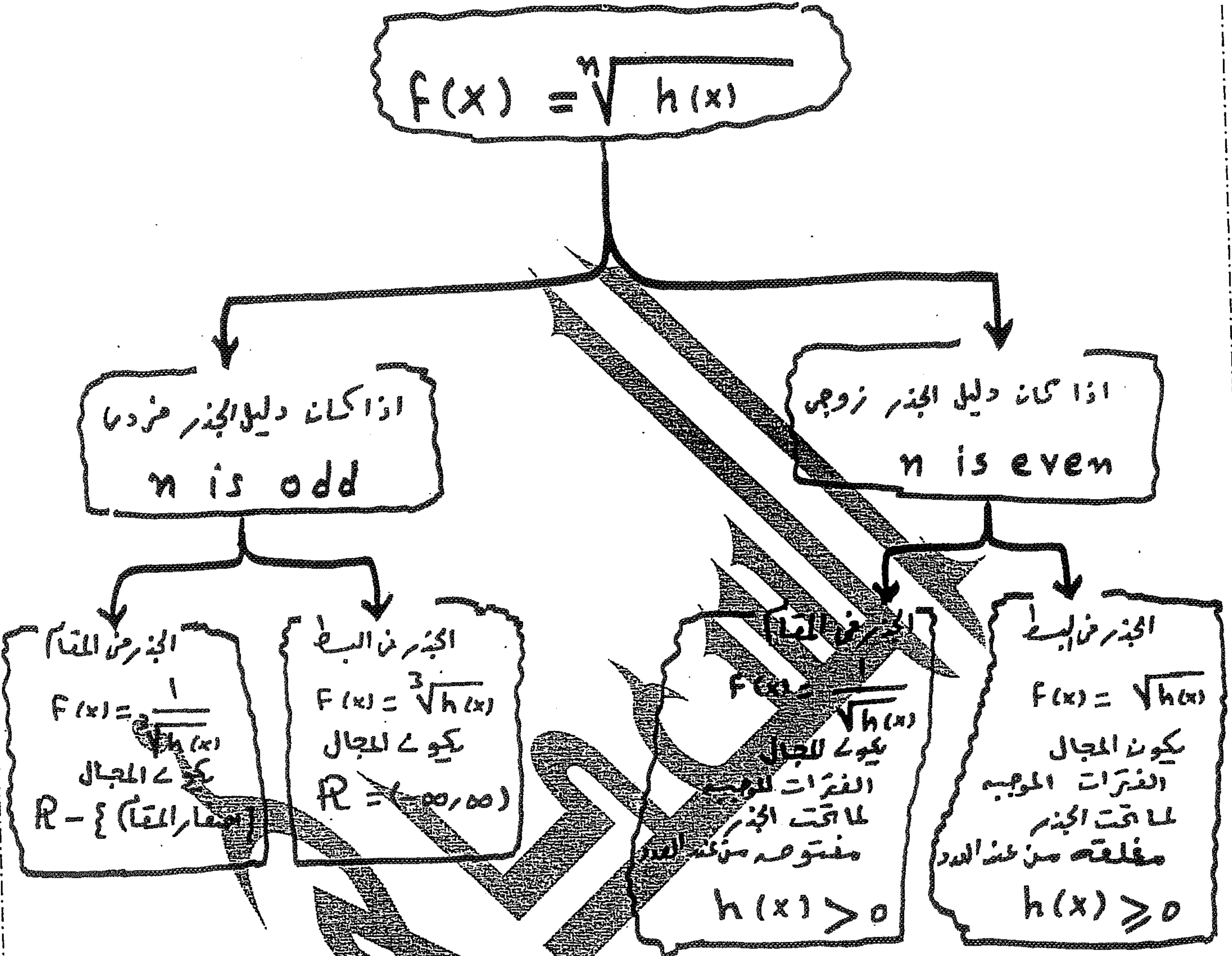
$\downarrow DF = R - \{4\}$



$= (-\infty , 4) \cup (4 , \infty)$

* لإيجاد أصفار المقام
نضع المقام = ٠
$\Rightarrow x - 4 = 0$
$x = 4$

* مجال الدالة الجذرية (Radical)



$f(x) = \sqrt[n]{h(x)}$

اذا كان دليل الجذر فرديا
n is odd

الجذر في المقام
$F(x) = \frac{1}{\sqrt[3]{h(x)}}$
يكون المجال
$R - \{$ (أصفار المقام)

الجذر في البسط
$F(x) = \sqrt[3]{h(x)}$
يكون المجال
$R = (-\infty, \infty)$

اذا كان دليل الجذر زوجي
n is even

الجذر في المقام
$F(x) = \frac{1}{\sqrt{h(x)}}$
يكون المجال
الفترات الموجبة
لما تحت الجذر
مفتوحة من عند الصفر
$h(x) > 0$

الجذر في البسط
$F(x) = \sqrt{h(x)}$
يكون المجال
الفترات الموجبة
لما تحت الجذر
مغلقة من عند الصفر
$h(x) \geq 0$

Find the domain of the functions:

مواصفات كثيرة الحدود
- * خالية من X تحت الجذر
- * ,, ,, X ذات أس سالب
- * ,, ,, X ذات أس كسر
- * ,, ,, X بالمقام.

* $f(x) = 2x^3 + x^2 - 5x + 1$
- polynomial كثيرة حدود
- degree 3
- coefficients: 2 , 1 , −5 , 1
- Domain : $R = (-\infty, \infty)$

Example :

Find the domain of the functions:

① $F(x) = \dfrac{2x+1}{4x-8}$

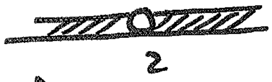
* اصفار المقام

$4x - 8 = 0$

$4x = 8 \quad \boxed{\div 4}$

$x = 2$

$\Rightarrow DF = R - \{2\}$



$= (-\infty, 2) \cup (2, \infty)$

② $F(x) = \dfrac{5}{x^2 - 4}$

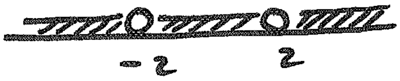
* اصفار المقام

$x^2 - 4 = 0$

$x^2 = 4 \quad \text{by} \sqrt{\ }$

$x = \pm 2$

$\Rightarrow DF = R - \{-2, 2\}$



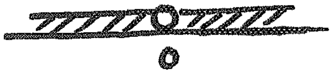
$= (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

③ $F(x) = \dfrac{3x^2 - 1}{x}$

* اصفار المقام

$x = 0$

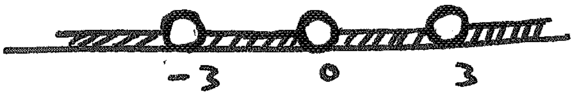
$\Rightarrow DF = R - \{0\}$



$= (-\infty, 0) \cup (0, \infty)$

④ $F(x) = \frac{x^2-1}{x^3-9x}$

$\Rightarrow DF = R - \{0, 3, -3\}$



$= (-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)$

* اصفار المقام

$x^3 - 9x = 0$

$x(x^2-9) = 0$

$\underline{\underline{x = 0}}$ | $x^2 - 9 = 0$

$x^2 = 9$

$\underline{\underline{x = \pm 3}}$

---

⑤ $F(x) = \frac{2x+1}{x^2+4}$

$\Rightarrow DF = R$

$= (-\infty, \infty)$

* المقام ليس له اصفار

$x^2 +$ عدد

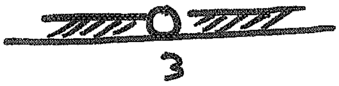
دائماً ليس له اصفار

---

⑥ $F(x) = (2x-6)^{-2}$

$F(x) = \frac{1}{(2x-6)^2}$

$\Rightarrow DF = R - \{3\}$



$= (-\infty, 3) \cup (3, \infty)$

* اصفار المقام

$(2x-6)^2 = 0$

$2x - 6 = 0$

$2x = 6$

$x = 3$

---

⑦ $F(x) = 3x^{-2} \Rightarrow \quad = \frac{3}{x^2} \Rightarrow DF = R - \{0\}$

Absolute value

① $F(x) = |x^2 - 2x - 8|$

$DF = R = (-\infty, \infty)$

---

② $F(x) = \dfrac{2x-1}{|x-3|}$

$DF = R - \{3\}$

$= (-\infty, 3) \cup (3, \infty)$

* أصفار المقام

$|x-3| = 0$

$x - 3 = 0$

$x = 3$

---

③ $F(x) = \dfrac{2x+1}{|x|-2}$

$DF = R - \{-2, 2\}$

$= (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

* أصفار المقام

$|x| - 2 = 0$

$|x| = 2$

$x = \pm 2$

---

④ $F(x) = \dfrac{x^2-4}{|x|+2}$

∵ المقام ليس له أصفار

$\therefore DF = R = (-\infty, \infty)$

* أصفار المقام

$|x| + 2 = 0$

$|x| = -2$

discard

مرفوض لأنه

$|x| \geq 0$

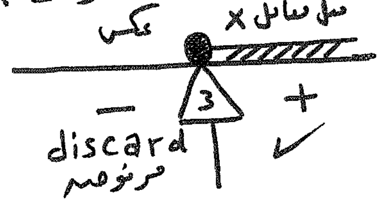
Radical function

① $F(x) = \sqrt{2x-6}$

$2x - 6 = 0$

$2x = 6$ ÷2

$x = 3$

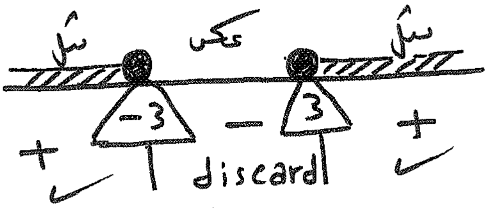


∴ $DF = [3, \infty)$

* جذر تربيعى
دليل الجذر $n = 2$
زوجى even
∴ المجال هو $2x-6 \geqslant 0$
اى الفترات الموجبه لما تحت الجذر
مغلقه من عند العدد

---

② $F(x) = \sqrt{x^2-9}$



∴ $DF = (-\infty, -3] \cup [3, \infty)$

ممكن كتابه المجال بشكل آخر وهو

$DF = R - (-3, 3)$

اى الخط كله ماعدا الفتره السالبه

* الجذر تربيعى
∴ المجال هو الفترات الموجبه
لما تحت الجذر ومغلقه
من عند العدد

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

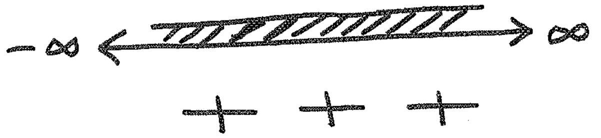
---

③ $F(x) = \dfrac{1}{\sqrt{x^2-9}}$

* نفس حل المثال السابق ولكن الجذر فى المقام
∴ تكون الفترات مفتوحه حيث $x^2-9 > 0$

∴ $DF = (-\infty, -3) \cup (3, \infty)$

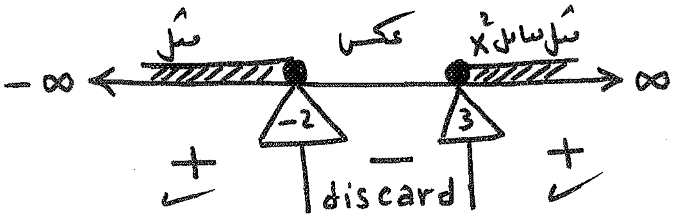
④ $F(x) = \sqrt{x^2+9}$



$\therefore DF = R = (-\infty, \infty)$

* مجموع المربعين
$x^2 + 9$
$x^2 +$ عدد
دائماً كميه موجبه
على خط الأعداد كاملاً

⑤ $F(x) = \sqrt{x^2 - x - 6}$



$\therefore DF = (-\infty, -2] \cup [3, \infty)$

* جذر تربيعي في البسط
∴ المجال هو الفترات
الموجبه مغلقه من عند العدد
$x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$
$x = 3 \mid x = -2$

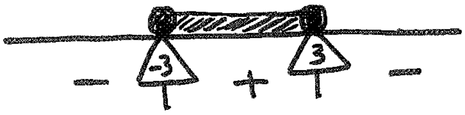
⑥ $F(x) = \dfrac{2x-1}{\sqrt{x^2 - x - 6}}$

نفس المثال السابق
ولكن الفترات مفتوحه من عند العدد

$\therefore DF = (-\infty, -2) \cup (3, \infty)$

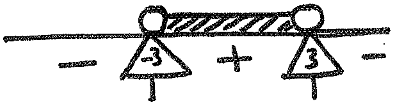
* جذر تربيعي في المقام
∴ المجال هو الفترات
الموجبه مفتوحه من عند العدد

⑦ $F(x) = \sqrt{9-x^2}$



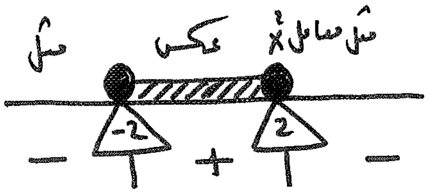
$DF = [-3, 3]$

⑧ $F(x) = \dfrac{2x}{\sqrt{9-x^2}}$



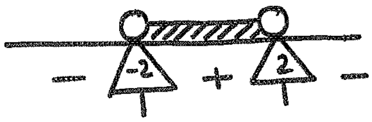
$DF = (-3, 3)$

⑨ $F(x) = \sqrt[4]{4-x^2}$



$DF = [-2, 2]$

⑩ $F(x) = \dfrac{2x}{\sqrt[4]{4-x^2}}$



نفس المثال رقم ⑨ ولكن الجذر في المقام

∵ الفترة تكون مفتوحة $4-x^2>0$

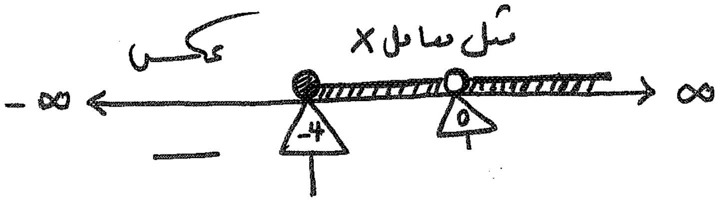
$\therefore DF = (-2, 2)$

---

⑪ $F(x) = \sqrt{x+4} + \dfrac{2}{x}$

فترات موجبه
$x+4 \geq 0$

اصفار المقام



$\therefore DF = [-4, 0) \cup (0, \infty)$

بشكل آخر $= [-4, \infty) - \{0\}$

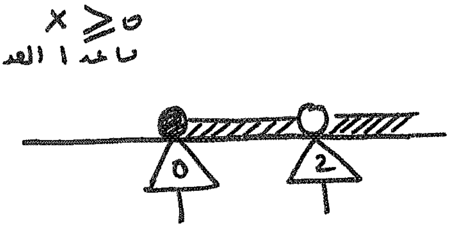
---

⑫ $F(x) = \dfrac{x - \sqrt{x}}{x - 2}$ $\longrightarrow$

الفترات الموجبه
لما تحت الجذر
ماعدا اصفار
المقام.

$x - 2 = 0$
$x = 2$

$x \geq 0$
ماعدا العدد 2 .



$DF = [0, 2) \cup (2, \infty)$

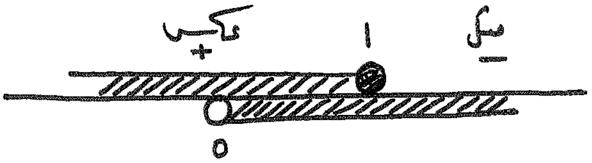
بشكل آخر $= [0, \infty) - \{2\}$

⑬ $F(x) = \frac{1}{\sqrt{x}} + \sqrt{1-x}$

* الفتره الموجبه مغلقه من العدد 1 لأنه الجذر في البسط

* الفتره الموجبه مفتوحه من عند العدد 0 لأنه الجذر في المقام.

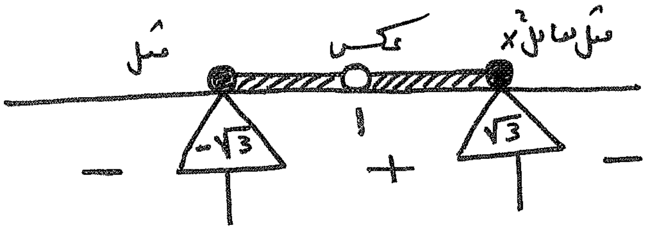
* ثم نوجد تقاطع الفترتين



$\therefore Df(x) = (0, 1]$

⑭ $F(x) = \frac{\sqrt{3-x^2}}{x-1}$

* المجال هو الفترات الموجبه لما تحت الجذر، مغلقه من عند العدد لوجود الجذر في البسط ما عدا أصفار المقام



* $3 - x^2 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

* أصفار المقام

$x - 1 = 0$

$x = 1$

$Df = [-\sqrt{3}, 1) \cup (1, \sqrt{3}]$

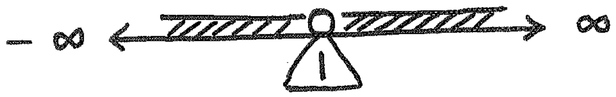
بشكل آخر $= [-\sqrt{3}, \sqrt{3}] - \{1\}$

⑮ $F(x) = \frac{\sqrt{x^2+1}}{x-1}$

* ما تحت الجذر x^2+1
كمية موجبة دائماً
لأنه مجموع مربعين
∴ مجال البسط هو
$R = (-\infty, \infty)$

* أصفار المقام
$X - 1 = 0$
$X = 1$

مجال الداله $f(x)$
هو مجال البسط
ماعدا أصفار المقام



$$\therefore DF = (-\infty, 1) \cup (1, \infty)$$
$$= R - \{1\}$$

---

⑯ $f(x) = \sqrt{\frac{x^2+1}{x-1}}$

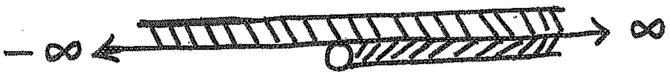
∵ نُعيد كتابة الداله بالشكل التالي
وهو توزيع الجذر على البسط والمقام

$$F(x) = \frac{\sqrt{x^2+1}}{\sqrt{x-1}}$$

* نوجد مجال البسط
وهو R
لوجود مجموع مربعين
تحت الجذر

* نوجد مجال المقام

* ثم نوجد تقاطع
المجالين
فيكون هو مجال
الداله $F(x)$



مفتوحة لوجود الجذر $\sqrt{x-1}$
في المقام.

$$DF = (1, \infty)$$

* في حالة الجذور التكعيبية

- اذا كان الجذر في البسط يكون المجال هو $R = (-\infty,\infty)$ →
- اذا كان الجذر في المقام يكون المجال هو $R - \{$اصفار المقام$\}$ →

⑰ $F(x) = \sqrt[3]{x^2-9}$

- الجذر التكعيبي في البسط

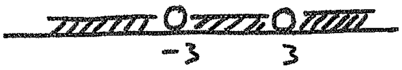
∴ المجال هو ← $Df = R = (-\infty,\infty)$

---

⑱ $F(x) = \dfrac{2x}{\sqrt[3]{x^2-9}}$

- الجذر التكعيبي في المقام

∴ المجال هو $R - \{$اصفار المقام$\}$



∴ $Df = R - \{-3, 3\}$

بشكل آخر $= (-\infty,-3) \cup (-3,3) \cup (3,\infty)$

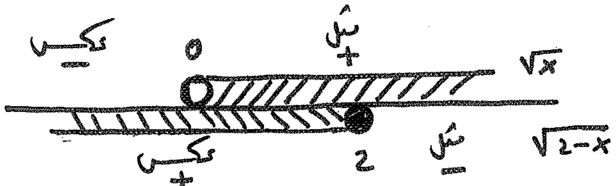
* اصفار المقام
$x^2-9=0$
$x^2=9$
$x^2=\pm 3$

---

⑲ $F(x) = \sqrt{x} + \sqrt{2-x}$

* نوجد مجال $\sqrt{x}$ ، مجال $\sqrt{2-x}$

ثم نوجد تقاطع المجالين فيكون هو مجال الداله



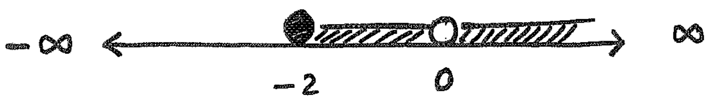
الجزء المشترك من التظليل هو مجال الداله $f(x)$

$DF = (0, 2]$

⑳ $F(x) = \frac{2}{x} + \sqrt{x+2}$

* نوجد مجال الجذر / $\sqrt{x+2}$

ثم نحذف منه اصفار (المقام) $x=0$



$DF = [-2,0) \cup (0,\infty)$

بشكل آخر $= [-2, \infty) - \{0\}$

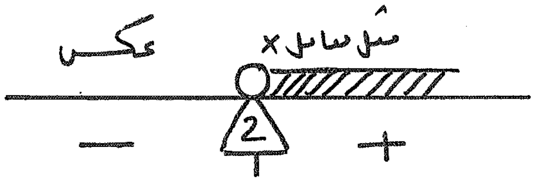
㉑ $F(x) = \ln(x-2)$

* الداله اللوغاريتميه مجالها هو الفترات الموجبه المفتوحه دائماً

لما داخل ln

$x-2=0$

$x=2$



$\therefore DF = (2, \infty)$

㉒ $F(x) = \sqrt{2 - \sqrt{x}}$

* نوجد تقاطع الفترتين :

ما تحت الجذر الأصغر ، ما تحت الجذر الأكبر

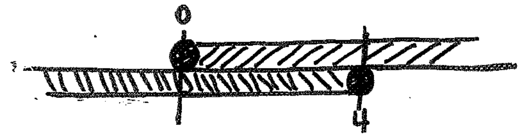
$x \geq 0$

$2 - \sqrt{x} \geq 0$

$-\sqrt{x} \geq -2$

$\sqrt{x} \leq 2$

$x \leq 4$



مجال الدالة F(x) هو الجزء المشترك في التظليل .

$\therefore \; DF = [0, 4]$

㉓ $F(x) = \sqrt{\sqrt{x} - 2}$

* نوجد تقاطع الفترتين :

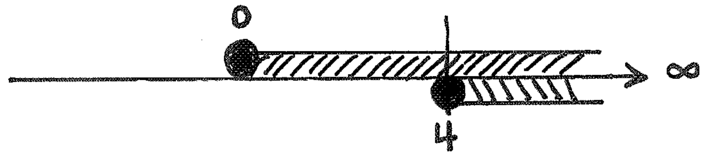
ما تحت الجذر الأصغر ، ما تحت الجذر الأكبر

$x \geq 0$

$\sqrt{x} - 2 \geq 0$

$\sqrt{x} \geq 2$

$x \geq 4$

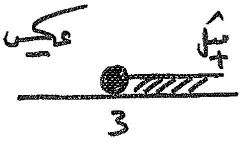


مجال الدالة F(x) هو الجزء المشترك في التظليل .

$\therefore \; DF = [4, \infty)$

㉔ $$F(x) = \frac{\sqrt{2x-6}}{\sqrt[3]{x^2-25}}$$

* نوجد مجال البسط $\sqrt{2x-6}$ ←



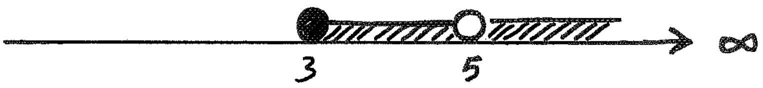
و نستبعد منه أصفار المقام

$x^2 - 25 = 0$

$x^2 = 25$

$x = \pm 5$

-5 أصلاً غير موجوده في مجال الجذر $\sqrt{2x-6}$ لكن 5 موجوده في مجال الجذر $\sqrt{2x-6}$ فنستبعد لأنها تمثل صفر للمقام .



Find the domain and range:

① $y = x^2$

كثيره حدود درجه ثانيه

* Domain $= (-\infty, \infty)$
* Range $= [0, \infty)$

---

② $y = \sqrt{x}$ داله جذريه

* Domain $= [0, \infty)$ الفتره الموجبه
* Range $= [0, \infty)$

---

③ $y = \sqrt{4-x}$ داله جذريه

* Domain $= (-\infty, 4]$ الفتره الموجبه
* Range $= [0, \infty)$

---

④ $y = \sqrt{1-x^2}$ داله جذريه

* Domain $= [-1, 1]$ الفتره الموجبه
* Range $= [0, 1]$

كيفيه ايجاد الـ Range

① اذا كانت فتره المجال بها
سالب وموجب نعوض بطرفي
فتره المجال وكذلك الصفر
في الداله المعطاه
ثم نأخذ اصغر وأكبر النتائج
فتكون هي فتره المدى

② اذا كانت فتره المجال
كلها موجبه أو كلها سالبه
نعوض بطرفي فتره المجال
في الداله المعطاه
فتكون النتائج هي فتره المدى

③ في حاله الرسم
المدى هو الفترات
التي تناظر الرسم على محور y

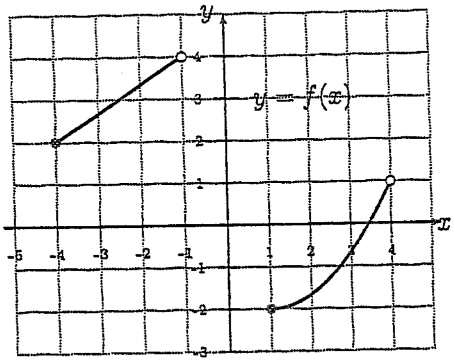
The accompanying figure shows the graph of $y = f(x)$. Then the domain of f is

(a) $[-4, -1) \cup [1, 4)$

(b) $[-4, -1] \cup (1, 4]$

(c) $[-4, -1) \cup (1, 4)$

(d) $[-4, -1] \cup (1, 4]$



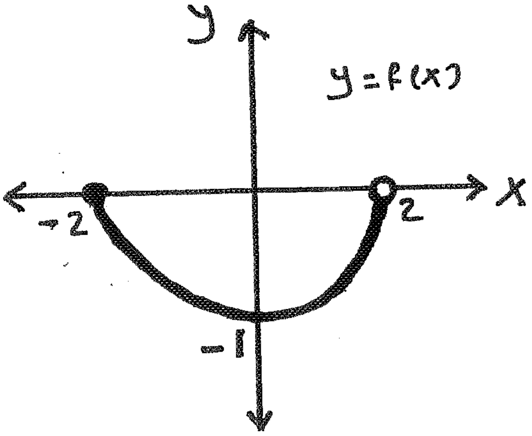
* Domain المجال

$[-4, -1) \cup [1, 4)$ هو ما يأخذ الرسم على محور x

* Range اما اذا طلب المدى

هو ما يأخذ الرسم على محور y

$[2, 4) \cup [-2, 1)$



* Domain

$[-2, 2)$ من محور x

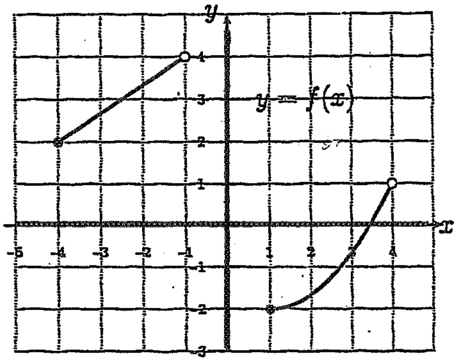
* Range

$[-1, 0]$ من محور y

Find The Domain ⟶ ماياخذ المنحنى على محور X
and Range ⟶ ماياخذ المنحنى علا محور y
for the functions :

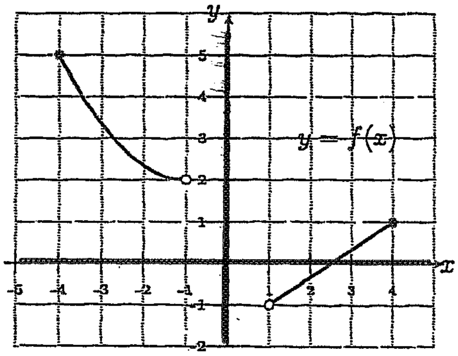
① Domain = [-4, -1) U [1, 4)

Range = [2, 4) U [-2, 1)



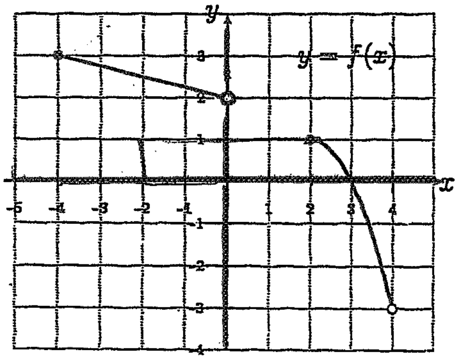
② Domain = [-4, -1) U (1, 4]

Range = (2, 5] U (-1, 1]



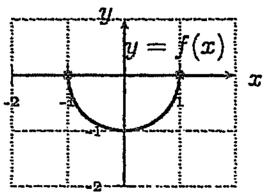
③ Domain = [-4, 0) U [2, 4)

Range = (2, 3] U (-3, 1]



④ Domain = [-1, 1]

Range = [-1, 0]



Note : المدى في حالة الدوال الجذرية على الصور الآتية .

① $y = \sqrt{ax+b}$

* داله جذرية أعلى محور X

Range $= [0, \infty)$

---

② $y = -\sqrt{ax+b}$

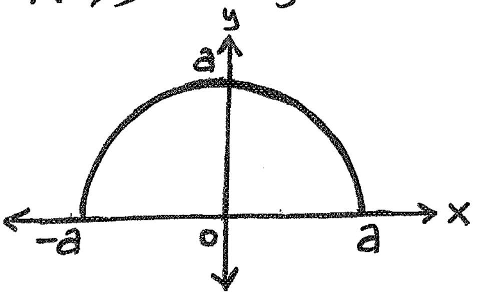
* داله جذرية أسفل محور X

Range $= (-\infty, 0]$

---

③ $y = \sqrt{a^2 - x^2}$

* نصف دائره اعلى محور X

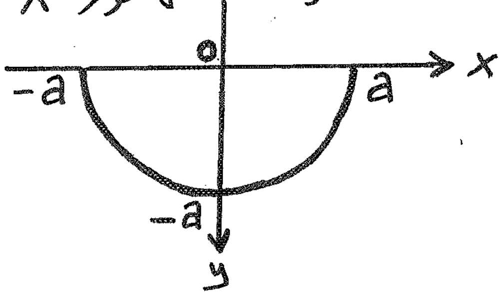


Range $= [0, a]$

---

④ $y = -\sqrt{a^2 - x^2}$

* نصف دائره أسفل محور X



Range $= [-a, 0]$

* If: The domain of $y = f(x)$ is $\underline{\underline{[-2, 6]}}$

Find the domain of $g(x)$ where

① $g(x) = f(x-2)$

Domain $g(x)$ $\xrightarrow{\text{اضافه } (+2) \text{ لـ } Df(x)}$ $[0, 8]$

---

② $g(x) = f(x+2)$

Domain $g(x)$ $\xrightarrow{\text{اضافه } (-2) \text{ لـ } Df(x)}$ $[-4, 4]$

---

③ $g(x) = f(2x)$

Domain $g(x)$ $\xrightarrow{\text{قسمه } Df(x) \text{ على } ②}$ $[-1, 3]$

---

④ $g(x) = f\left(\frac{x}{2}\right)$

Domain $g(x)$ $\xrightarrow{\text{ضرب } Df(x) \text{ في } ②}$ $[-4, 12]$

••• Types of Functions . انواع الدوال

① Polynomials كثيرات الحدود

$$F(x) = a_n X^n + a_{n-1} X^{n-1} + \cdots\cdots + a_1 X + a_0$$

* degree $= n$ أس

* Coefficients المعاملات : $a_n , a_{n-1} , \cdots , a_1 + a_0$

* Domain $f = R = (-\infty , \infty)$

Example :

$$F(x) = 3x^4 - 2x^3 + x - 1$$

* degree $= 4$ (quartic function)

* Coefficients : $a_4 = 3$, $a_3 = -2$, $a_2 = 0$, $a_1 = 1$, $a_0 = -1$

* Domain $F = R = (-\infty , \infty)$

- $f(x) = ax^3 + bx^2 + cx + d$ (Cubic fun.)

تكعيبيه.

$F(x) = 2x^3 + x^2 - x + 3$

* degree = 3 (cubic)
* coefficients : 2 , 1 , -1 , 3
* Domain f = R = $(-\infty , \infty)$

- $f(x) = ax^2 + bx + c$ (quadratic function)

تربيعيه

$F(x) = x^2 - 3x + 2$ (quadratic)

* degree = 2
* coefficients : 1 , -3 , 2
* Domain f = R = $(-\infty , \infty)$

- $f(x) = ax + b$ (Linear function)

$f(x) = -3x + 2$ (linear)

خطيه

* degree =
* Coefficints :
* Domain f =

- $f(x) = a$ (constant function)

$f(x) = 2$

ثابته

* degree = 0 * coefficints : 2 * D f = R = $(-\infty, \infty)$

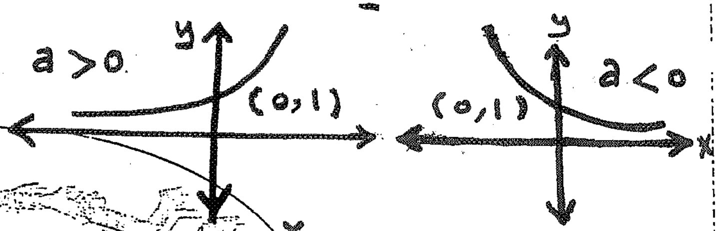
② Power function داله القوى

$F(x) = x^a$ where a is postive integer.

- $f(x) = x^3$, $f(x) = x^2$, $F(x) = x$ are power functions.

③ Exponential function داله أسيه

- $F(x) = a^x$

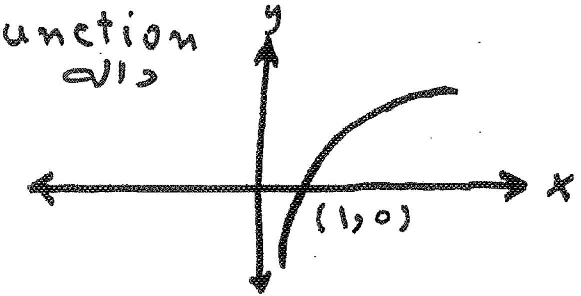


$F(x) = 2^x$ $F(x) = (\frac{1}{2})^x$

are exponential function

* Domain = R = $(-\infty, \infty)$ * Range = $(0, \infty)$

④ Logarithm function
داله لوغاريتميه



$F(x) = \log_a x$

* Domain $(0, \infty)$ * Range $(-\infty, \infty)$

⑤ Trigonometric functions دوال مثلثيه

- $f(x) = \sin x$ $f(x) = \cos x$ $f(x) = \tan x$

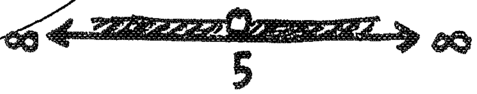
⑥ Rational function الداله الكسريه

$$f(x) = \frac{h(x)}{g(x)}$$

where $h(x)$ ، $g(x)$ are polynomial and $g(x) \neq 0$

$f(x) = \frac{x^2}{2x-10}$ rational function

$D\,f = R - \{5\}$



$= (-\infty, 5) \cup (5, \infty)$

⑦ Radical function الداله الجذريه

$F(x) = \sqrt[3]{x^2-1}$ ، $f(x) = \sqrt{x^2-1}$

⑧ Algebric Function الداله الجبريه

$* F(x)=\sqrt{x^2+3}$ $* F(x) = \frac{x-2}{\sqrt{x}+1}$ $* F(x) = \sqrt{x} - x^2$ $* F(x)= X^2 + \frac{X}{\sqrt{X-1}}$

Example :

1) The polynomial $f(x)=3x^2-2x+5$ is

A linear B quadratic C cubic D quartic

2) The polynomial $f(x)=-3x^4-2x^3+5x-1$ is

A linear B quadratic C cubic D quartic

3) The polynomial $f(x)=2x+5$ is

A linear B quadratic C cubic D quartic

4) The polynomial $f(x)=5x^3-2x+1$ is

A linear B quadratic C cubic D quartic

5) The polynomial $f(x)=\sqrt{13}$ is

A linear B quadratic C cubic D constant

6) The zeros of x^2-2x-8 are

A $-4,2$ B $-2,4$ C $2,4$ D $-4,-2$

7) The zeros of $f(x)=x^3+3x^2-x-3$ are

A $-3,-1,1$ B $-3,-1$ C $-1,1$ D $-3,1$

8) The zeros of $f(x)=x^4-4x^2+3$ are

A $\sqrt{3},1$ B $\sqrt{3},\pm 1$ C $\pm\sqrt{3},\pm 1$ D $\pm\sqrt{3},1$

9) The zeros of $f(x)=(x^2-4)^2$ are

A 2 B ± 2 C -2 D ± 4

10) The zeros of $f(x)=x(x-1)^2(x+2)^3$ are

A $-2,1$ B $-2,0,1$ C $-2,0$ D $-1,0,2$

11) The zeros of the function $f(x)=5x^2+3x-2$ are

A $-1,\frac{2}{5}$ B $-1,-\frac{2}{5}$ C $1,\frac{2}{5}$ D $-\frac{2}{5},1$

12) The leading coefficient of $f(x)=5x^4-3x^5-2x+1$ is

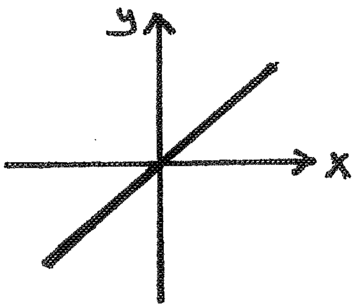
A -3 B 3 C -5 D 5

13) The leading term of $f(x)=5x^4-3x^5-2x+1$ is

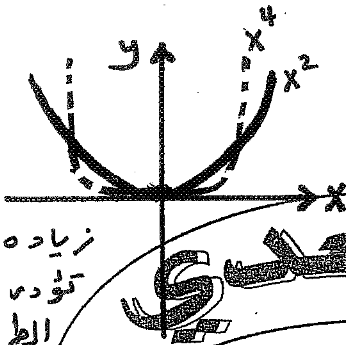
A $3x^5$ B $-3x^5$ C $-5x^5$ D $5x^4$

رسم الدوال المشهوره :

① $y = x$

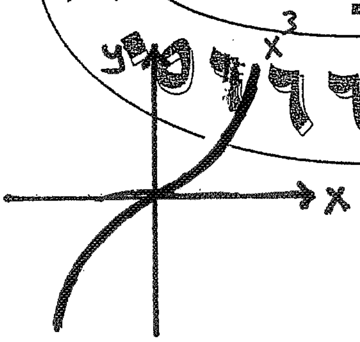


② $y = x^2$

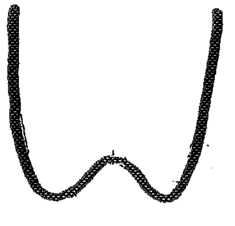


زياده الأس
تؤدي إلى اقتراب
الطرف العلويا للمنحنى
من محور y

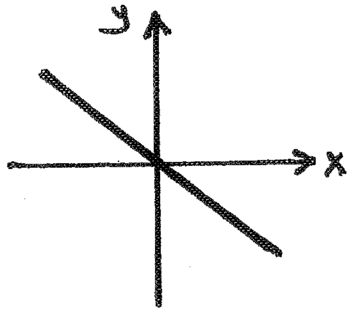
③ $y = x^3$



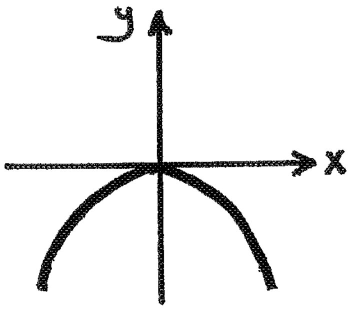
④ $y = x^4$



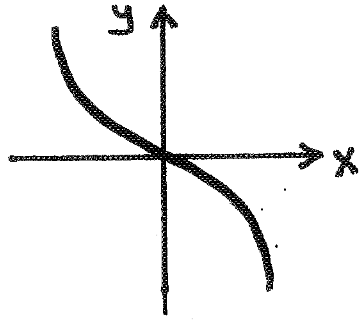
⑤ $y = -x$



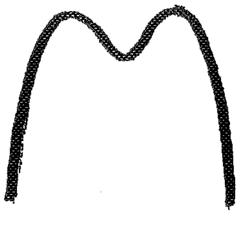
⑥ $y = -x^2$



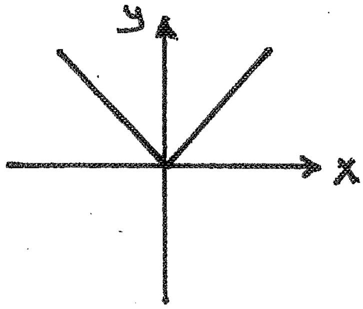
⑦ $y = -x^3$



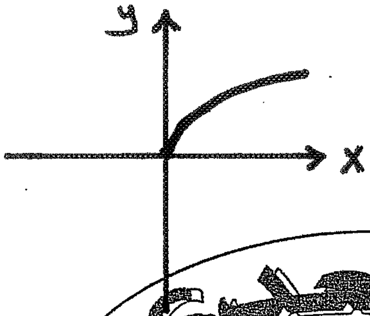
⑧ $y = -x^4$



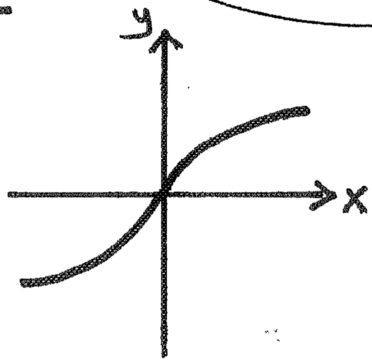
⑨ $y = |x|$



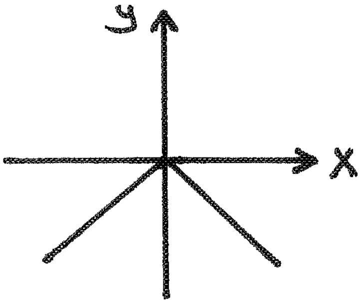
⑩ $y = \sqrt{x}$



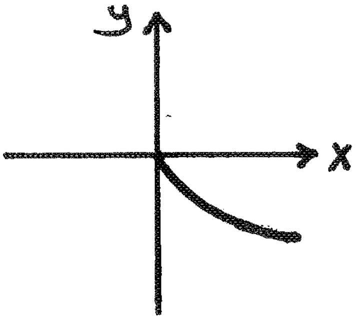
⑪ $y = \sqrt[3]{x}$



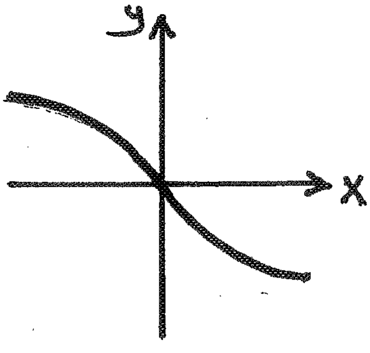
⑫ $y = -|x|$



⑬ $y = -\sqrt{x}$



⑭ $y = -\sqrt[3]{x}$



••• Increasing and decreasing :

* $f(x)$ is increasing if $x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$

* $f(x)$ is decreasing if $x_2 > x_1 \Rightarrow f(x_2) < f(x_1)$

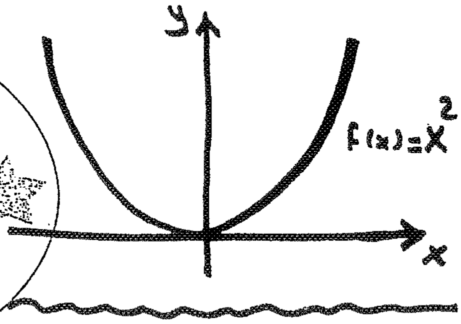
* $f(x)$ is constant if $x_2 > x_1 \Rightarrow f(x_2) = f(x_1)$

Example:

- $f(x) = x^2$

$F(x)$ is increasing in $[0, \infty)$

$F(x)$ is decreasing in $(-\infty, 0]$

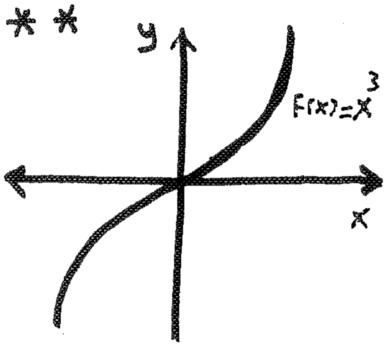


- $F(x) = x^3$

** في حاله الداله التكعيبيه

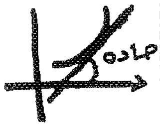
- اذا كان معامل x^3 موجب تكون الداله تزايديه .
- " " " x^3 سالب " " تناقصيه .

$F(x)$ is increasing in $(-\infty, \infty)$



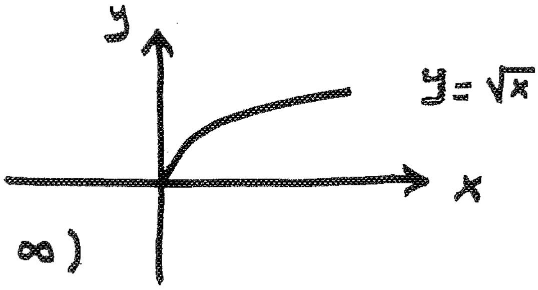
ملحوظه هامه :

* اذا كان المماس للمنحنى يصنع زاويه حاده مع محور X تكون الداله تزايديه



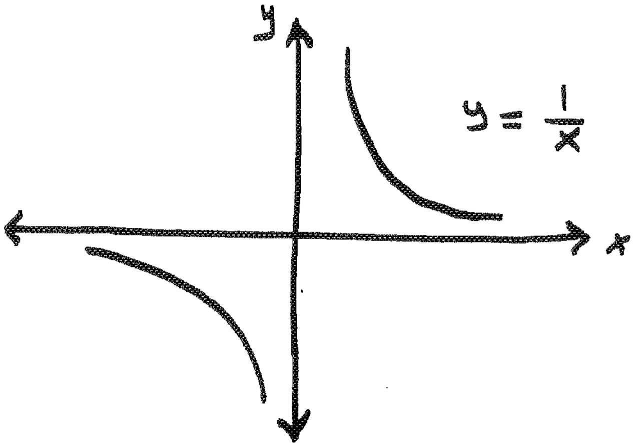
* " " " " " " " " منفرجه " " " تناقصيه

$f(x) = \sqrt{x}$



f(x) is increasing in $[0, \infty)$

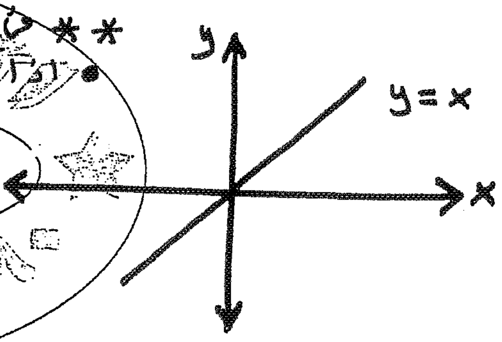
- $F(x) = \frac{1}{x}$



F(x) is decreasing
in $R - \{0\}$
or $(-\infty, 0) \cup (0, \infty)$

** في حاله الداله درجه أوليا
• اذا كان معامل x موجب تكون الداله تزايديه ،

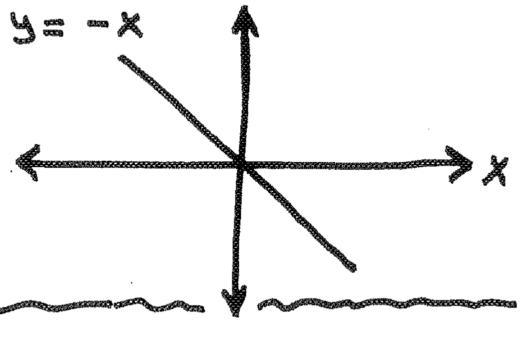
- $f(x) = x$



f(x) is increasing
in $(-\infty, \infty)$

** في حاله الداله درجه أوليا
• اذا كان معامل x سالب تكون الداله تناقصيه ،

- $F(x) = -x$

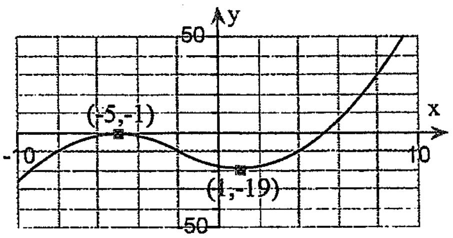


F(x) is decreasing
in $(-\infty, \infty)$

Note

$y = kx$ where $k \neq 0$

x and y are proportional متناسبه



On which intervals is the function increasing?

○ A $(-5,-1)$ and $(1,-19)$

○ B $(-\infty,-5]$ and $[-2,1]$

☑ C $(-\infty,-5]$ and $[1,\infty)$

○ D $[-5,1]$ and $[1,\infty)$

On which interval is the function decreasing?

○ A $[1,\infty)$

○ B $(-5,-1)$

☑ C $[-5,1]$

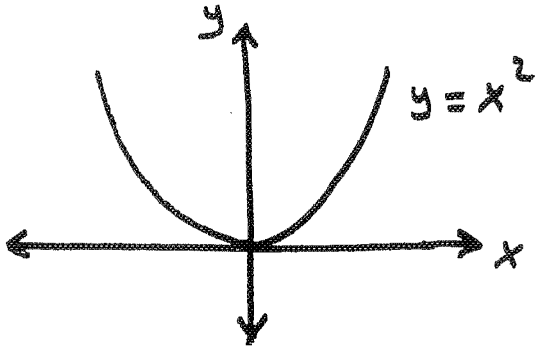
○ D $(-\infty,-5]$

••• Even and odd function

• Even function :

$F(-x) = F(x)$

symmetric about y-axis



Notes

F(x) is even

اذا كانت

الزوجي مثل اشاره (+)

* كل الأسس زوجيه من كثيره الحدود

$F(x) = 3x^4 + x^2$ ، $F(x) = 3$ ، $F(x) = -5$

$F(x) = x^2 - 5$

* ضرب داله زوجيه X داله زوجيه يعطي داله زوجيه

$f(x) = (3x^4 + x^2) \cdot (x^2 - 5)$

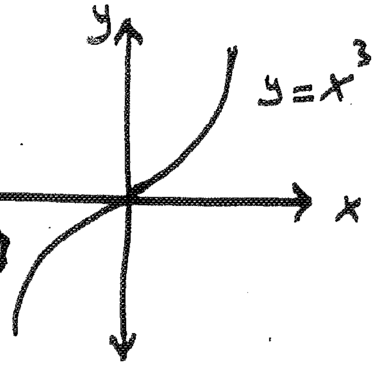
* ضرب داله فرديه X داله فرديه يعطي داله زوجيه

$F(x) = (x^3 + x) \cdot (x^3 - 2x^5)$

* قسمه دالتين: زوجيتين أو فرديتين يعطي داله زوجيه

$f(x) = \dfrac{x^2 + 5 \rightarrow \oplus}{3x^4 + x^2 \rightarrow \oplus} \quad \dfrac{}{\oplus} = \oplus$ even ، $f(x) = \dfrac{x^3}{2x^3 + x}$ even

Odd function :
$F(-x) = -F(x)$
symmetric about origin $(0,0)$



Notes

F(x) is odd
اذا كانت

الفردى مثل اشاره (–)

* كل الأســـس فرديه فى كثيره الحدود $\quad F(x) = 2x^3 + x$

* ضرب دالتين احداهما فرديه والاخرى زوجيه يعطى داله فرديه

$$F(x) = (2x^3+x)(x^2+1)$$

* قسمه دالتين احداهما فرديه والاخرى زوجيه يعطى داله فرديه

$$F(x) = \frac{2x^3+x \rightarrow \ominus}{x^2+1 \rightarrow \oplus} = \ominus \quad \text{odd}$$

$$F(x) = \frac{x^2+1 \rightarrow \oplus}{2x+x^3 \rightarrow \ominus} = \ominus \quad \text{odd}$$

Note : If F(x) is odd function

$$F(0) = 0$$

The curve of the function passes in origin point

neither even nor odd لا زوجية ولا فردية

ليست متماثلة حول محور y .

و ـــ ـــ نقطة الأصل .

$F(-x) \neq f(x)$

$F(-x) \neq -F(x)$.

Notes :

* اذا كانت مكونة من جمع أو طرح حدود مختلفة

$f(x) = 2x^3 + x^2$

* اذا كان البسط مختلط أو المقام مختلط أو كلاهما.

$F(x) = \frac{2x^3+x^2}{x^2+1}$ ، $f(x) = \frac{x^2+5}{x+3}$

$F(x) = \frac{x + x^2}{x^3+1}$

Notes

* دالة القيمة المطلقة دالة زوجية.

$F(x) = |\quad\quad|$

$F(x) = |x|$ ، $F(x) = |x^3|$ ، $F(x) = |x^3+x-1|$

are even function .

* كل الدوال المثلثية فردية ما عدا cos ، sec فهما زوجية

$F(x) = \sin x$ ، $\csc x$ ، $\tan x$ ، $\cot x \Rightarrow$ **odd**

$F(x) = \cos x$ ، $\sec x \longrightarrow$ **even**.

Example

Determine if the Function is even, odd, or neither.

① $f(x) = \underbrace{\underset{\overset{\downarrow}{\text{فردي}}}{x} \cdot \underset{\overset{\downarrow}{\text{فردي}}}{\sin x}}_{\text{زوجي}} - \underset{\overset{\downarrow}{\text{زوجي}}}{x^2} \Rightarrow$ even Function

② $f(x) = \underset{\overset{\downarrow}{\text{زوجي}}}{|x|} + \underbrace{\underset{\overset{\downarrow}{\text{فردي}}}{x} \cdot \underset{\overset{\downarrow}{\text{فردي}}}{\sin x}}_{\text{زوجي}} \Rightarrow$ even Function

③ $f(x) = \underbrace{\underset{\overset{\downarrow}{\text{فردي}}}{x} \cdot \underset{\overset{\downarrow}{\text{زوجي}}}{\cos x}}_{\text{فردي}} + \underset{\overset{\downarrow}{\text{فردي}}}{x} \Rightarrow$ Odd Function

④ $f(x) = \underset{\overset{\downarrow}{\text{زوجي}}}{5} + \underset{\overset{\downarrow}{\text{زوجي}}}{x^2} - \underset{\overset{\downarrow}{\text{زوجي لوجود التربيع}}}{\sin^2 x} \Rightarrow$ even Function

(أي حد ثابت يمثل حد زوجي)

كل التمنيات بالنجاح والتوفيق

السعدي

CH. 1.3

5

Notes

- التركيز على المفاهيم الأساسية.
- شرح أبواب المنهج حسب الخطة.
- أمثلة توضيحية وتدريبات.
- نماذج اختبارات.

السعدي

رياضيات ١١٠
Math.110

جمال السعدي

استاذ الرياضيات والإحصاء للمرحلة الجامعية

0566664790

1.3

New Functions From old Functions

- In this section we start with the basic functions and obtain new functions by :

ازاحه ، شـ ، انعكاس

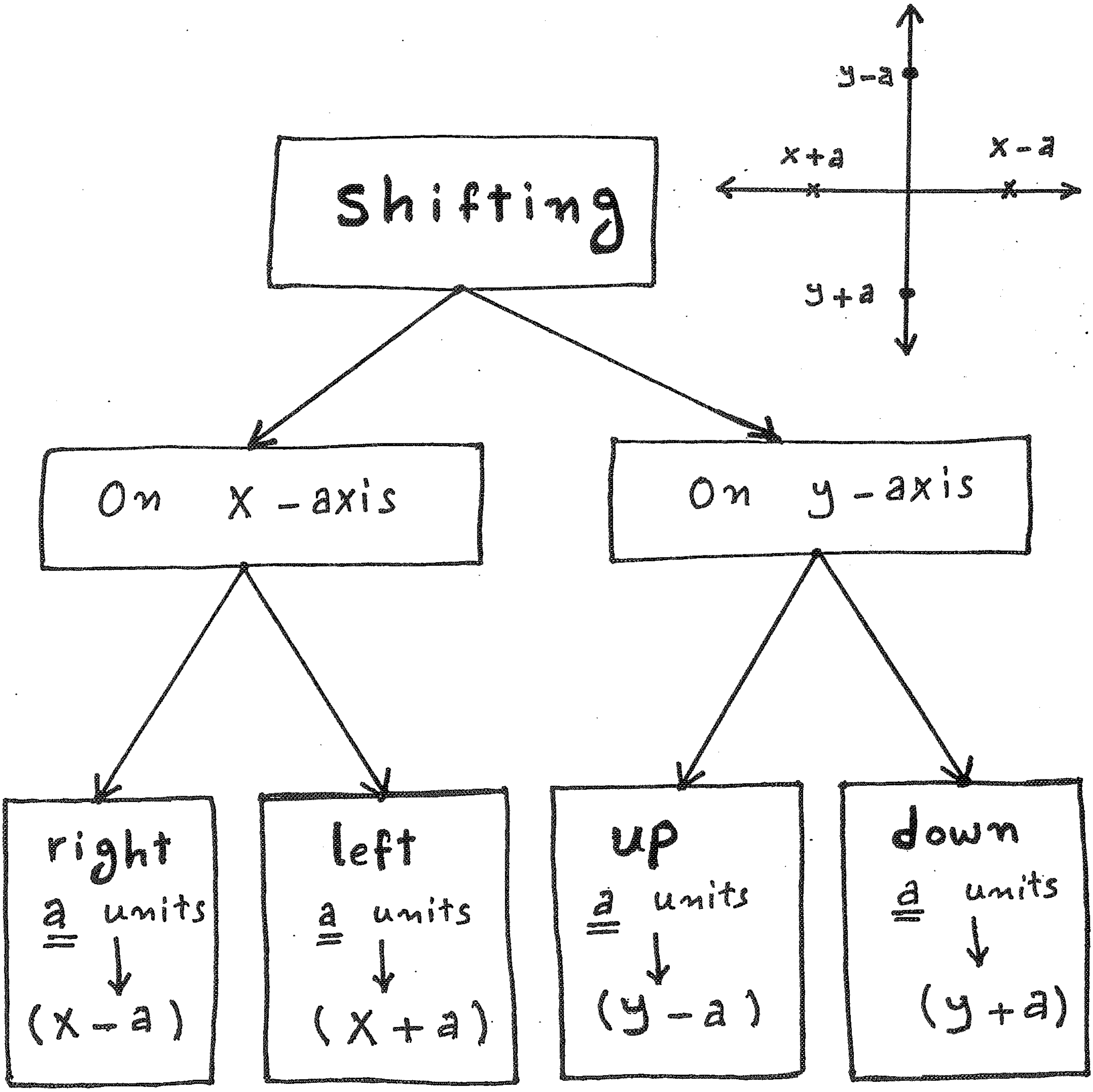
Shifting , Stretching and reflecting

- And we also show :

Combine pairs of functions

تحصيل

by composition .



Write an equation for a function that has

The shape of $y = x^2$, but upside-down and shifted right 7 units.

Which of the following is the equation of the function?

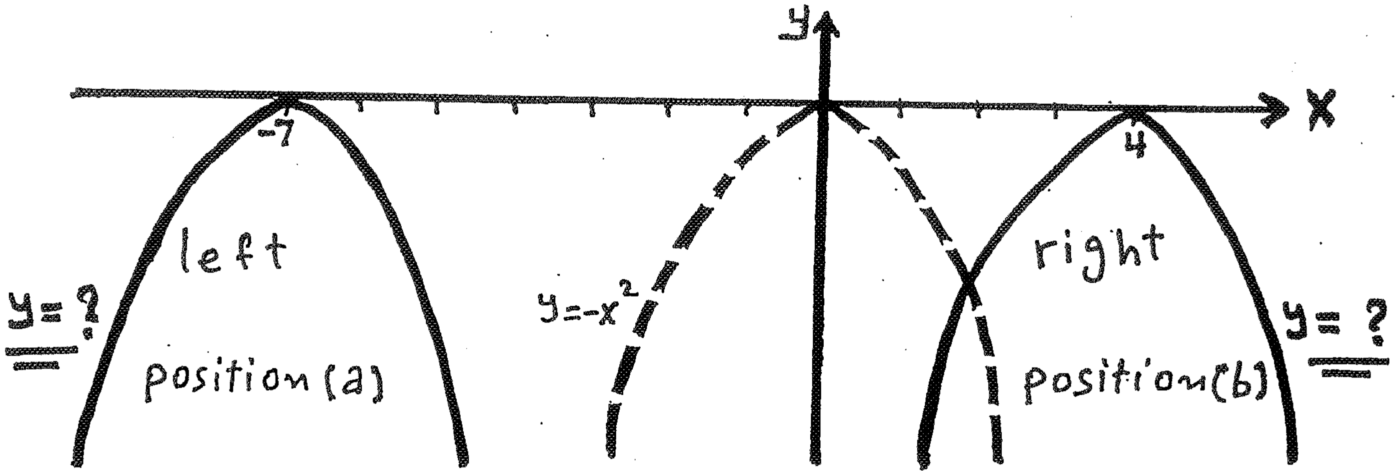
○ A. $y = -x^2 + 7$

○ B. $y = -x^2 - 7$

○ C. $y = -(x+7)^2$

☑ D. $y = -(x-7)^2$

The accompanying figure shows the graph of $y = -x^2$ shifted to two new positions.
write equations for the new graphs.



solution

position (a)
$y = -(x+7)^2$

position(b)
$y = -(x-4)^2$

---

* Give an equation for the shifted graph.

$x^2 + y^2 = 49$ Down 3 , left 2.

solution

الدائره تحركت لليسار وحدتين
ولأسفل ثلاث وحدات

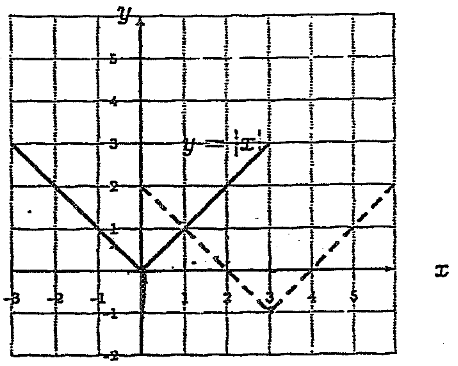
equ. : $(x+2)^2 + (y+3)^2 = 49$

Find An equation for shifted to the new position :

① $y = |x|$ x تحركت يمين 3 ∴ $\boxed{x-3}$

$y+1 = |x-3|$ y 〃 لأسفل 1 ∴ $\boxed{y+1}$

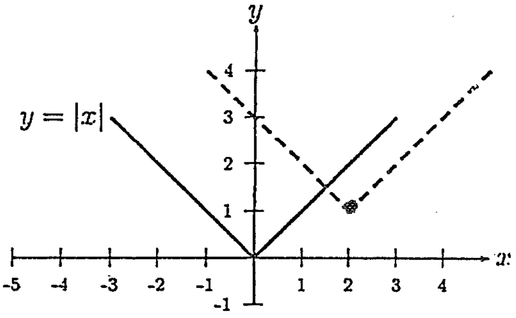
$y = |x-3| - 1$



② $y = |x|$ x تحركت يمين 2 ∴ $\boxed{x-2}$

$y-1 = |x-2|$ y 〃 لأعلى 1 ∴ $\boxed{y-1}$

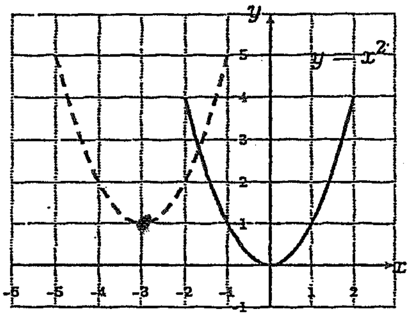
$y = |x-2| + 1$



③ $y = x^2$ x تحركت يسار 3 ∴ $\boxed{x+3}$

$y-1 = (x+3)^2$ y 〃 لأعلى 1 ∴ $\boxed{y-1}$

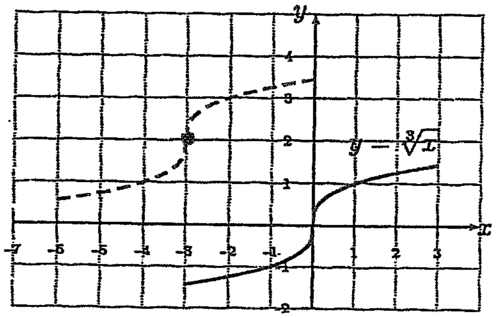
$y = (x+3)^2 + 1$



④ $y = \sqrt[3]{x}$ x تحركت يسار 3 ∴ $\boxed{x+3}$

$y-2 = \sqrt[3]{x+3}$ y 〃 لأعلى 2 ∴ $\boxed{y-2}$

$y = \sqrt[3]{x+3} + 2$

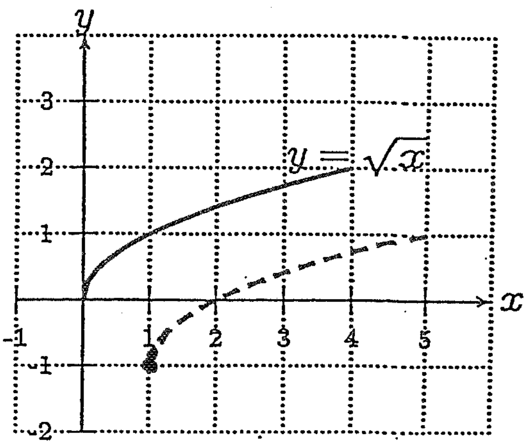


⑤ $y = \sqrt{x}$ x تحركت يمين 1 ∴ $\boxed{x-1}$

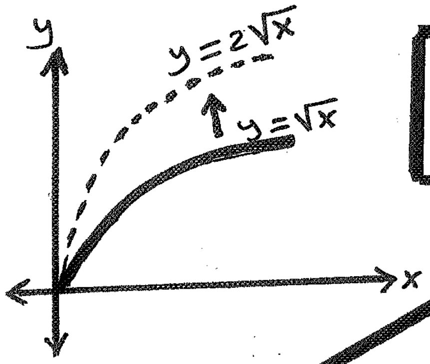
y 〃 لأسفل 1 ∴ $\boxed{y+1}$

$y+1 = \sqrt{x-1}$

$y = \sqrt{x-1} - 1$



Vertical



stretch by c

$y = c\, f(x)$

ضرب الداله المعطاه فى العدد c

Example :

For the function

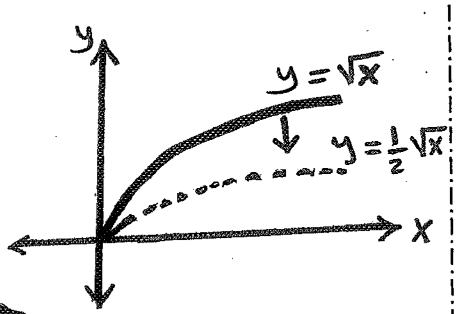
$y = x^2 - 1$

Find the equation for stretch vertical by a factor of 3

Solution

$y = 3 \cdot (x^2 - 1)$

$y = 3x^2 - 3$



compress by c

$y = \frac{1}{c}\, f(x)$

ضرب الداله المعطاه فى $\frac{1}{c}$

Example :

For the function

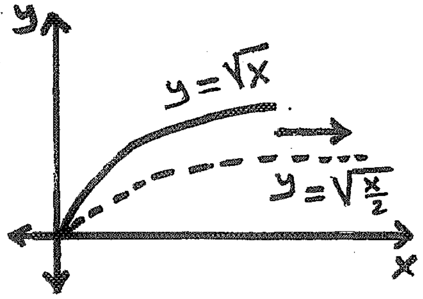
$y = 6x^2 - 1$

Find the equation for compress vertical by a factor of 2

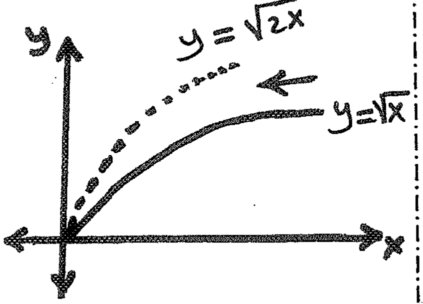
Solution

$y = \frac{1}{2}(6x^2 - 1)$

$y = 3x^2 - \frac{1}{2}$



Horizontal



Stretch by c

$$y = f\left(\frac{x}{c}\right)$$

استبدال x ب $\frac{x}{c}$

compress by c

$$y = f(cx)$$

استبدال x ب cx

Example:

If: $y = x^2 - 1$

stretch horizontal by a factor of $\underline{\underline{3}}$

The new fun. is

$$y = \left(\frac{x}{3}\right)^2 - 1$$

$$y = \frac{x^2}{9} - 1$$

Example:

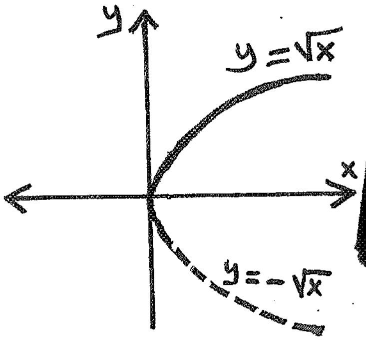
If: $y = x^2 - 1$

Compress horizontal by a factor of $\underline{\underline{5}}$

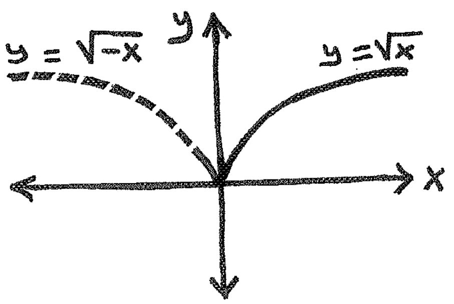
The new fun. is

$$y = (5x)^2 - 1$$

$$y = 25x^2 - 1$$



Reflected



انعكاس الداله حول محور x

انعكاس الداله حول محور y

about x-axis

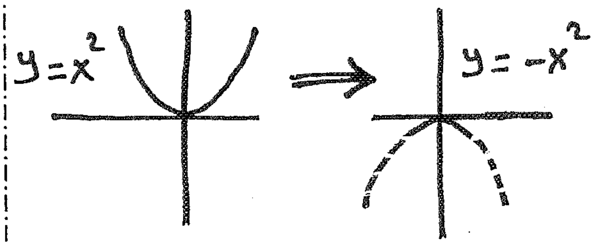
$y = -f(x)$

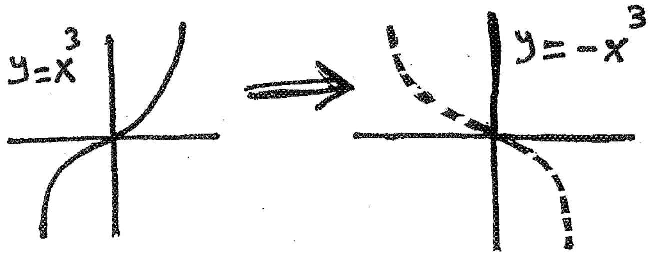
ضرب الداله المعطاه فى سالب

about y-axis

$y = f(-x)$

استبدال x ب $-x$





Example

If: $y = x^2 - 1$

is reflected across x-axis

The new fun.

is $y = -(x^2 - 1)$

$y = -x^2 + 1$

Example:

If: $y = x^3 + 2x^2$

is reflected across y-axis

The new fun.

$y = (-x)^3 + 2(-x)^2$

$y = -x^3 + 2x^2$

If: $F(x)$ الداله الأصليه

Domain $F = [x_1 , x_2]$ على محور x

** هام : اذا حدث تغير فى x يكون التغير فى المجال ،

Range $F = [y_1 , y_2]$ على محور y

** هام : اذا حدث تغير فى y يكون التغير فى المدى .

New function	New domain	New range
$F(x \overset{\downarrow}{\pm} c)$ ازاحه على محور X	$[x_1 \overset{\downarrow}{\mp} c , x_2 \overset{\downarrow}{\mp} c]$ ملاحظه عكس الاشارات	$[y_1 , y_2]$
$F(x) \overset{\downarrow}{\pm} c$ ازاحه على محور y	$[x_1 , x_2]$	$[y_1 \overset{\downarrow}{\pm} c , y_2 \overset{\downarrow}{\pm} c]$ ملاحظه الأشارات كما هما
$F(cx)$ ضرب x فى c	$[\frac{x_1}{c} , \frac{x_2}{c}]$ ملاحظه قسمه x على c	$[y_1 , y_2]$
$F(\frac{x}{c})$ قسمه x على c	$[cx_1 , cx_2]$ ملاحظه ضرب x فى c	$[y_1 , y_2]$
$F(-x)$ انعكاس حول محور y	$[-x_2 , -x_1]$ ضرب الفتره فى سالب مع ملاحظه تبديل الحدود	$[y_1 , y_2]$
$c F(x)$	$[x_1 , x_2]$	$[cy_1 , cy_2]$

Example

If: $y = f(x)$ where Domain $= [-2, 4]$

and Range $= [0, 6]$

Find the Domain and Range

For the new functions :

① $y = f(x+2)$

الحركة على محور X

∴ التغير في المجال

$D = [-2-2, 4-2] = [-4, 2]$ بإضافة 2 - للمجال الأصلي

$R = [0, 6]$ كما هو

② $y = f(x-2)$

$D = [-2+2, 4+2] = [0, 6]$ بإضافة 2+ للمجال الأصلي

$R = [0, 6]$ كما هو

③ $y = f(x) + 1$

الحركة على محور y

∴ التغير في المدى

$D = [-2, 4]$ كما هو

$R = [0+1, 6+1] = [1, 7]$

④ $y = f(x) - 1$

$D = [-2, 4]$ كما هو

$R = [0-1, 6-1] = [-1, 5]$

تابع

المجال الأصلي ← $[-2,4]$

المدى الأصلي ← $[0,6]$

⑤ $y = f(2x)$ ضرب x في 2

∴ قسمة المجال الأصلي على 2 ← $D = [-1,2]$

$R = [0,6]$ كما هو

⑥ $y = f(\frac{x}{2})$ قسمة x على 2

∴ ضرب المجال الأصلي في 2 ← $D = [-4,8]$

$R = [0,6]$ كما هو

---

⑦ $y = f(-x)$ استبدال x بـ -x

ضرب فترة المجال الأصلي في سالب مع ملاحظة تبديل الحدود ← $D = [-4,2]$

$R = [0,6]$ كما هو

⑧ $y = 2f(x)$ ضرب الداله في 2

$D = [-2,4]$ كما هو

∴ ضرب المدى في 2 ← $R = [0,12]$

---

⑨ $y = f(x+1)+4$ نطرح من المجال 1 ، نضيف للمدى 4

$D = [-3,3]$

$R = [4,10]$

⑩ $y = f(x-2)+3$ نضيف 2 للمجال ، نضيف للمدى 3

$D = [0,6]$

$R = [3,9]$

Combining Functions

we look at the main ways functions are combined or transformed to form new functions.

* Combining Functions Algebraically

● $(F \overset{+}{\underset{\times}{-}} g)(x) = F(x) \overset{+}{\underset{\times}{-}} g(x)$ جمع طرح ضرب

$D(F \overset{+}{\underset{\times}{-}} g) = D_F \cap D_g$ المجال هو تقاطع مجال الدالتين

●● $\left(\frac{F}{g}\right)(x) = \frac{F(x)}{g(x)}$ where $g(x) \neq 0$ قسمة

$D\left(\frac{F}{g}\right) = D_F \cap D_g \overset{\text{ماعدا}}{-} \{\overset{\text{اصفار المقام}}{g(x) = 0}\}$

المجال هو تقاطع مجال الدالتين ماعدا اصفار المقام.

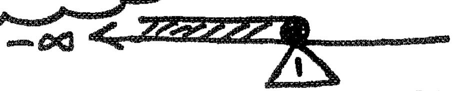
Example: If: $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$

Find: ① $(F+g)(x)$ ② $(F-g)(x)$

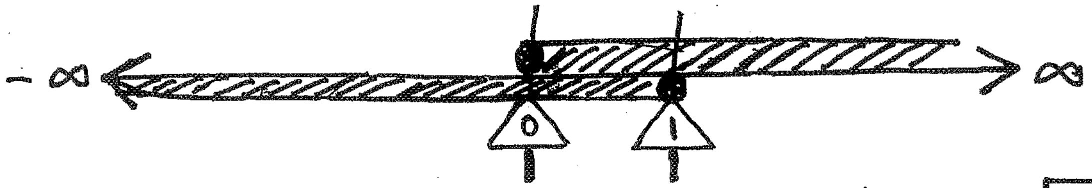
③ $(F\cdot g)(x)$ ④ $\left(\frac{F}{g}\right)(x)$

and Domain of each.

solution



$Df=[0,\infty)$ $Dg=(-\infty,1]$



① $(f+g)(x) = f(x)+g(x)=\sqrt{x}+\sqrt{1-x}$

② $(F-g)(x) = F(x)-g(x)=\sqrt{x}-\sqrt{1-x}$

③ $(F\cdot g)(X)=F(x)\cdot g(x)=\sqrt{x}\cdot\sqrt{1-x}$

** $D(F \overset{+}{\underset{\times}{-}} g) = DF \cap Dg = [0,\infty)\cap(-\infty,1]=[0,1]$

④ $\left(\frac{F}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\sqrt{x}}{\sqrt{1-x}}$

** $D\left(\frac{f}{g}\right)=DF\cap Dg-\{\text{أصفار المقام}\} \quad =[0,1)$

Example :

If : $F(x) = 3x - 3$ ، $g(x) = x^2 - 3x - 4$

Find : ① $f+g$ ② $F-g$ ③ $F \cdot g$ ④ $\frac{F}{g}$

and The domain of each.

(solution)

① $(F+g)(x) = (3x-3) + (x^2-3x-4) = x^2 - 7$

② $(F-g)(x) = (3x-3) - (x^2-3x-4) = -x^2+6x+1$

③ $(F \cdot g)(x) = (3x-3) \cdot (x^2-3x-4) = 3x^3 - 12x^2 - 3x + 12$

* $D(F \overset{\pm}{\div} g) = DF \cap Dg = R \cap R = R = (-\infty, \infty)$

④ $\left(\frac{F}{g}\right)(x) = \frac{3x-3}{x^2-3x-4}$

* $D\left(\frac{F}{g}\right) = DF \cap Dg - \overset{\text{اصفار المقام}}{\{4,-1\}}$

$= R - \{4,-1\}$

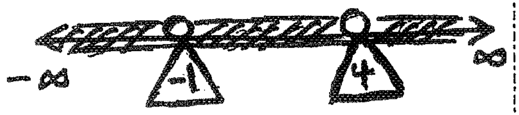
اصفار المقام

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x = 4} \; \| \; \boxed{x = -1}$

$= (-\infty, -1) \cup (-1, 4) \cup (4, \infty)$



For the given functions, find the domain of f, g, and f + g, and find (f + g)(x).

$f(x) = x - 9,\ g(x) = \sqrt{x+3}$

What is the domain of f?

○ A $(9,\infty)$ ● B $(-\infty,\infty)$

○ C $[9,\infty)$ ○ D $(-\infty,9) \cup (9,\infty)$

What is the domain of g?

○ A $(-3,\infty)$ ○ B $(-\infty,-3) \cup (-3,\infty)$

○ C $(-\infty,\infty)$ ● D $[-3,\infty)$

What is the domain of f + g?

○ A $(-3,\infty)$ ○ B $(-\infty,-3) \cup (-3,\infty)$

● C $[-3,\infty)$ ○ D $(-\infty,\infty)$

$(f+g)(x) = x - 9 + \sqrt{x+3}$

Find the domain and graph the function.

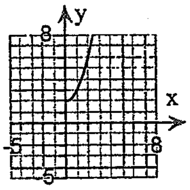
$f(x) = \sqrt{x-2}$

Choose the domain of the function.

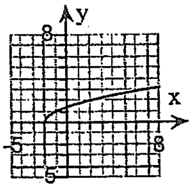
● A $[2,\infty)$ ○ B $(2,\infty)$ ○ C $[-2,\infty)$ ○ D $(0,\infty)$

Choose the correct graph of the function.

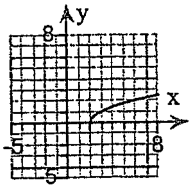
○ A



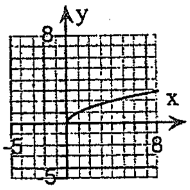
○ B



● C



○ D



Composition of function تحصيل الدوال

If: f and g are functions

The composition function

- $(f \circ g)(x) = f(g(x))$
- $D(f \circ g) = D_{\text{الناتج}} \cap D_{g}$ الداله الثانيه

Example

If: $f(x) = \sqrt{x-1}$ and $g(x) = x^2+1$

Find ① $(f \circ g)(x)$

نعوض بالداله الأخيره مكانه X في الداله الأولى

$= \sqrt{x^2 + \cancel{1} - \cancel{1}}$

$= \sqrt{x^2}$

$= |x|$

② $(g \circ f)(x)$

نعوض بالداله الأخيره مكان X في الداله الأولى

$= \cancel{\sqrt{x-1}}^{\cancel{2}} + 1$

$= x - \cancel{1} + \cancel{1}$

$= x$

If: $F(x) = x^2$ ، $g(x) = \sqrt{x}$ ، $h(x) = \sin x$

Find : ① $(F \circ g)(x)$

$= \sqrt{x}^{\,\cancel{2}}$

$= x$

② $(h \circ g)(x)$

$= \sin\sqrt{x}$

③ $(F \circ h)\left(\frac{3\pi}{2}\right)$

$\frac{3\pi}{2} = 270$

بالتعويض بـ 270 في داله h
ثم الناتج يعوض به في داله F

$(\sin 270)^2 = (-1)^2 = 1$

④ $(F \circ h)\left(\frac{\pi}{3}\right)$

$\frac{\pi}{3} = 60$

بالتعويض بـ 60 في داله h ثم الناتج
يعوض به في داله F

$= (\sin 60)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

⑤ $(F \circ g \circ h)(x)$

بالتعويض بالداله h مكان x في الداله g
ثم التعويض بالناتج مكان x في الداله F

$\sqrt{\sin x}^{\,\cancel{2}} \longleftarrow \sqrt{\sin x}$

∴ الناتج النهائي = $\sin x$

⑥ $(g \circ f)(4)$

بالتعويض بالعدد 4
في الداله F ثم الناتج يعوض به في g

∴ الناتج النهائي = 4

⑦ $(g \circ g)(x)$

بالتعويض بالداله g مكان x
في الداله g نفسها

∴ الناتج النهائي = $\sqrt{\sqrt{x}}$

If : $f(x) = 4 + x^5$ and $g(x) = (x-4)^{\frac{1}{5}}$

Find :

① $(g \circ f)(x)$

بالتعويض بـ داله f مكان x في داله g

$= (\cancel{4} + x^5 - \cancel{4})^{\frac{1}{5}}$

$= (x^5)^{\frac{1}{5}}$

$= x$

② $D(g \circ f)$

$= D_{\text{الناتج}} \cap D_{\text{الداله الأخيره}}$

$= (-\infty, \infty) \cap (-\infty, \infty)$

$= (-\infty, \infty) = R$

If : $f(x) = \sqrt{x-4}$ and $g(x) = 3x - 1$

Find: ① $(f \circ f)(x)$

بالتعويض بالداله F مكان X في داله F

$= \sqrt{\sqrt{x-4} - 4}$

② $(g \circ f)(x)$

بالتعويض بالداله f مكان x في داله g

$= 3\sqrt{x-4} - 1$

If: $f(x) = \dfrac{x}{x+1}$, $g(x) = x^{10}$ and $h(x) = x+3$

Find : $(f \circ g \circ h)(x)$

بالتعويض بداله $h(x)$ مكانه x في داله g
ثم التعويض بالناتج في داله $F(x)$

الناتج $= \dfrac{(x+3)^{10}}{(x+3)^{10}+1}$

If: $F(x) = \cos^2(x+2)$ find f , g and h
where $F = f \circ g \circ h$

المعطى هو ناتج التحصيل , والمطلوب هو ايجاد الدوال الثلاثه

$f(x) = x^2$, $g(x) = \cos x$, $h(x) = x+2$

السؤال يأتي في الأختبار على شكل أختيارات

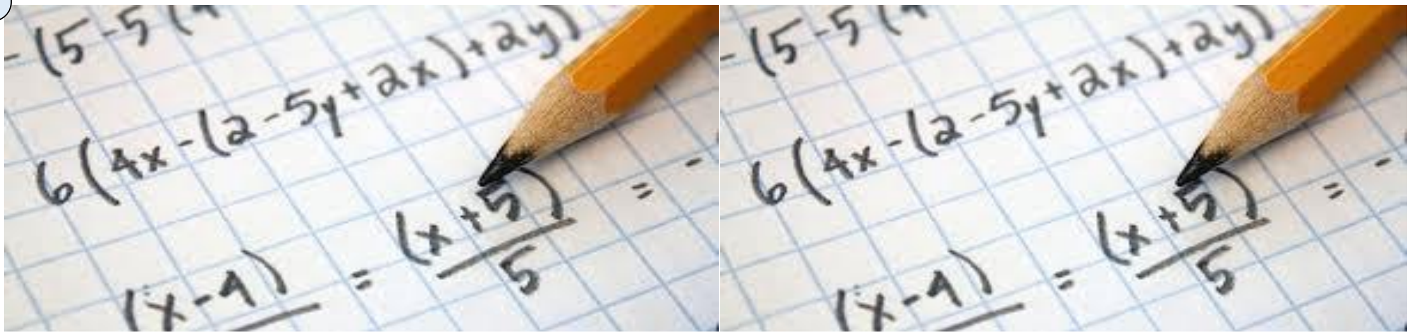
نجرب الاختيارات : الأختيار الذي يكون

تحصيل دواله الثلاثة f ، g ، h
هو $F(x) = \cos^2(x+2)$ يكون هو الاختيار الصحيح

كل التمنيات بالنجاح والتوفيق

السعدي

MATH 110



CH 1

1.1+1.2+1.3

Function

محـــمد عـمـران

ريــاضـــــيات واحــــــصاء

www.3mran2016.wordpress.com

0507017098-0580535304

Function

Four way to represent a function

1-verbally	**لفظيا**
By adscription in word	وصفها بالكلمات
2-Numerically	**عدديا**
By (table of value)	
3-visually	**بصريا**
By a graph	
4-algebrically	**جبريا**
by explicit formula	صيغه صريحه

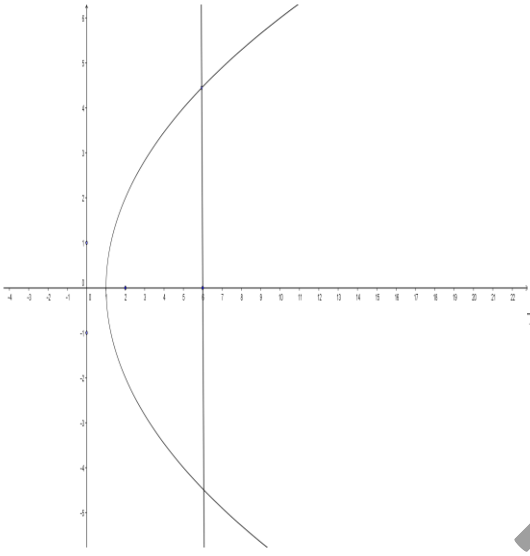
A function: f

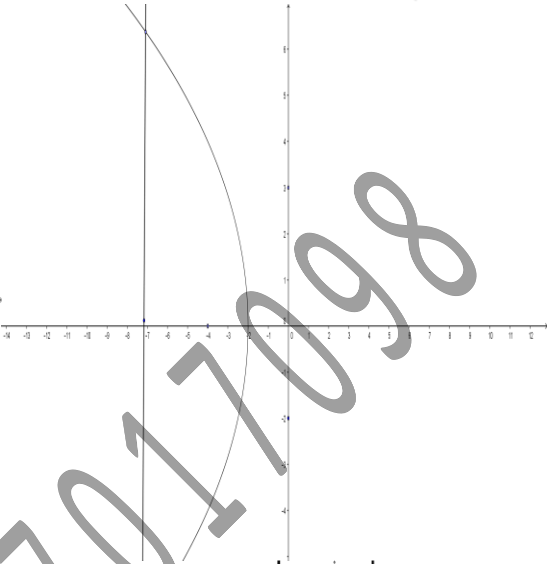
From asset A to asset B is relation that assigns to each element X in the set A exactly one element in the set B

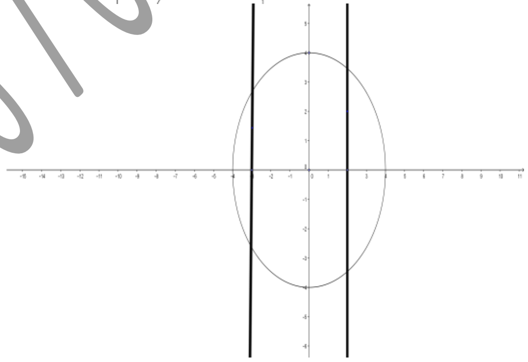
الدالة: هي علاقة تربط بين كل عنصر من مجموعه غير حاليه Aبعنصر واحد فقط من المجموعة B

The vertical line test   اختبار الخط الرأسي

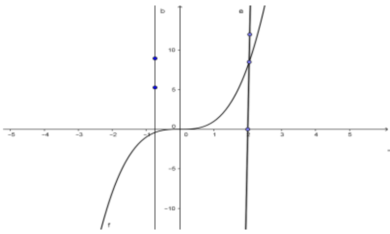
إذا رسمنا خط راسي وقطع المنحنى في أكثر من نقطه فان المنحنى لا يمثل داله كما بالرسم

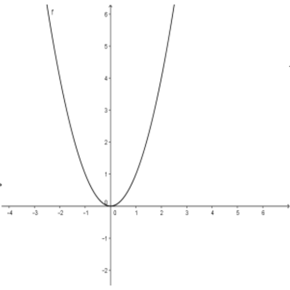


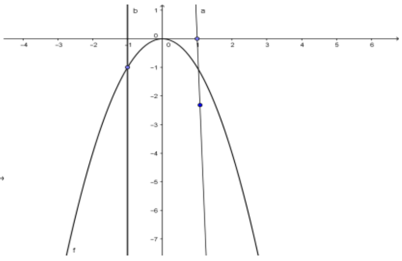




اذا كان الخط الراسي يمر بنقطه واحده فالمنحنى يمثل داله





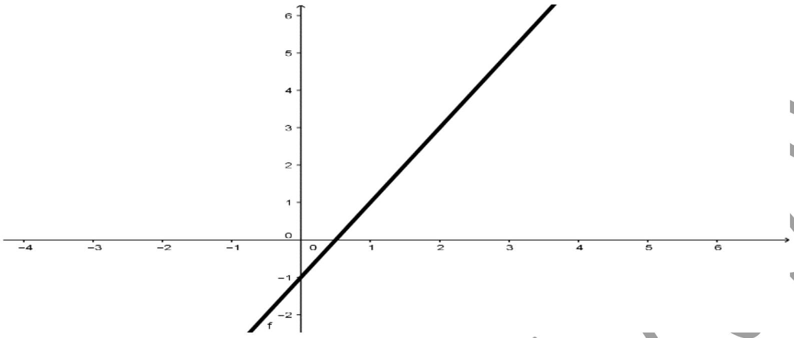


$sketch\ the\ following\ graph$

$1) f(x) = 2x - 1$

sol

من الداله نجد ان الميل 2 والجزء المقطوع من محور y هو 1-

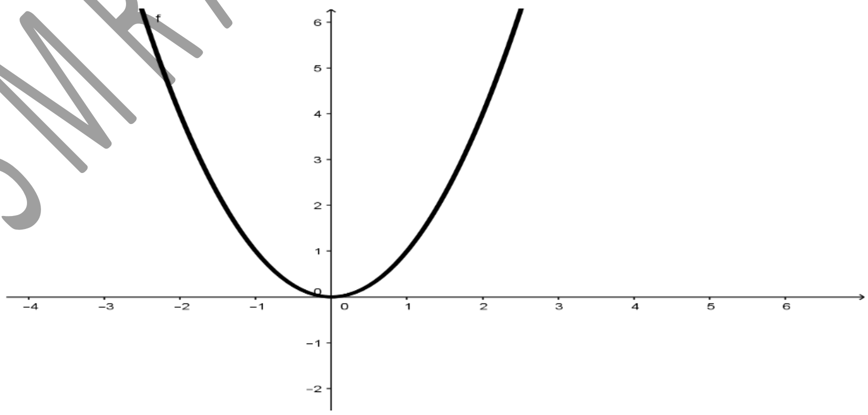


$2)\ f(x) = x^2$

sol

للرسم نعوض ببعض النقاط

X	-2	-1	0	1	2
F(x)	**4**	**1**	**0**	**1**	4



Example

$$if\ f(x) = \begin{cases} 1 - x & if\ x \leq -1 \\ x^2 & if\ x > -1 \end{cases}$$

$$evaluate\ f(-1), f(-2), f(0) and\ sketch\ it$$

$$sol$$

$$f(-1) = 1 - (-1) = 1 + 1 = 2$$

$$f(-2) = 1 - (-2) = 1 + 2 = 3$$

$$f(0) = 0^2$$

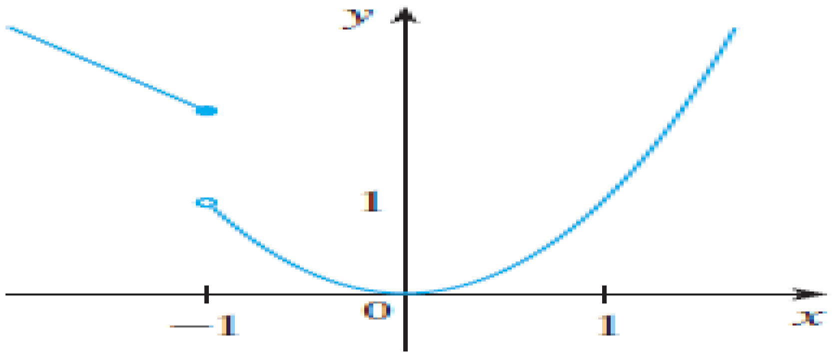
للرسم نعوض ببعض النقاط

at $x \leq -1$

x	F(x)
-1	2
-2	3
-3	4
-4	5
-5	6

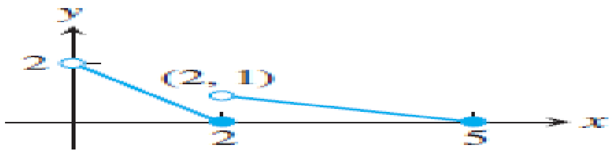
at $x > 1$

x	F(x)
-1	1
0	0
1	1
2	4
3	9



$Example$

$find\ formula\ of function$



Sol

للحل نجدان لدينا مستقيمان

الأول $0 < x \le 2$

الثانى $2 < x \le 5$

لايجاد معادلتهم نحدد نقطتين على كل مستقيم ونوجد معادلته

الأول $(0,2),(2,0)$

معادلته $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$$\frac{y-2}{x-0} = \frac{0-2}{2-0}$$

$$\frac{y-2}{x} = \frac{-2}{2} = -1$$

$$y-2 = -x$$

$$y = -x+2$$

الثانى $(2,1),(5,0)$

معادلته $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$$\frac{y-1}{x-2} = \frac{0-1}{5-2}$$

$$\frac{y-1}{x-2} = \frac{-1}{3}$$

$$y-1 = -\frac{1}{3}(x-2)$$

$$y-1 = -\frac{1}{3}x + \frac{2}{3}$$

$$y = -\frac{1}{3}x + \frac{2}{3} + 1$$

$$y = -\frac{1}{3}x + \frac{5}{3}$$

$$formula = \begin{cases} -x+2 & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3} & 2 < x \le 5 \end{cases}$$

Mathematical model
Type of function

1) polynomial function

دوال كثيرات الحدود

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_0$$

We called $a_0, a_1, \ldots\ldots\ldots\ldots\ldots\ldots . a_{n-1}, a_n$ is coefficient of polynomial

Such that $a_n \neq 0$ n≥0

and *n* is positive number n is degree of polynomial

$$Df = R = (-\infty, \infty)$$

Example

$f(x) = 5x^2 + 4x - 3$

Degree=…………..

$a_2 = \cdots ....$ $a_1 = \cdots ....$ $a_0 = \cdots ...$

Type of poly nominal	
1)$f(x) = a$	**-Polynomial of degree 0** **–constant function** داله ثابته
2) $f(x) = ax + b$	**-Polynomial of degree 1** **–linear function** داله خطيه
3) $f(x) = ax^2 + bx + c$	**-Polynomial of degree 2** **-quadratic function** داله تربيعيه
4) $ax^3 + bx^2 + cx + d$	**-polynomial of degree 3** **–cubic function** داله تكيبه
5) $f(x) = x^n$ $n \ \ postive \qquad f(x) = x^n$ $n \, negative \qquad f(x)\frac{1}{x^n} = x^{-n}$	**–power function** داله قوه

2) Rational function

الدالة الكسرية

Ratio of two polynomial

هي داله النسبة بين دالتي كثيرات الحدود

$$f(x) = \frac{p(x)}{Q(x)}$$

3) Root function

الدالة الجذرية

$$f(x) = \sqrt[n]{x^m}$$

4) Algebraic function

الدالة الجبرية

Algebraic is a function constructed from polynomial using algebraic operations (addition-subtraction-multiplication-division)

الدالة الجبرية هي داله تتكون من كثيرات الحدود باستخدام العمليات الجبرية (الجمع-الطرح-الضرب-القسمة)

Example

1) $f(x) = (x+2)\sqrt{x+4} - 2x$

2) $f(x) = \dfrac{x+\sqrt{x}}{x+1}$

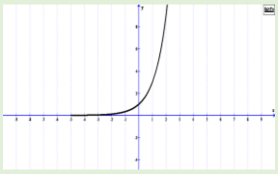
3) $f(x) = \dfrac{\sqrt{x^2+1}}{x+4}$

5) exponential function

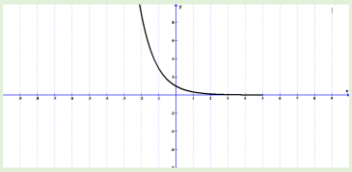
الدالة الأسية

$f(x) = a^x \quad a \text{ is constant}$

$a > 1$



$0 < a < 1$



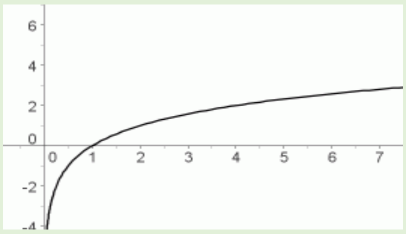
$$D_f = R = (-\infty, \infty) \qquad R_f = (0, \infty)$$

6) Logarithmic function

$$f(x) = \log_a x$$

$$D_f = (0, \infty)$$

$$R_f = (-\infty, \infty)$$

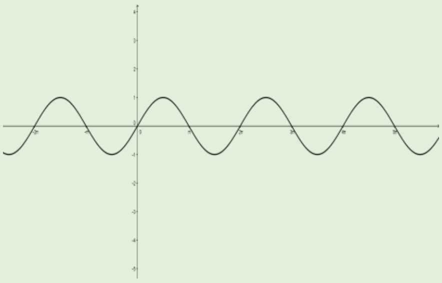


7) Trigonometric function

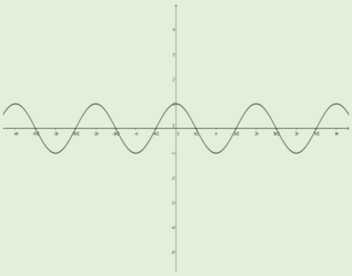
الدوال المثلثية

As sin x cos x tan x

Sin x



Cos x



$$D_f = R = (-\infty, \infty)$$

$$R_f = [-1, 1]$$

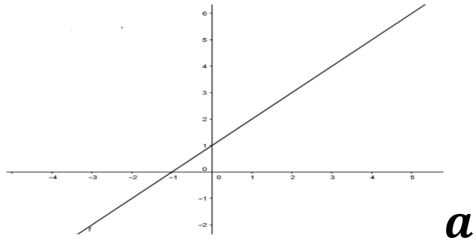
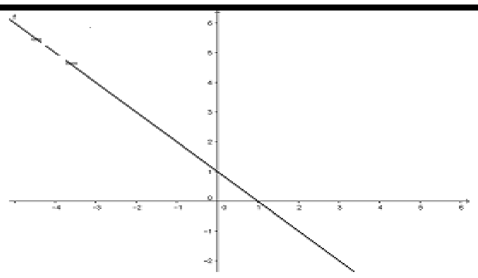
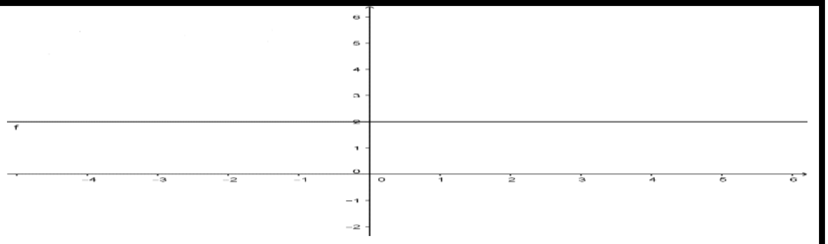
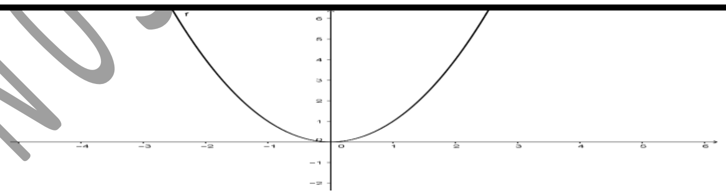
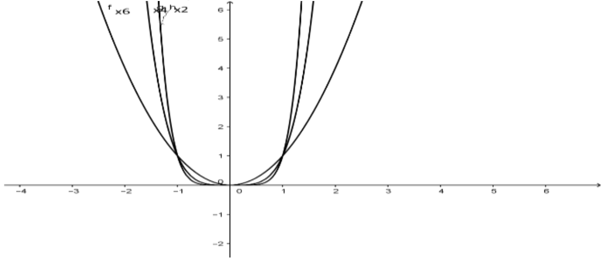
$$-1 \leq \sin x \leq 1$$

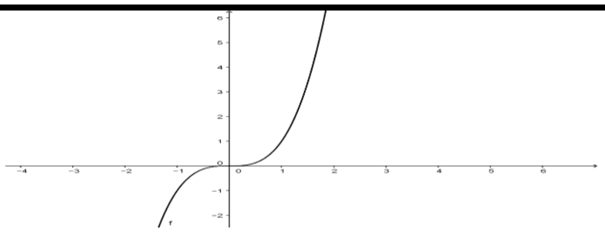
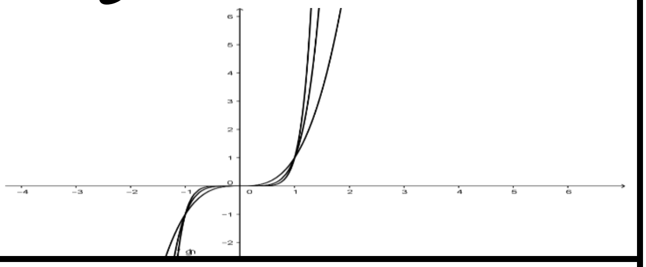
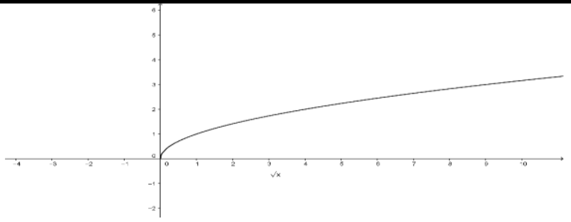
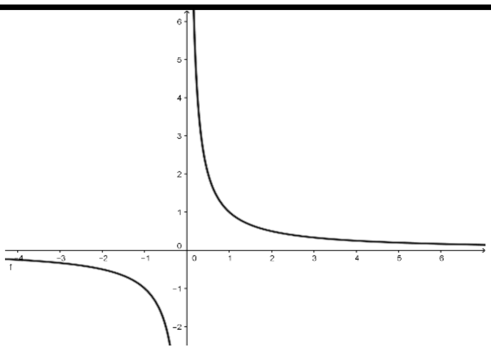
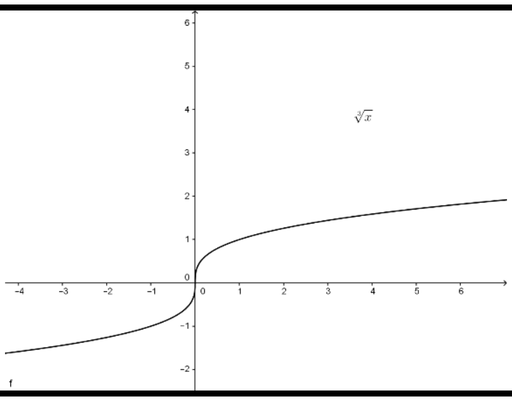
$$-1 \leq \cos x \leq 1$$

Classify the following function

1	$f(x) = 4^x$	Exponenatioanl
2	$f(x) = x^6 - x^2 + 2x$	Polynomial
3	$f(x) = \log_4 x$	Lograthim
4	$f(x) = \frac{x-1}{x^3+1}$	Rational
5	$f(x) = \cos\left(x - \frac{\pi}{3}\right)$	Trigometric
6	$f(x) = x^4$	Power
7	$f(x) = \frac{\sqrt{x}+1}{x^2-1}$	Algebric
8	$f(x) = 8$	Constant/polynomial
9	$f(x) = 9 - x^2$	Quadrtic/polynomial
10	$f(x) = 7$	Constant/polynomial
11	$f(x) = \tan x$	Trigonmetric
12	$f(x) = \sqrt{4 + x^2}$	Root
13	$f(x) = 2x + \frac{2-\sqrt{x}}{\sqrt{x^2+1}}$	Algebric
14	$f(x) = 2^x$	Exponentional
15	$f(x) = 20^{-x}$	Exponentional
16	$f(x) = \frac{x^2-3}{x^2+1}$	Rational

Summary of standard function

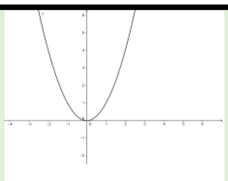
1	$y = ax + b$ $D_f = R$ $R_f = R$	 $a > 0$
2	$y = ax + b$ $D_f = R$ $R_f = R$	 $a < 0$
3	$y = ax + b$ $y = b$ $D_f = R$ A=0 Y=b $R_f = b$	
4	$y = x^2$ $D_f = R$ $R_f = R$	
5	$y = x^n$ ℕ is even positive numbers الاس عدد صحيح زوجى موجب	لاحظ كلما زاد الاس اقترب المنحنى من y 

6	$y = x^3$ $D_f = \mathbb{R}$ $R_f = \mathbb{R}$	
7	$y = x^n$ ℕ is positive number الاس عدد موجب فردى	لاحظ كلما زاد الاس اقترب المنحنى من محور y 
8	$y = \sqrt{x}$ $D_f = [0, \infty)$ $R_f = [0, \infty)$	
9	$y = \frac{1}{x}$ Reciprocal function $D_f = \mathcal{R} - \{0\}$ $R_f = \mathcal{R} - \{0\}$	
10	$y = \sqrt[3]{x}$ $D_f = \mathcal{R}$ $R_f = \mathcal{R}$	

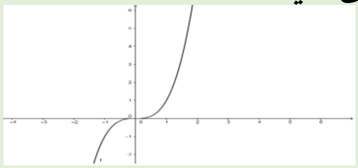
Even and odd function
الدوال الزوجية والفردية

1) Even function

If $f(x) = f(-x) \to even\ function$

1	 لاحظ ان نهايتي المنحنى في اتجاه واحد **F(x) is an even function if symmetric about y-axis**
2	**إذا كانت الدالة ثابته أي انها تساوى عدد بدون x** **Ex :** $f(x) = 5$
3	**داله القيمة المطلقة داله زوجيه** $f(x) = \lvert x + 2 \rvert$
4	**داله** $\cos x$
5	إذا كانت جميع اسس **x** اعداد زوجيه **Ex :** $f(x) = 3x^6 - 5x^4 + x^2 - 3$

Odd function

1	**If** $f(x) = -f(x) \to oddfunction$ **لاحظ ان نهايتي المنحنى في اتجاهين متضادين** 
2	**F(x) is an odd function if symmetric about the origin point**
3	**داله sin(x), tan(x) دوال فرديه**
4	**إذا كانت جميع اسس x اعداد فرديه** **Example** $f(x) = 5x^3 - 2x$

قاعده الجمع والطرح

فردى ± فردى =فردى
زوجي ± زوجي = زوجي
فردى ± زوجي = ليست زوجيه ولا فرديه neither odd nor even
زوجي ± فردى= ليست زوجيه ولا فرديه neither odd nor even

قاعده الضرب والقسمة

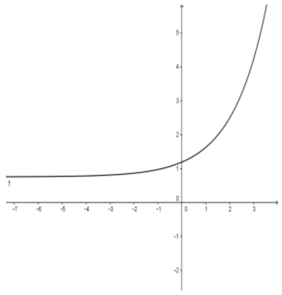
فرديه × او÷ فرديه = زوجيه
زوجيه × او ÷ زوجيه = زوجيه
زوجيه × او ÷ فرديه = فرديه
فرديه × او ÷ زوجيه = فرديه
إذا تشابها في الضرب او القسمة فالناتج داله زوجيه
إذا اختلافا في الضرب او القسمة فالناتج داله فرديه

Determine each function is odd or even or neither

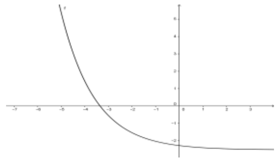
1	$f(x) = x^5 + x$	Odd
2	$f(x) = 1 - x^4$	Even
3	$f(x) = 2x - x^2$	Neither
4	$f(x) = x^2(x^2 + 1)$	Even
5	$f(x) = x(x^2 + 4)$	Odd
6	$f(x) = \frac{3x}{x^6 + 9}$	Odd
7	$f(x) = \|x\|$	Even
8	$f(x) = \frac{x^2 + 1}{\|x\|}$	Even
9	$f(x) = x(x^3 + x)$	Even
10	$f(x) = x^4 - x^2$	Even
11	$f(x) = x^5 + x^3 - 3x$	Odd
12	$f(x) = \frac{x^2 - 1}{x^3 + 1}$	Neither
13	$f(x) = x^5 - x^2 - 3x + 7$	Niether

Increasing and decreasing function

1) $if \quad f(x_1) < f(x_2)$
Where ever $x_1 < x_2$
Then f(x) is increasing function
داله متزايدة



2)$if \ f(x_1) < f(x_2)$
When ever $x_1 > x_2$
Then f(x) is decreasing function
داله متناقصة



A function that is increasing or decreasing on Interval is called monotonic on Interval

Note

في حاله داله الدرجة الاولى

$$\boldsymbol{f(x) = ax + b}$$

if a >0 معامل xموجب

Then f(x) is increasing in $\Re$=$(-\infty\ ,\infty)$

-if a < 0 معامل xسالب

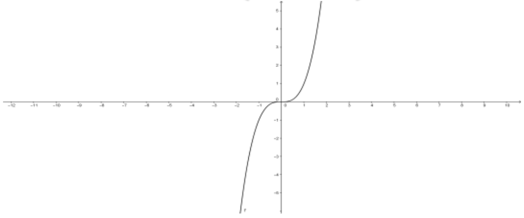
Then f(x) is decreasing in $\Re$ =$(-\infty, \infty)$

في حاله داله الدرجة الثالثة

$f(x) = x^3$

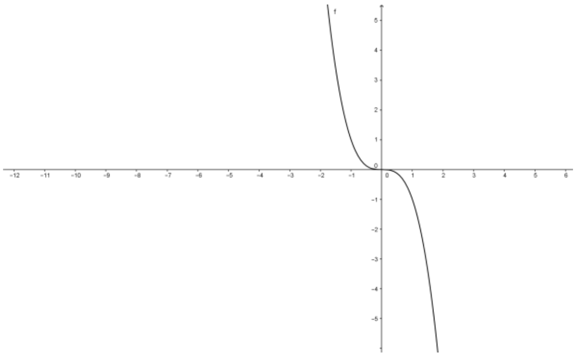
إذا كان معامل x^3 موجب فالدالة متزايدة في

$\mathcal{R} = (-\infty, \infty)$



إذا كان معامل x^3 سالب فالدالة متناقصة في

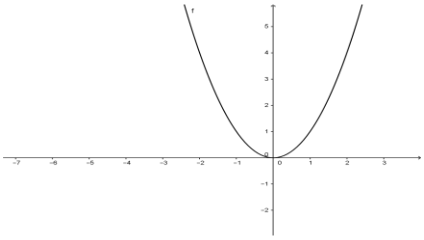
$\mathcal{R} = (-\infty, \infty)$



في حاله داله الدرجة الثانية

$f(x) = ax^2$

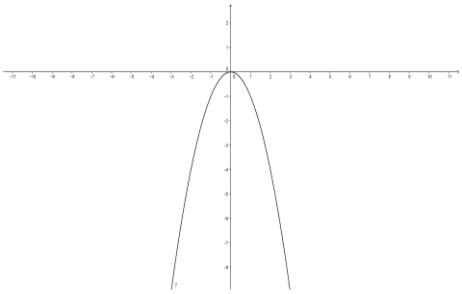
a>0 إذا كان معامل x^2 موجب



F(x) is decreasing in $(-\infty, 0]$

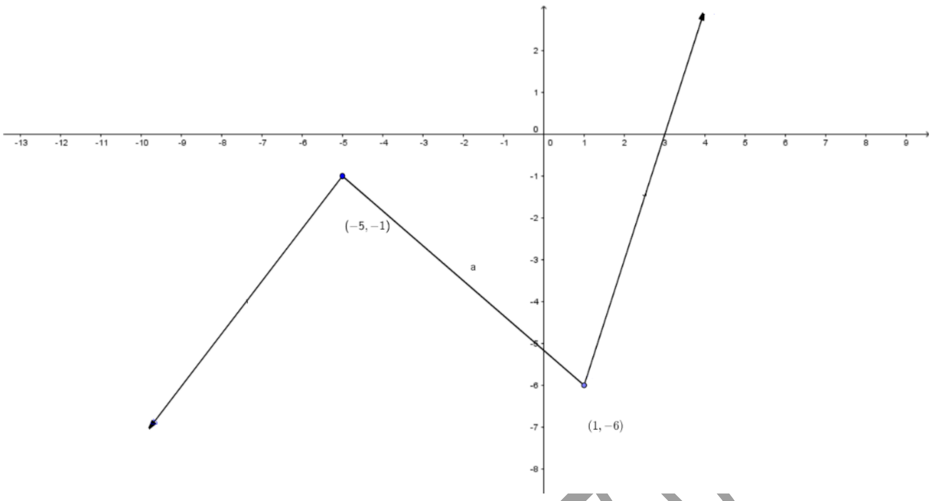
F(x) is increasing in $[0, \infty)$

a<0 إذا كان معامل x^2 موجب



F(x) is decreasing in $[0, \infty)$

F(x) is increasing in $(-\infty, 0]$



On which intervals is the function increasing

1) $(-5,-1) \cap \ (1,-6)$ 2)$(-\infty,-5] \ \cup [-2,-1]$

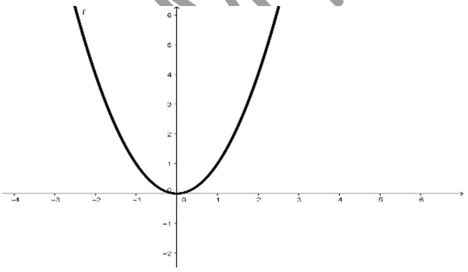
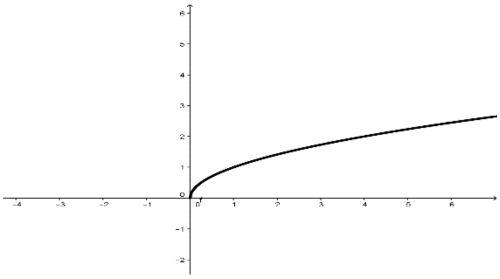
3) $(-\infty,-5) \cup (1,\infty)$ 4) $[-5,-1] \cup [1,\infty)$

On which Intervals the function decreasing

1) $[1,\infty)$ 2) (-5,1)

3) $[-5,-1]$

Show that if the function increasing or decreasing

1) $\boldsymbol{f(x) = x^2}$ **Sol**  $\boldsymbol{decreasing\ (-\infty,0)}$ $\boldsymbol{increasing(0,\infty)}$	**2)** $\boldsymbol{f(x) = \sqrt{x}}$ **Sol**  $\boldsymbol{increasing\ (0,\infty)}$

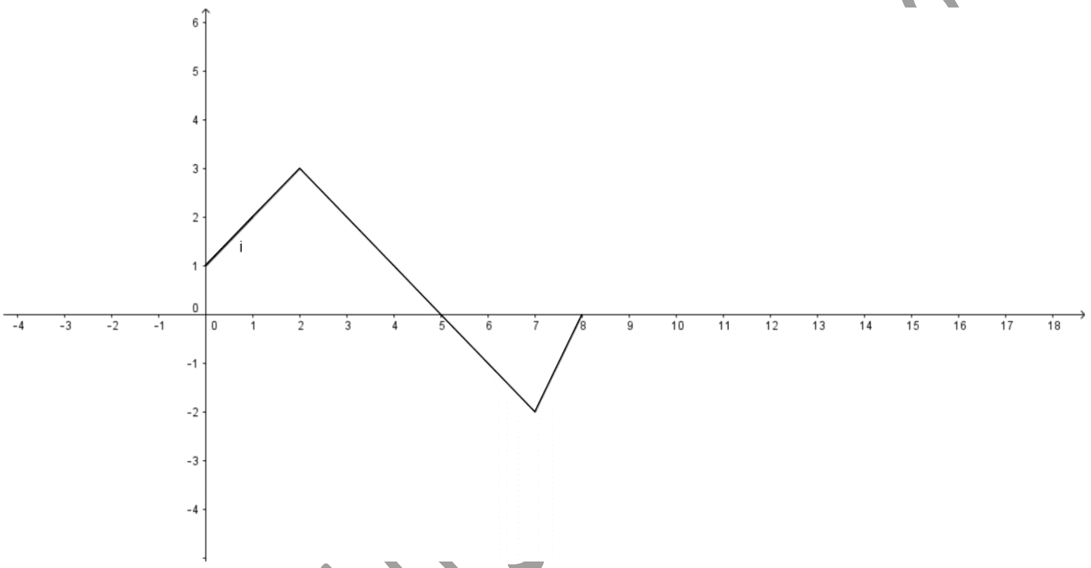
Domain and range of function

مجال الدالة D_f

هي مجموعه الاعداد الحقيقة التي يمكن ان تأخذها x

مدى الدالة R_f

هي كل قيم f(x)



In the graph

1- Find the value of f (1), f (5)?

sol

$$f(1) = 2 \qquad f(5) = 0$$

2- What are the domain and rang of F?

Sol

$$D_f = [0, 8]$$
$$R_f = [-2, 3]$$

Domain function

1) polynomial **داله كثيرات الحدود**

هي الدالة الخالية من الكسور او الجذور

$$D_f = \mathcal{R} = (-\infty, \infty)$$

Ex

$$f(x) = 3x^2 - 7x + 1$$
$$f(x) = 5$$
$$f(x) = 3x + 2$$

2) Rational function **الدالة الكسرية**

$$D_f = \mathcal{R} - \{\text{اصفار المقام}\}$$

Ex

Find domain

$$f(x) = \frac{x^2 + 2x}{x + 1}$$

Sol

المقام $\neq 0$

$$x + 1 \neq 0$$
$$x \neq -1$$
$$D_F = \mathcal{R} - \{-1\} = (-\infty, -1) \cup (-1, \infty)$$

3) Domain of root function with odd root $\sqrt[3]{\blacksquare}, \sqrt[5]{\blacksquare}$

مجال الدالة الجذرية ودليل الجذر فردى $\sqrt[3]{\blacksquare}, \sqrt[5]{\blacksquare}$

في المقام	في البسط
$D_f = \mathcal{R} - \{\text{اصفار المقام}\}$	$D_f = \mathcal{R} = (-\infty, \infty)$

Ex

Find domain

1) $f(x) = \sqrt[3]{x^2 + x - 1}$

Sol

جذر تكيبى في البسط مجاله R

$D_f = \mathcal{R}$

2) $f(x) = \dfrac{3x + 1}{\sqrt[3]{x - 4}}$

Sol

$R - \{\text{اصفار المقام}\}$ جذر تكيبى في المقام مجاله

$x - 4 \neq 0$

$x \neq 4$

$D_f = \mathcal{R} - \{4\} = (-\infty, 4) \cup (4, \infty)$

4) Domain of root function with even root $\sqrt{\blacksquare}, \sqrt[4]{\blacksquare}$

مجال الدالة الجذرية ودليل الجذر زوجي $\sqrt{\blacksquare}, \sqrt[4]{\blacksquare}$

فى المقام	فى البسط
ماتحت الجذر>صفر	ماتحت الجذر≥ صفر

Ex

Find domain

1)$f(x) = \sqrt{x - 2}$

Sol

جذر تربيعى بالبسط

$x - 2 \geq 0$

$x \geq 2$

$D_f\ [2, \infty)$

2) $f(x) = \dfrac{x}{\sqrt{x - 1}}$

Sol

جذر تربيعى في المقام

$x - 1 > 0$

$x > 1$

$D_f = (1, \infty)$

ملحوظه

اذا كانت الدالة تحت الجذر من الدرجة الثانية

1) نوجد اصفار الدالة
2) ندرس اشاره الدالة على خط الاعداد
3) مجال الدالة هو الفترات الموجبة

Ex

Find domain

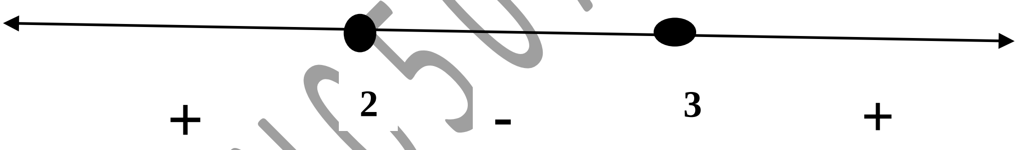
$$f(x)=\sqrt{x^2-5x+6}$$

Sol

$$x^2-5x+6\geq 0$$
$$x^2-5x+6=0$$
$$(x-2)(x-3)=0$$

$x-2=0$
$x=2$

$x-3=0$
$x=3$



$D_f=(-\infty,2]\cup[3,\infty)$

المدي

Rang

لإيجاد مدى الدالة
نعوض بطرفي المجال **domain**
وإذا كان ناتج التعويض متشابه نعوض بالصفر
وتكون أكبر قيمه وأصغر قيمه هى المدى **Rang**

Find domain and rang of the following function	
1	$f(x) = 3x^3 - 4x^2 + 5$ **Sol** **كثيرات الحدود مجالها R ومداها R** $D_f = \Re\ (-\infty, \infty) \qquad R_f = \Re = (-\infty, \infty)$
2	$f(x) = \frac{x+1}{x^2 - 3x + 2}$ **Sol** **داله كسريه مجالها** $R - \{$**اصفار المقام**$\}$ $x^2 - 3x + 2 \neq 0$ $(x-2)(x-1) \neq 0$ $x - 2 \neq 0 \quad \mid \quad x - 1 \neq 0$ $x \neq 2 \quad \mid \quad x \neq 1$ $D_f = R - \{1, 2\} = (-\infty, 1) \cup (1, 2) \cup (2, \infty)$
3	$f(x) = \sqrt[3]{x-1}$ **Sol** **جذرتكعيبى بالبسط مجاله R** $D_f = \mathcal{R} = (-\infty, \infty)$
4	$f(x) = \frac{x+7}{\sqrt[3]{x+4}}$ **Sol** **جذر تكعيبى بالمقام مجاله** $R - \{$**اصفار المقام**$\}$ $x + 4 \neq 0$ $x \neq -4$ $D_f = R - \{4\} = (-\infty, 4) \cup (4, \infty)$
5	$f(x) = 3$ **sol** **داله ثابته مجالها R** **ومداها 3**

6

$f(x) = \sqrt{x+2}$

Sol

$$x + 2 \geq 0$$
$$x \geq -2$$
$$D_f = [-2, \infty)$$

لايجاد المدى

$$f(-2) = \sqrt{-2+2} = 0$$
$$f(\infty) = \sqrt{\infty + 2} = \infty$$
$$R_f = [0, \infty)$$

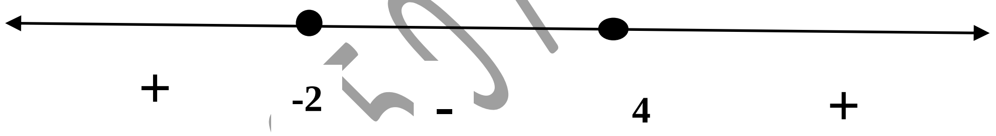
7

$f(x) = \sqrt{x^2 - 2x - 8}$

Sol

$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$

$$x - 4 = 0 \qquad x + 2 = 0$$
$$x = 4 \qquad x = -2$$



$$D_f = (-\infty, -2] \cup [4, \infty)$$

8

$f(x) = \frac{1}{\sqrt{2-x}} + 5$

Sol

$$2 - x > 0$$
$$-x > -2 \quad \boxed{-1}$$
$$x < 2$$
$$D_f = (-\infty, 2)$$

المدى

$$f(-\infty) = \frac{1}{\sqrt{2-(-\infty)}} + 5 \; = \frac{1}{\infty} + 5 = 5$$
$$f(-2) = \frac{1}{\sqrt{2-2}} + 5 = \frac{1}{0} + 5 = \infty + 5 = \infty$$
$$R_f = (5, \infty)$$

9

$$f(x)=\frac{\sqrt{4-x^2}}{x-1}$$

Sol

البسط

$$4-x^2\geq 0$$

$$-x^2\geq -4 \quad \boxed{\div -1}$$

$$x^2\leq 4$$ باخذ الجذر

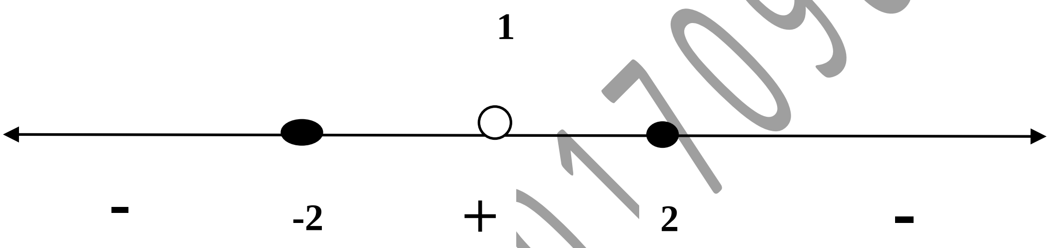
$$|x|\leq 2$$

$$-2\leq x\leq 2$$

المقام

$$x-1\neq 0$$

$$x\neq 1$$



$$D_f=[-2,1)\cup(1,2]$$

10

$$f(x)=\frac{x-\sqrt{x}}{x-2}$$

Sol

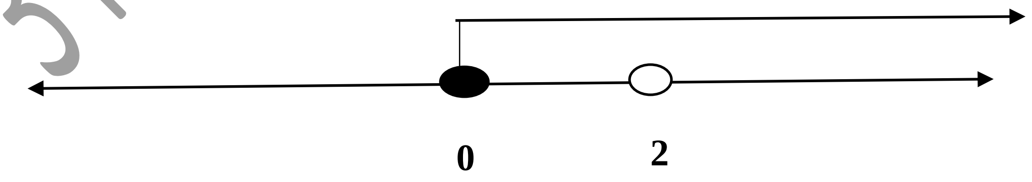
البسط

$$x\geq 0$$

المقام

$$x-2\neq 0$$

$$x\neq 2$$



$$D_f=[0,2)\cup(2,\infty)$$

11

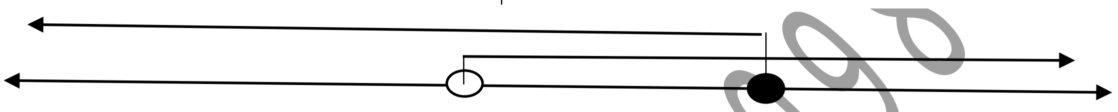
$$f(x) = \frac{1}{\sqrt{x}} + \sqrt{1-x}$$

Sol

$$\frac{1}{\sqrt{x}}$$
$$x > 0$$

$+$

$$\sqrt{1-x}$$
$$1 - x \geq 0$$
$$-x \geq -1 \quad \boxed{\div\ -1}$$
$$x \leq 1$$



$$D_f = (0, 1]$$

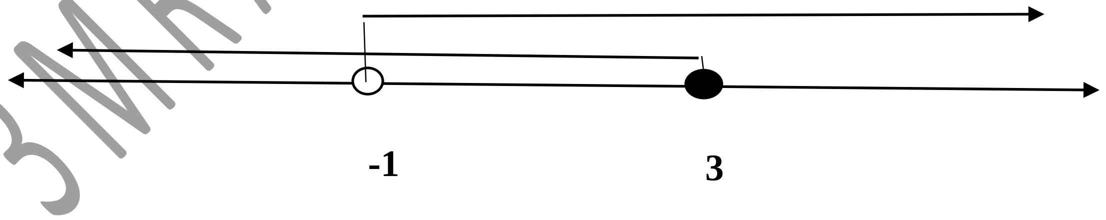
12

$$f(x) = \frac{\sqrt{3-x}}{\sqrt[4]{x+1}}$$

Sol

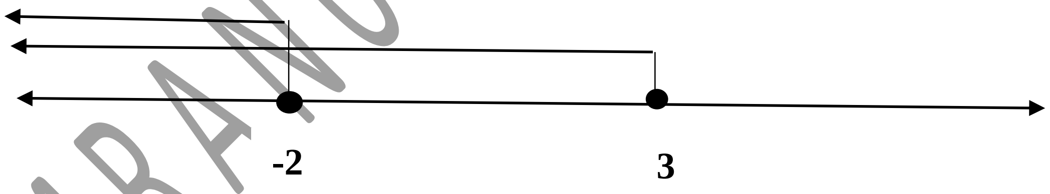
البسط
$$3 - x \geq 0$$
$$-x \geq -3 \quad \boxed{\div\ -1}$$
$$x \leq 3$$

المقام
$$x + 1 > 0$$
$$x > -1$$



$$D_f = (-1, 3]$$

13	$f(x) = \frac{36 - x^2}{x - 5}$ **Sol** $x - 5 \neq 0$ $x \neq 5$ $D_f = R - \{5\} \ = (-\infty, 5) \cup (5, \infty)$
14	$f(x) \ = \frac{x^2 - 3}{x^2 + 1}$ **Sol** $x^2 + 1 \neq 0$ $x^2 \neq -1$ $\boxed{\text{غيرحقيقى}}$ $D_f = R$
15	$f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ **Sol** البسط $x \geq 0$ المقام $x^2 + 1 > 0$ $x^2 > -1$ $\boxed{\text{غيرحقيقى}}$ $D_f \ = [0, \infty)$
16	$f(x) = \frac{x + \|x\|}{x}$ **Sol** $x \neq 0$ $D_f = R - \{0\} \ = (-\infty, 0) \cup (0, \infty)$ لايجاد المدى نعد نتعريف القيمه المطلقه $x = 0$ - $\frac{x - x}{x} = \frac{0}{x} = 0$ + $\frac{x + x}{x} = \frac{2x}{x} = 2$ $Rang = R_f = \{0, 2\}$

17	$f(x) = \begin{cases} x & 0 \leq x \leq 2 \\ 2 - x & 2 < x \leq 6 \end{cases}$ **Sol** $D_f = [0, 6]$ المدى $f(0) = 0$ $f(6) = 2 - 6 = -4$ $Rang = R_f = [-4, 0]$
18	$f(x) = \lvert 3x - 6 \rvert$ **Sol** $D_f = R = (-\infty, \infty)$ المدى $Rang = R_f = R = (-\infty, \infty)$
19	$f(x) = \sqrt{3 - x} + \sqrt{2 + x}$ **Sol** + $\sqrt{3 - x}$ $3 - x \geq 0$ $-x \geq -3 \quad \boxed{\div -1}$ $x \leq 3$ $\sqrt{2 + x}$ $2 + x \geq 0$ $x \geq -2$  $D_f = (-\infty, -2]$

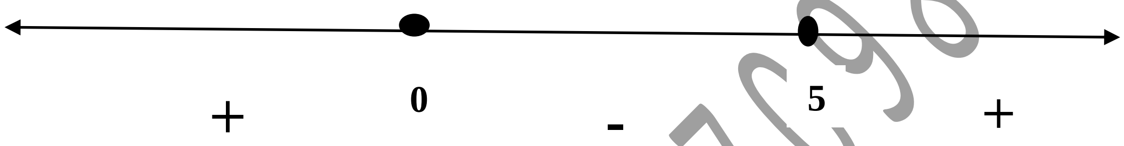
20

$$f(x)=\frac{1}{\sqrt[4]{x^2-5x}}$$

Sol

$$x^2-5x\geq 0$$
$$x^2-5x=0$$
$$x(x-5)=0$$

$$x=0 \quad\Big|\quad \begin{array}{c} x-5=0\\ x=5\end{array}$$



$$D_f=(-\infty,0]\cup[5,\infty)$$

21

$$f(x)=\frac{x+1}{1+\frac{1}{x+1}}$$

Sol

$$1+\frac{1}{x+1}\neq 0$$
$$\frac{1}{x+1}\neq -1$$
$$x+1\neq -1$$
$$x\neq -1-1$$
$$x\neq -2$$
$$D_f=R-\{-2\}=(-\infty,-2)\cup(-2,\infty)$$

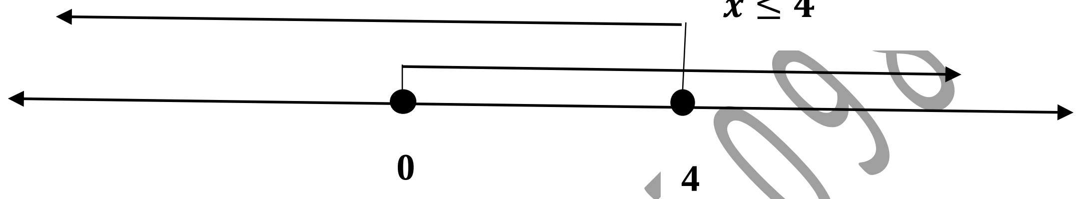
22

$f(x) = \sqrt{2-\sqrt{x}}$

Sol

$$\sqrt{x}$$
$$x \geq 0$$

$$\sqrt{2-\sqrt{x}}$$
$$2-\sqrt{x} \geq 0$$
$$-\sqrt{x} \geq -2 \quad \boxed{\div -1}$$
$$\sqrt{x} \geq 2 \boxed{\text{بالتربيع}}$$
$$x \leq 4$$



$$D_f = [0,4]$$

المدى

$$f(0) = \sqrt{2-\sqrt{0}} = \sqrt{2}$$
$$f(4) = \sqrt{2-\sqrt{4}} = \sqrt{2-2} = 0$$
$$Rang = R_f = [0,\sqrt{2}]$$

23

$f(x) = \sqrt{1-x^2}$

Sol

$$1-x^2 \geq 0$$
$$-x^2 \geq -1 \quad \boxed{\div -1}$$
$$x^2 \leq 1 \boxed{\text{باخذ الجذر التربيعى}}$$
$$|x| \leq 1$$
$$-1 \leq x \leq 1$$
$$D_f = [-1,1]$$

المدى

$$f(-1) = \sqrt{1-(-1)^2} = \sqrt{1-1} = 0$$
$$f(1) = \sqrt{1-1^2} = \sqrt{1-1} = 0$$
$$f(0) = \sqrt{1-0} = \sqrt{1} = 1$$
$$Rang = R_f = [0,1]$$

24

$f(x) = \sqrt{x^2 - 2}$

Sol

$$x^2 - 2 \geq 0$$

$$x^2 \geq 2 \quad \boxed{\text{باخذ الجذر التربيعى}}$$

$$|x| \geq \sqrt{2}$$

$$x \geq \sqrt{2} \quad \Big| \quad x \leq -\sqrt{2}$$

$$D_f = (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$$

المدى

$$f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 - 2} = \sqrt{2 - 2} = 0$$

$$f(-\sqrt{2}) = \sqrt{(-\sqrt{2})^2 - 2} = \sqrt{2 - 2} = 0$$

$$f(0) = \sqrt{0 - 2} = \sqrt{-2} \text{ غير معرف } = \infty$$

$$Rang = R_f = [0, \infty)$$

25

$f(x) = 9 - x^2$

Sol

$$\text{كثيرات حدود } D_f = R = (-\infty, \infty)$$

المدى

$$f(\infty) = 9 - \infty^2 = -\infty$$

$$f(-\infty) = 9 - (-\infty)^2 = -\infty$$

$$f(0) = 9 - 0 = 9$$

$$Rang = R_f = (-\infty, 9]$$

26

$f(x) = 2 - \sqrt{x}$

Sol

$$x \geq 0 \quad D_f = [0, \infty)$$

المدى

$$f(0) = 2 - \sqrt{0} = 2$$

$$f(\infty) = 2 - \sqrt{\infty} = 2 - \infty = -\infty$$

$$Rang = R_f = (-\infty, 2]$$

28

$f(x)\frac{x}{|x|}$

Sol

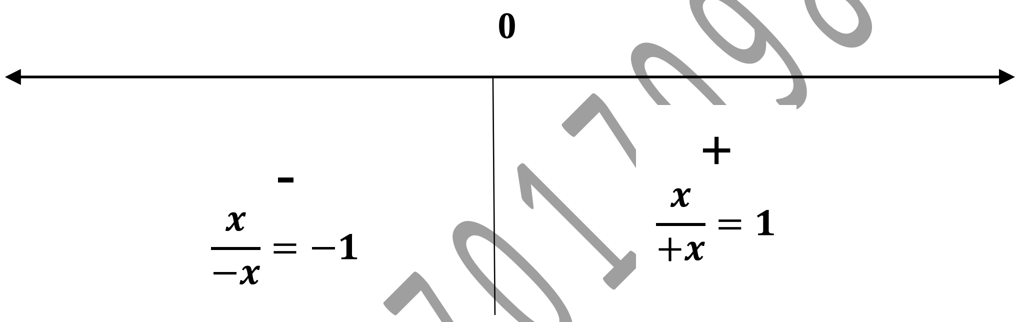
$$|x| \neq 0$$

$$x \neq 0$$

$$D_f = R - \{0\} \;= (-\infty, 0) \cup (0, \infty)$$

لايجاد المدى نعد تعريف القيمه المطلقه

$$x = 0$$



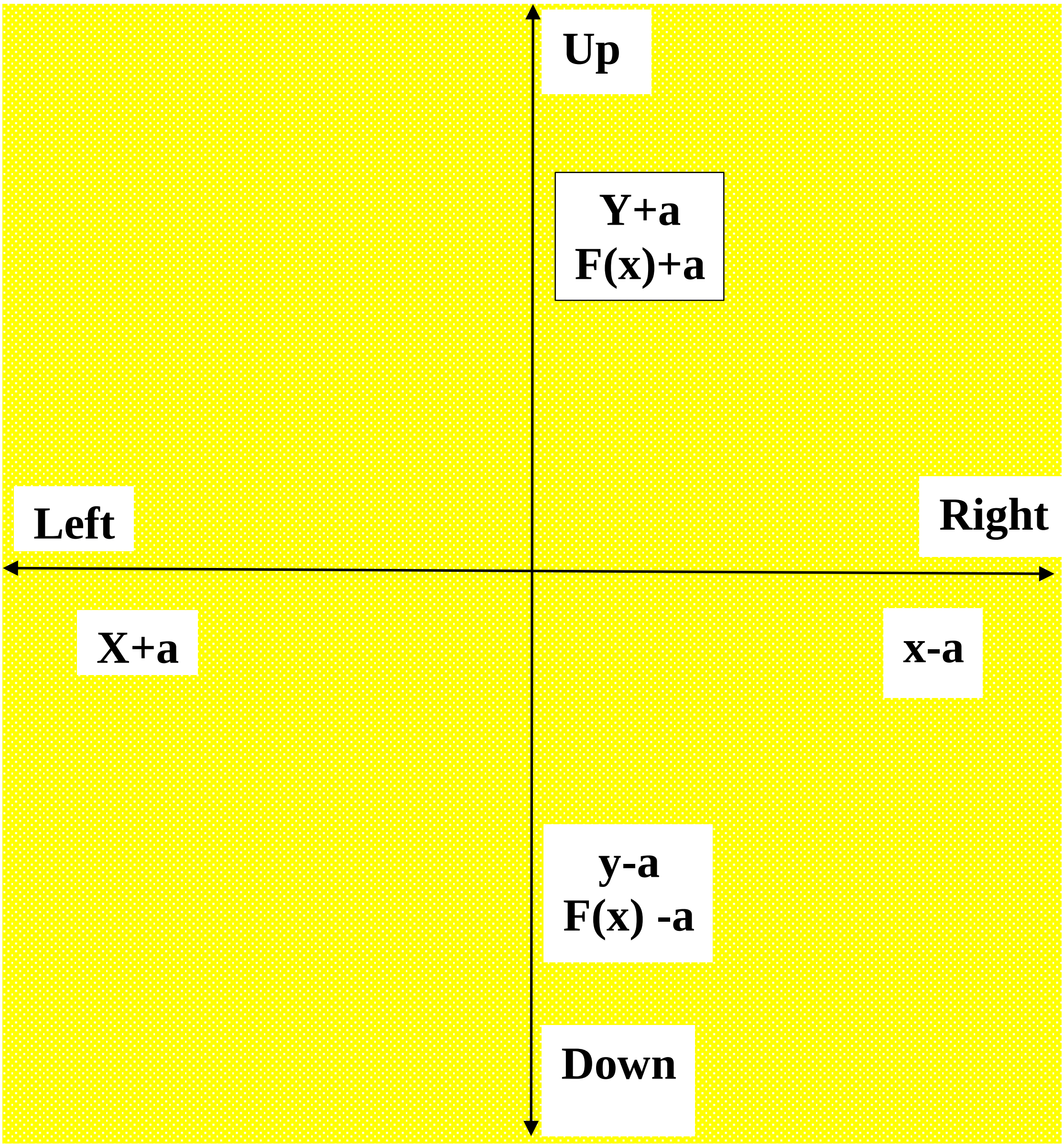
$$\frac{x}{-x} = -1$$

$$\frac{x}{+x} = 1$$

$$Rang = R_f = \{-1, 1\}$$

New function from old function

Shifting



On x-axis

-right with a units → x-a

- left with a units →x+a

On y-axis

-up with a units →y+a

-Down with a units →y-a

Stretch and compress
التمدد والضغط

vertical
راسى

strech
by c
y=cf(x)
نضرب
الداله فى
c

compress
by c
y=$\frac{1}{c}$ $f(x)$
ضرب الداله
فى $\frac{1}{c}$ اوقسمه
الداله على c

horizonatal
افقى

strech by
c y=f$\left(\frac{1}{c}\right)$
استبدال كل
1/c بx

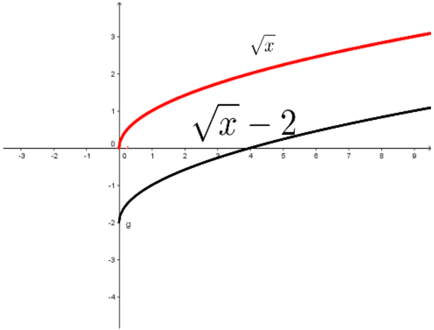
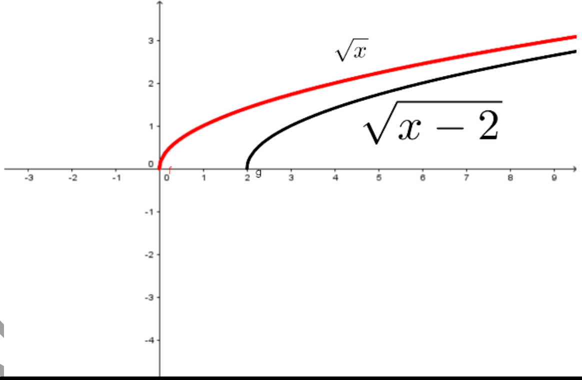
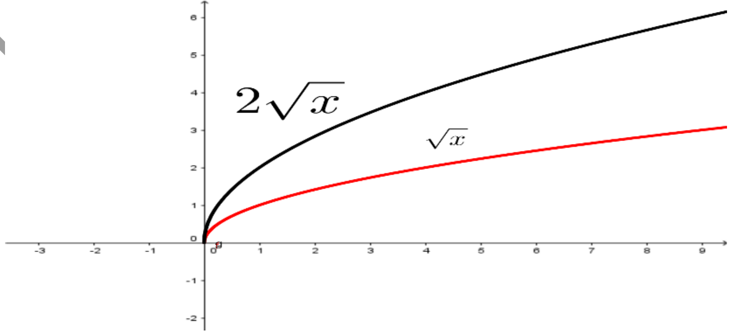
compress
by c
y=f(cx)
نستبدل كل x
ب c

Reflecting

aboyt x-axis
y=-f(x)
نضرب الداله فى 1-

about y- axis
y=f(-x)
نستبدل كل x ب -x

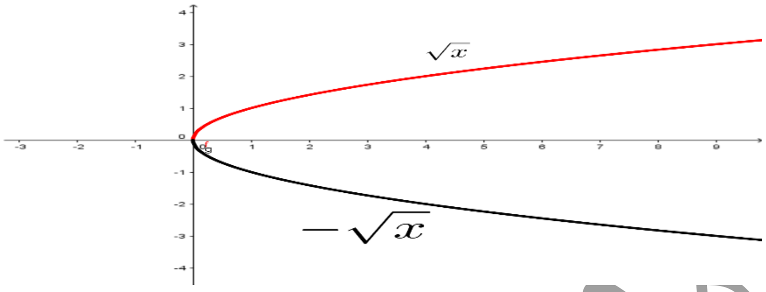
Given the graph of $y=\sqrt{x}$ use transformation to graph

1	$y=\sqrt{x}-2$ **Sol** **Shifted down <u>2</u> unite** 
2	$y=\sqrt{x-2}$ **sol** **shifted right <u>2</u> unites** 
3	$y=2\sqrt{x}$ **Sol** **Strech vertically <u>2</u> units** 

4 $y = -\sqrt{x}$

Sol

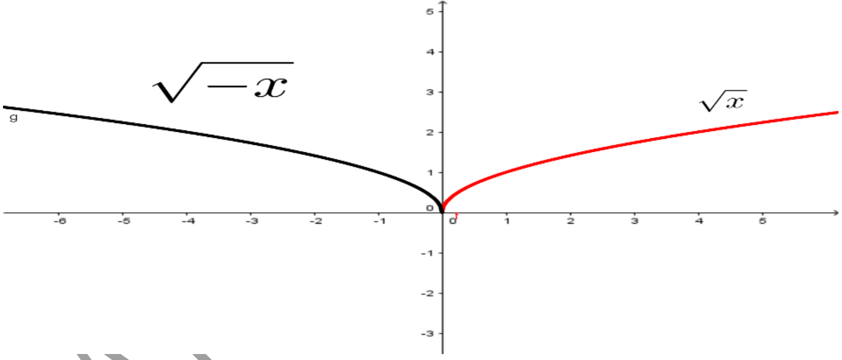
Reflected about x- axis



5 $y = \sqrt{-x}$

Sol

Reflected about y – axis



Explain how each graph is obtained from the graph $y = f(x)$	
1	$y = f(x) + 8$ *Sol* **Shifted up 8 units**
2	$y = f(x + 8)$ Sol **Shifted left 8 units**
3	$y = 8f(x)$ *Sol* **Strech vertically 8 units**
4	$y = f(8x)$ Sol **Compress horizonitally 8 units**
5	$y = -f(x) - 1$ *Sol* **Raflected about x- axis and then shifted down 1 unit**
6	$y = 8f\left(\frac{1}{8}\right)$ Sol **Strech horizonitally 8 units and then strech vertically 8 units**

If $y = x^2$ given an equation for the new function if

1	Shifted 3 units up words Sol $new\ function = x^2 + 3$
2	Shifted 2 unites to right Sol $new\ function = (x-2)^2 = x^2 - 4x + 4$
3	Shifted 3 units to left Sol $new\ function = (x+3)^2 = x^2 + 6x + 9$
4	Shifted 4 units to down Sol $new\ function = x^2 - 4$

If $y = x^2 - 1$ given an equation for the new function if

1	Stretched vertically by a factor 3 Sol $newfunction = 3(x^2 - 1) = 3x^2 - 1$
2	Compress vertically by a factor 2 Sol $new\ function = \frac{1}{2}(x^2 - 1) = \frac{1}{2}x^2 - \frac{1}{2}$
3	Stretch horizontally by a factor 4 Sol $new\ function = (\frac{x}{4})^2 - 1 = \frac{x^2}{16} - 1$
4	Compresses horizontally by a factor 5 Sol $new\ function = (5x)^2 - 1 = 25x^2 - 1$
5	Reflect the graph about x-axis Sol $new\ function = -(x^2 - 1) = -x^2 + 1$
6	Reflect the graph about y-axis Sol $new\ function = (-x)^2 - 1 = x^2 - 1$

Combination of function
تركيب الدوال

Given two function $f(x)$ and $g(x)$ and their domains are D_f and D_g

1) $(f+g)(x) = f(x) + g(x)$
2) $(f-g)(x) = f(x) - g(x)$
3) $(f.g)(x) = f(x).g(x)$
4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ *such that* $g(x) \neq 0$

Note

* $D_{f+g} = D_{f-g} = D_{f.g} = D_f \cap D_g$
* $D_{f/g} = D_f \cap D_g - \{\text{اصفار المقام}\}$

او مجال الدالة الناتجة من تركيب القسمة

If $\boldsymbol{f(x) = x^3 + 2x^2}$ and $\boldsymbol{g(x) = 3x^2 - 1}$

Find combination and their domain

1) $(\mathbf{f+g})(\mathbf{x})$

sol

$$(f+g)(x) = x^3 + 2x^2 + 3x^2 - 1 = x^3 + 5x^2 - 1$$

$$D_{f+g} = R$$

$$D_f = R \quad D_g = R$$
$$D_f \cap D_g = R$$

2)(f-g)(x)

Sol

$$(f-g)(x) = x^3 + 2x^2 - (3x^2 - 1)$$
$$= x^3 + 2x^2 - 3x^2 + 1 = x^3 - x^2 + 1$$
$$D_{f-g} = R$$

3) (f.g)(x)

Sol

$$(f.g)(x) = (x^3 + 2x^2)(3x^2 - 1)$$
$$= 3x^5 - x^3 + 6x^4 - 2x^2$$
$$= 3x^5 + 6x^4 - x^3 - 2x^2$$
$$D_{f.g} = R$$

4) (f/g) (x)

Sol

$$\left(\frac{f}{g}\right)(x) = \frac{x^3 + 2x^2}{3x^2 - 1}$$

لايجاد المدى المقام ≠ صفر

$$3x^2 - 1 \neq 0$$
$$3x^2 \neq 1 \quad \boxed{\div 3}$$
$$\frac{3x^2}{3} \neq \frac{1}{3}$$
$$x^2 \neq \frac{1}{3} \quad \boxed{\text{باخذ الجذر}}$$
$$x \neq \pm\sqrt{\frac{1}{3}} \qquad x \neq \pm\frac{1}{\sqrt{3}}$$
$$D_{\frac{f}{g}} = R - \left\{\pm\frac{1}{\sqrt{3}}\right\}$$

$$f(x)=\sqrt{3-x}\qquad\qquad g(x)=\sqrt{x^2-1}$$

Find

1) (f+g)(x)

Sol

$(f+g)(x)=\sqrt{3-x}+\sqrt{x^2-1}$

$D_{f+g}=[3,\infty)$

2)(f-g)(x)

Sol

$(f-g)(x)=\sqrt{3-x}-\sqrt{x^2-1}$

$D_{f-g}=[3,\infty)$

3) (f.g)(x)

sol

$(f.g)(x)=\left(\sqrt{3-x}\right)\left(\sqrt{x^2-1}\right)$

$=\sqrt{(3-x)(x^2-1)}$

$=\sqrt{3x^2-3-x^3+x}$

$=\sqrt{-x^3+3x^2+x-3}$

$D_{f.g}=[3,\infty)$

4) (f/g) (x)

Sol

$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{3-x}}{\sqrt{x^2-1}}$

$D_{\frac{f}{g}}=[3,\infty)$

D_f

$3-x\geq 0$

$-x\geq -3\quad \boxed{\div -1}$

$\frac{-x}{-1}\leq -\frac{3}{-1}$

$x\leq 3$

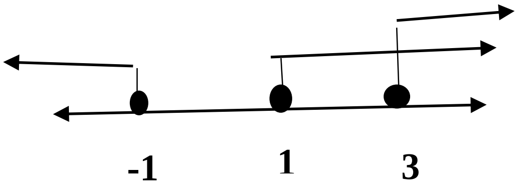
D_g

$x^2-1\geq 0$

$x^2\geq 1\quad$ باخذ الجذر

$|x|\geq 1$

$x\leq -1 \quad\Big|\quad x\geq 1$



$\boxed{D_f\cap D_g=[3,\infty)}$

Composite function
تحصيل الدوال

- (FoG) (x) =f(g(x))
- (GoF)(x) =g(f(x))

$D_{fog} =$ مجال الدالة الناتجة تقاطع مجال الدالة الثانية g(x)

$D_{gof} =$ مجال الدالة الناتجة تقاطع مجال الدالة الثانية $f(x)$

Example

If $f(x)$=x^2 $g(x) = x - 3$ find

1) fog(x) **2) gof(x)**

And their domain

Sol

$$D_f = R \quad D_g = R \quad D_f \cap D_g = R$$

$$1) fog(x) = f(g(x)) = f(x-3) = (x-3)^2$$
$$= x^2 - 6x + 9$$
$$D_{fog} = R$$

$$2) gof(x) = g(f(x)) = g(x^2)$$
$$= x^2 - 3$$
$$D_{gof} = R$$

If $f(x) = \sqrt{x}$ $\quad\quad\quad$ $g(x) = \sqrt{2-x}$

Find each function and its domain

1)fog $\quad$ 2)gof $\quad$ 3) fof $\quad$ 4)gog

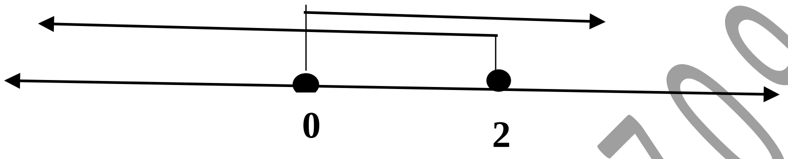
Sol

$$D_f \quad x \geq 0$$

$$D_g$$

$$2 - x \geq 0 \qquad -x \geq -2 \quad \boxed{\div -1}$$

$$\frac{-x}{-1} \leq \frac{-2}{-1} \qquad x \leq 2$$



$$\boxed{D_f \cap D_g = [0,2]}$$

$$1)fog(x) = f(g(x)) = f(\sqrt{2-x}) = \sqrt{\sqrt{2-x}} = \sqrt[4]{2-x}$$

$$D_{fog} = [0,2]$$

$$2)gof(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$$

$$D_{gof} = [0,2]$$

$$3)fof(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$$

$$D_{fof} = [0,\infty)$$

$$4)gog(x) = g(g(x)) = g(\sqrt{2-x}) = \sqrt{2-\sqrt{2-x}}$$

$$D_{gog}$$

$$2 - x \geq 0$$

$$-x \geq -2 \quad \boxed{\div \ -1}$$

$$\frac{-x}{-1} \leq \frac{-2}{-1}$$

$$x \leq 2$$

$$2-\sqrt{2-x}\ \geq 0$$

$$-\sqrt{2-x}\ \geq -2\ \boxed{\div -1}$$

$$\frac{-\sqrt{2-x}}{-1} \leq \frac{-2}{-1}$$

$\sqrt{2-x} \leq 2$ بالتربيع

$$2-x \geq 4$$

$$-x \geq 4-2$$

$$-x \geq 2\ \boxed{\div\ -1}$$

$$\frac{-x}{-1} \leq \frac{2}{-1}$$

$$x \leq -2$$

$$D_{gog} = (-\infty, -2]$$

If

$f(x) = \dfrac{x}{x+1}$ $\qquad g(x) = x^{10}$ $\qquad h(x) = x + 3$

Find

1)fohog **2) fogoh**

Sol

$$1)fohog = f\big(h(g)\big)(x) = f\left(h(x^{10})\right) = f(x^{10}+3)$$

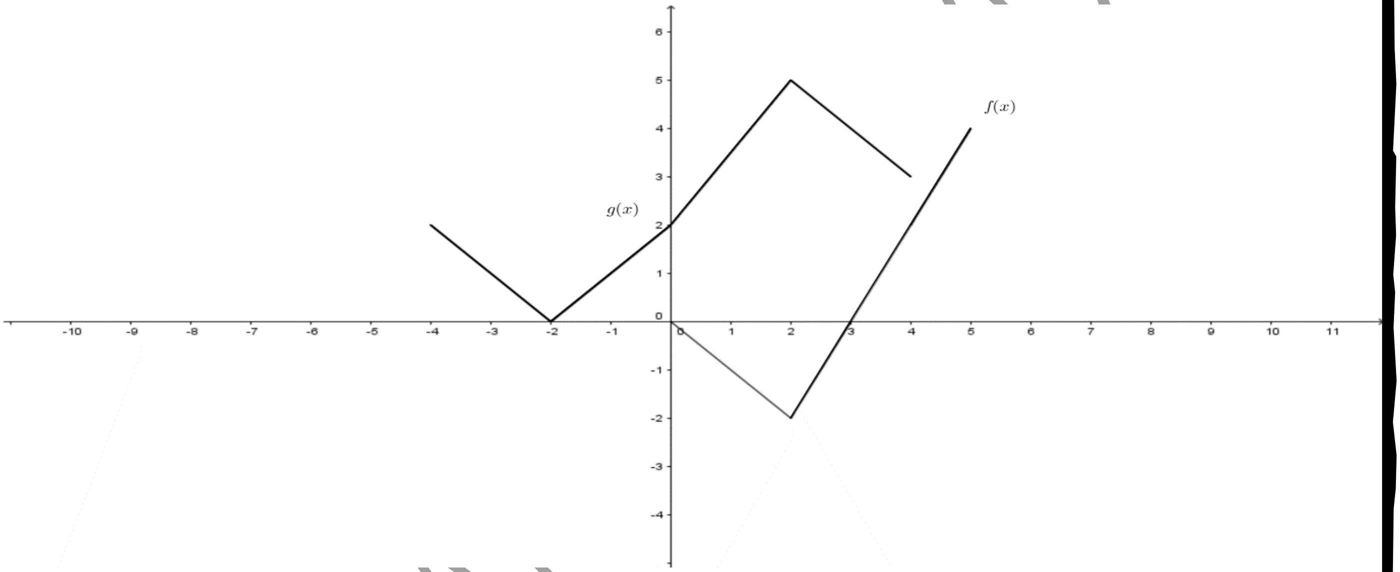
$$= \frac{x^{10}+3}{x^{10}+3+1} = \frac{x^{10}+3}{x^{10}+4}$$

$$2)fogoh = f\big(g(h)\big)(x) = f\big(g(x+3)\big) = f[(x+3)^{10}]$$

$$= \frac{(x+3)^{10}}{(x+3)^{10}+1}$$

x	1	2	3	4	5	6
F(x)	3	1	4	2	2	5
G(x)	6	3	2	1	2	3

Use the table to evaluate each expression	
1) f(g(1))=**f(6)=5**	2) g(f(1))=**g(3)=2**
3) g(g(1))=**g(6)=3**	4) (gof)(3)=**g(f(3))=g(4)=1**
5) f(f(1))=**f(3)=4**	6)(fog)(6)=**f(g(6))=f(3)=4**



Use the graph of f and g to evaluate

1)f(g(2)) =f(4)=1

2) g(f(0))=g(0)=2

3) (fog) (0)=f(g(0))=f(2)=-2

4) gof (6)=g(f(6))=g(4)=3

5) (gog)(-2)=g(g(-2))=g(0)=2

6) (fof)(4)=f(f(4))=f(1)=-1

Find fog ,gof , fof and gog

And their domain if

$$1)f(x) = 1 - 3x \qquad\qquad g(x) = cosx$$

Sol

$$D_f = R \quad D_g = R \quad D_f \cap D_g = R$$

$$-fog(x) = f\left(g(x)\right) = f(cosx) = 1 - 3cosx$$

$$D_{fog} = R$$

$$-gof(x) = g\left(f(x)\right) = g(1 - 3x) = cos\,(1 - 3x)$$

$$D_{gof} = R$$

$$-fof(x) = f\left(f(x)\right) = f(1 - 3x) = 1 - 3(1 - 3x) = 1 - 3 + 9x$$

$$= -2 + 9x$$

$$D_{fof} = R$$

$$-gog(x) = g\left(g(x)\right) = g(cos\,x) = cos\,(cosx)$$

$$D_{gog} = R$$

$$2)f(x)=\sqrt{x} \qquad\qquad g(x)=x^2$$

Sol

$$D_f \quad x\geq 0 \quad D_f=[0,\infty)$$

$$D_g=R$$

$$D_f\cap D_g=[0,\infty)$$

$$-fog(x)=f\big(g(x)\big)=f(x^2)=\sqrt{x^2}\;=x$$

$$D_{fog}=[0,\infty)$$

* * *

$$-gof(x)=g\big(f(x)\big)=g\big(\sqrt{x}\,\big)=(\sqrt{x}\,)^2=x$$

$$D_{gof}=[0,\infty)$$

* * *

$$-fof(x)=f\big(f(x)\big)=f\big(\sqrt{x}\big)=\sqrt{\sqrt{x}}=\sqrt[4]{x}$$

$$D_{fof}=[0,\infty)$$

* * *

$$-gog(x)=g\big(g(x)\big)=g(x^2)=(x^2)^2=x^4$$

$$D_{gog}=R$$

Function	Domain	Range
1) Polynomial كثيرة حدود	$\mathbb{R} = (-\infty, \infty)$ for all degrees	
من الدرجة الصفرية of zero degree ex: $f(x) = C$		$\{C\}$
من الدرجة الأولى of degree one or odd		$\mathbb{R}$ //
من الدرجة الثانية of degree two :		
i) x^2		$[0, \infty)$
ii) $x^2 - a$		$[-a, \infty)$
iii) $x^2 + a$		$[a, \infty)$
iiii) $a - x^2$		$(-\infty, a]$
2) $\sqrt[3]{f(x)}$	$\mathbb{R}$	
3) $\sqrt{f(x)}$	$f(x) \geq 0$	The range of an even root i always [0,∞
i) $\sqrt{x^2 - a^2}$	$(-\infty, -a] \cup [a, \infty)$	$[0, \infty)$
ii) $\sqrt{a^2 - x^2}$	$[-a, a]$	$[0, a]$
iii) $\sqrt{x^2 + a^2}$	$\mathbb{R}$	$[a, \infty)$
iiii) $\sqrt{x + a}$	$[-a, \infty)$	$[0, \infty)$
iiiii) $\sqrt{x - a}$	$[a, \infty)$	$[0, \infty)$
iiiiii) $\sqrt{a - x}$	$(-\infty, +a]$	$[0, \infty)$

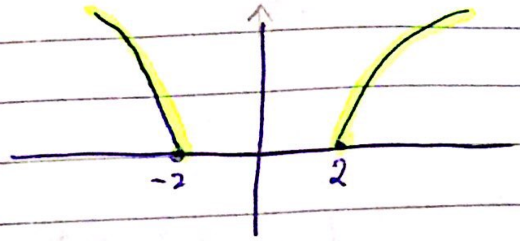
rational

	Function	Domain	Range
4) الكسرية دائماً موجب و لا يساوي صفر	Fraction i) $\frac{f(x)}{\boxed{x^2+a^2}}$ where $f(x)$ is Polynomial	$\mathbb{R}$	
	ii) $\frac{f(x)}{g(x)}$	$\mathbb{R} - \{\text{zeros of } g(x)\}$	
	iii) $\frac{f(x)}{\sqrt{g(x)}}$	$g(x) > 0$ لانها كانت الجذر وفي المقام في نفس الوقت	
5)	Absolute value of Polynomail القيمة المطلقة لكثيرة حدود	$\mathbb{R}$	$[0,\infty)$
6)	You have to know how to find the range & domain of a Piecewise defined function like example 9 page 20		

Domain & Range

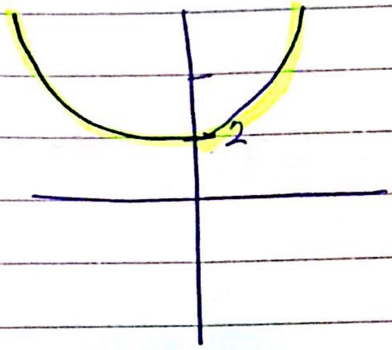
	D	R
(i) $\sqrt{x^2-a^2}$	$[-\infty, -a] \cup [a, \infty]$	$[0, \infty)$

$\sqrt{x^2-4}$



(ii) $\sqrt{x^2+a^2}$ $\longrightarrow$ $D_f = \mathbb{R}$

Range $= [a, \infty]$

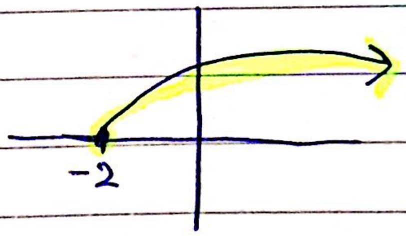


$\sqrt{x^2+4}$

(iii) $\sqrt{x+a}$

$D_f = [-a, \infty)$

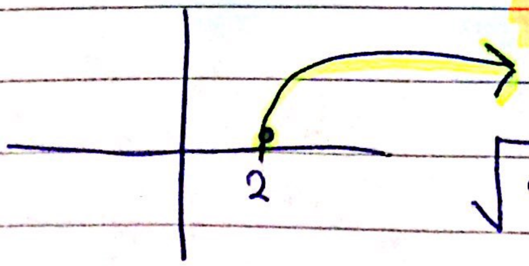
Range $= [0, \infty$



$\sqrt{x+2}$

(iiii) $\sqrt{x-a}$ $D_f = [a, \infty)$

Range $[0, \infty)$

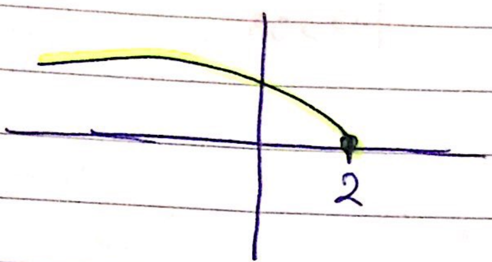


$\sqrt{x-2}$

(ii) $\sqrt{a-x}$

$D_f = (-\infty, a]$

Range $[0, \infty)$



$\sqrt{2-x}$

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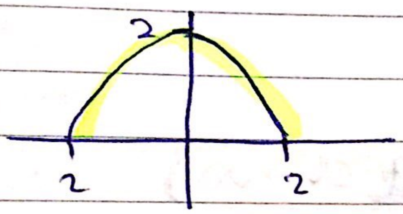
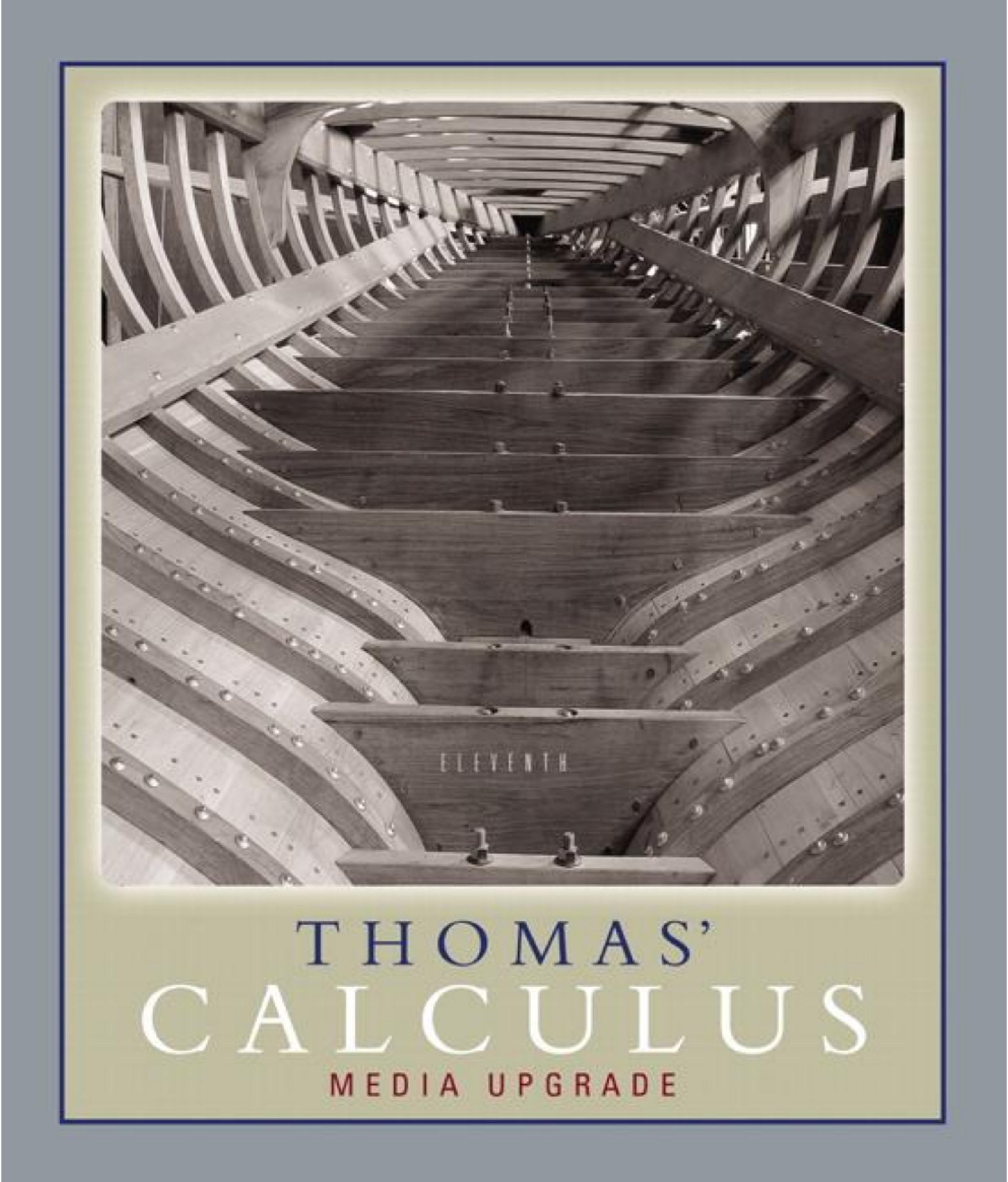
* $\sqrt{a^2-x^2}$

Domain : $[a, -a]$

Range $[0, a]$





ELEVENTH

THOMAS'
CALCULUS
MEDIA UPGRADE

Chapter 1

Preliminaries

1.1

Real Numbers and the Real Line

Rules for Inequalities

If a, b, and c are real numbers, then:

1. $a < b \Rightarrow a + c < b + c$

2. $a < b \Rightarrow a - c < b - c$

3. $a < b$ and $c > 0 \Rightarrow ac < bc$

4. $a < b$ and $c < 0 \Rightarrow bc < ac$
Special case: $a < b \Rightarrow -b < -a$

5. $a > 0 \Rightarrow \dfrac{1}{a} > 0$

6. If a and b are both positive or both negative, then $a < b \Rightarrow \dfrac{1}{b} < \dfrac{1}{a}$

TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x \mid a < x < b\}$	Open	
	$[a, b]$	$\{x \mid a \leq x \leq b\}$	Closed	
	$[a, b)$	$\{x \mid a \leq x < b\}$	Half-open	
	$(a, b]$	$\{x \mid a < x \leq b\}$	Half-open	
Infinite:	(a, ∞)	$\{x \mid x > a\}$	Open	
	$[a, \infty)$	$\{x \mid x \geq a\}$	Closed	
	$(-\infty, b)$	$\{x \mid x < b\}$	Open	
	$(-\infty, b]$	$\{x \mid x \leq b\}$	Closed	
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open and closed	

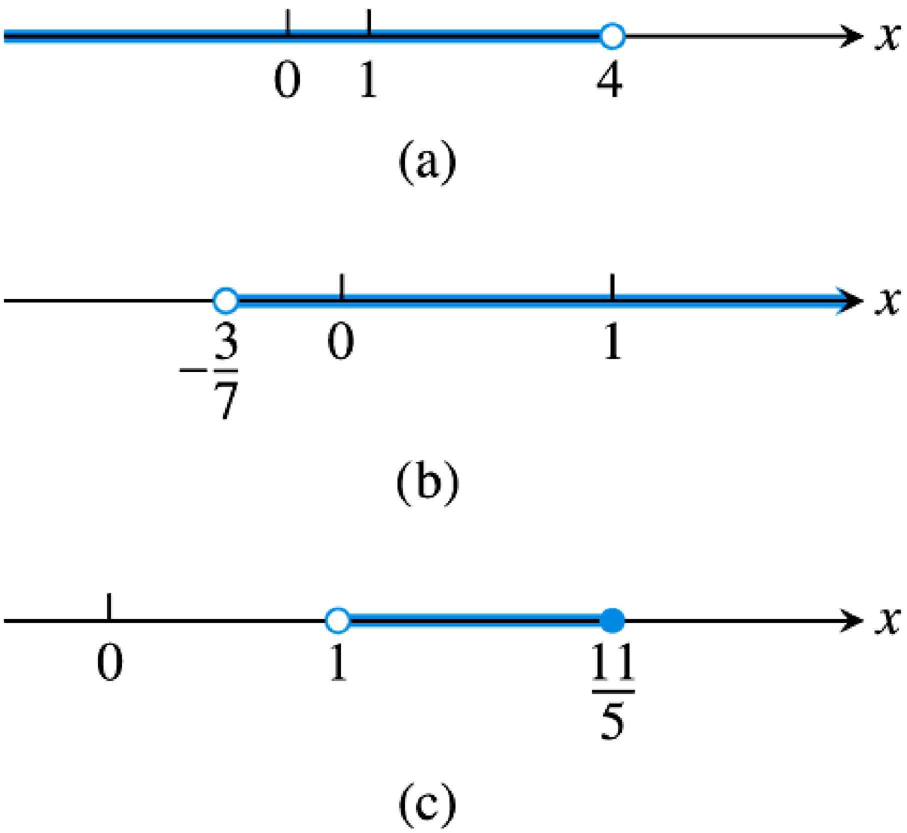


FIGURE 1.1 Solution sets for the inequalities in Example 1.

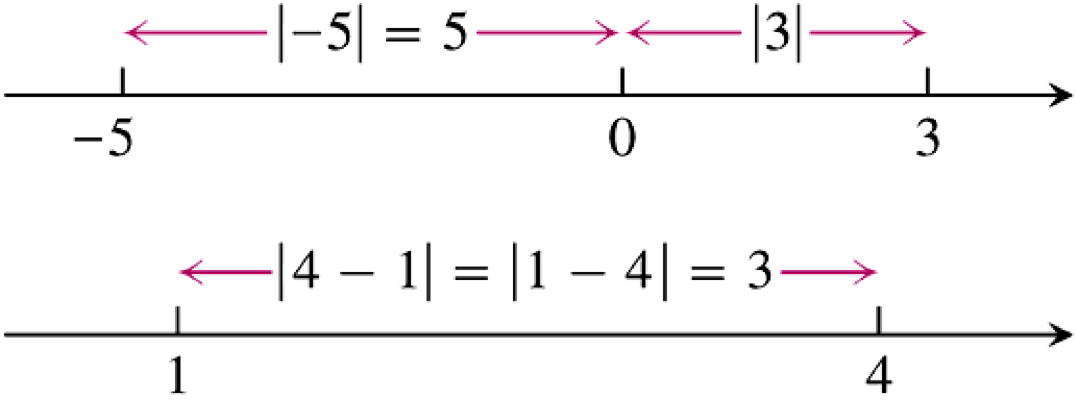


FIGURE 1.2 Absolute values give distances between points on the number line.

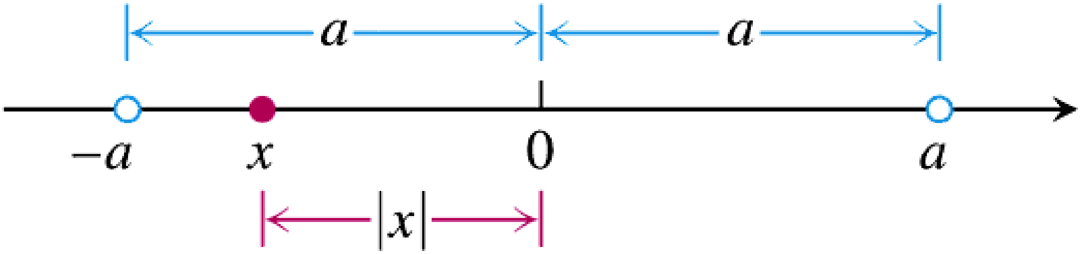
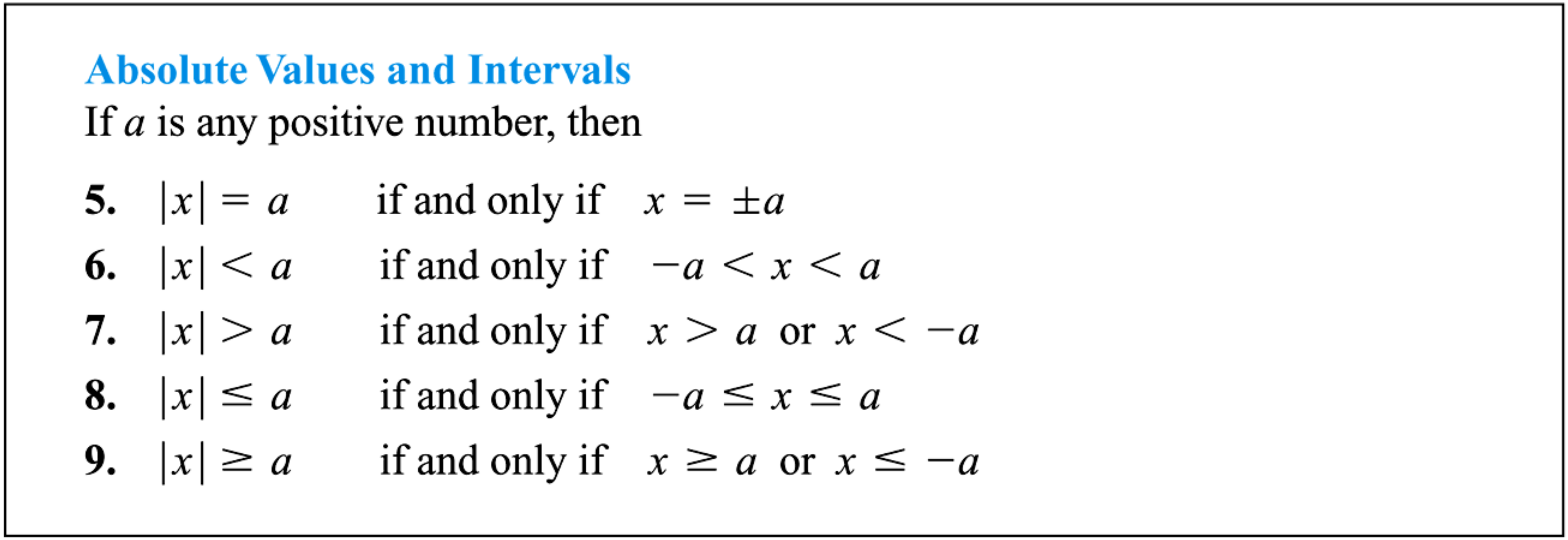


FIGURE 1.3 $|x| < a$ means x lies between $-a$ and a.



Absolute Values and Intervals

If a is any positive number, then

5. $|x| = a$ if and only if $x = \pm a$

6. $|x| < a$ if and only if $-a < x < a$

7. $|x| > a$ if and only if $x > a$ or $x < -a$

8. $|x| \leq a$ if and only if $-a \leq x \leq a$

9. $|x| \geq a$ if and only if $x \geq a$ or $x \leq -a$

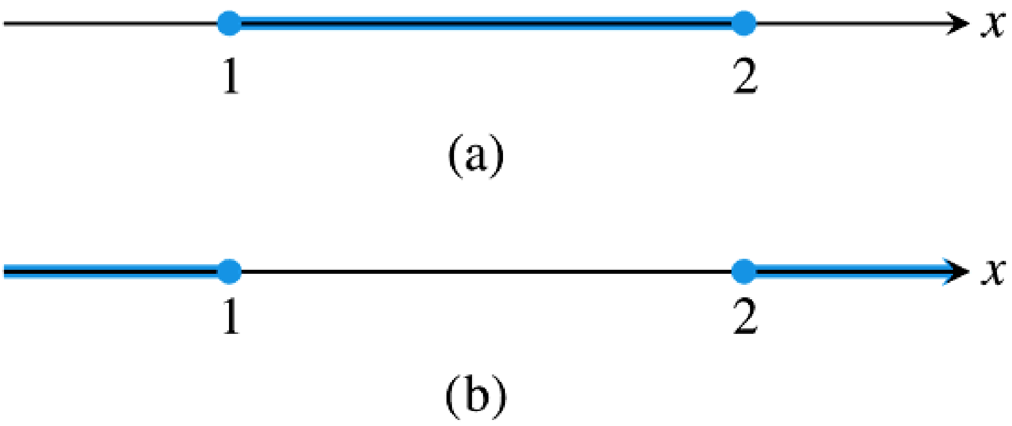


FIGURE 1.4 The solution sets (a) [1, 2] and (b) $(-\infty, 1] \cup [2, \infty)$ in Example 6.

1.2

Lines, Circles and Parabolas

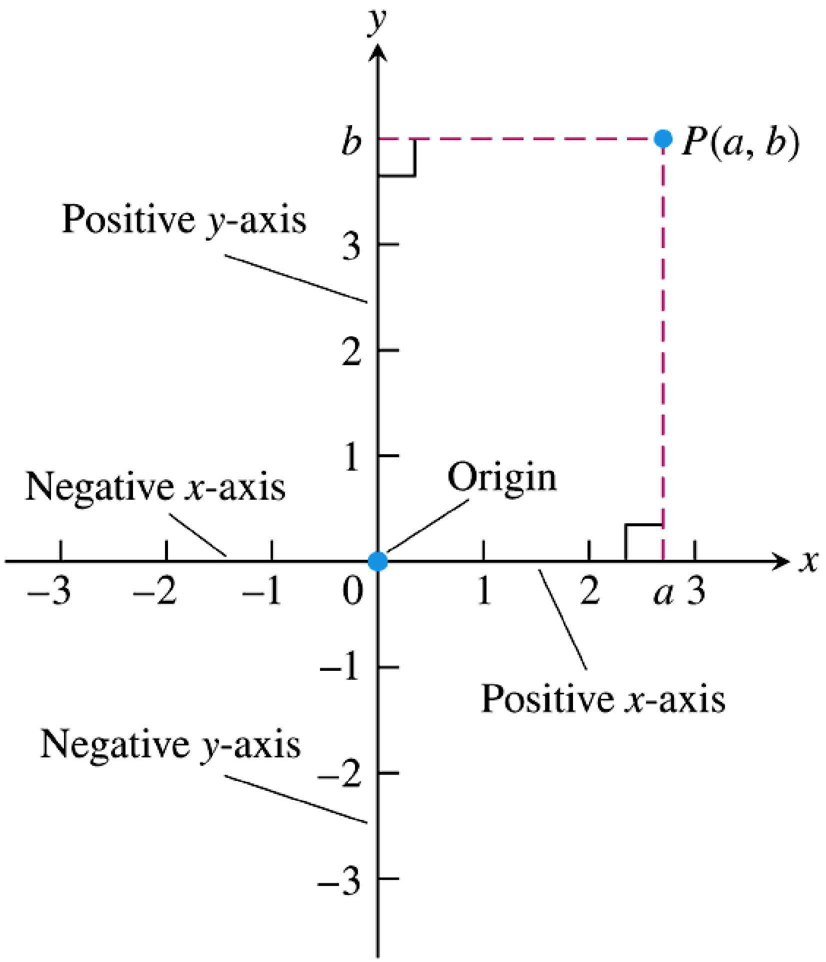


FIGURE 1.5 Cartesian coordinates in the plane are based on two perpendicular axes intersecting at the origin.

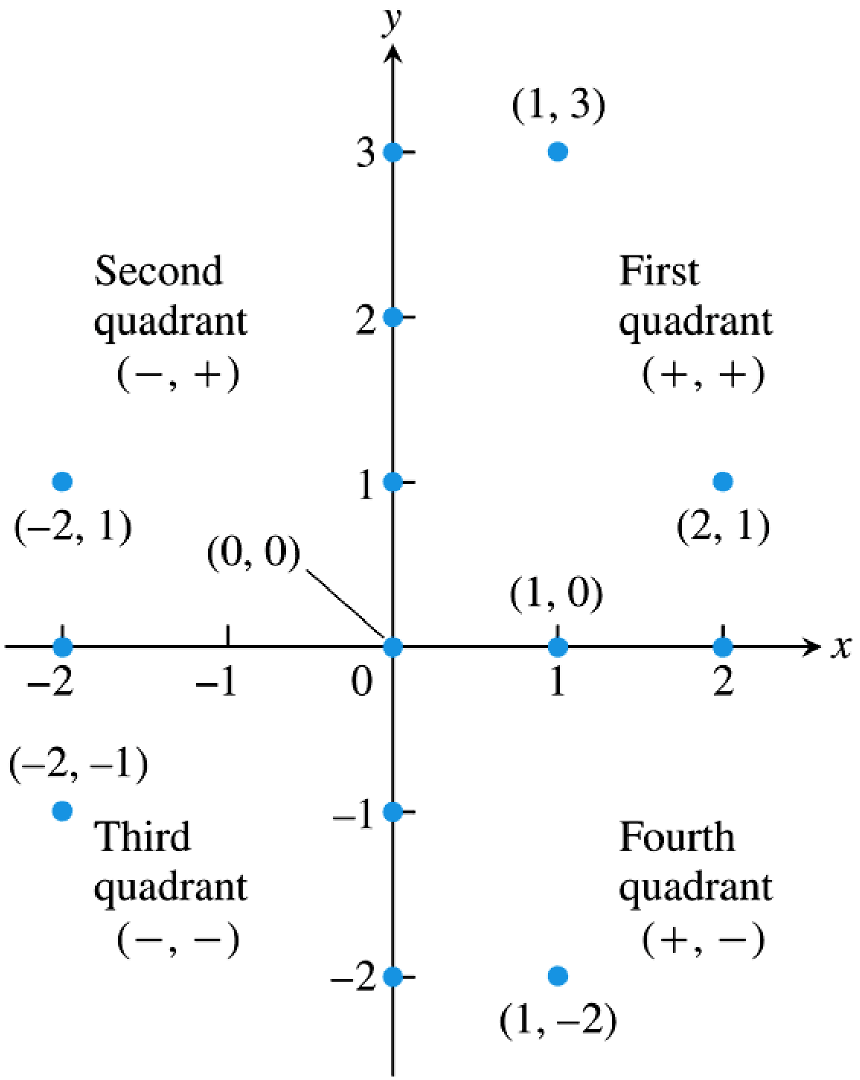


FIGURE 1.6 Points labeled in the *xy*-coordinate or Cartesian plane. The points on the axes all have coordinate pairs but are usually labeled with single real numbers, (so (1, 0) on the *x*-axis is labeled as 1). Notice the coordinate sign patterns of the quadrants.

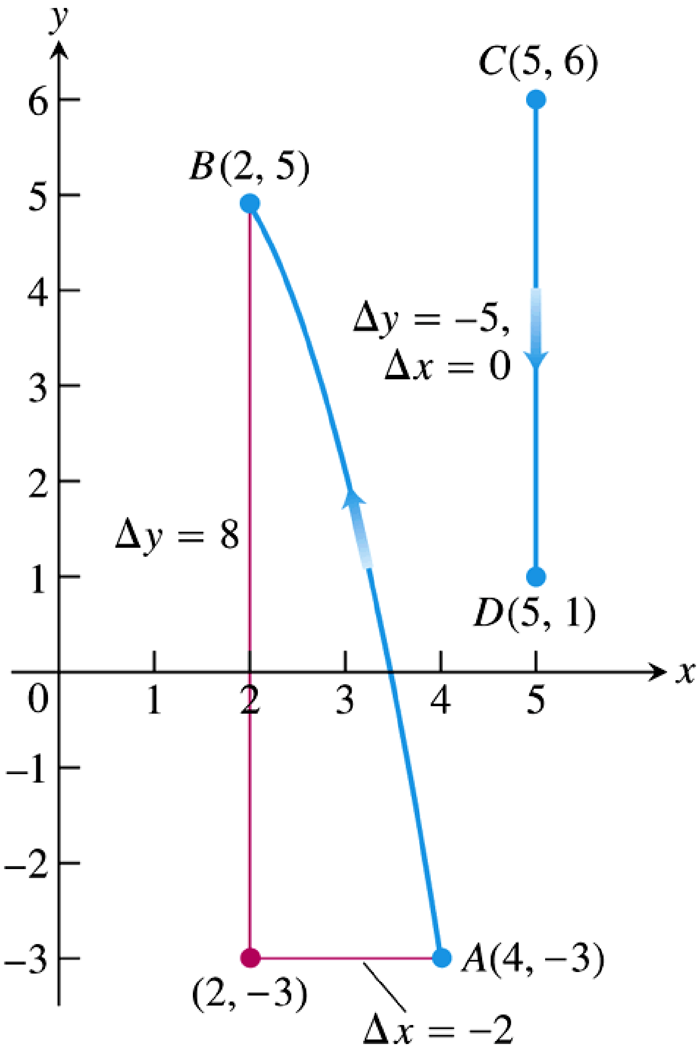


FIGURE 1.7 Coordinate increments may be positive, negative, or zero (Example 1).

DEFINITION **Slope**

The constant

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

is the **slope** of the nonvertical line $P_1 P_2$.

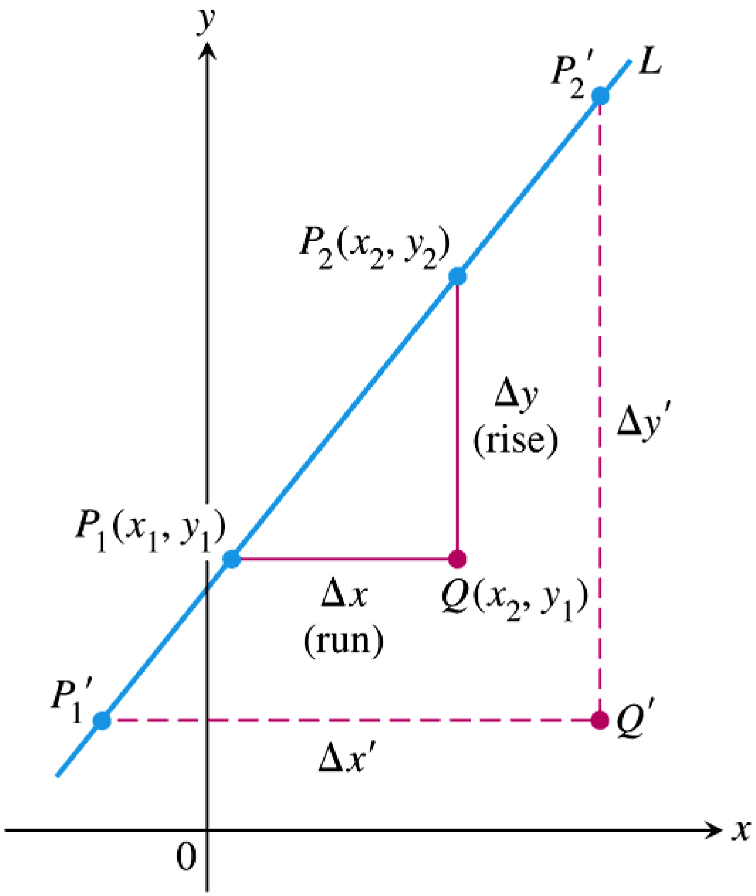


FIGURE 1.8 Triangles P_1QP_2 and $P_1'Q'P_2'$ are similar, so the ratio of their sides has the same value for any two points on the line. This common value is the line's slope.

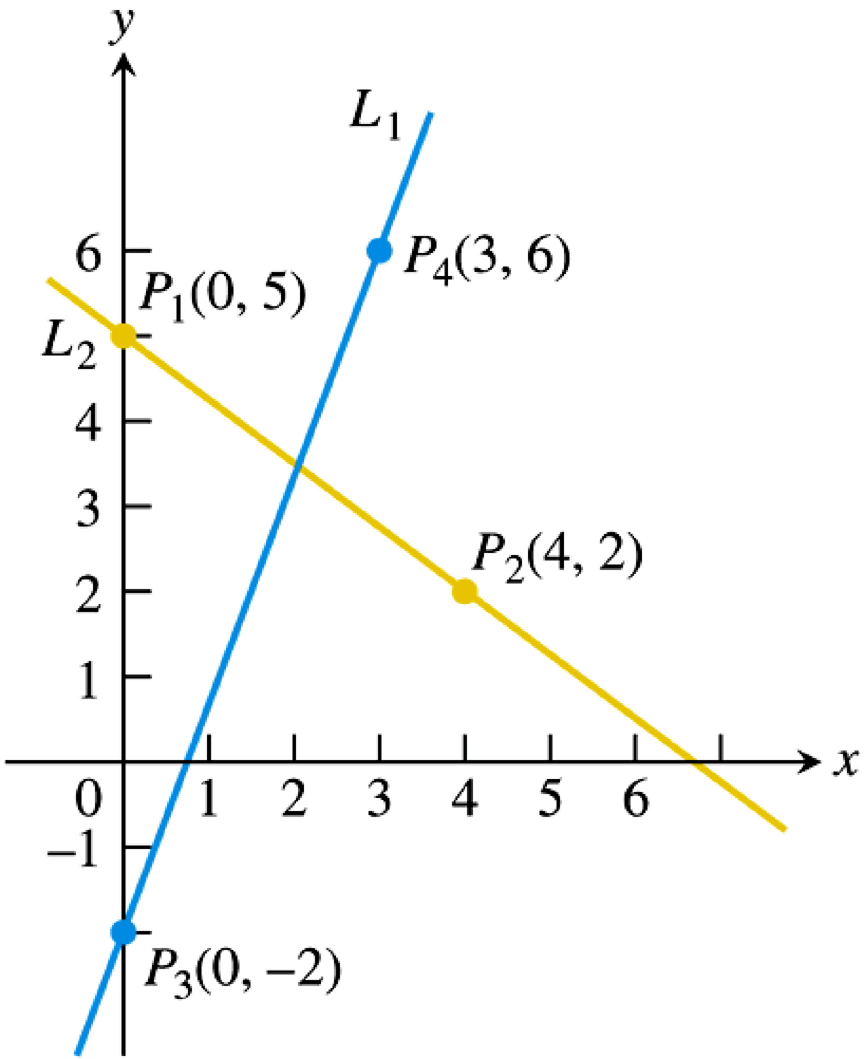


FIGURE 1.9 The slope of L_1 is

$$m = \frac{\Delta y}{\Delta x} = \frac{6 - (-2)}{3 - 0} = \frac{8}{3}.$$

That is, y increases 8 units every time x increases 3 units. The slope of L_2 is

$$m = \frac{\Delta y}{\Delta x} = \frac{2 - 5}{4 - 0} = \frac{-3}{4}.$$

That is, y decreases 3 units every time x increases 4 units.

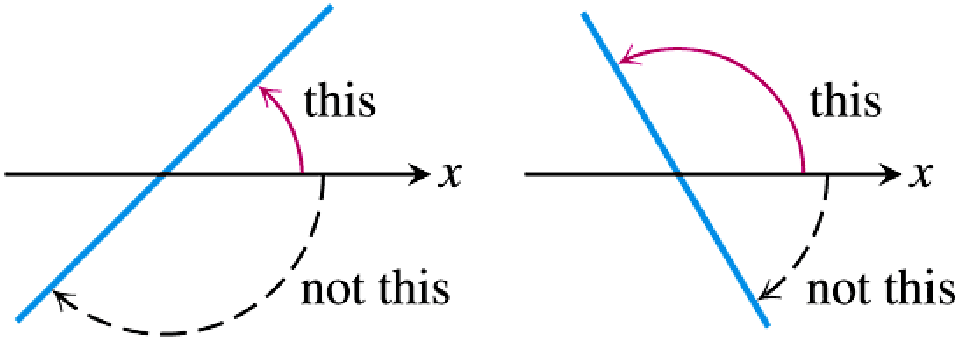
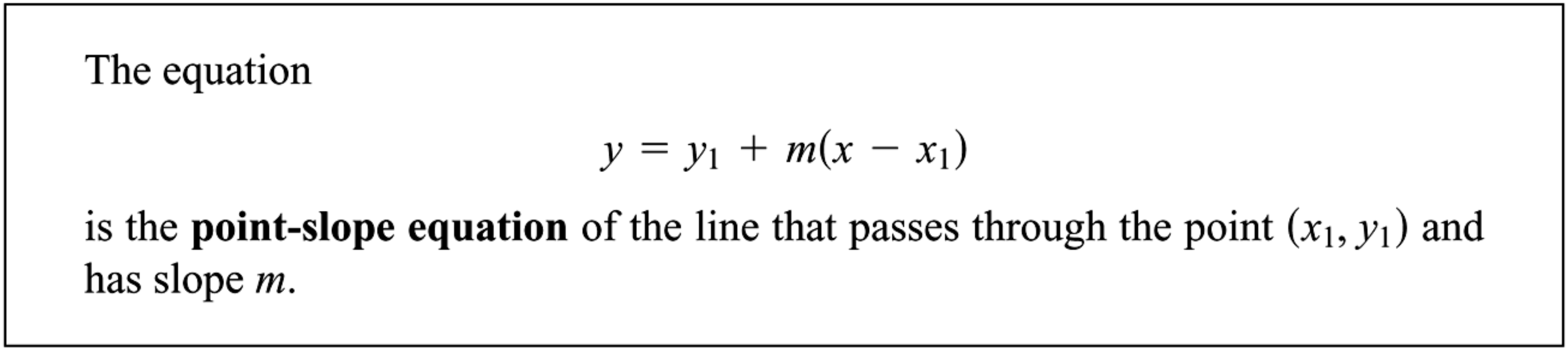


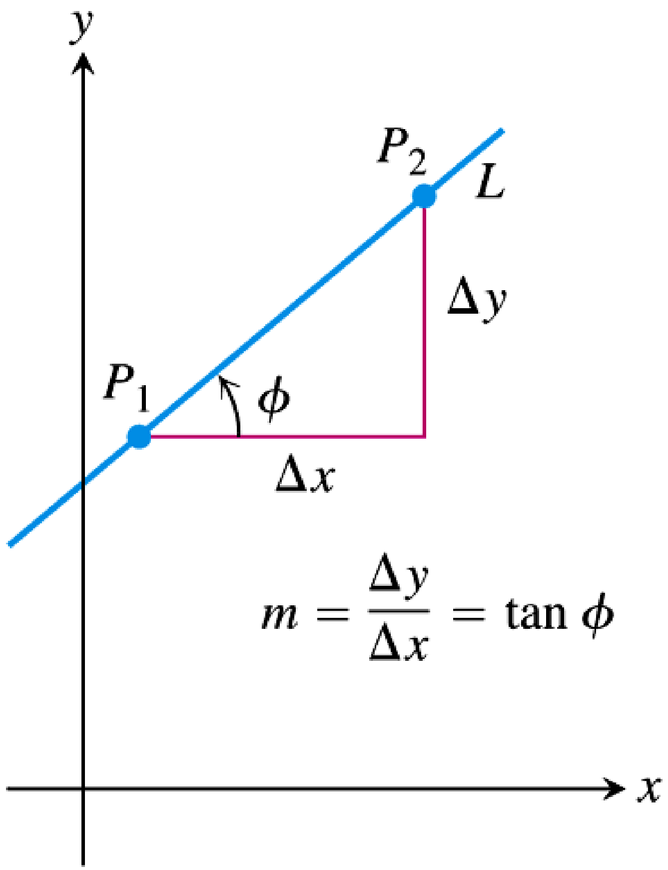
FIGURE 1.10 Angles of inclination are measured counterclockwise from the x-axis.



The equation

$$y = y_1 + m(x - x_1)$$

is the **point-slope equation** of the line that passes through the point (x_1, y_1) and has slope m.



$$m = \frac{\Delta y}{\Delta x} = \tan \phi$$

FIGURE 1.11 The slope of a nonvertical line is the tangent of its angle of inclination.

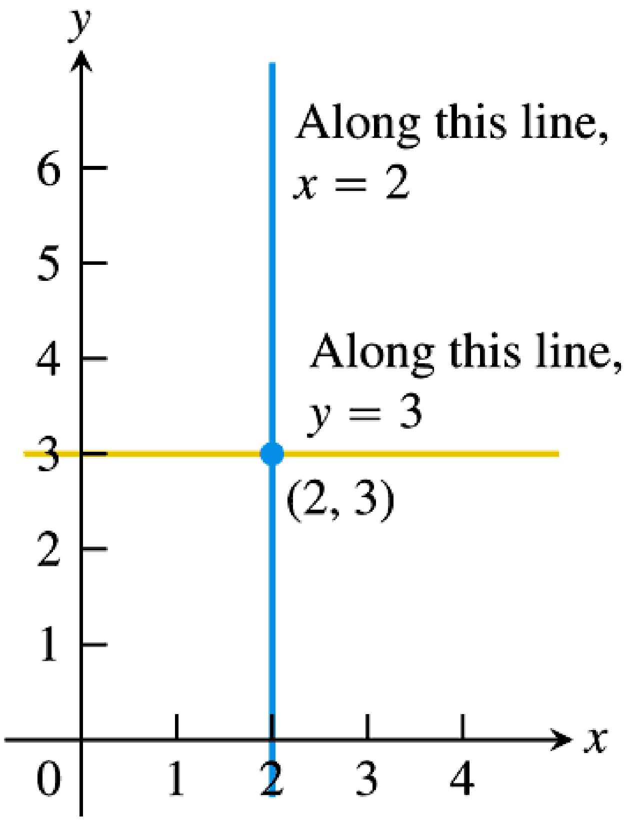


FIGURE 1.12 The standard equations for the vertical and horizontal lines through (2, 3) are $x = 2$ and $y = 3$.

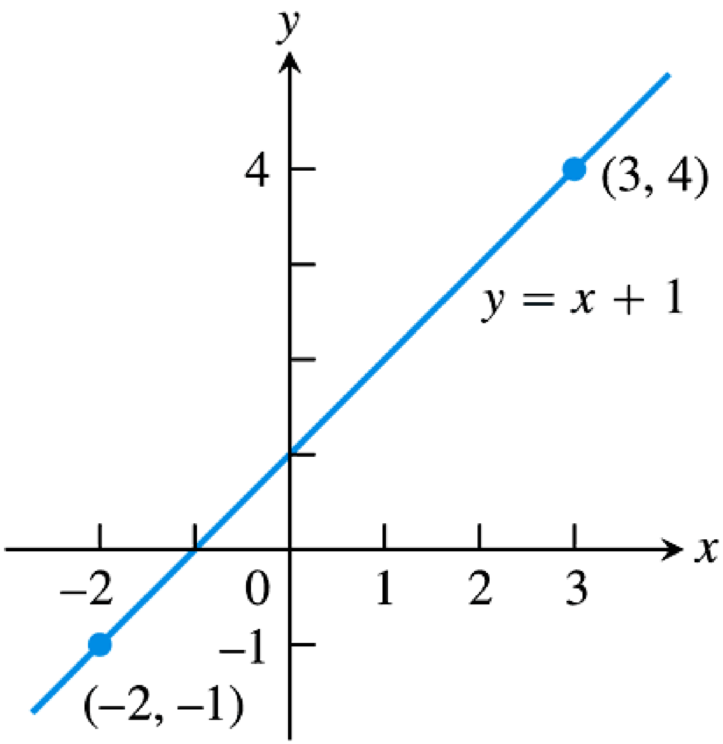


FIGURE 1.13 The line in Example 3.

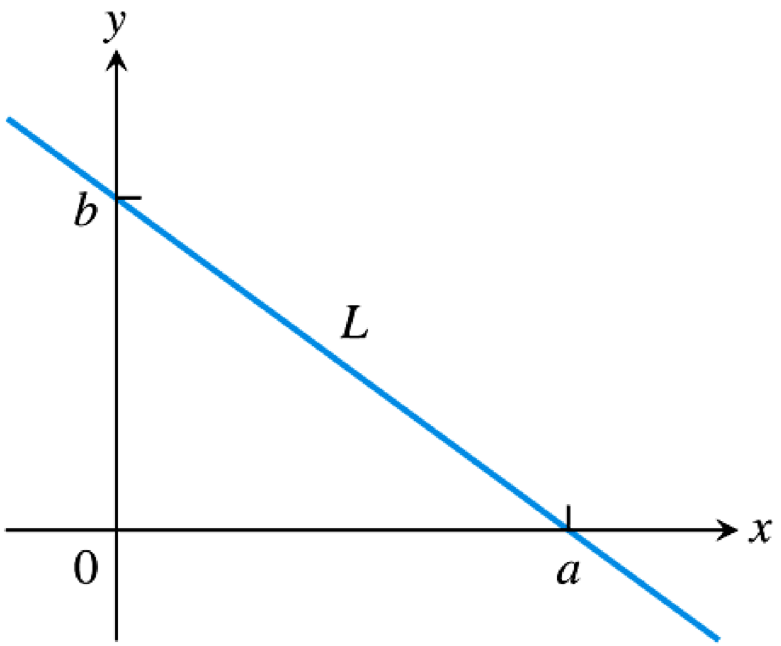


FIGURE 1.14 Line L has x-intercept a and y-intercept b.

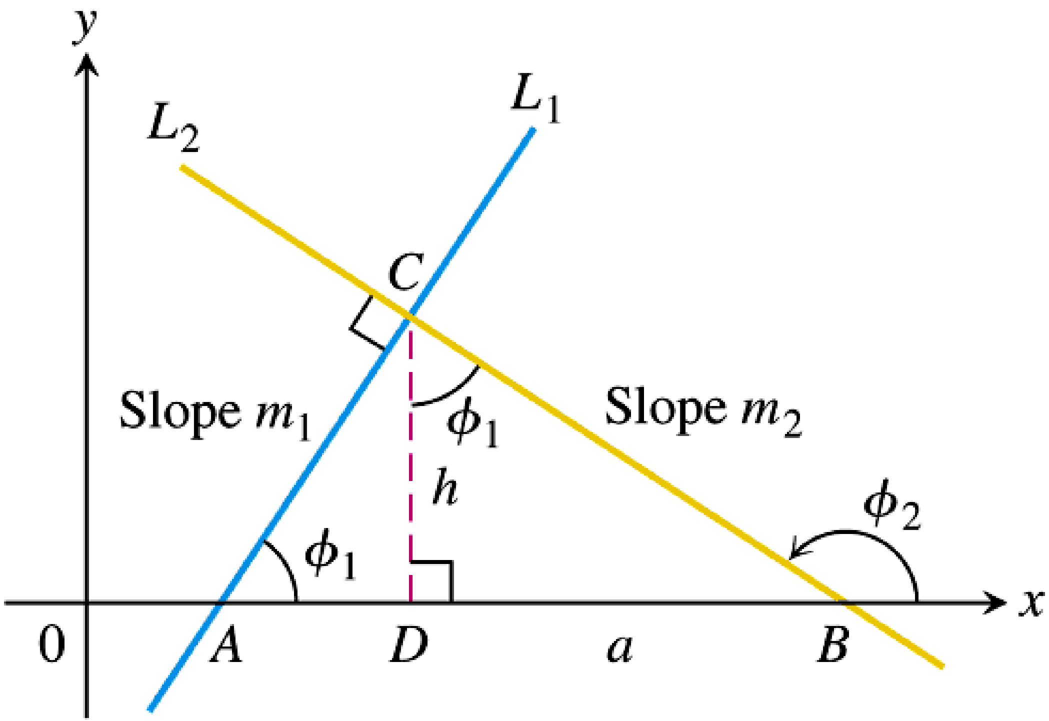


FIGURE 1.15 ΔADC is similar to ΔCDB. Hence ϕ_1 is also the upper angle in ΔCDB. From the sides of ΔCDB, we read $\tan \phi_1 = a/h$.

Distance Formula for Points in the Plane

The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

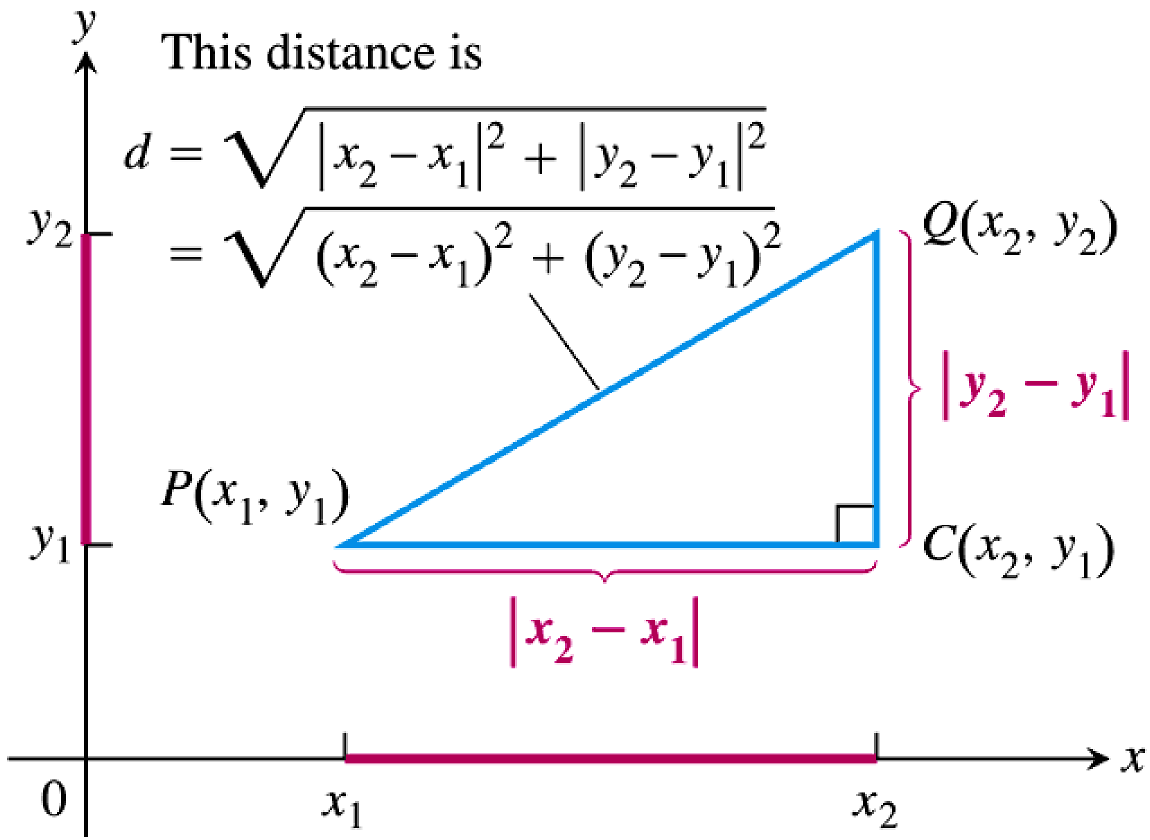


FIGURE 1.16 To calculate the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$, apply the Pythagorean theorem to triangle *PCQ*.

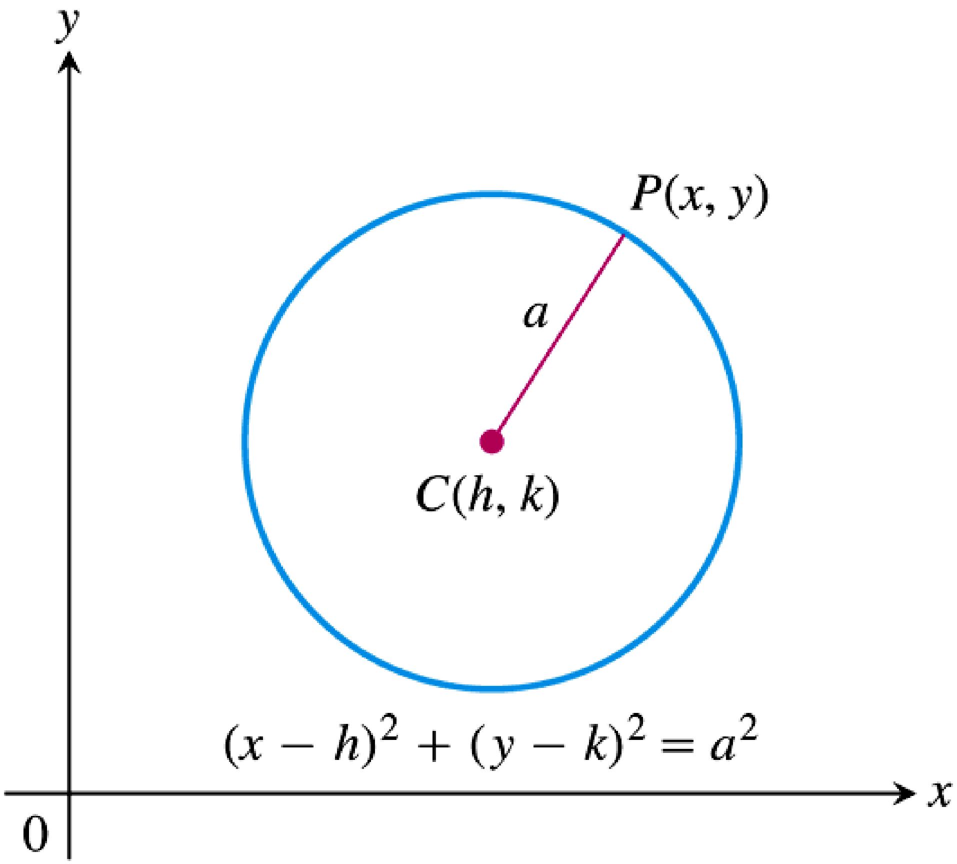


FIGURE 1.17 A circle of radius a in the xy-plane, with center at (h, k).

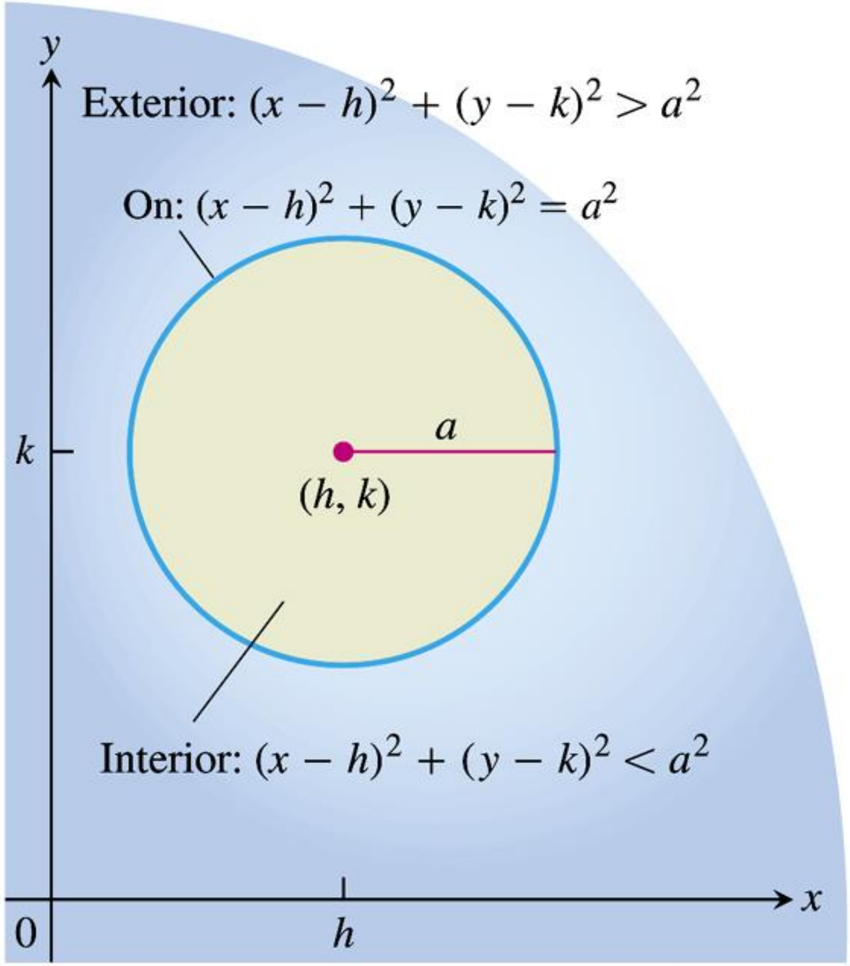


FIGURE 1.18 The interior and exterior of the circle $(x - h)^2 + (y - k)^2 = a^2$.

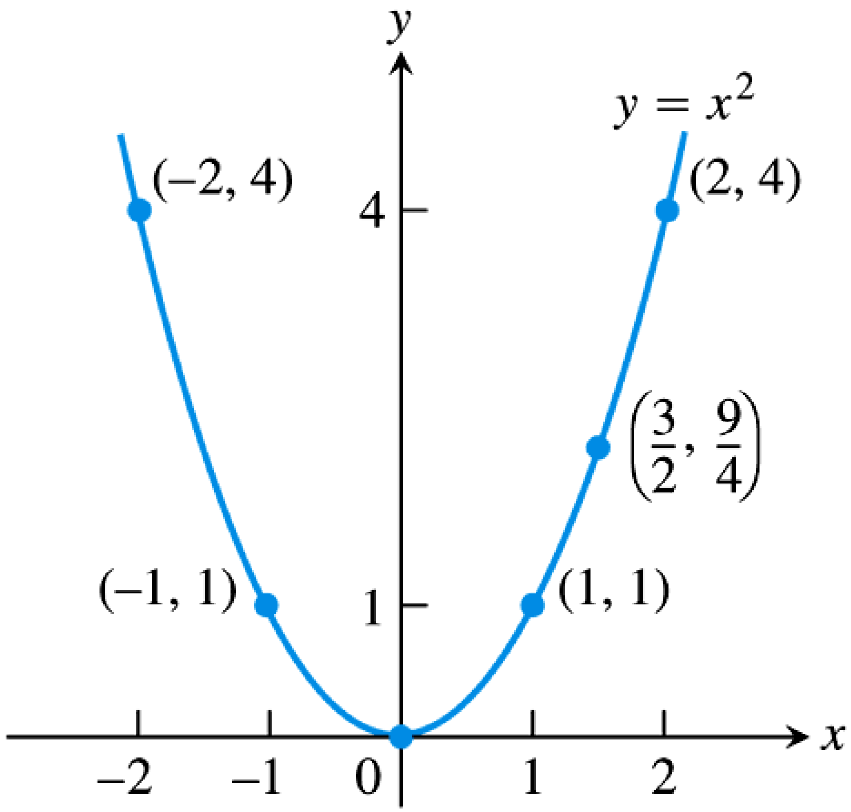


FIGURE 1.19 The parabola $y = x^2$ (Example 8).

The Graph of $y = ax^2 + bx + c, \quad a \neq 0$

The graph of the equation $y = ax^2 + bx + c$, $a \neq 0$, is a parabola. The parabola opens upward if $a > 0$ and downward if $a < 0$. The **axis** is the line

$$x = -\frac{b}{2a}. \tag{2}$$

The **vertex** of the parabola is the point where the axis and parabola intersect. Its x-coordinate is $x = -b/2a$; its y-coordinate is found by substituting $x = -b/2a$ in the parabola's equation.

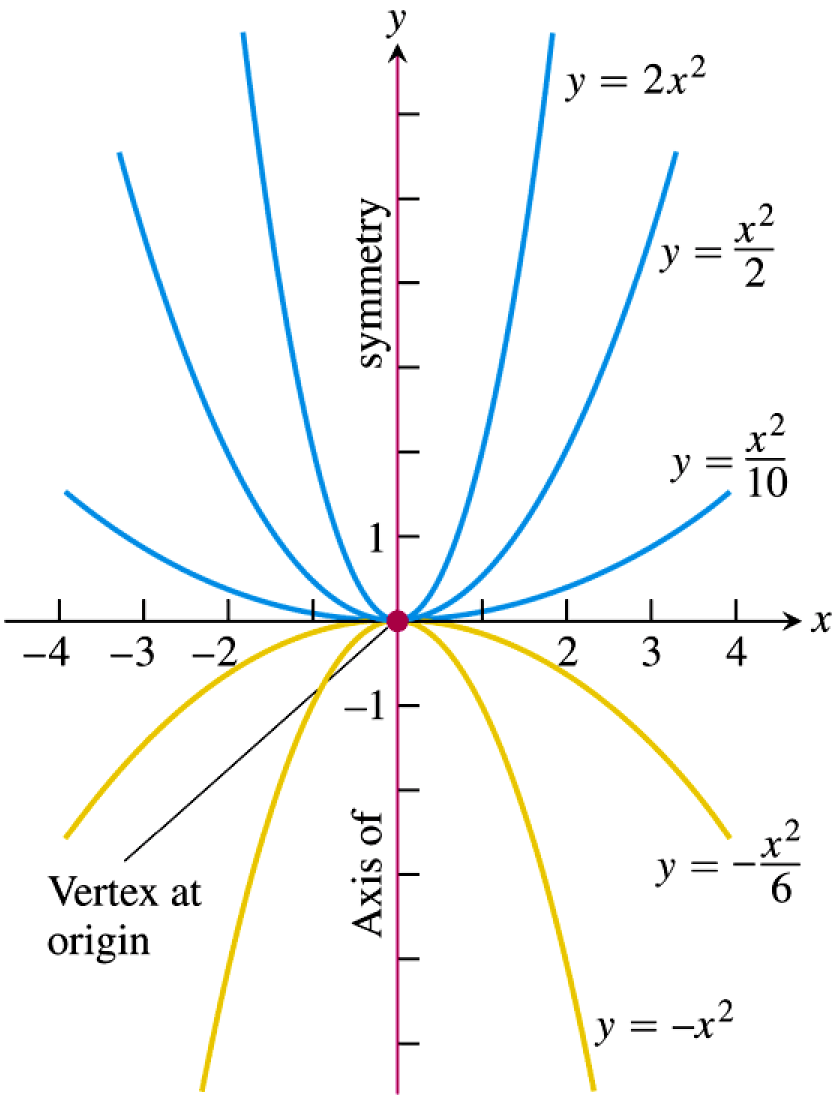


FIGURE 1.20 Besides determining the direction in which the parabola $y = ax^2$ opens, the number a is a scaling factor. The parabola widens as a approaches zero and narrows as $|a|$ becomes large.

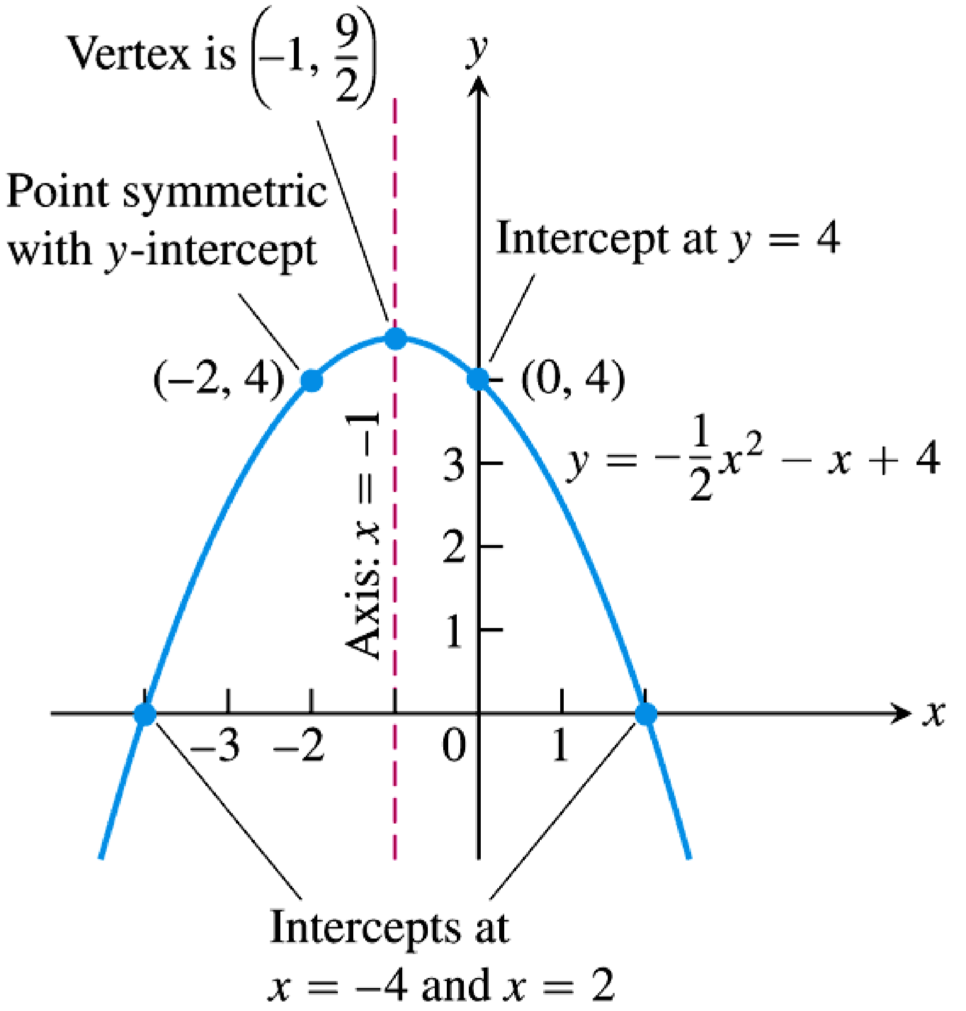


FIGURE 1.21 The parabola in Example 9.

1.3

Functions and Their Graphs

DEFINITION **Function**

A **function** from a set D to a set Y is a rule that assigns a *unique* (single) element $f(x) \in Y$ to each element $x \in D$.

FIGURE 1.22 A diagram showing a function as a kind of machine.

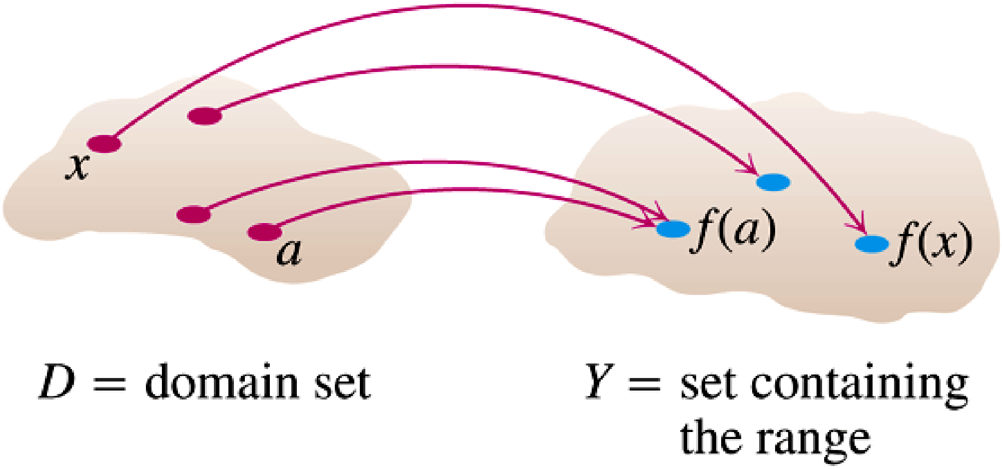


FIGURE 1.23 A function from a set D to a set Y assigns a unique element of Y to each element in D.

Function	**Domain (x)**	**Range (y)**
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

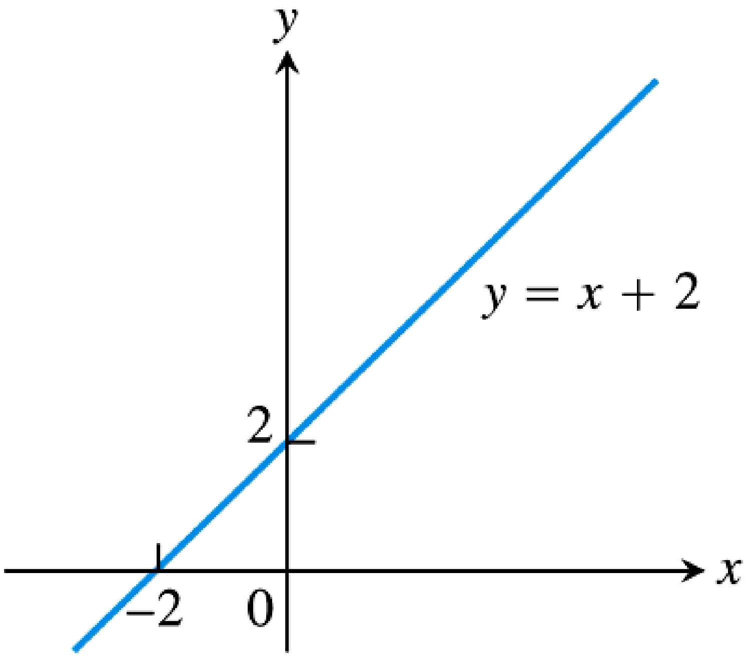


FIGURE 1.24 The graph of $f(x) = x + 2$ is the set of points (x, y) for which y has the value $x + 2$.

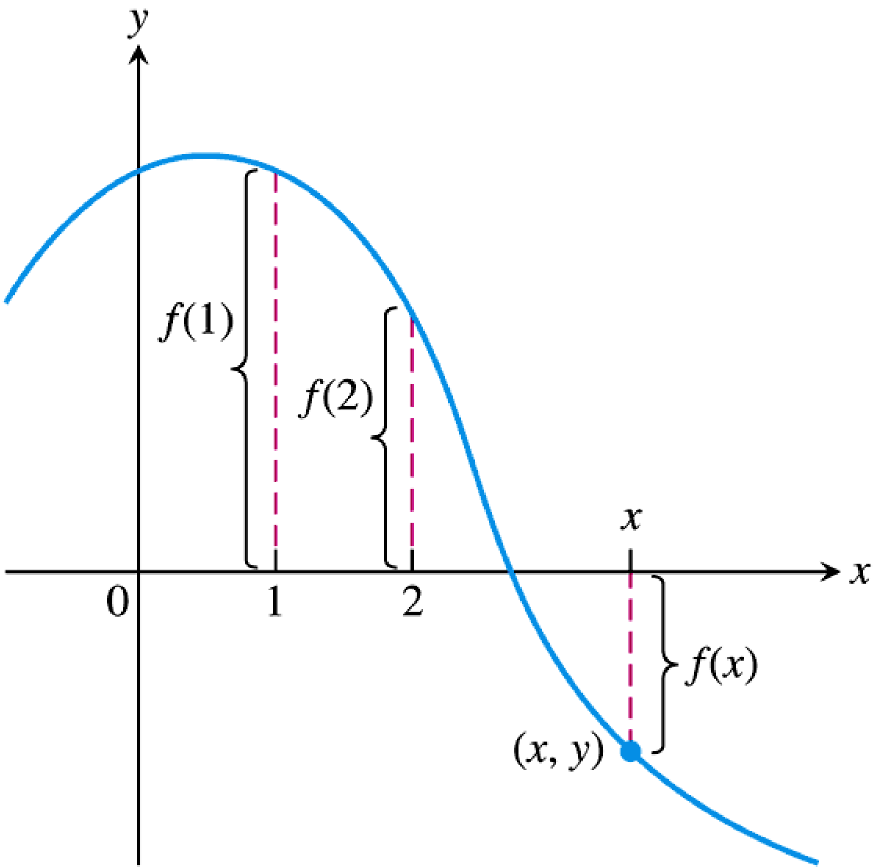


FIGURE 1.25 If (x, y) lies on the graph of f, then the value $y = f(x)$ is the height of the graph above the point x (or below x if $f(x)$ is negative).

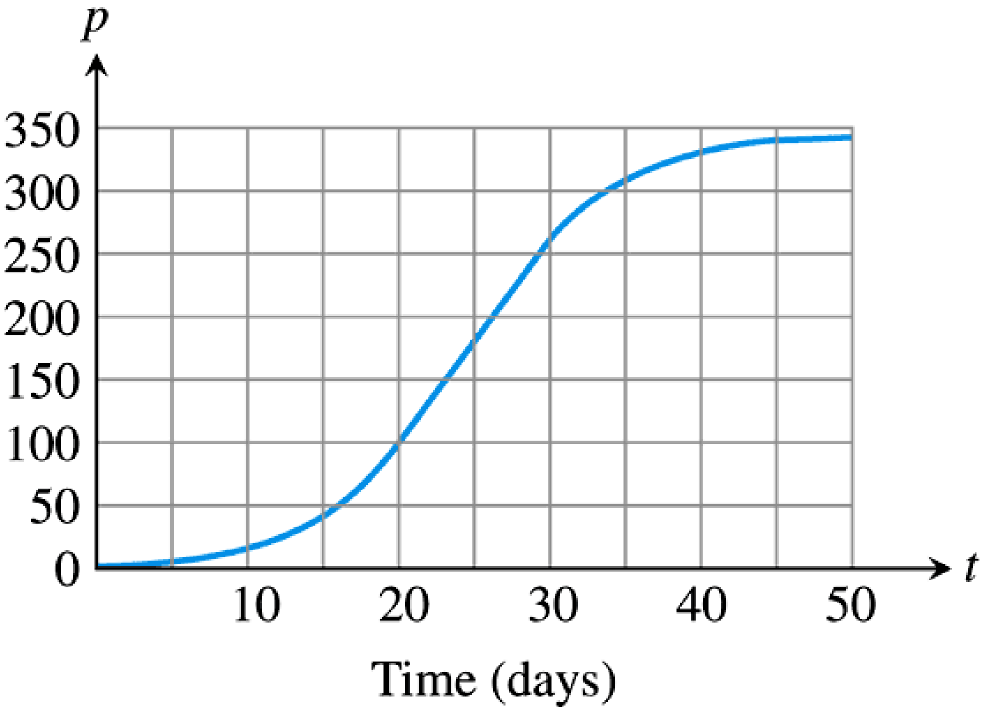


FIGURE 1.26 Graph of a fruit fly population versus time (Example 3).

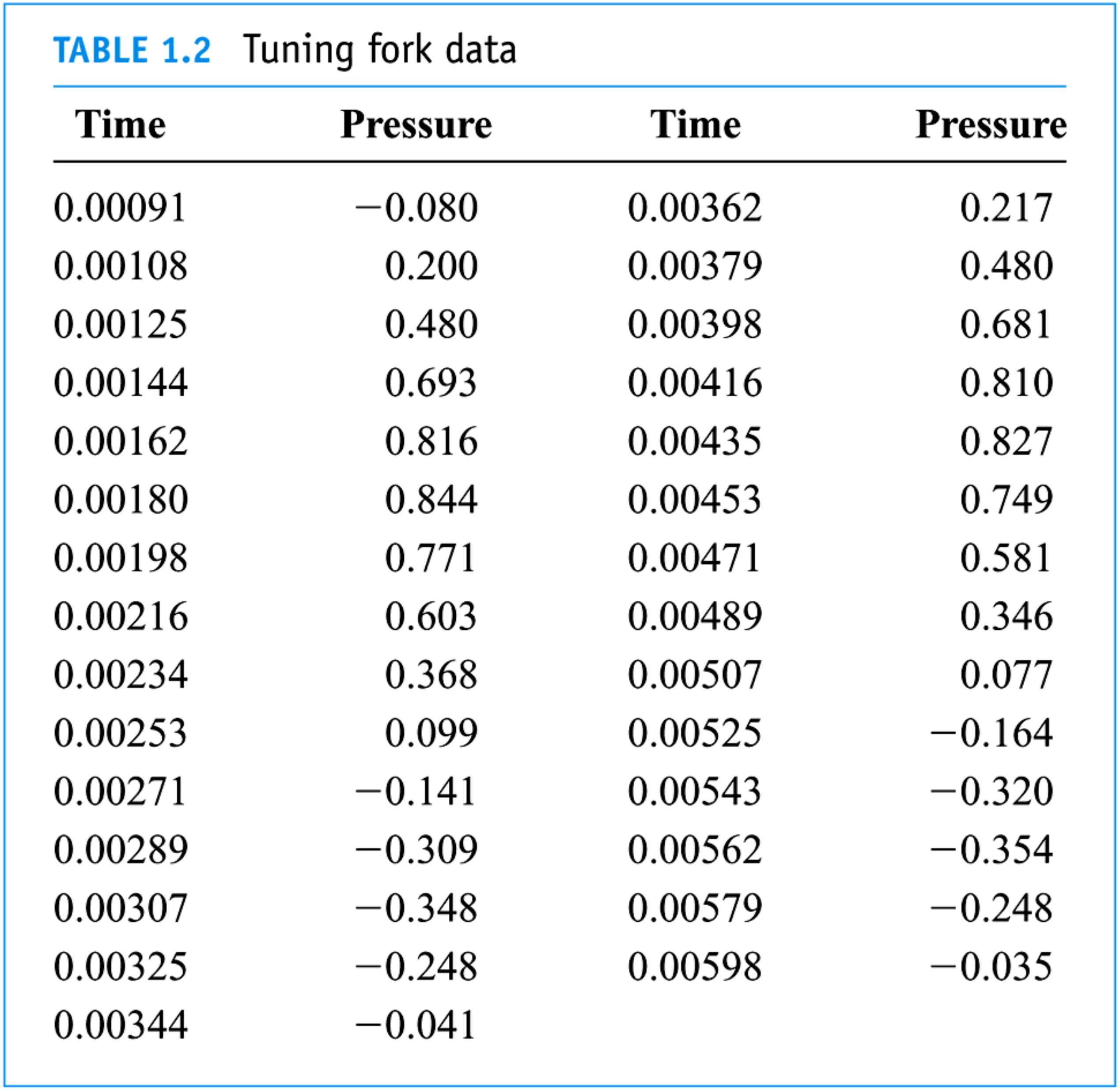


TABLE 1.2 Tuning fork data

Time	Pressure	Time	Pressure
0.00091	−0.080	0.00362	0.217
0.00108	0.200	0.00379	0.480
0.00125	0.480	0.00398	0.681
0.00144	0.693	0.00416	0.810
0.00162	0.816	0.00435	0.827
0.00180	0.844	0.00453	0.749
0.00198	0.771	0.00471	0.581
0.00216	0.603	0.00489	0.346
0.00234	0.368	0.00507	0.077
0.00253	0.099	0.00525	−0.164
0.00271	−0.141	0.00543	−0.320
0.00289	−0.309	0.00562	−0.354
0.00307	−0.348	0.00579	−0.248
0.00325	−0.248	0.00598	−0.035
0.00344	−0.041		

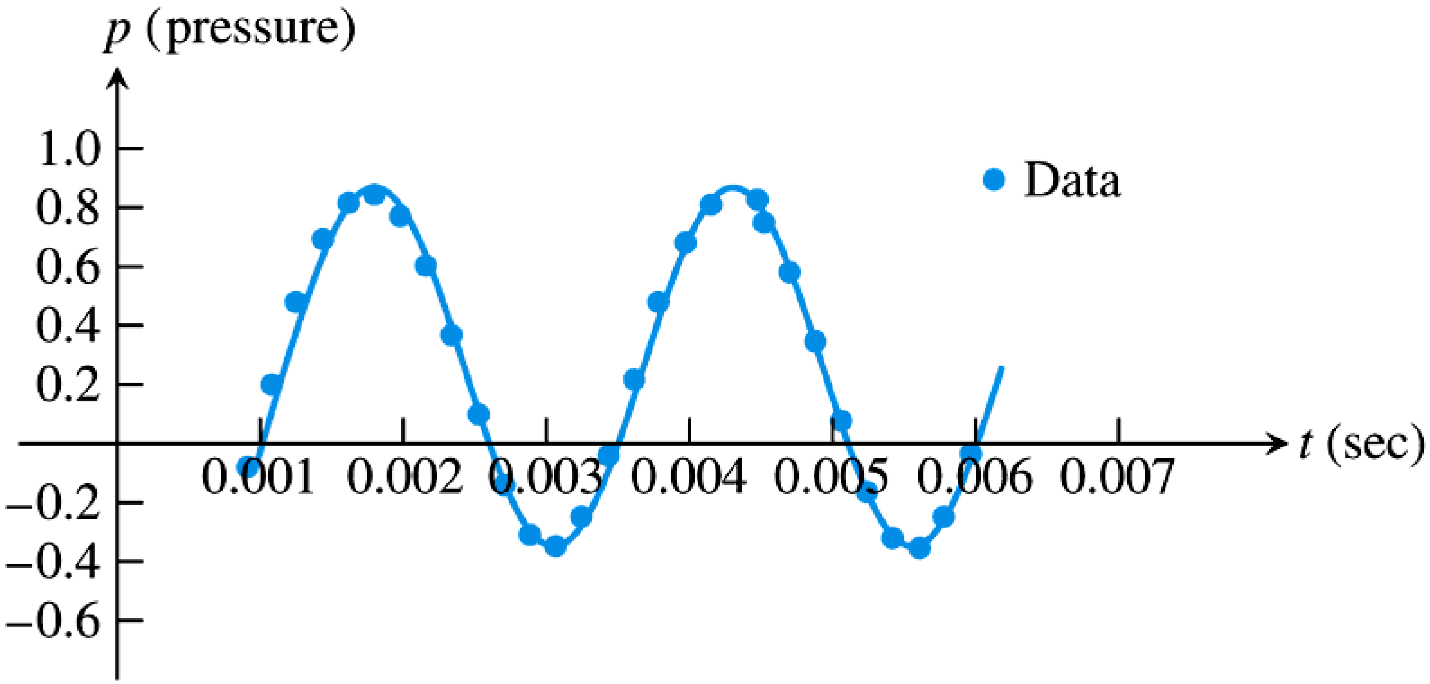
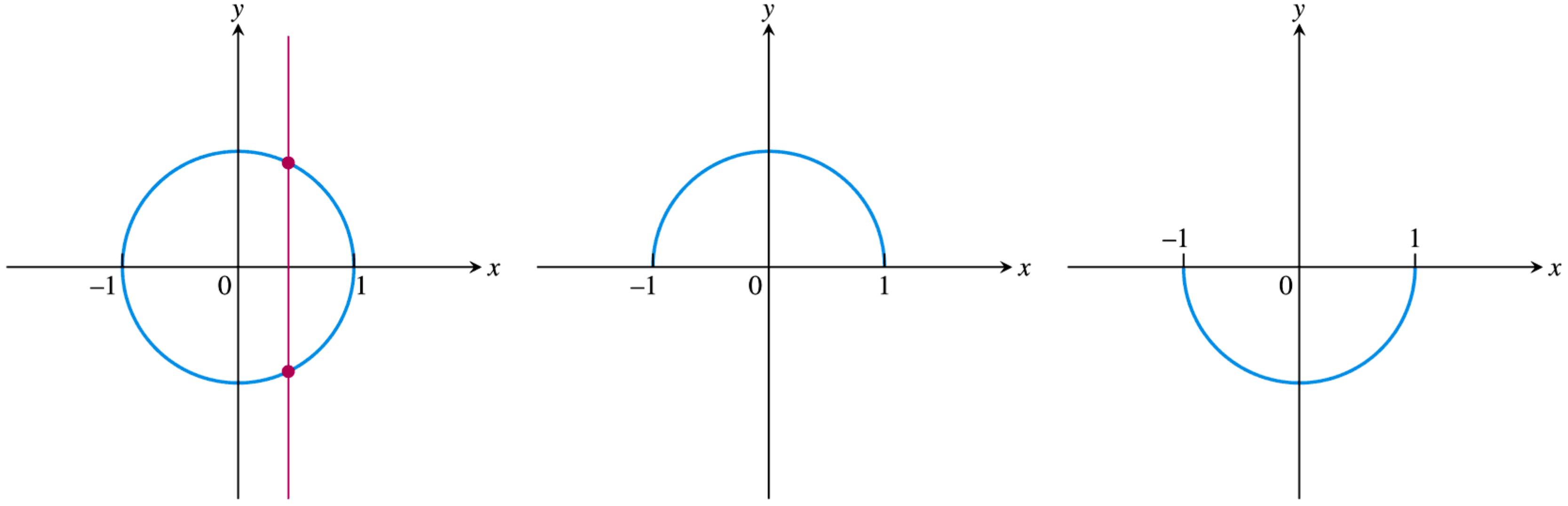


FIGURE 1.27 A smooth curve through the plotted points gives a graph of the pressure function represented by Table 1.2.



(a) $x^2 + y^2 = 1$ (b) $y = \sqrt{1 - x^2}$ (c) $y = -\sqrt{1 - x^2}$

FIGURE 1.28 (a) The circle is not the graph of a function; it fails the vertical line test. (b) The upper semicircle is the graph of a function $f(x) = \sqrt{1 - x^2}$. (c) The lower semicircle is the graph of a function $g(x) = -\sqrt{1 - x^2}$.

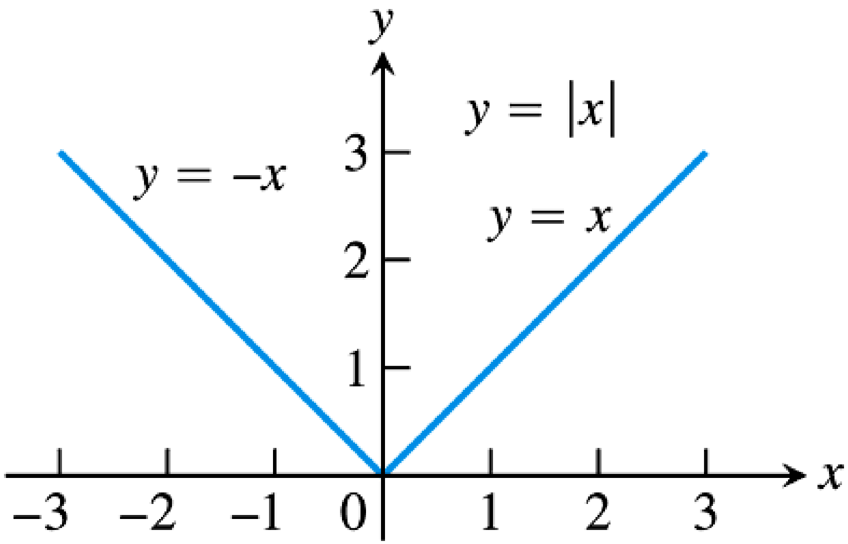


FIGURE 1.29 The absolute value function has domain $(-\infty, \infty)$ and range $[0, \infty)$.

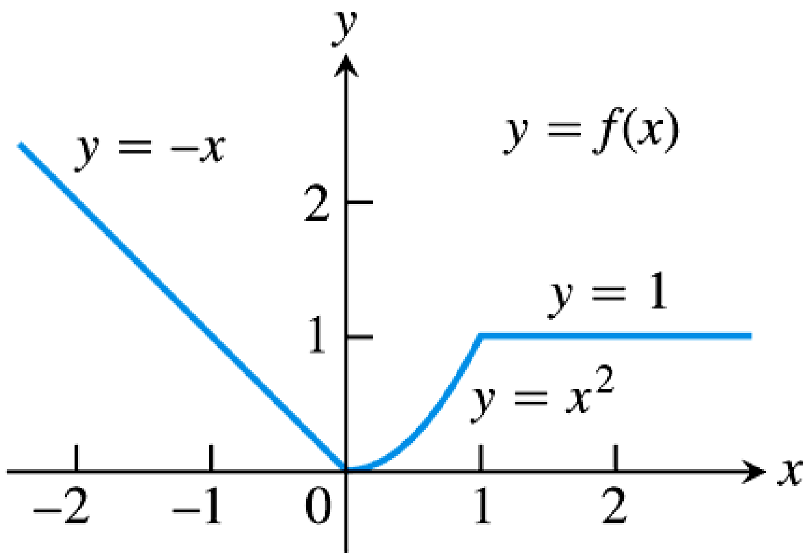


FIGURE 1.30 To graph the function $y = f(x)$ shown here, we apply different formulas to different parts of its domain (Example 5).

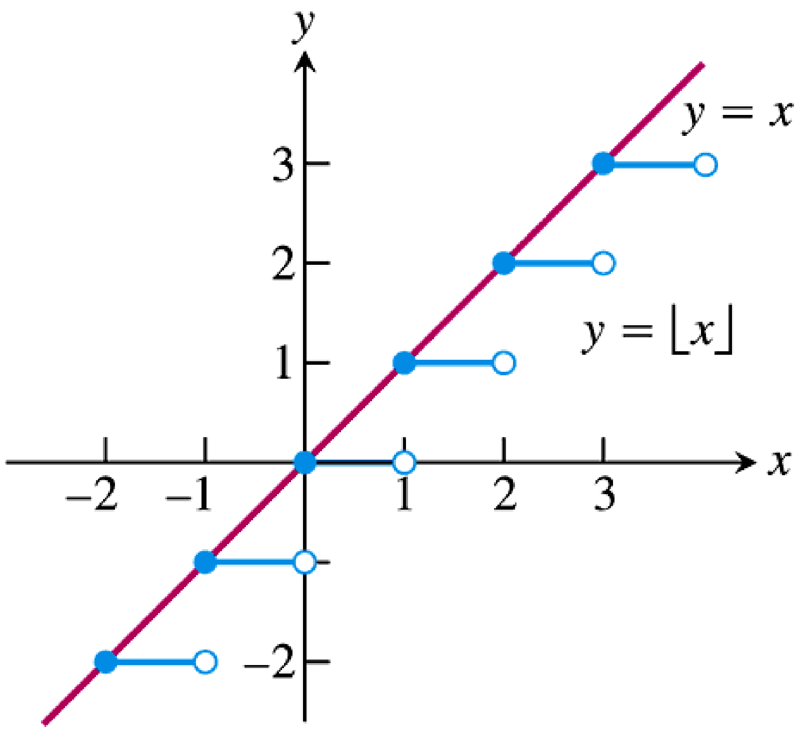


FIGURE 1.31 The graph of the greatest integer function $y = \lfloor x \rfloor$ lies on or below the line $y = x$, so it provides an integer floor for x (Example 6).

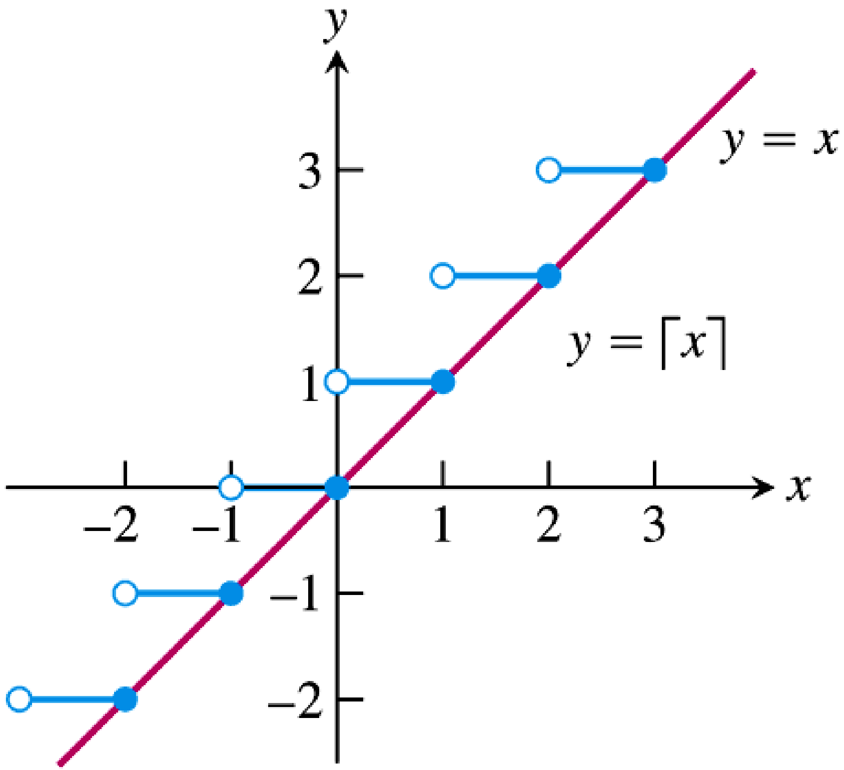


FIGURE 1.32 The graph of the least integer function $y = \lceil x \rceil$ lies on or above the line $y = x$, so it provides an integer ceiling for x (Example 7).

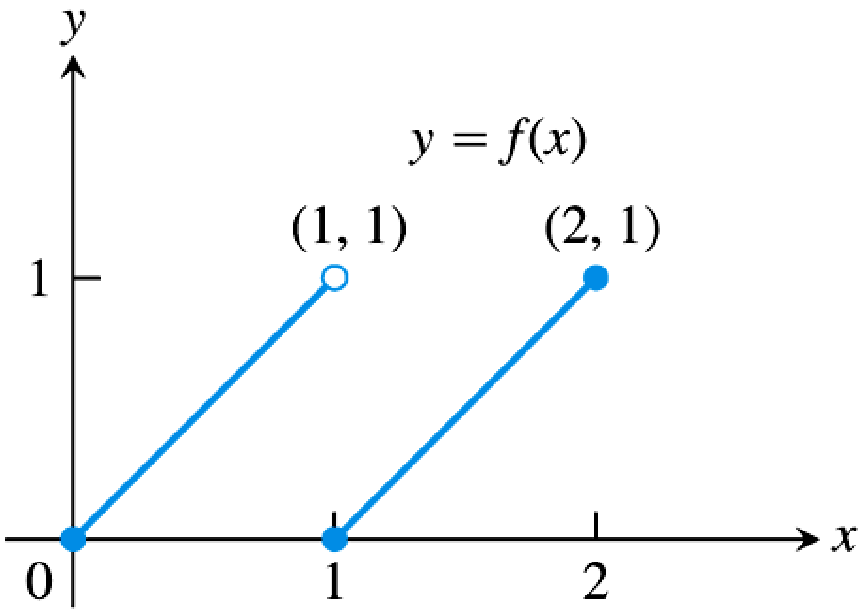


FIGURE 1.33 The segment on the left contains (0, 0) but not (1, 1). The segment on the right contains both of its endpoints (Example 8).

1.4

Identifying Functions; Mathematical Models

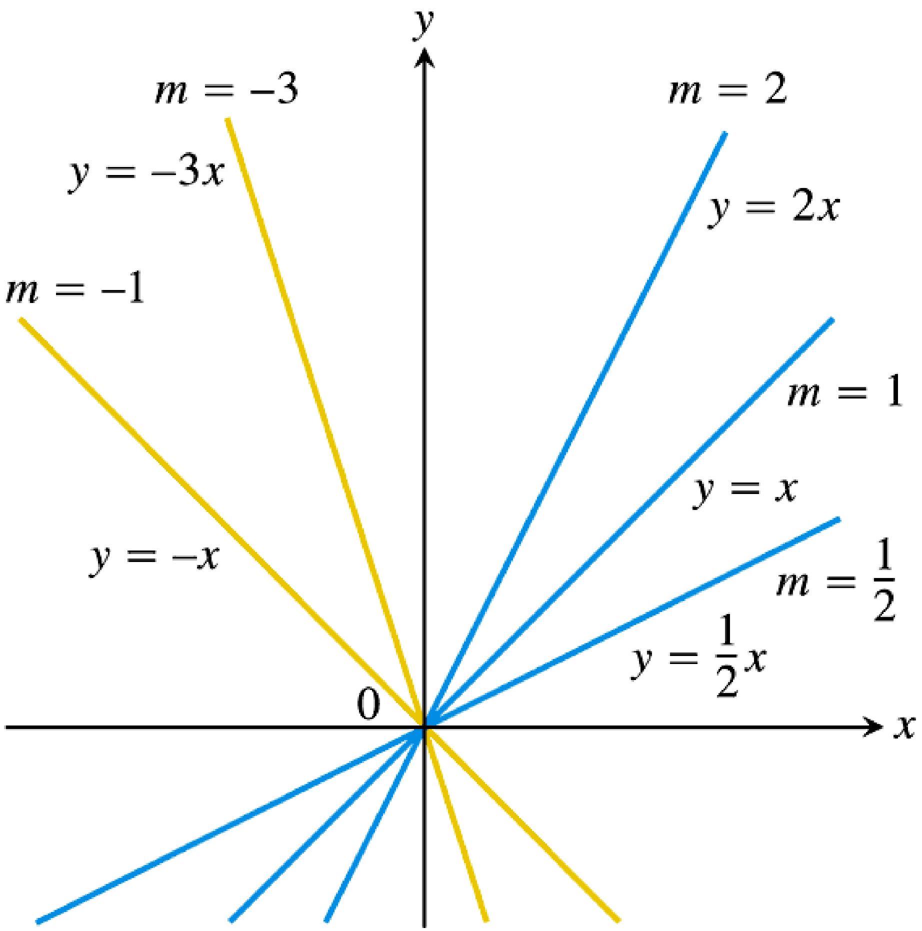


FIGURE 1.34 The collection of lines $y = mx$ has slope m and all lines pass through the origin.

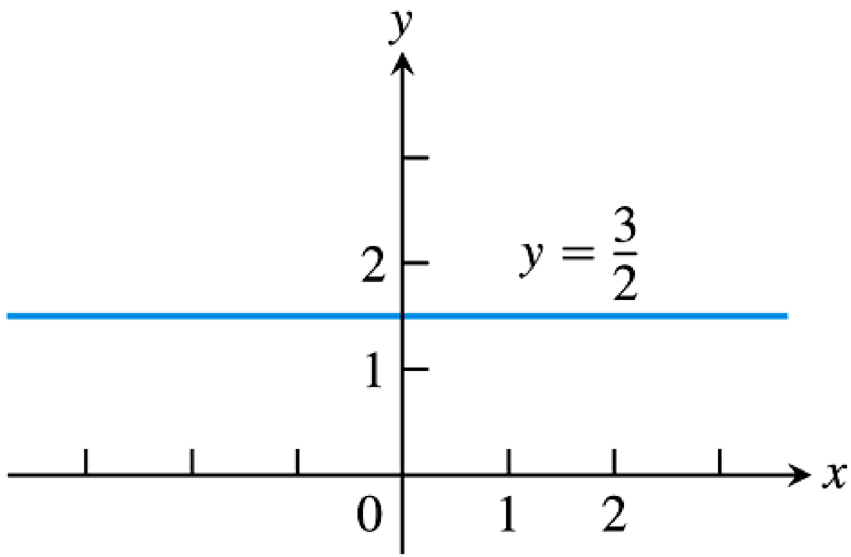


FIGURE 1.35 A constant function has slope $m = 0$.

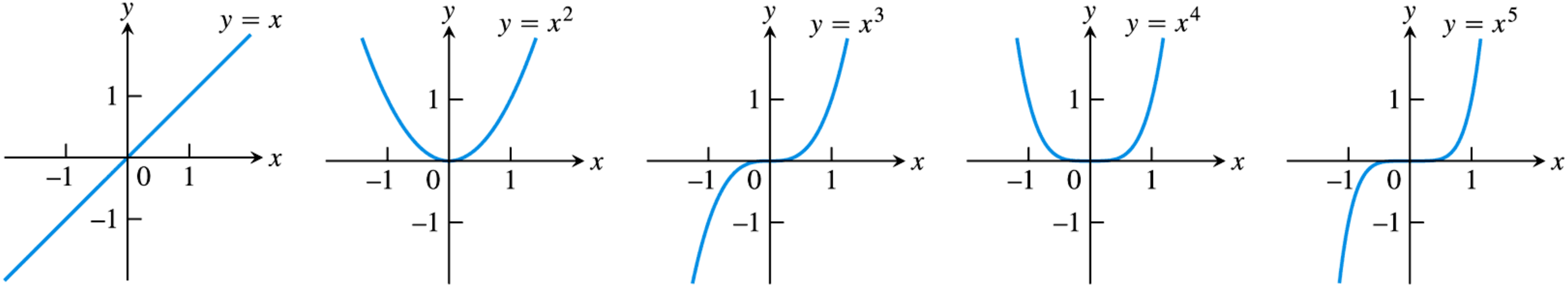


FIGURE 1.36 Graphs of $f(x) = x^n$, $n = 1, 2, 3, 4, 5$ defined for $-\infty < x < \infty$.

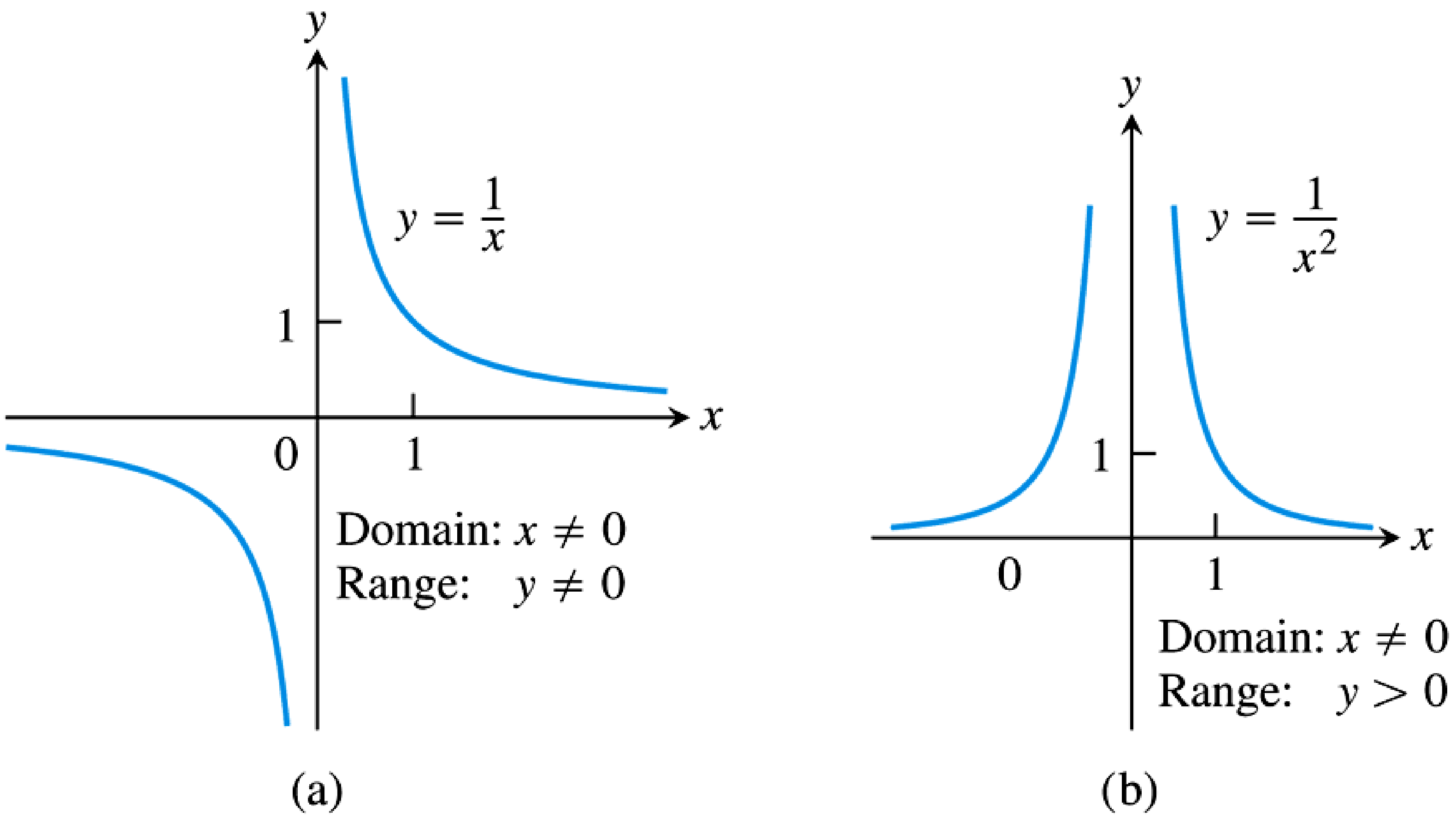


FIGURE 1.37 Graphs of the power functions $f(x) = x^a$ for part (a) $a = -1$ and for part (b) $a = -2$.

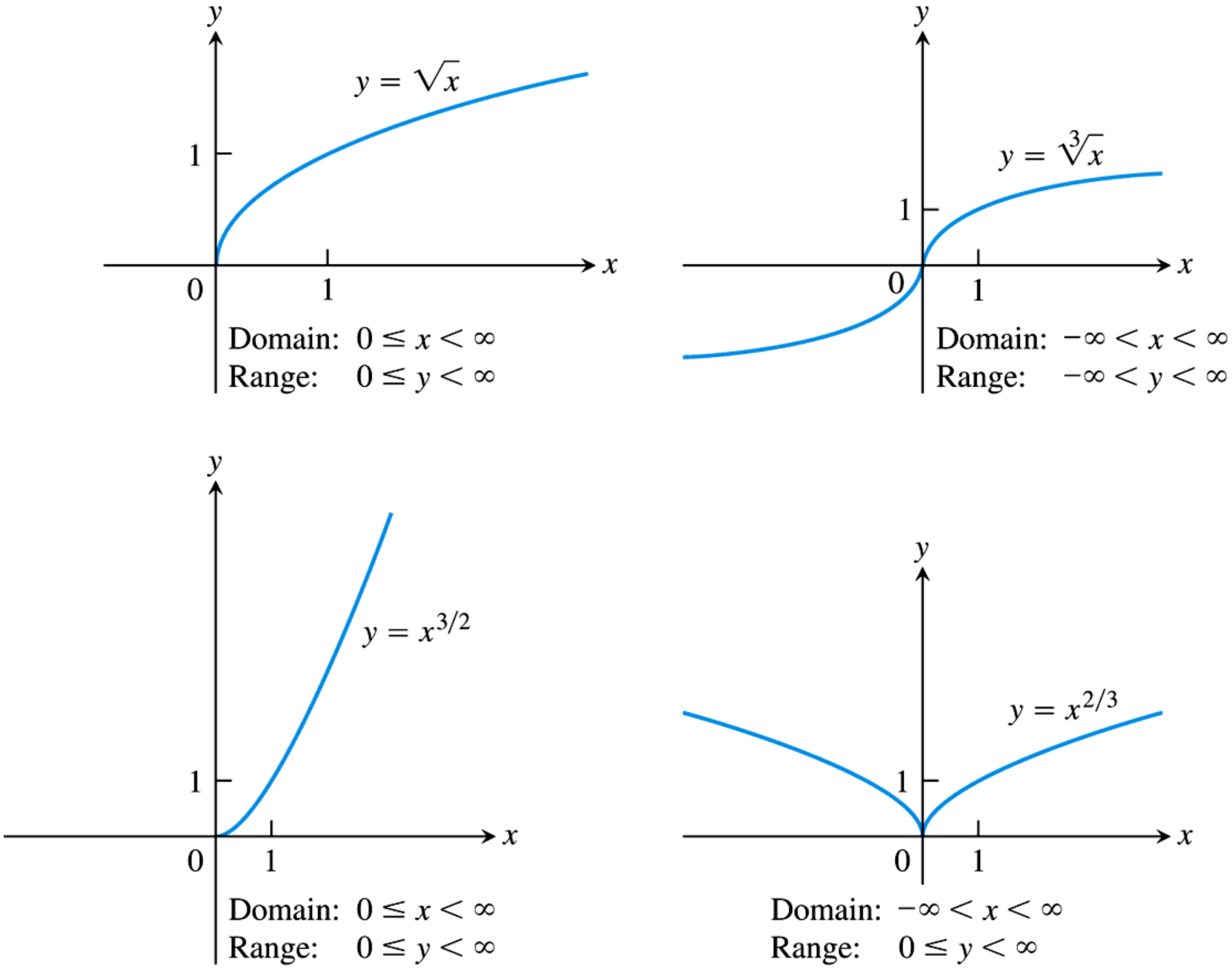


FIGURE 1.38 Graphs of the power functions $f(x) = x^a$ for $a = \frac{1}{2}, \frac{1}{3}, \frac{3}{2}$, and $\frac{2}{3}$.

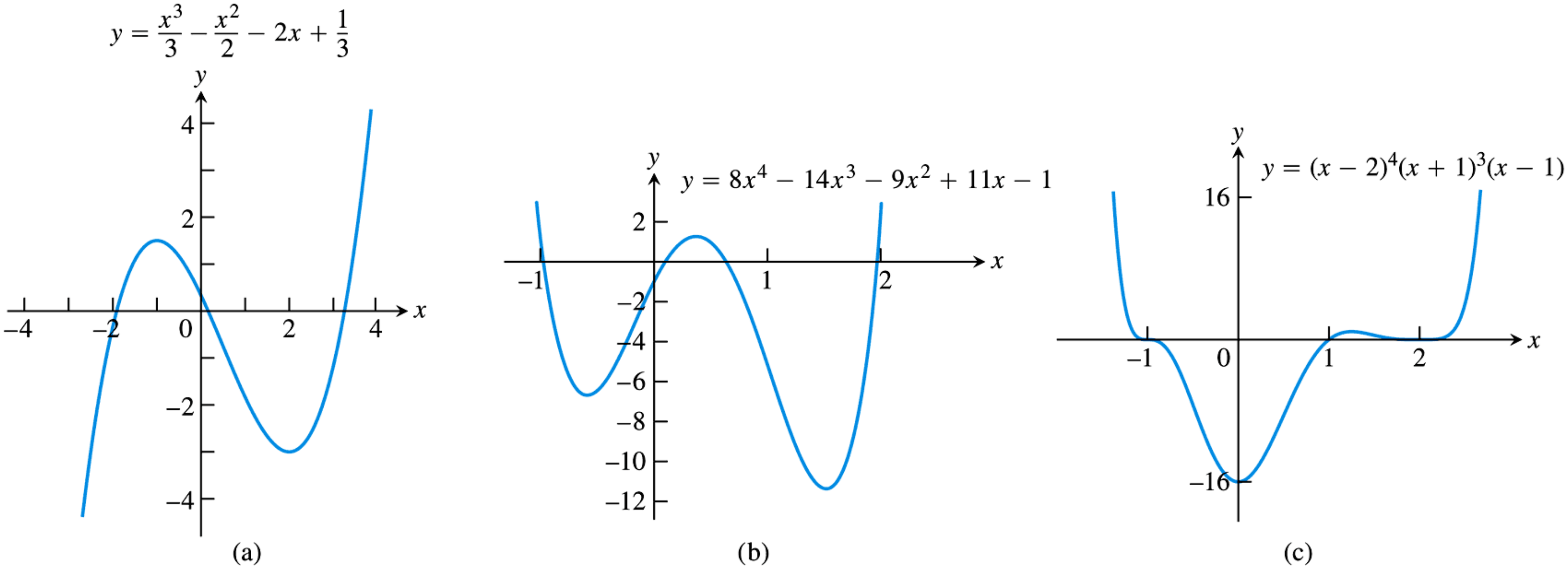


FIGURE 1.39 Graphs of three polynomial functions.

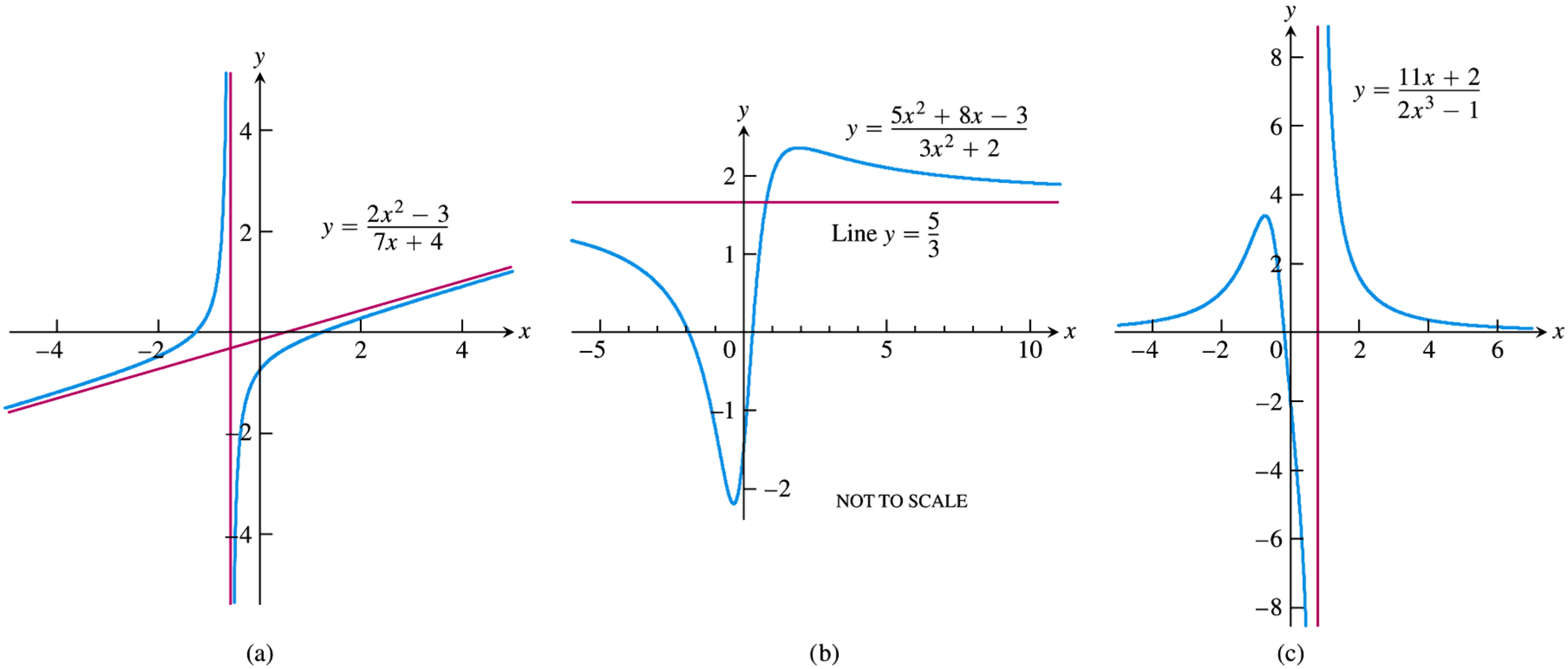


FIGURE 1.40 Graphs of three rational functions.

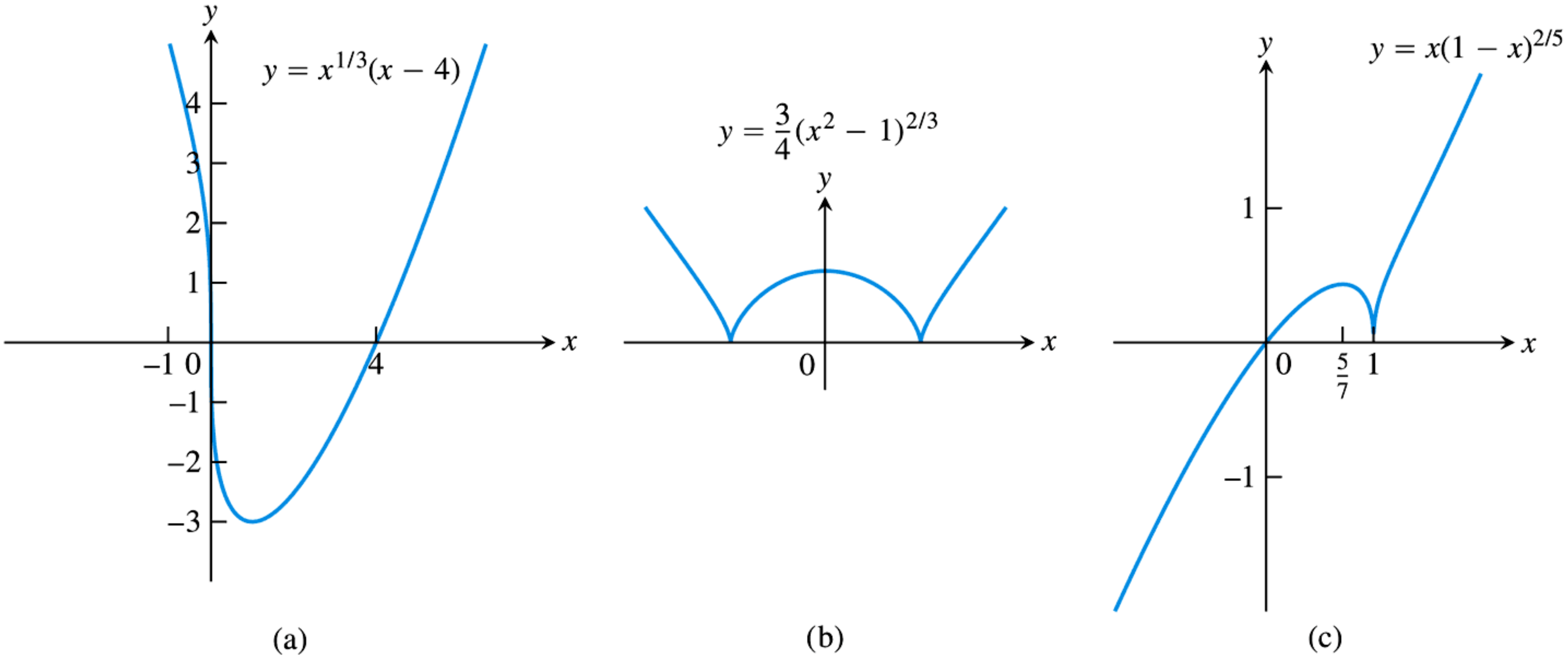
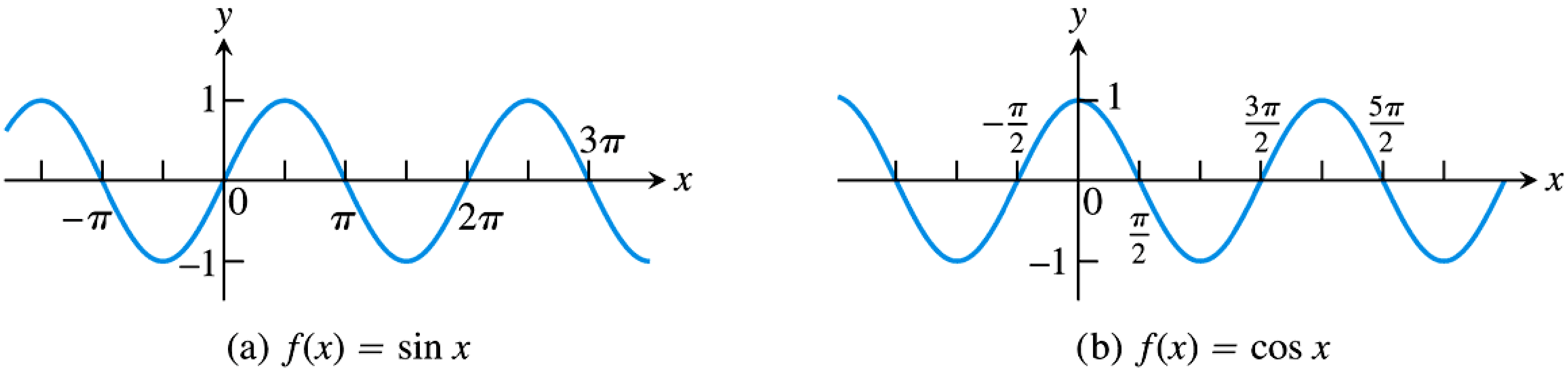


FIGURE 1.41 Graphs of three algebraic functions.



(a) $f(x) = \sin x$ (b) $f(x) = \cos x$

FIGURE 1.42 Graphs of the sine and cosine functions.

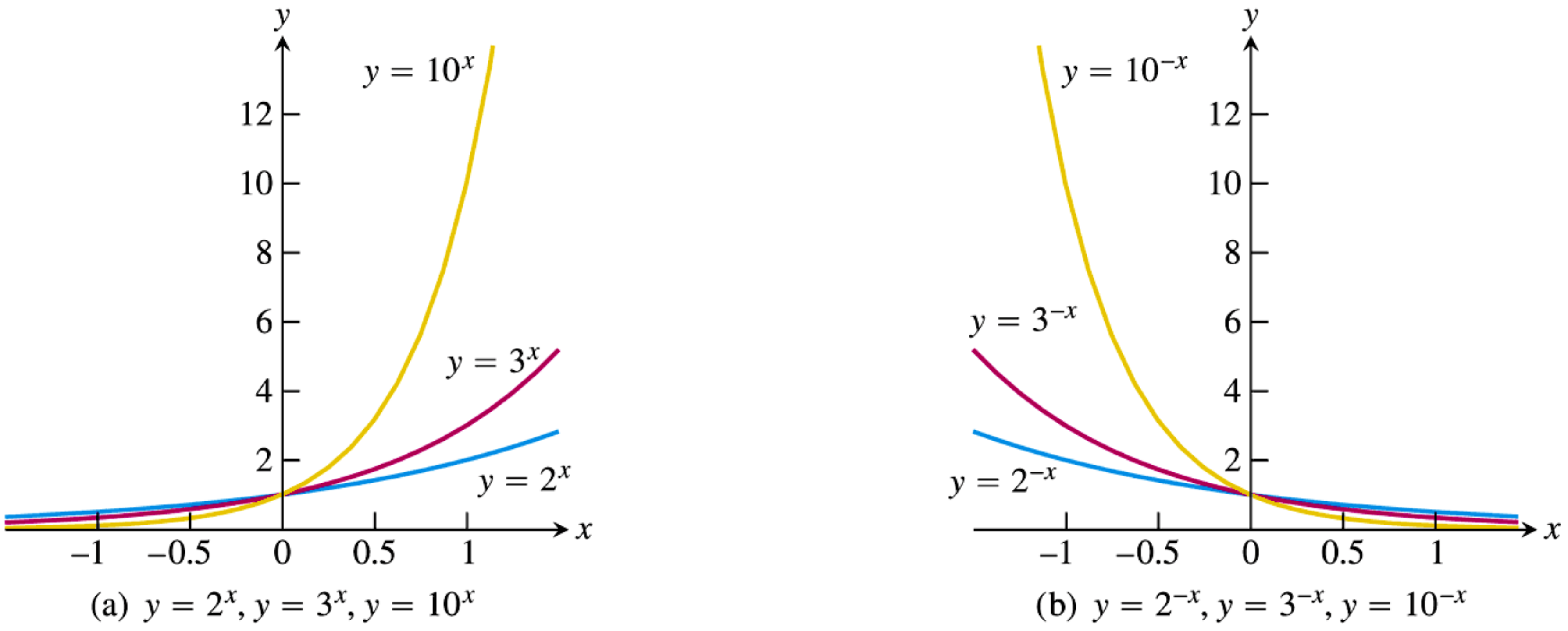


FIGURE 1.43 Graphs of exponential functions.

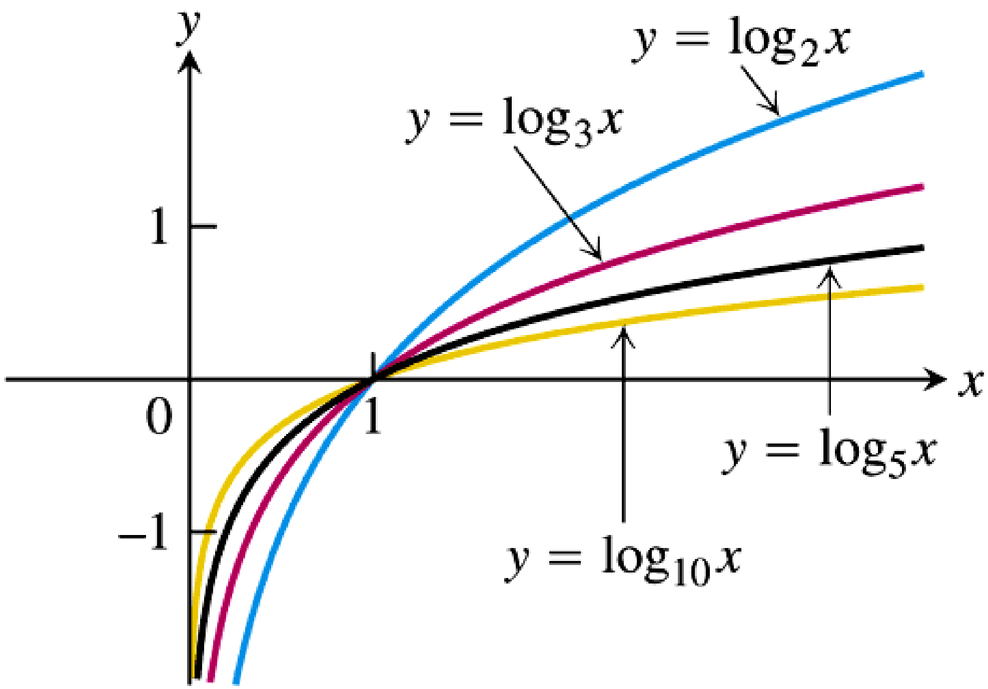


FIGURE 1.44 Graphs of four logarithmic functions.

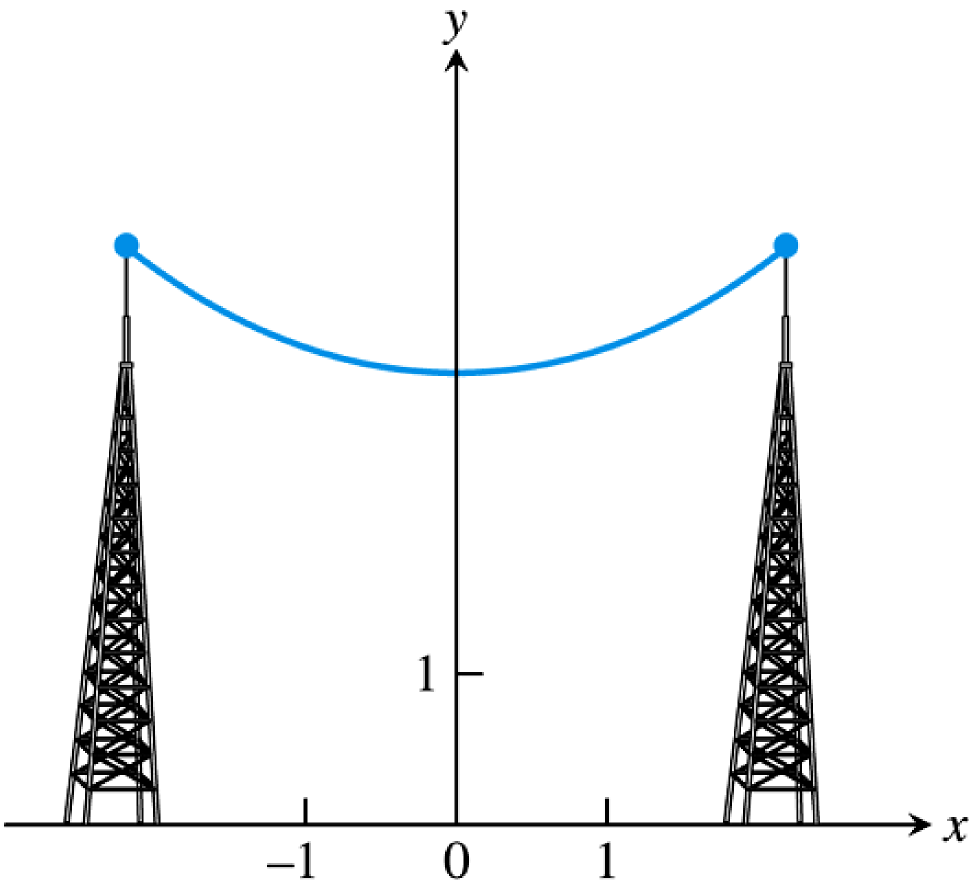
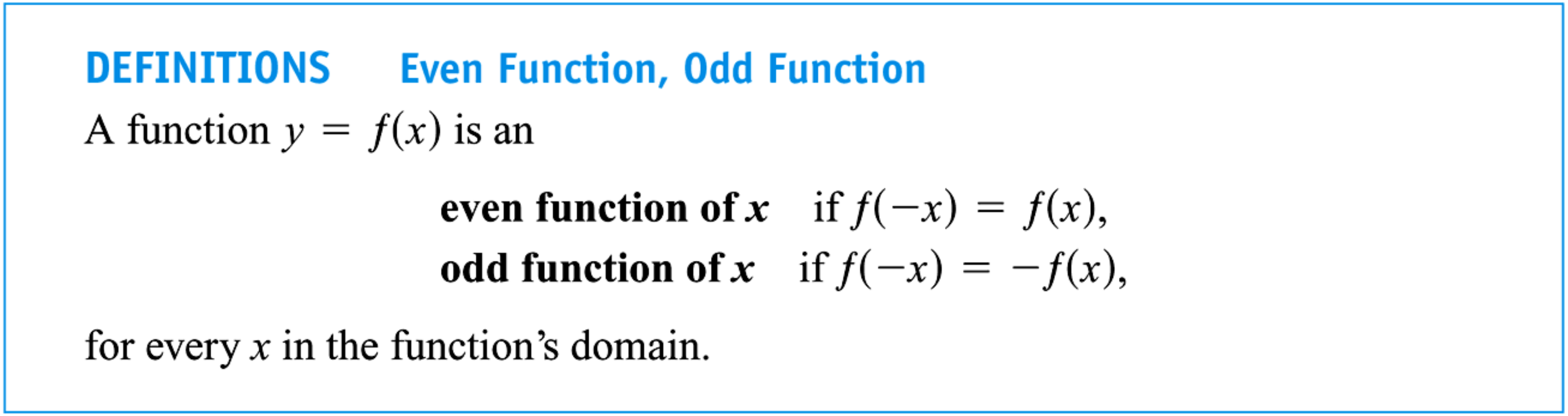


FIGURE 1.45 Graph of a catenary or hanging cable. (The Latin word *catena* means “chain.”)

Function	**Where increasing**	**Where decreasing**
$y = x^2$	$0 \leq x < \infty$	$-\infty < x \leq 0$
$y = x^3$	$-\infty < x < \infty$	Nowhere
$y = 1/x$	Nowhere	$-\infty < x < 0$ and $0 < x < \infty$
$y = 1/x^2$	$-\infty < x < 0$	$0 < x < \infty$
$y = \sqrt{x}$	$0 \leq x < \infty$	Nowhere
$y = x^{2/3}$	$0 \leq x < \infty$	$-\infty < x \leq 0$

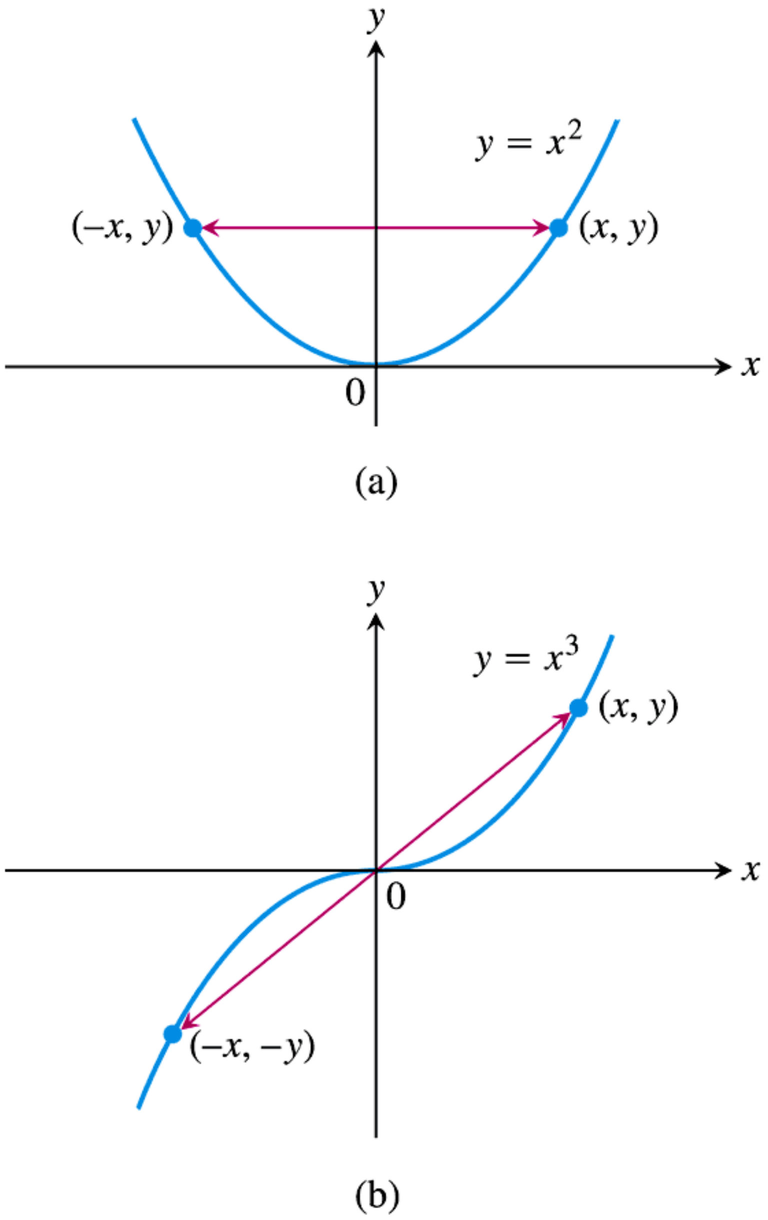


DEFINITIONS **Even Function, Odd Function**

A function $y = f(x)$ is an

$$\textbf{even function of } \boldsymbol{x} \quad \text{if } f(-x) = f(x),$$
$$\textbf{odd function of } \boldsymbol{x} \quad \text{if } f(-x) = -f(x),$$

for every x in the function's domain.



(a)

(b)

FIGURE 1.46 In part (a) the graph of $y = x^2$ (an even function) is symmetric about the y-axis. The graph of $y = x^3$ (an odd function) in part (b) is symmetric about the origin.

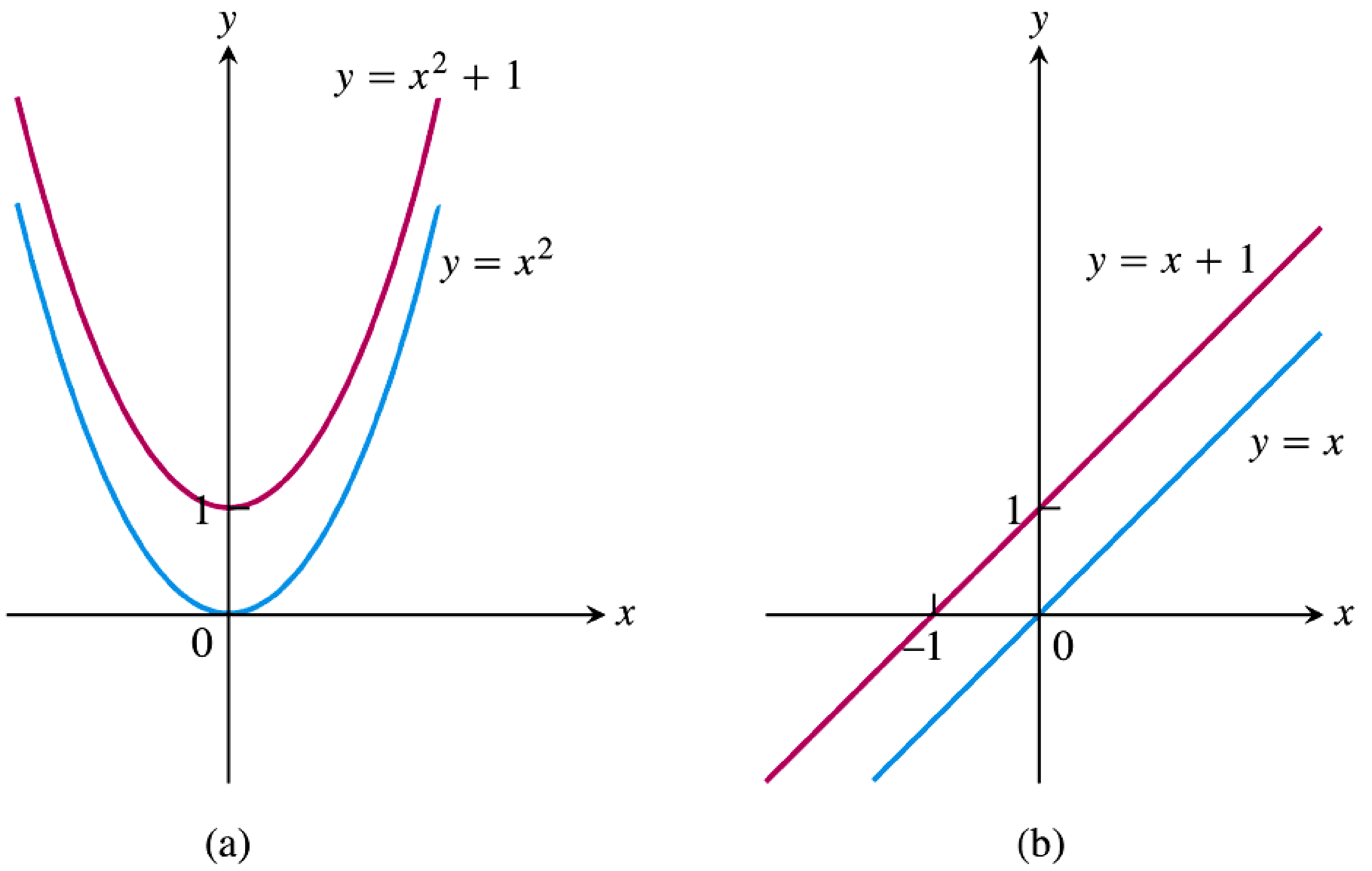
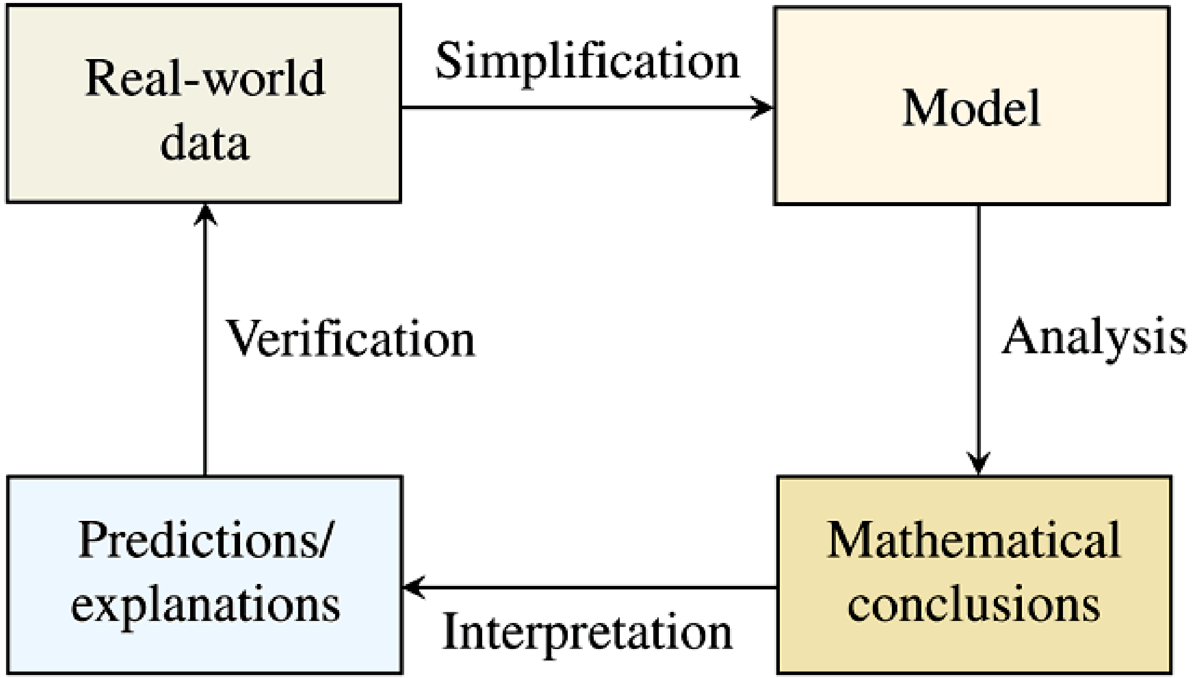


FIGURE 1.47 (a) When we add the constant term 1 to the function $y = x^2$, the resulting function $y = x^2 + 1$ is still even and its graph is still symmetric about the y-axis. (b) When we add the constant term 1 to the function $y = x$, the resulting function $y = x + 1$ is no longer odd. The symmetry about the origin is lost (Example 2).



Real-world data

Simplification

Model

Analysis

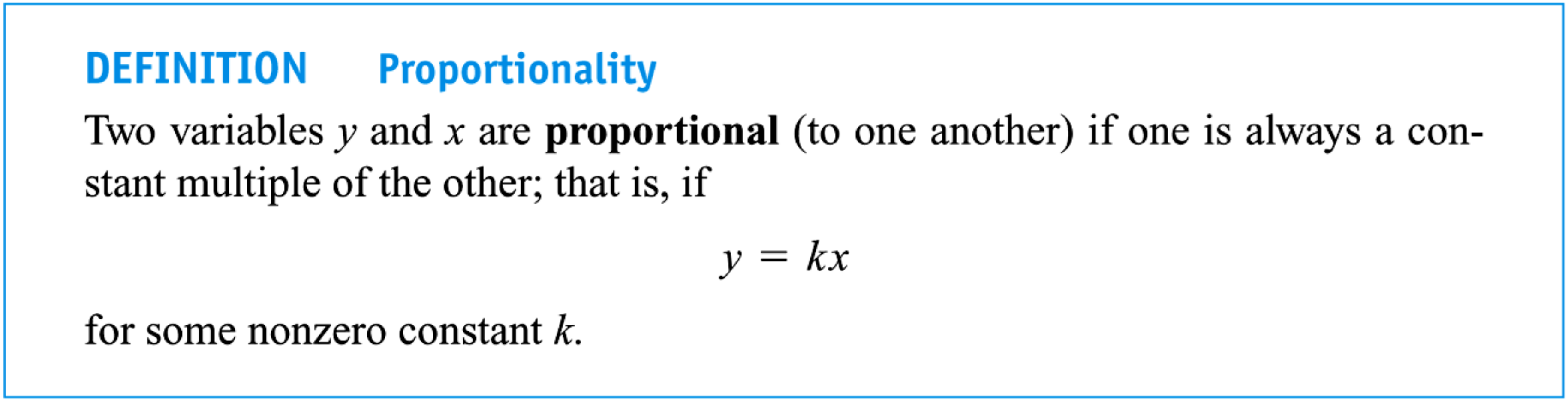
Mathematical conclusions

Interpretation

Predictions/ explanations

Verification

FIGURE 1.48 A flow of the modeling process beginning with an examination of real-world data.



DEFINITION Proportionality

Two variables y and x are **proportional** (to one another) if one is always a constant multiple of the other; that is, if

$$y = kx$$

for some nonzero constant k.

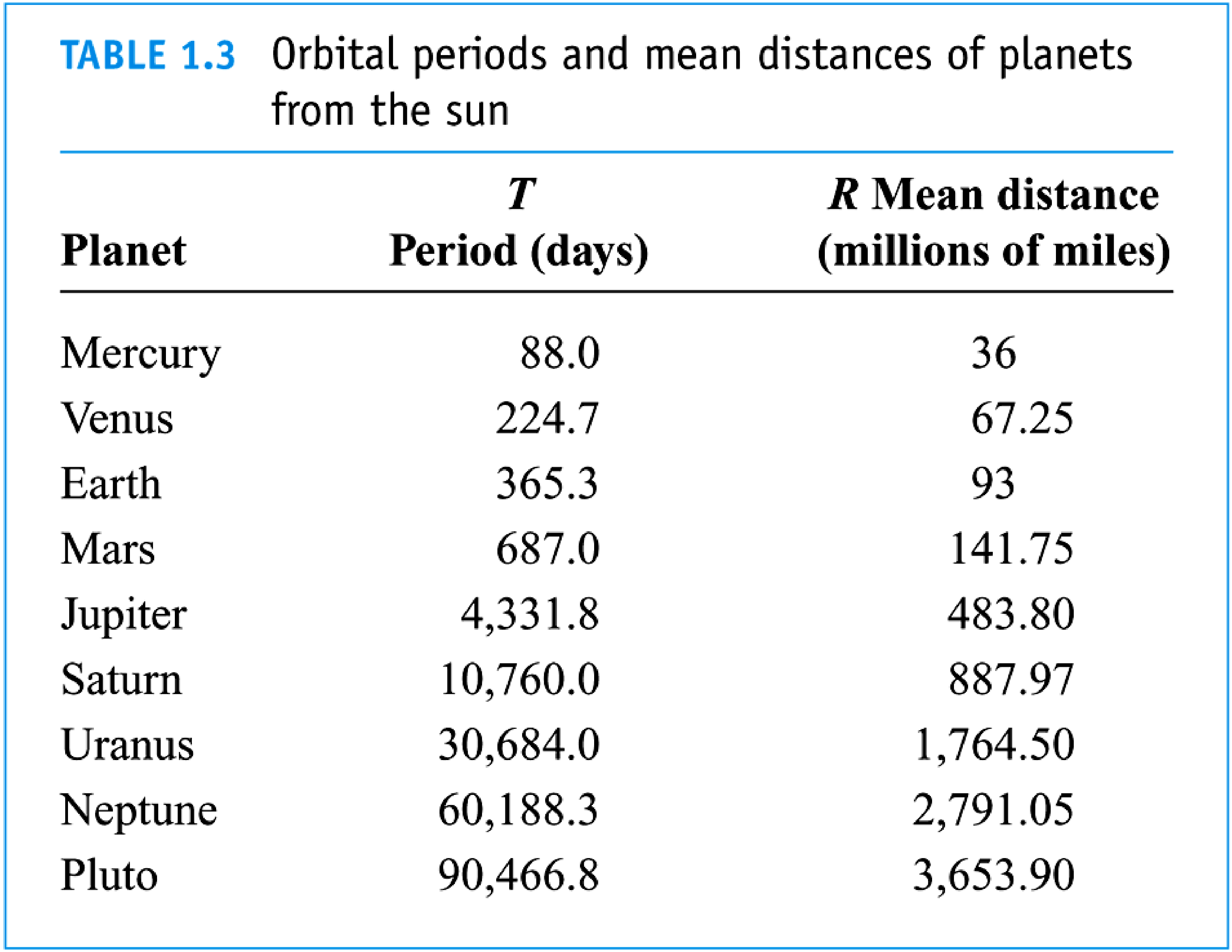


TABLE 1.3 Orbital periods and mean distances of planets from the sun

Planet	***T*** **Period (days)**	***R* Mean distance (millions of miles)**
Mercury	88.0	36
Venus	224.7	67.25
Earth	365.3	93
Mars	687.0	141.75
Jupiter	4,331.8	483.80
Saturn	10,760.0	887.97
Uranus	30,684.0	1,764.50
Neptune	60,188.3	2,791.05
Pluto	90,466.8	3,653.90

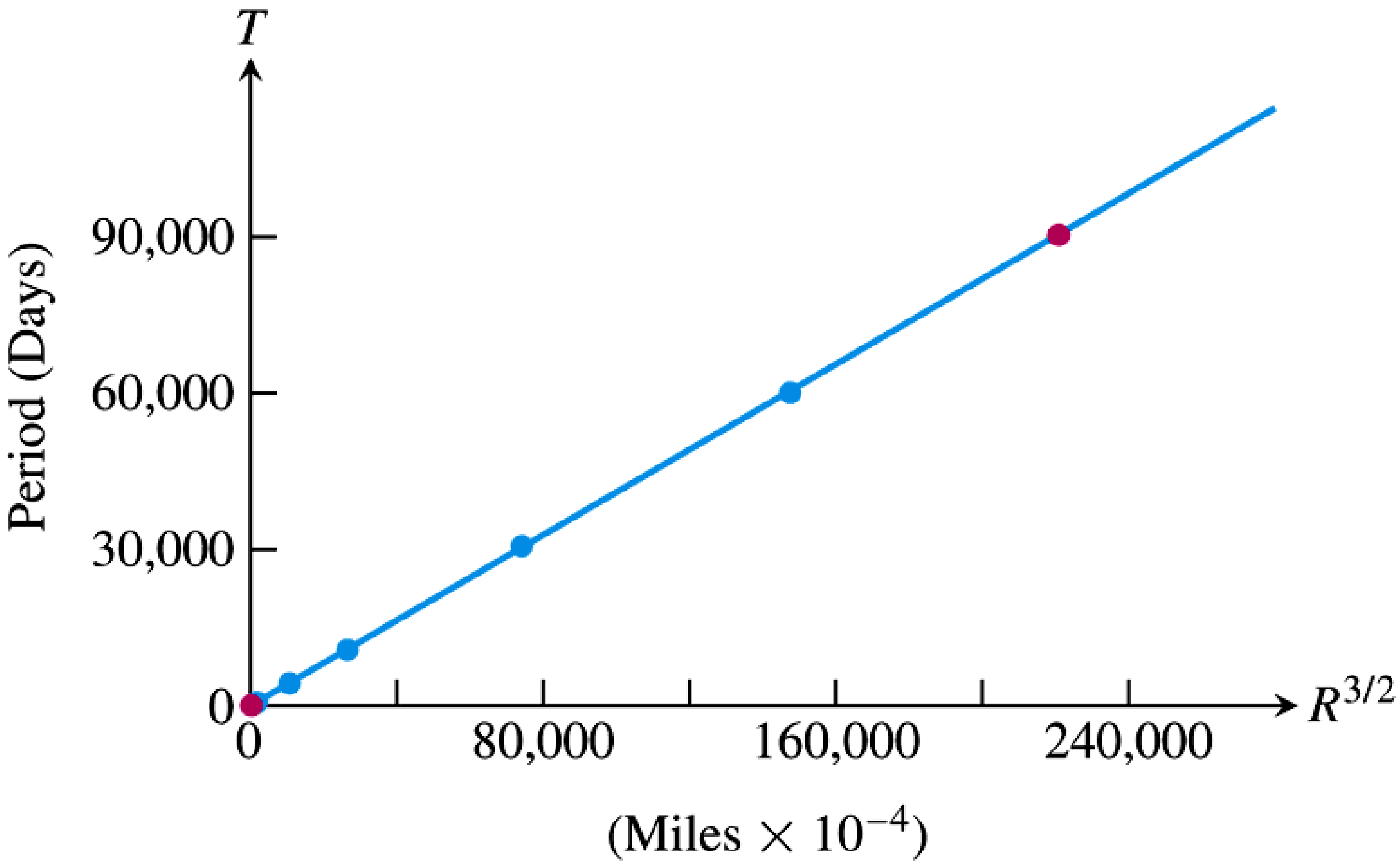


FIGURE 1.49 Graph of Kepler's third law as a proportionality: $T = 0.410R^{3/2}$ (Example 3).

1.5

Combining Functions;
Shifting and Scaling Graphs

Function	Formula	Domain
$f + g$	$(f + g)(x) = \sqrt{x} + \sqrt{1 - x}$	$[0, 1] = D(f) \cap D(g)$
$f - g$	$(f - g)(x) = \sqrt{x} - \sqrt{1 - x}$	$[0, 1]$
$g - f$	$(g - f)(x) = \sqrt{1 - x} - \sqrt{x}$	$[0, 1]$
$f \cdot g$	$(f \cdot g)(x) = f(x)g(x) = \sqrt{x(1 - x)}$	$[0, 1]$
f/g	$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \sqrt{\frac{x}{1 - x}}$	$[0, 1)$ ($x = 1$ excluded)
g/f	$\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \sqrt{\frac{1 - x}{x}}$	$(0, 1]$ ($x = 0$ excluded)

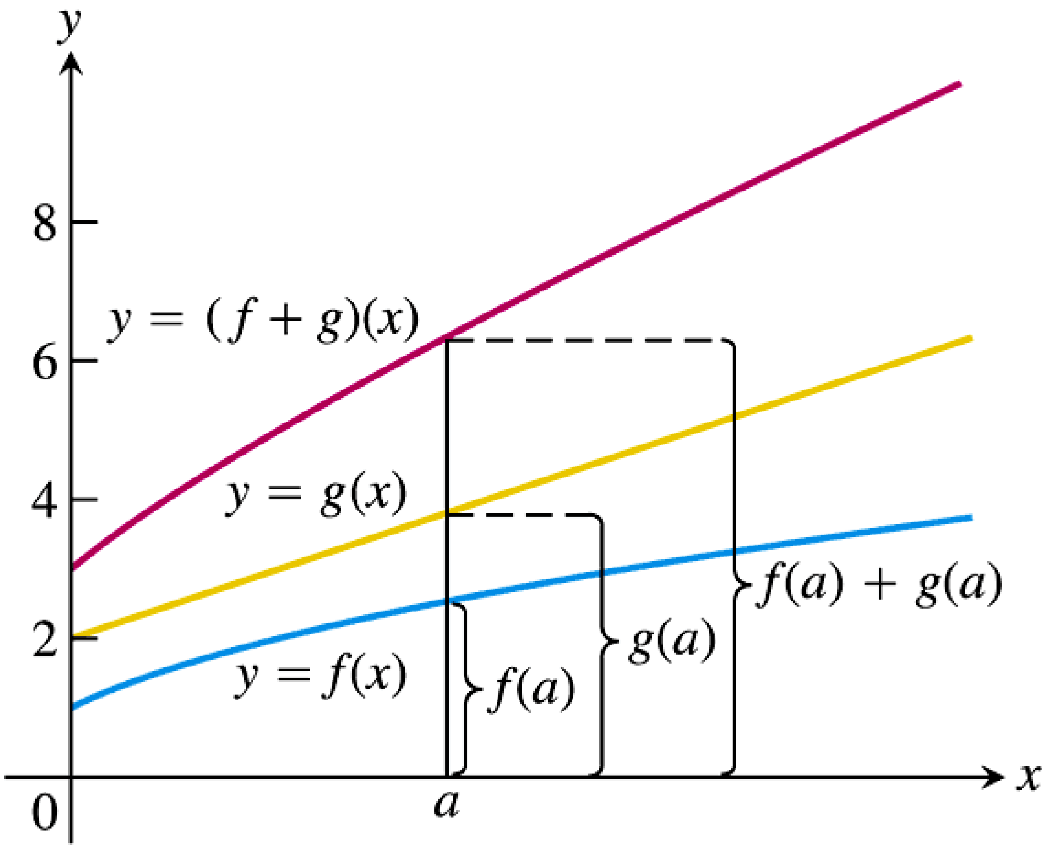


FIGURE 1.50 Graphical addition of two functions.

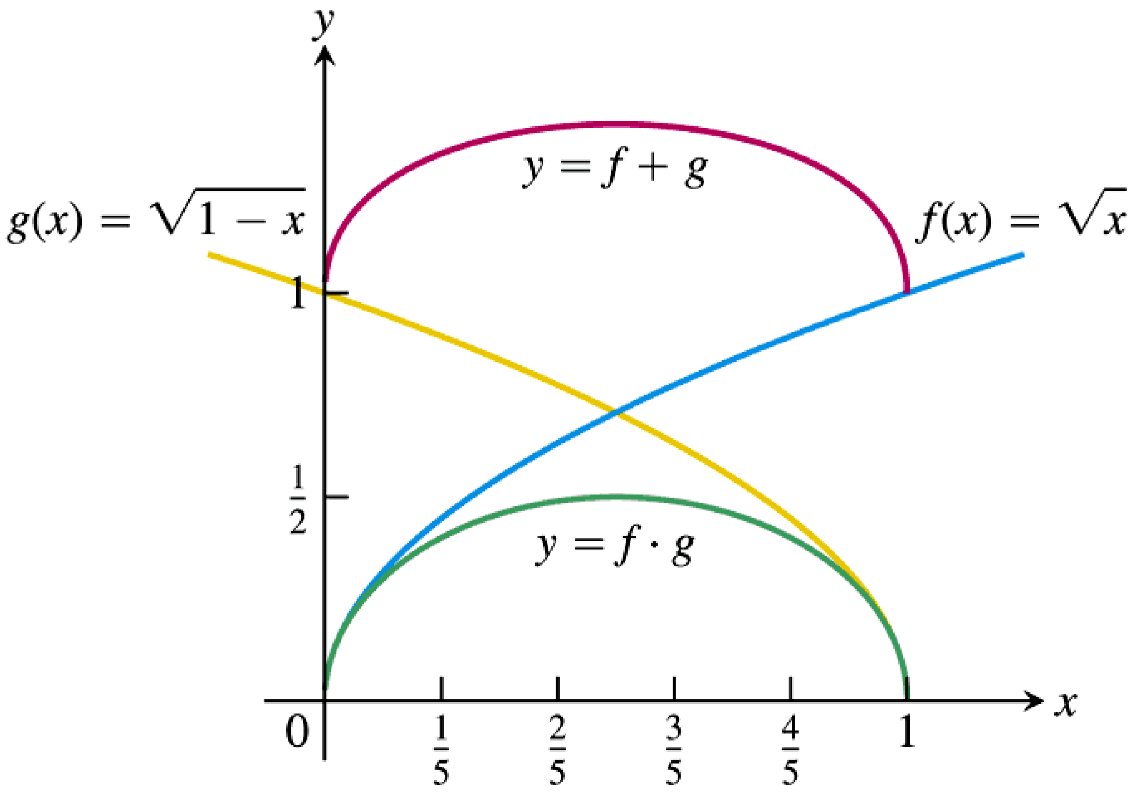
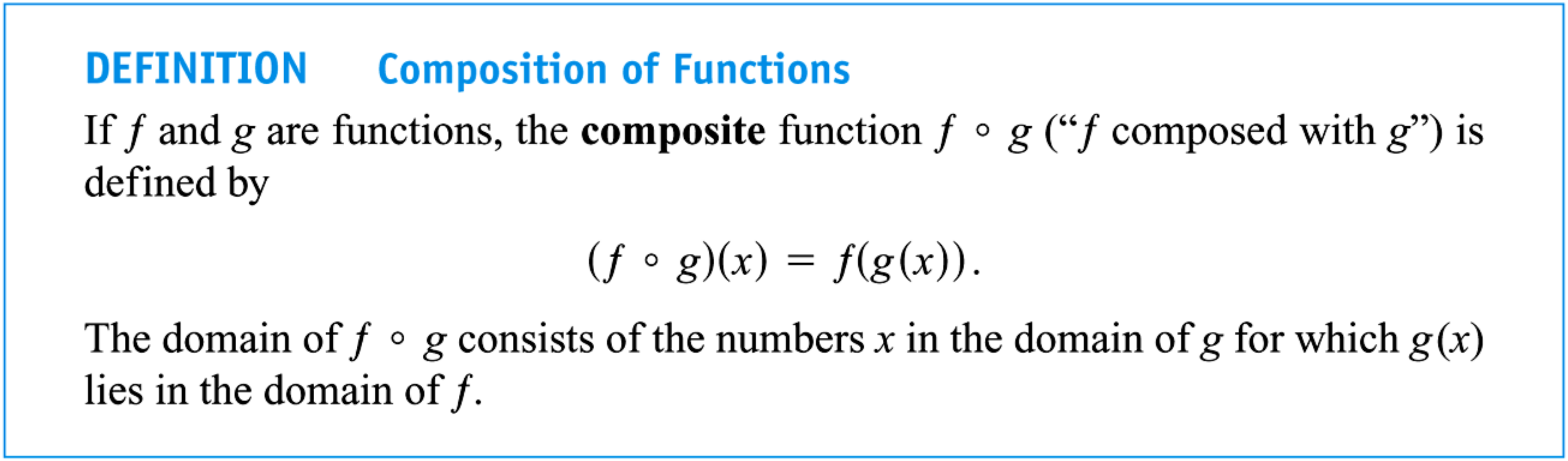


FIGURE 1.51 The domain of the function $f + g$ is the intersection of the domains of f and g, the interval [0, 1] on the x-axis where these domains overlap. This interval is also the domain of the function $f \cdot g$ (Example 1).



DEFINITION **Composition of Functions**

If f and g are functions, the **composite** function $f \circ g$ ("f composed with g") is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f.

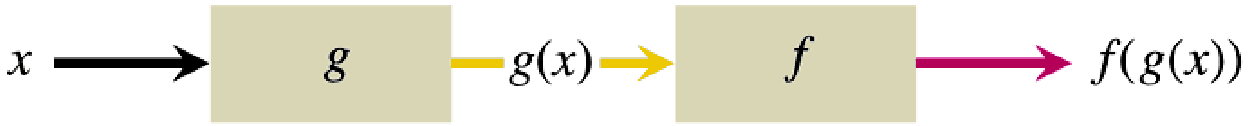


FIGURE 1.52 Two functions can be composed at x whenever the value of one function at x lies in the domain of the other. The composite is denoted by $f \circ g$.

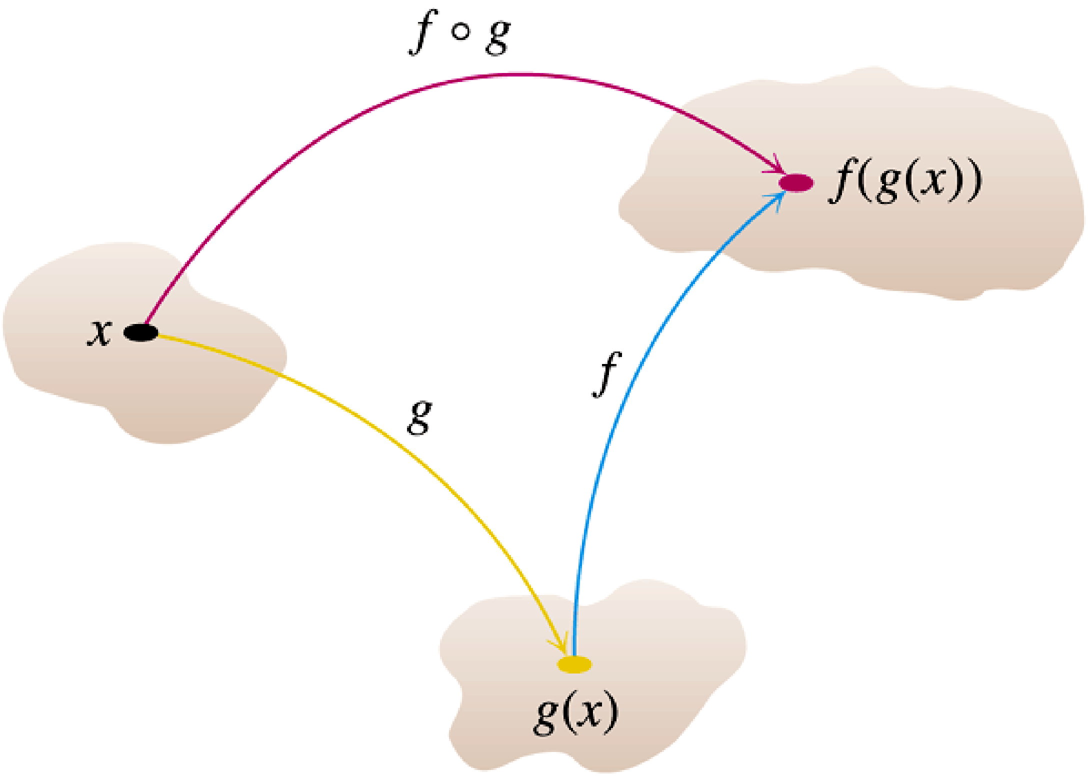
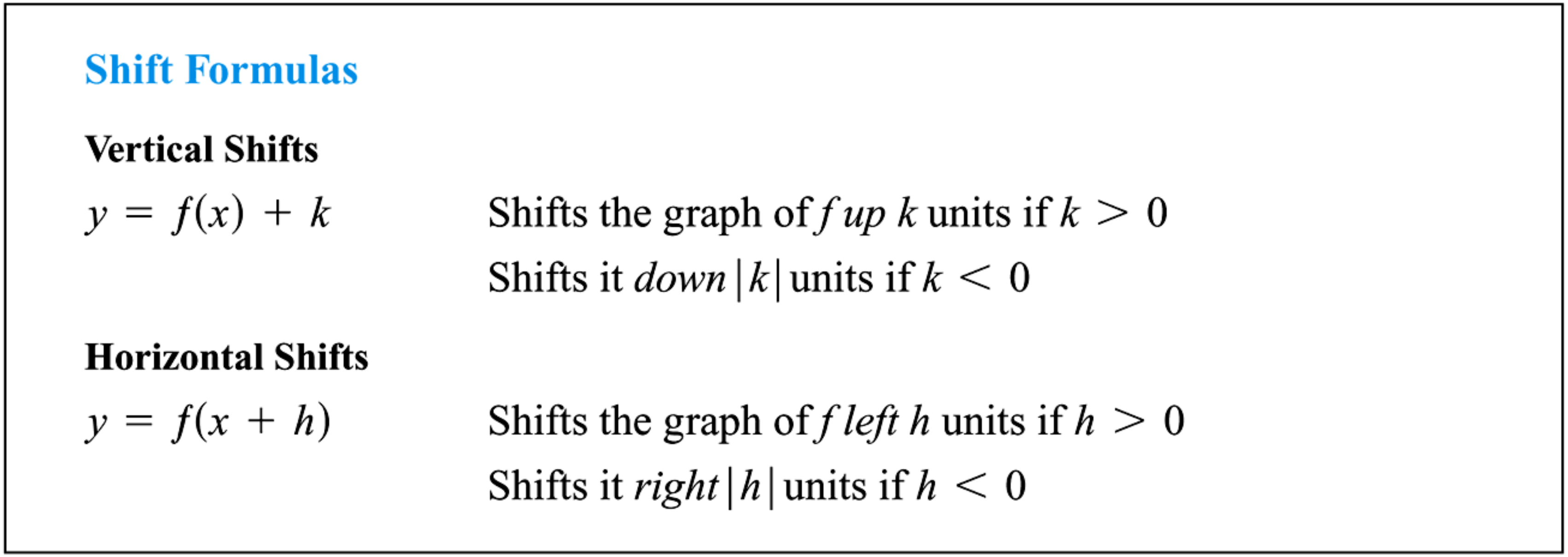


FIGURE 1.53 Arrow diagram for $f \circ g$.



Shift Formulas

Vertical Shifts

$y = f(x) + k$	Shifts the graph of f *up* k units if $k > 0$ Shifts it *down* $\lvert k \rvert$ units if $k < 0$

Horizontal Shifts

$y = f(x + h)$	Shifts the graph of f *left* h units if $h > 0$ Shifts it *right* $\lvert h \rvert$ units if $h < 0$

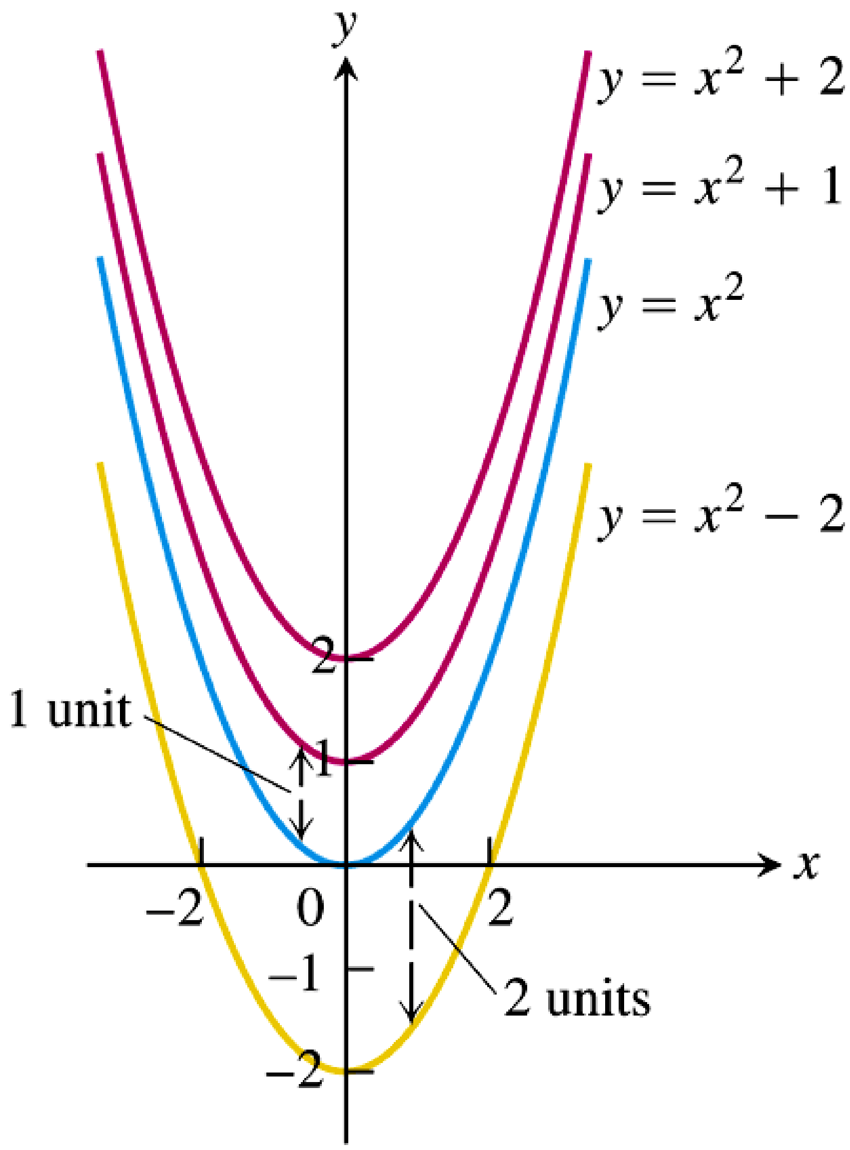


FIGURE 1.54 To shift the graph of $f(x) = x^2$ up (or down), we add positive (or negative) constants to the formula for f (Example 4a and b).

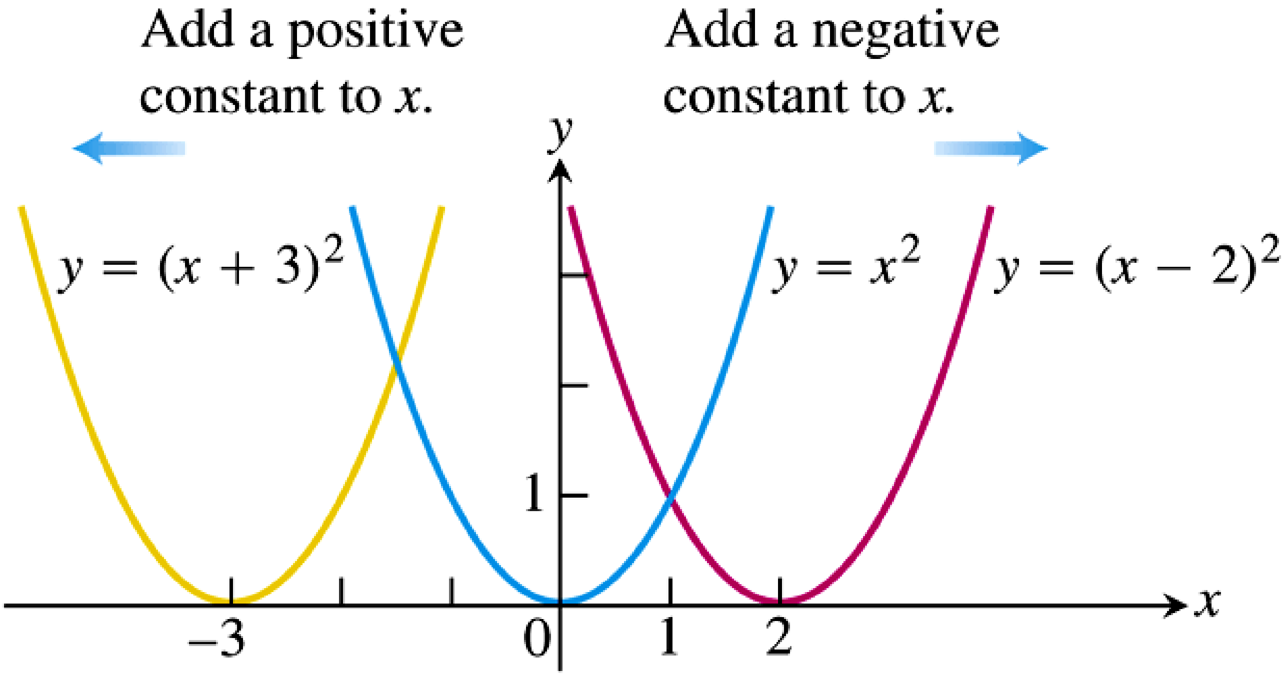


FIGURE 1.55 To shift the graph of $y = x^2$ to the left, we add a positive constant to x. To shift the graph to the right, we add a negative constant to x (Example 4c).

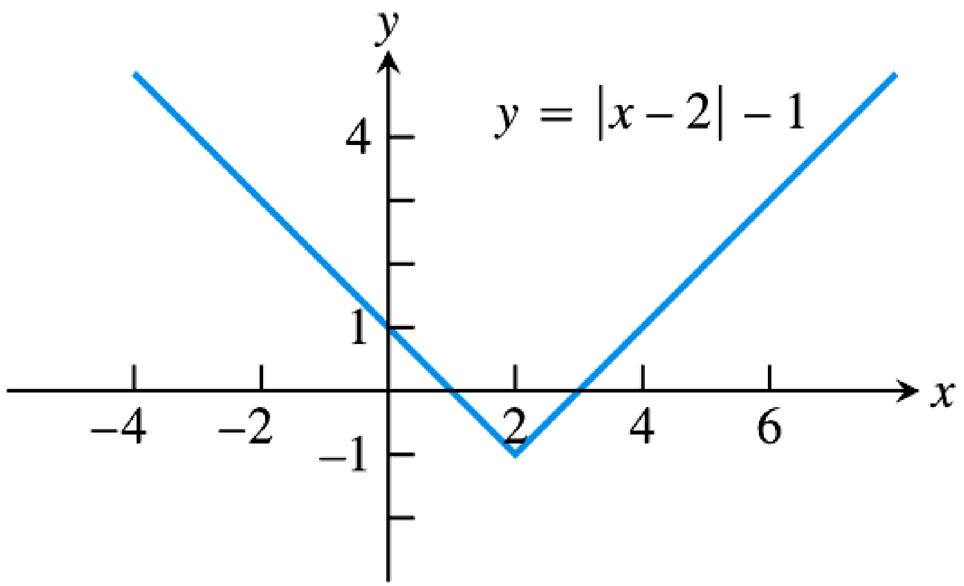


FIGURE 1.56 Shifting the graph of $y = |x|$ 2 units to the right and 1 unit down (Example 4d).

Vertical and Horizontal Scaling and Reflecting Formulas

For $c > 1$,

$y = cf(x)$	Stretches the graph of f vertically by a factor of c.
$y = \frac{1}{c} f(x)$	Compresses the graph of f vertically by a factor of c.
$y = f(cx)$	Compresses the graph of f horizontally by a factor of c.
$y = f(x/c)$	Stretches the graph of f horizontally by a factor of c.

For $c = -1$,

$y = -f(x)$	Reflects the graph of f across the x-axis.
$y = f(-x)$	Reflects the graph of f across the y-axis.

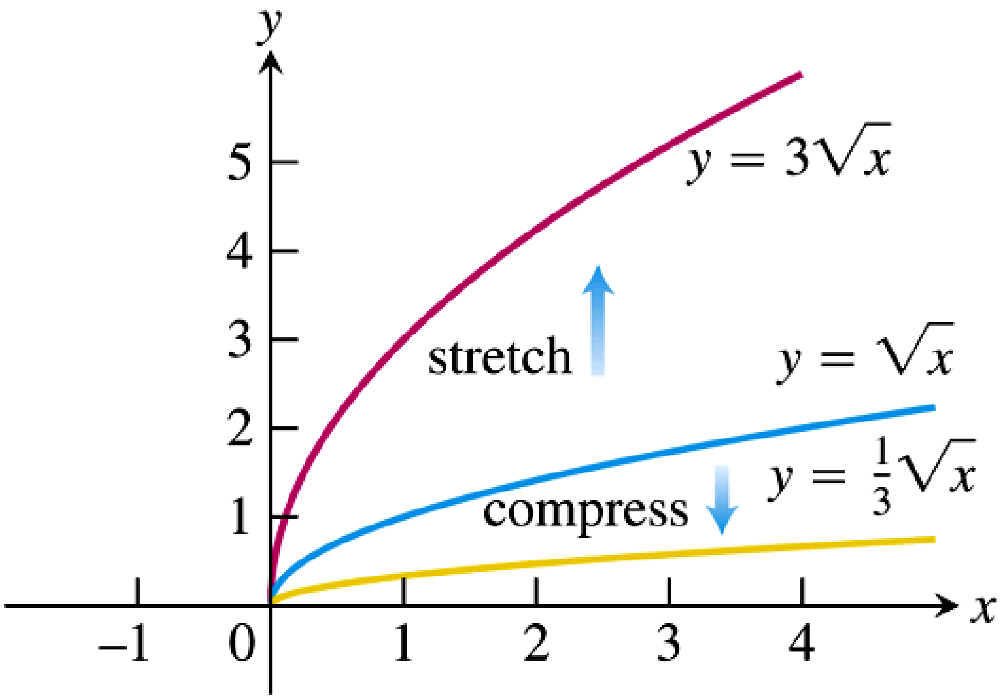


FIGURE 1.57 Vertically stretching and compressing the graph $y = \sqrt{x}$ by a factor of 3 (Example 5a).

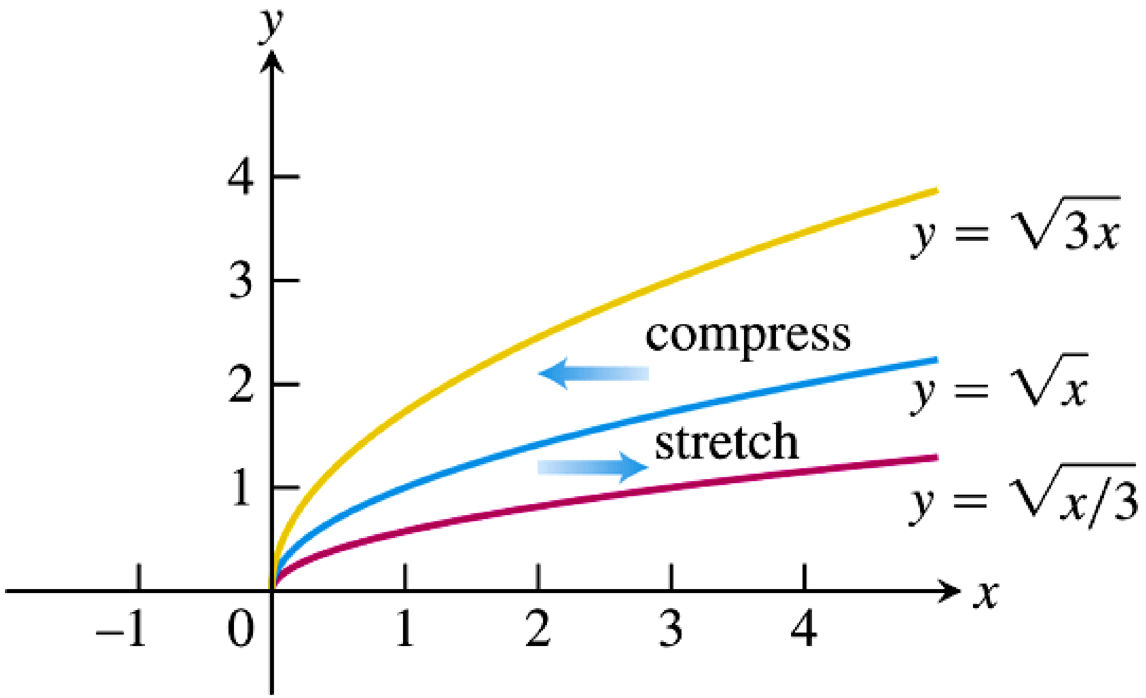


FIGURE 1.58 Horizontally stretching and compressing the graph $y = \sqrt{x}$ by a factor of 3 (Example 5b).

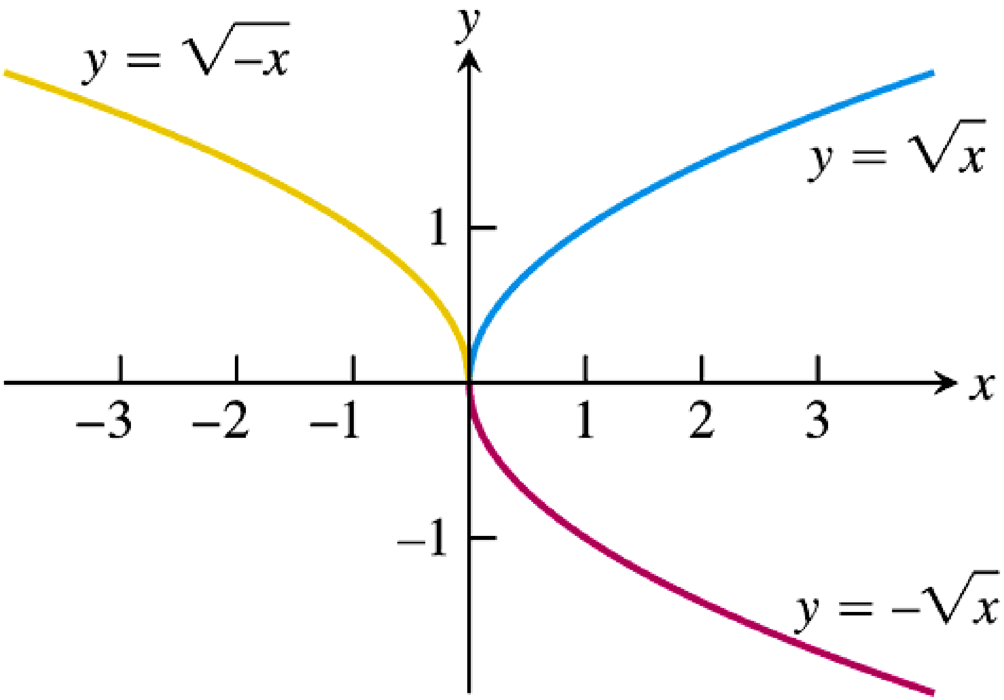


FIGURE 1.59 Reflections of the graph $y = \sqrt{x}$ across the coordinate axes (Example 5c).

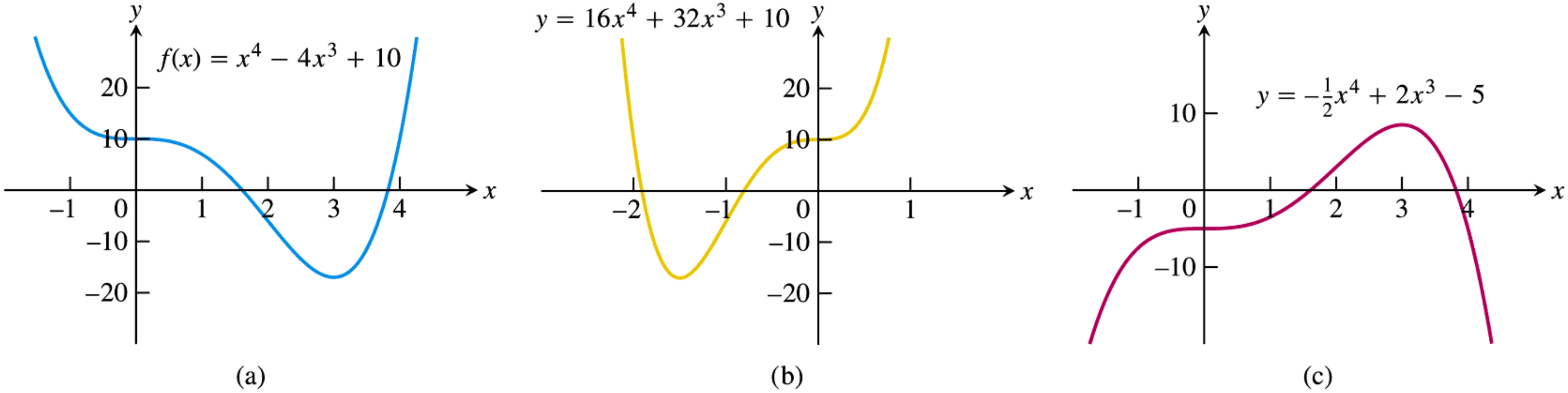


FIGURE 1.60 (a) The original graph of *f*. (b) The horizontal compression of $y = f(x)$ in part (a) by a factor of 2, followed by a reflection across the *y*-axis. (c) The vertical compression of $y = f(x)$ in part (a) by a factor of 2, followed by a reflection across the *x*-axis (Example 6).

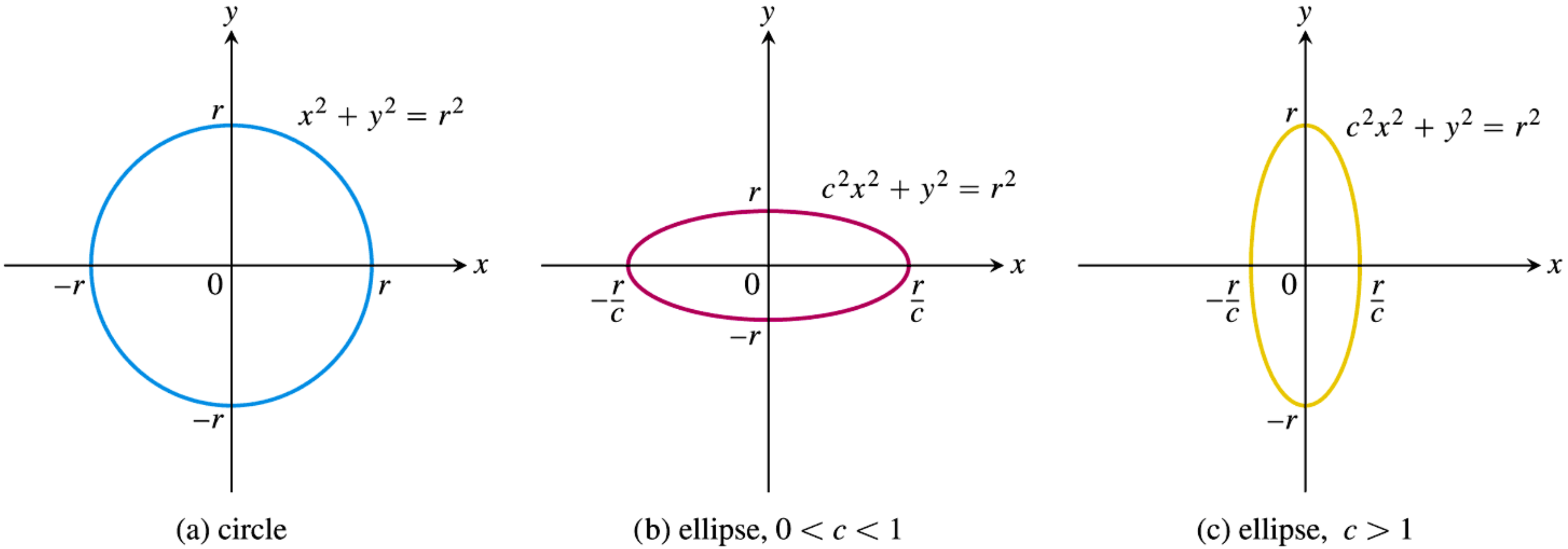


FIGURE 1.61 Horizontal stretchings or compressions of a circle produce graphs of ellipses.

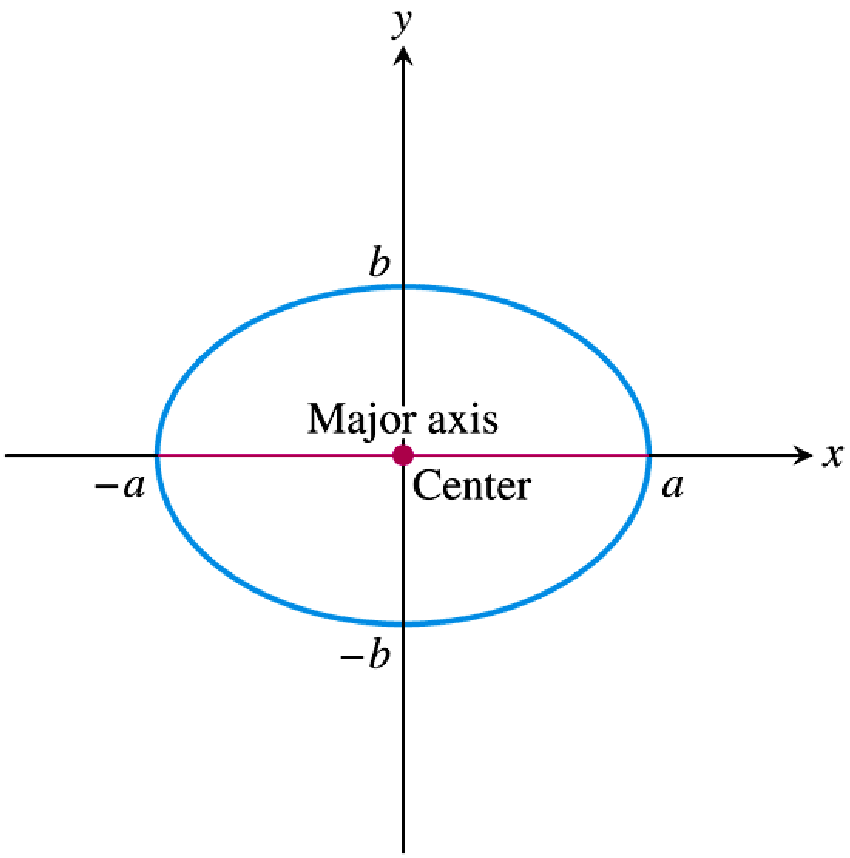


FIGURE 1.62 Graph of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$, where the major axis is horizontal.

1.6

Trigonometric Functions

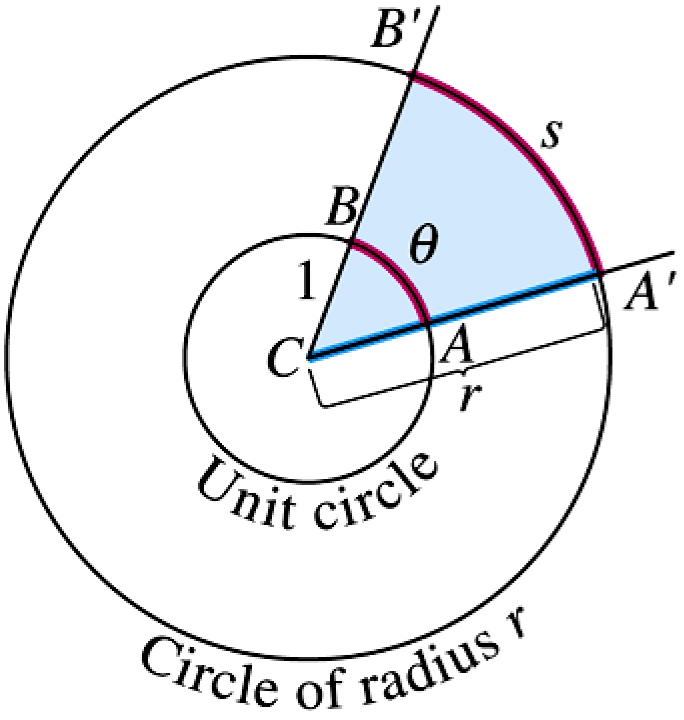
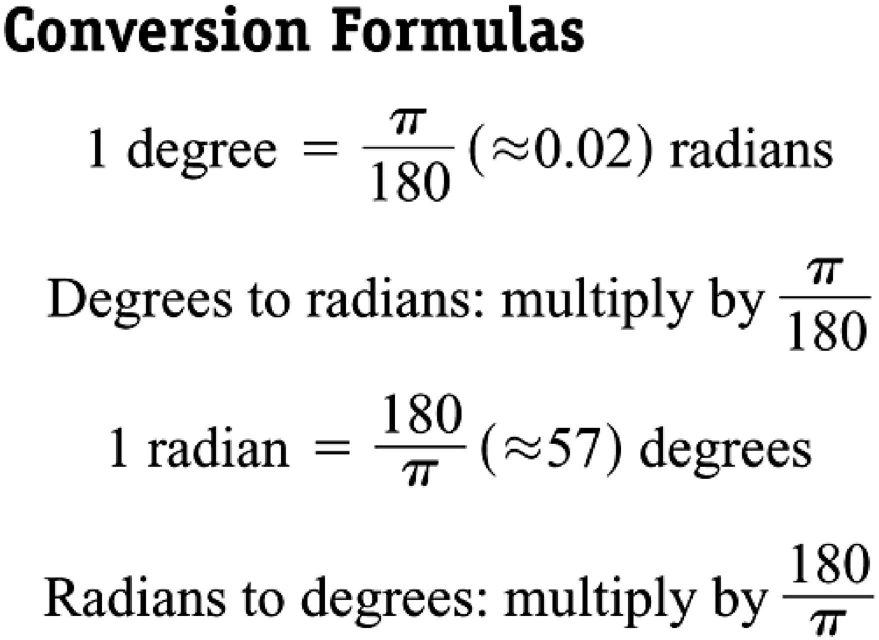


FIGURE 1.63 The radian measure of angle ACB is the length θ of arc AB on the unit circle centered at C. The value of θ can be found from any other circle, however, as the ratio s/r. Thus $s = r\theta$ is the length of arc on a circle of radius r when θ is measured in radians.



Conversion Formulas

$$1 \text{ degree } = \frac{\pi}{180} (\approx 0.02) \text{ radians}$$

$$\text{Degrees to radians: multiply by } \frac{\pi}{180}$$

$$1 \text{ radian } = \frac{180}{\pi} (\approx 57) \text{ degrees}$$

$$\text{Radians to degrees: multiply by } \frac{180}{\pi}$$

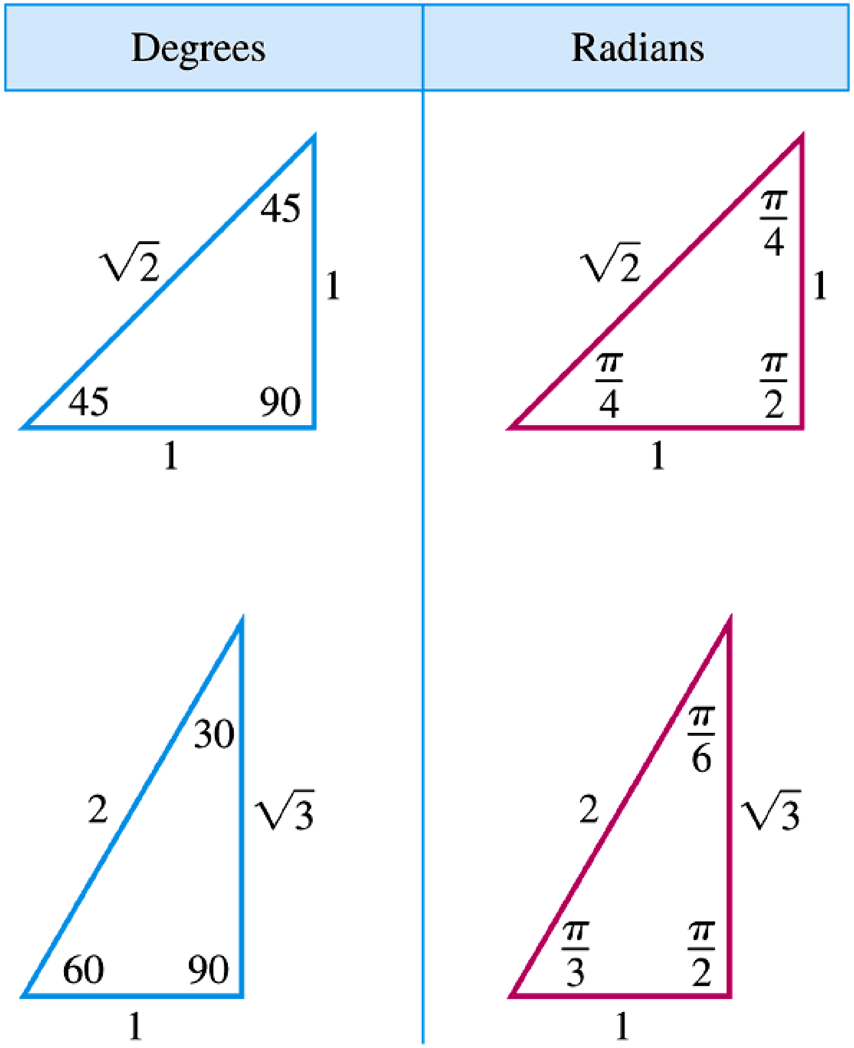


FIGURE 1.64 The angles of two common triangles, in degrees and radians.

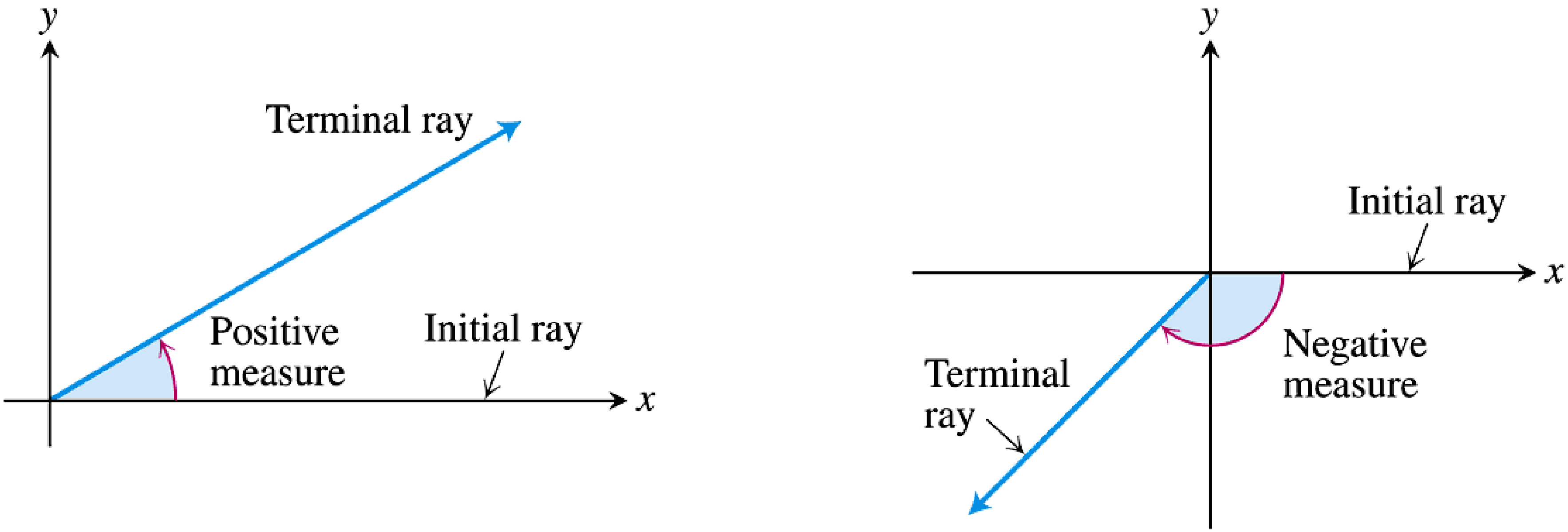


FIGURE 1.65 Angles in standard position in the xy-plane.

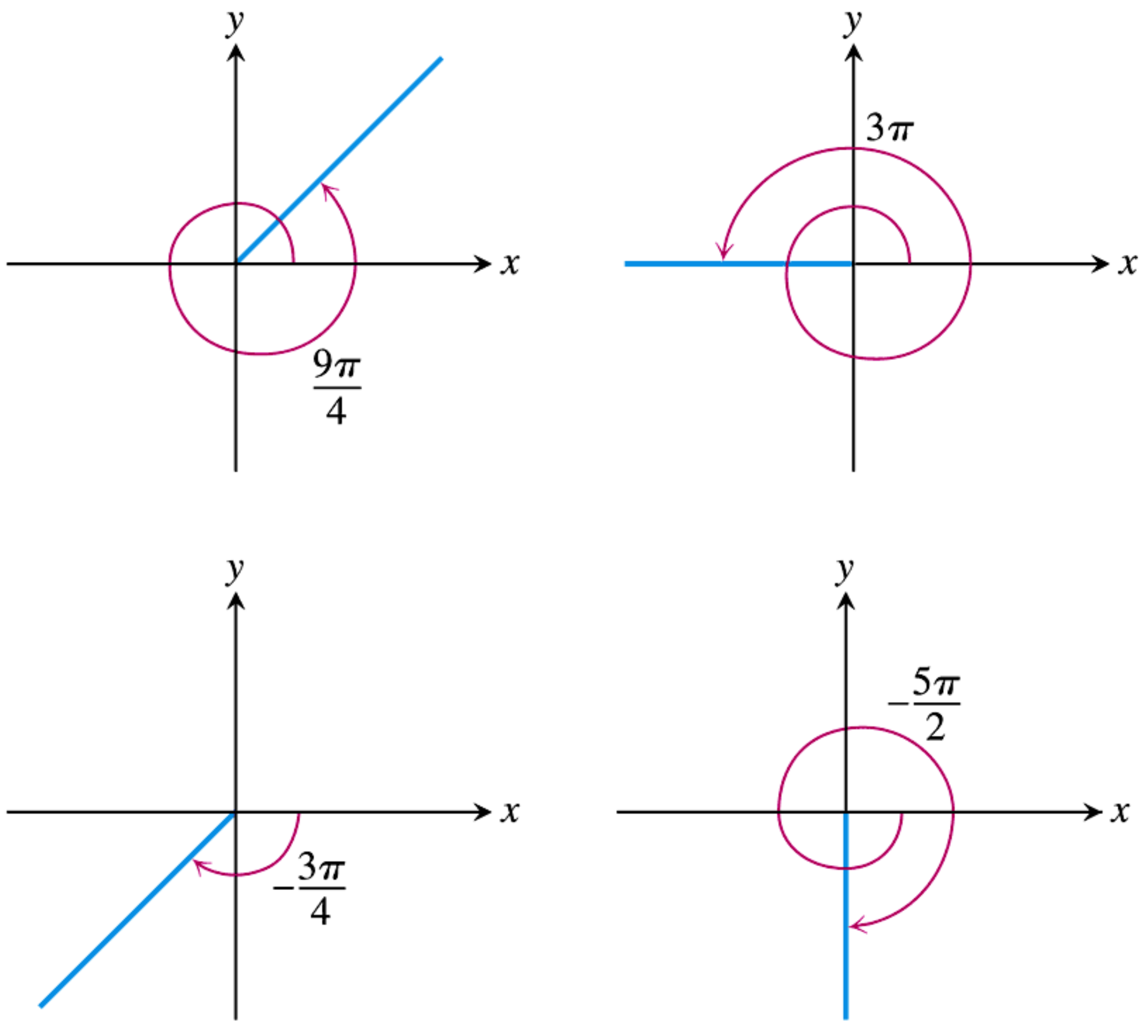
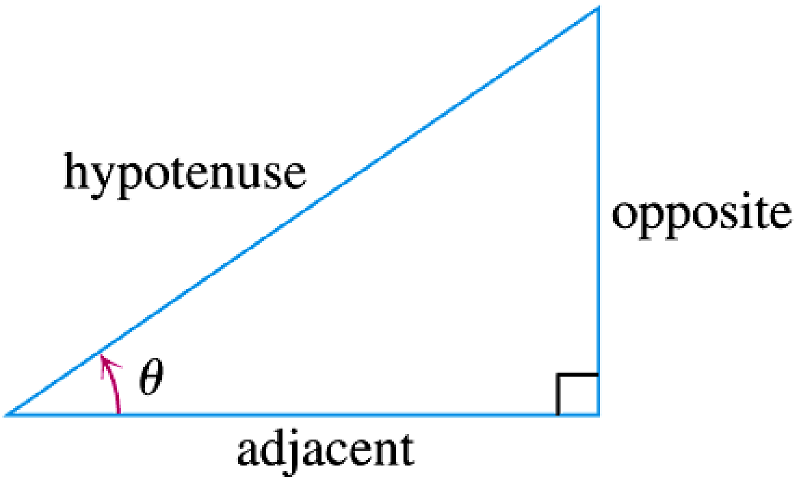


FIGURE 1.66 Nonzero radian measures can be positive or negative and can go beyond 2π.



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

FIGURE 1.67 Trigonometric ratios of an acute angle.

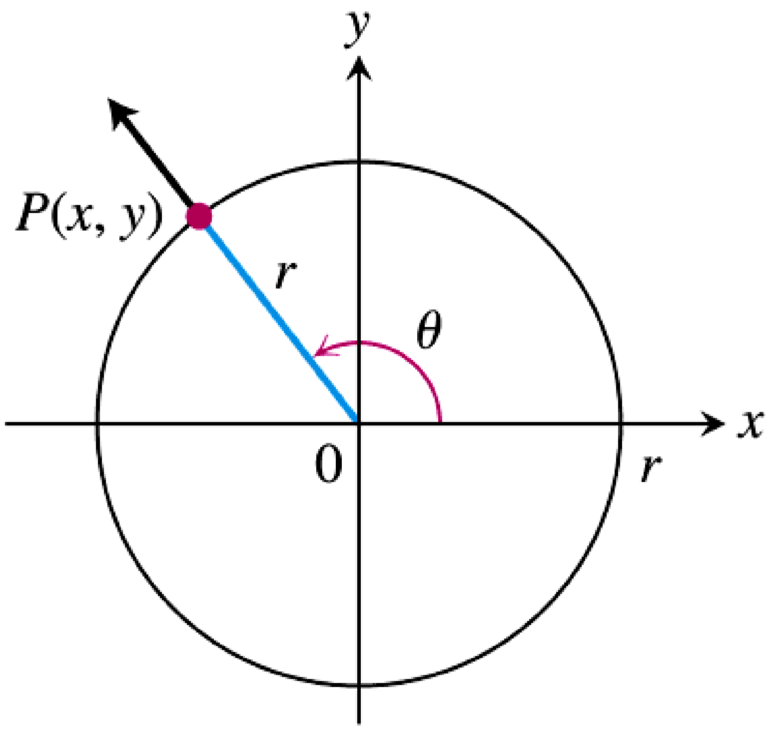


FIGURE 1.68 The trigonometric functions of a general angle θ are defined in terms of x, y, and r.

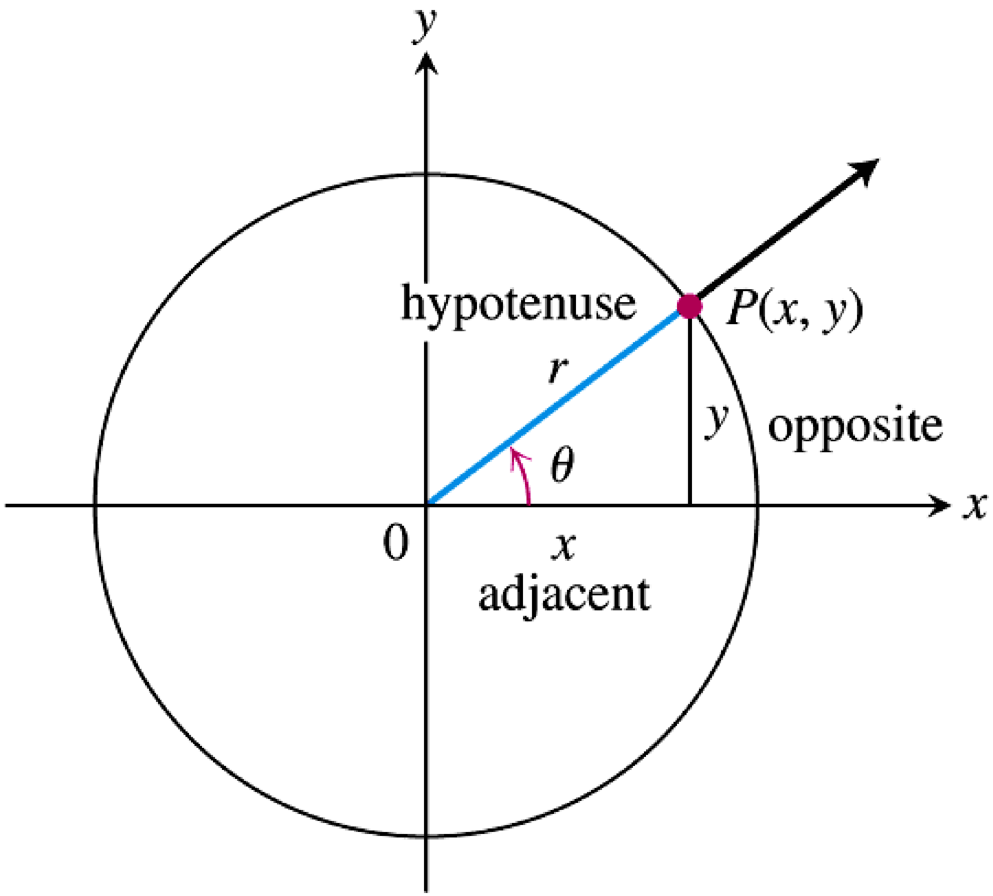


FIGURE 1.69 The new and old definitions agree for acute angles.

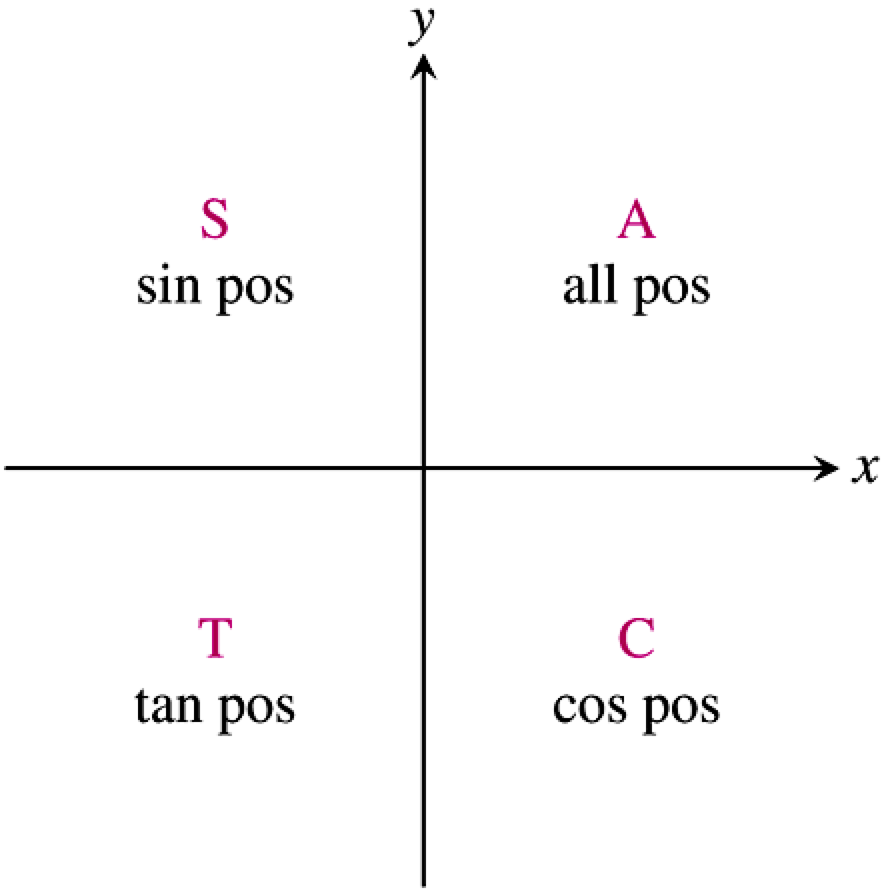


FIGURE 1.70 The CAST rule, remembered by the statement "All Students Take Calculus," tells which trigonometric functions are positive in each quadrant.

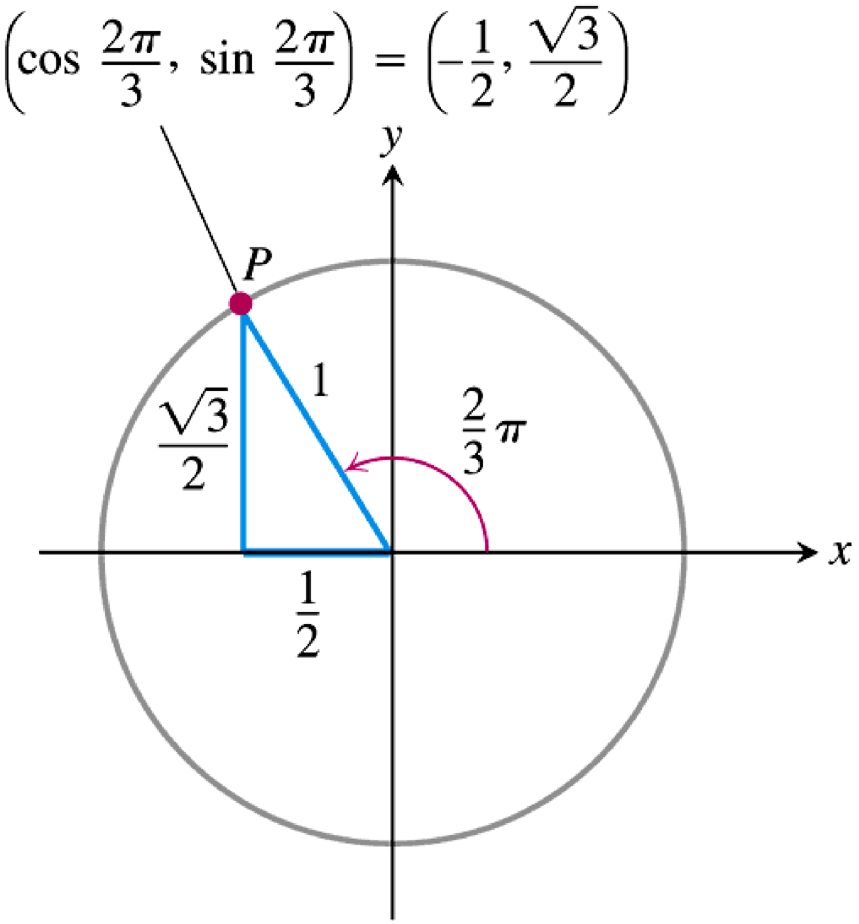


FIGURE 1.71 The triangle for calculating the sine and cosine of $2\pi/3$ radians. The side lengths come from the geometry of right triangles.

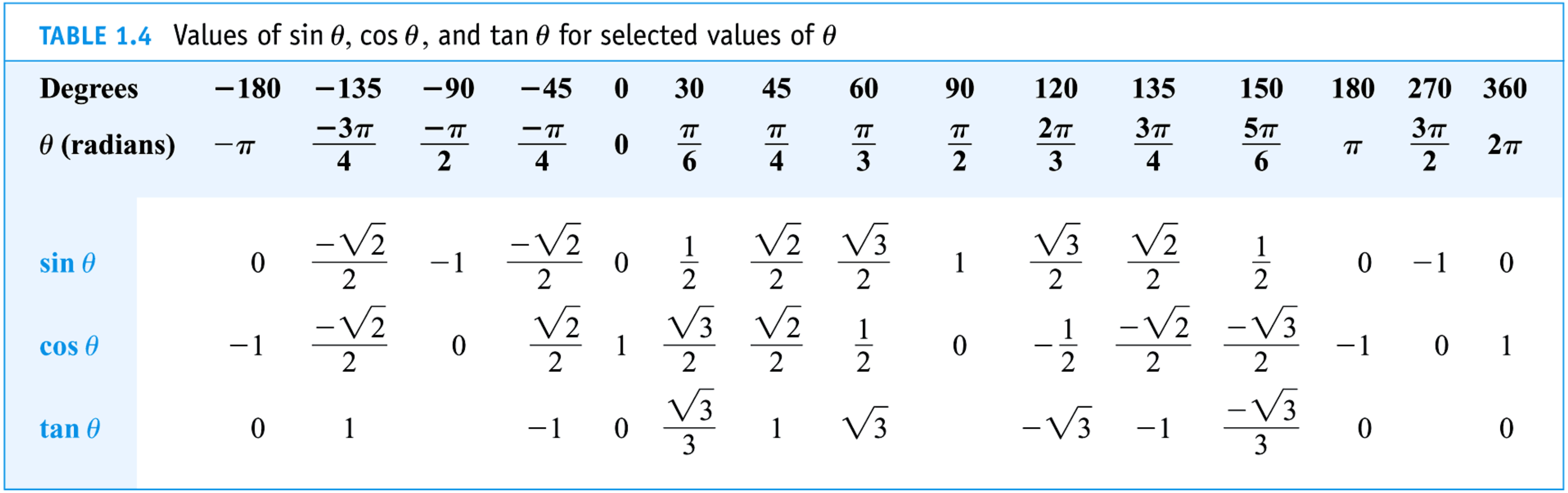


TABLE 1.4 Values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for selected values of θ

Degrees	**−180**	**−135**	**−90**	**−45**	**0**	**30**	**45**	**60**	**90**	**120**	**135**	**150**	**180**	**270**	**360**
θ (radians)	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	$\frac{-\sqrt{2}}{2}$	-1	$\frac{-\sqrt{2}}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\cos\theta$	-1	$\frac{-\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	0	1
$\tan\theta$	0	1		-1	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0		0

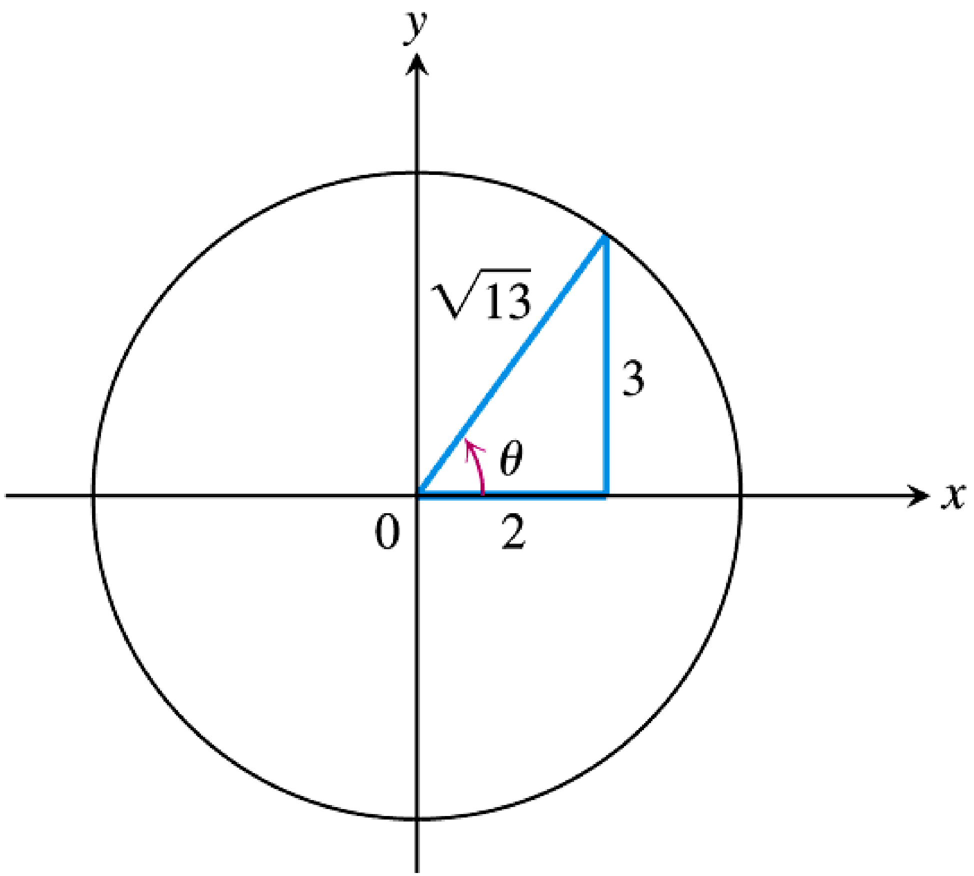
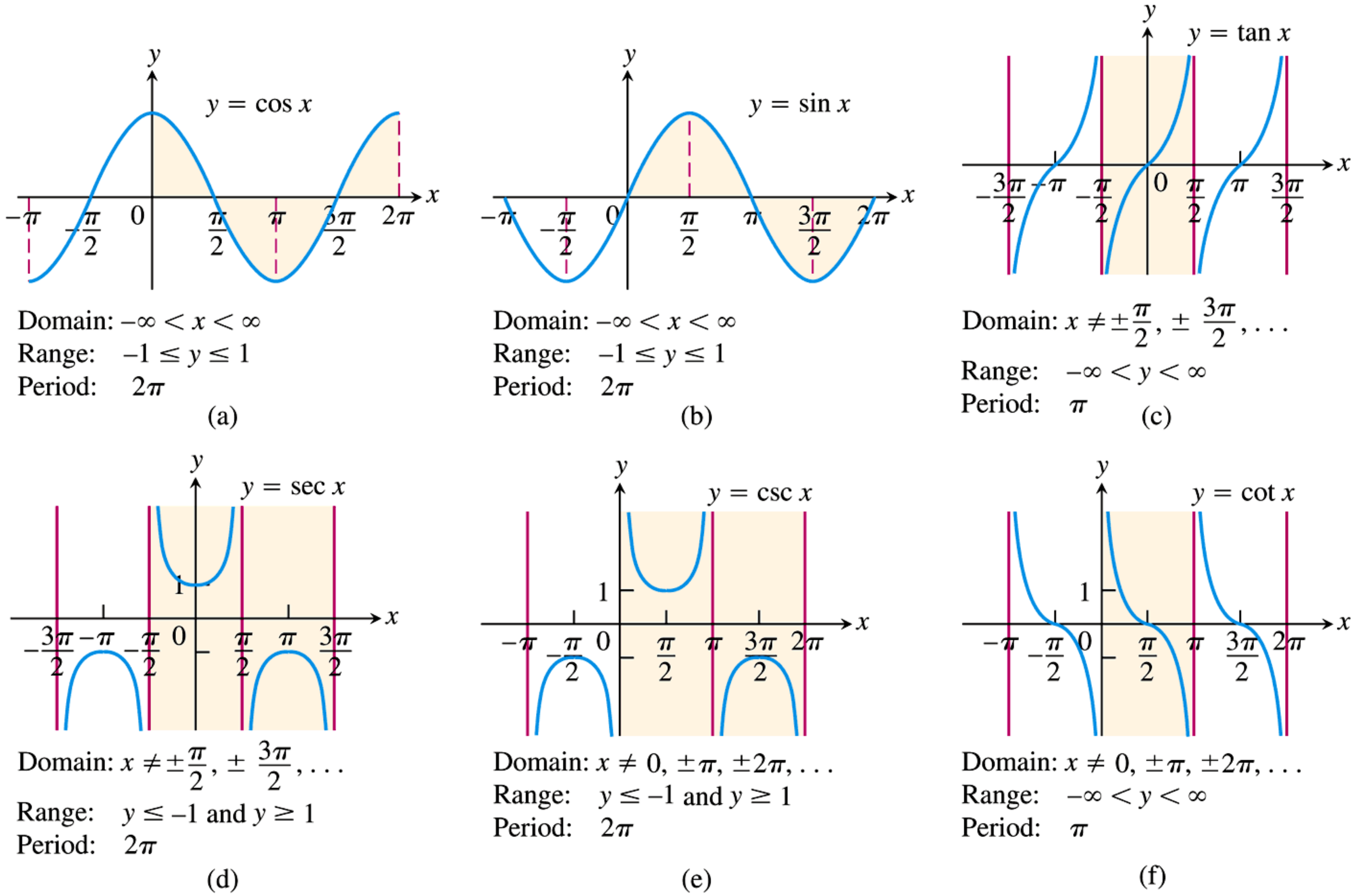


FIGURE 1.72 The triangle for calculating the trigonometric functions in Example 1.

DEFINITION Periodic Function

A function $f(x)$ is **periodic** if there is a positive number p such that $f(x + p) = f(x)$ for every value of x. The smallest such value of p is the **period** of f.



Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π
(a)

Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π
(b)

Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $-\infty < y < \infty$
Period: π
(c)

Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $y \le -1$ and $y \ge 1$
Period: 2π
(d)

Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $y \le -1$ and $y \ge 1$
Period: 2π
(e)

Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $-\infty < y < \infty$
Period: π
(f)

FIGURE 1.73 Graphs of the (a) cosine, (b) sine, (c) tangent, (d) secant, (e) cosecant, and (f) cotangent functions using radian measure. The shading for each trigonometric function indicates its periodicity.

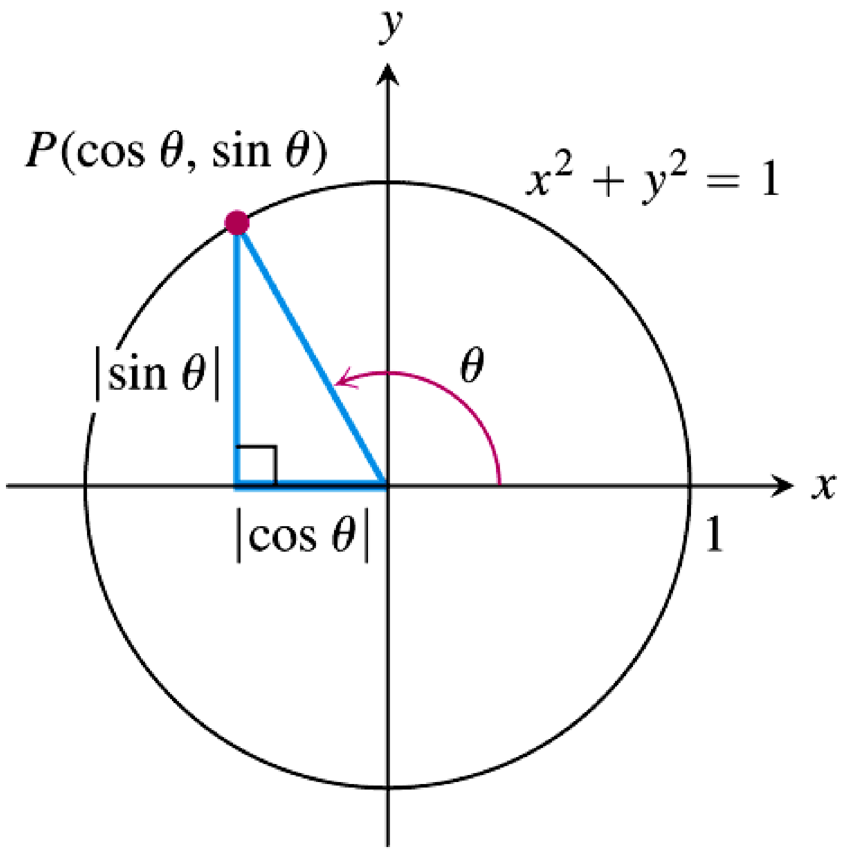


FIGURE 1.74 The reference triangle for a general angle θ.

Even	**Odd**
$\cos(-x) = \cos x$	$\sin(-x) = -\sin x$
$\sec(-x) = \sec x$	$\tan(-x) = -\tan x$
	$\csc(-x) = -\csc x$
	$\cot(-x) = -\cot x$

$$\cos^2 \theta + \sin^2 \theta = 1. \qquad (1)$$

$$1 + \tan^2 \theta = \sec^2 \theta.$$
$$1 + \cot^2 \theta = \csc^2 \theta.$$

Addition Formulas

$$\begin{aligned} \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \sin(A + B) &= \sin A \cos B + \cos A \sin B \end{aligned} \qquad (2)$$

Double-Angle Formulas

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta \tag{3}$$

Half-Angle Formulas

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \tag{4}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \tag{5}$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta\,. \tag{6}$$

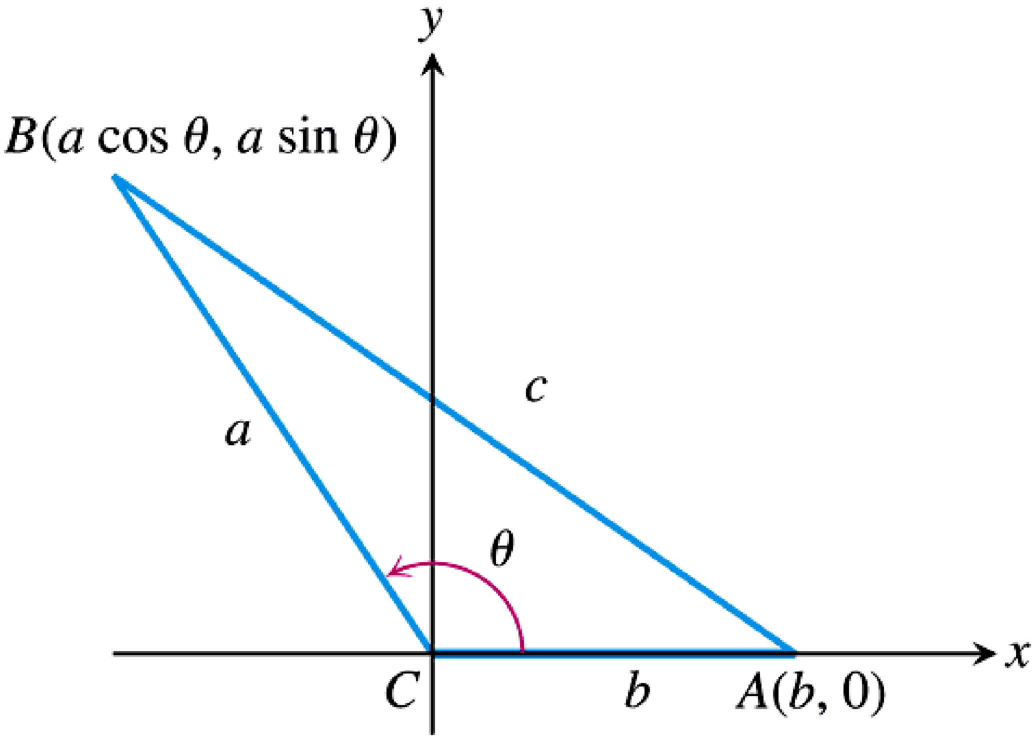
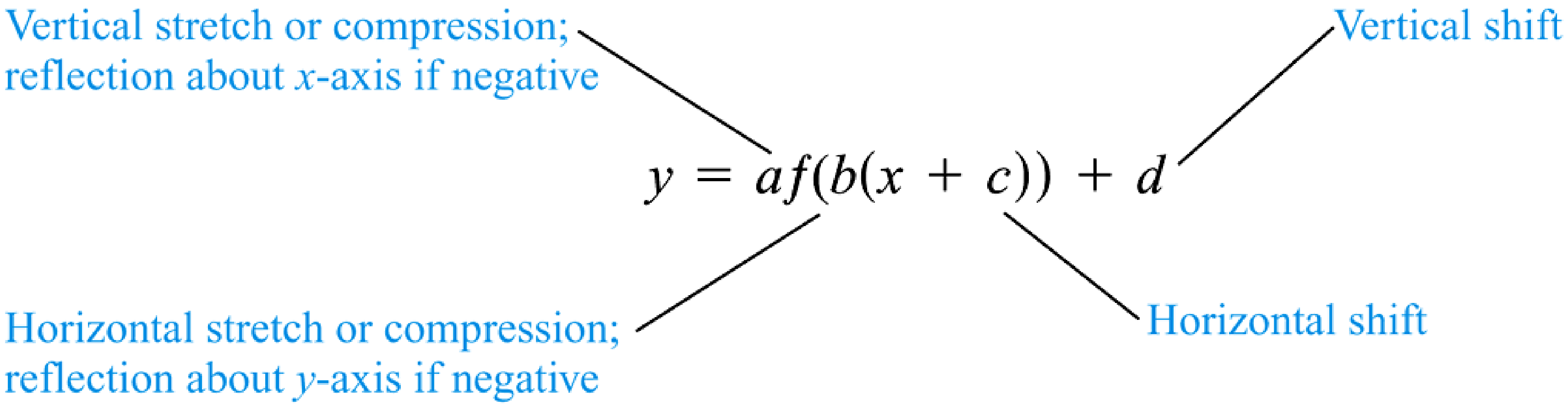


FIGURE 1.75 The square of the distance between A and B gives the law of cosines.



Vertical stretch or compression;
reflection about x-axis if negative

Vertical shift

$$y = af(b(x + c)) + d$$

Horizontal stretch or compression;
reflection about y-axis if negative

Horizontal shift

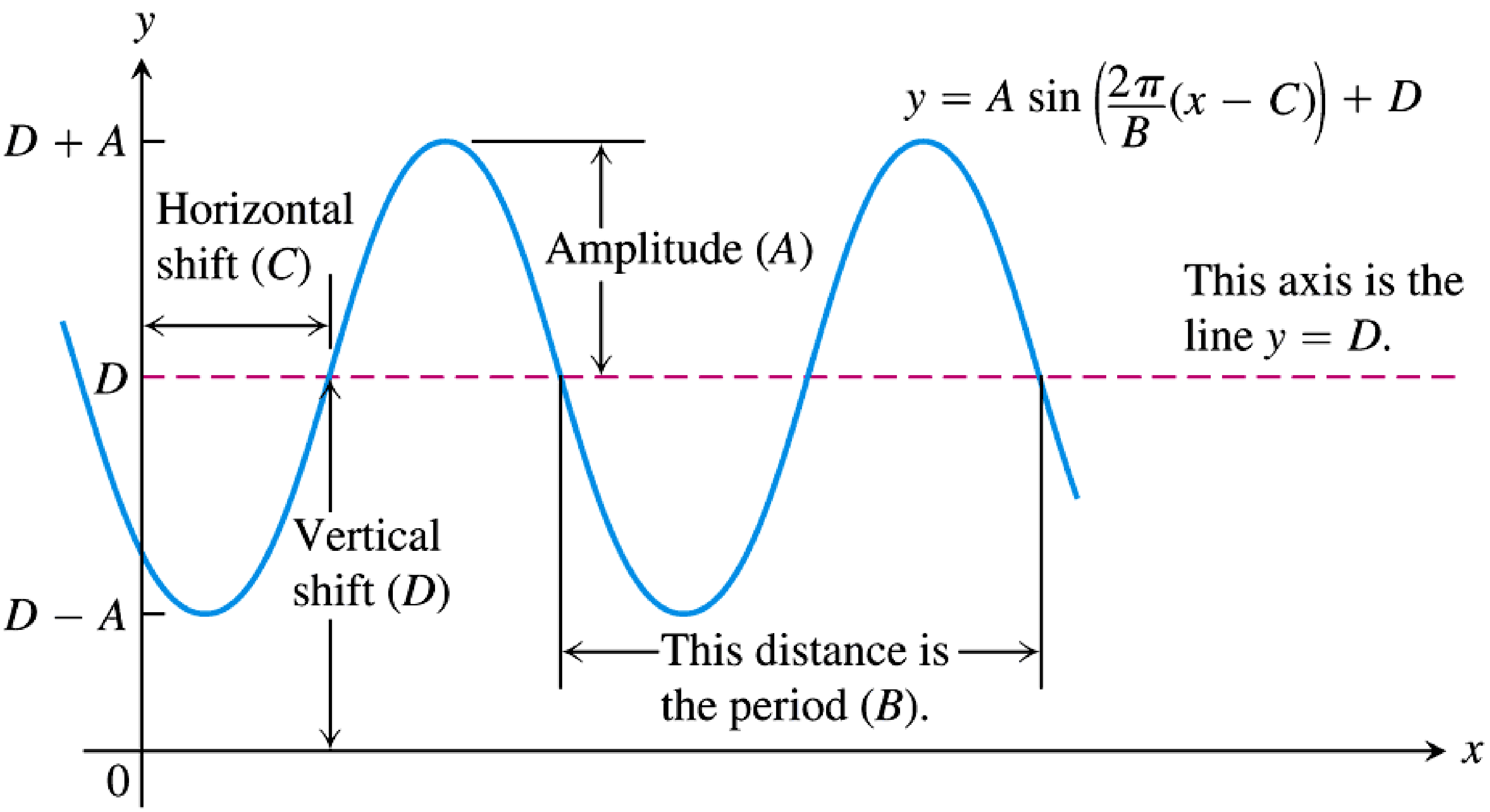


FIGURE 1.76 The general sine curve $y = A \sin[(2\pi/B)(x - C)] + D$, shown for A, B, C, and D positive (Example 2).

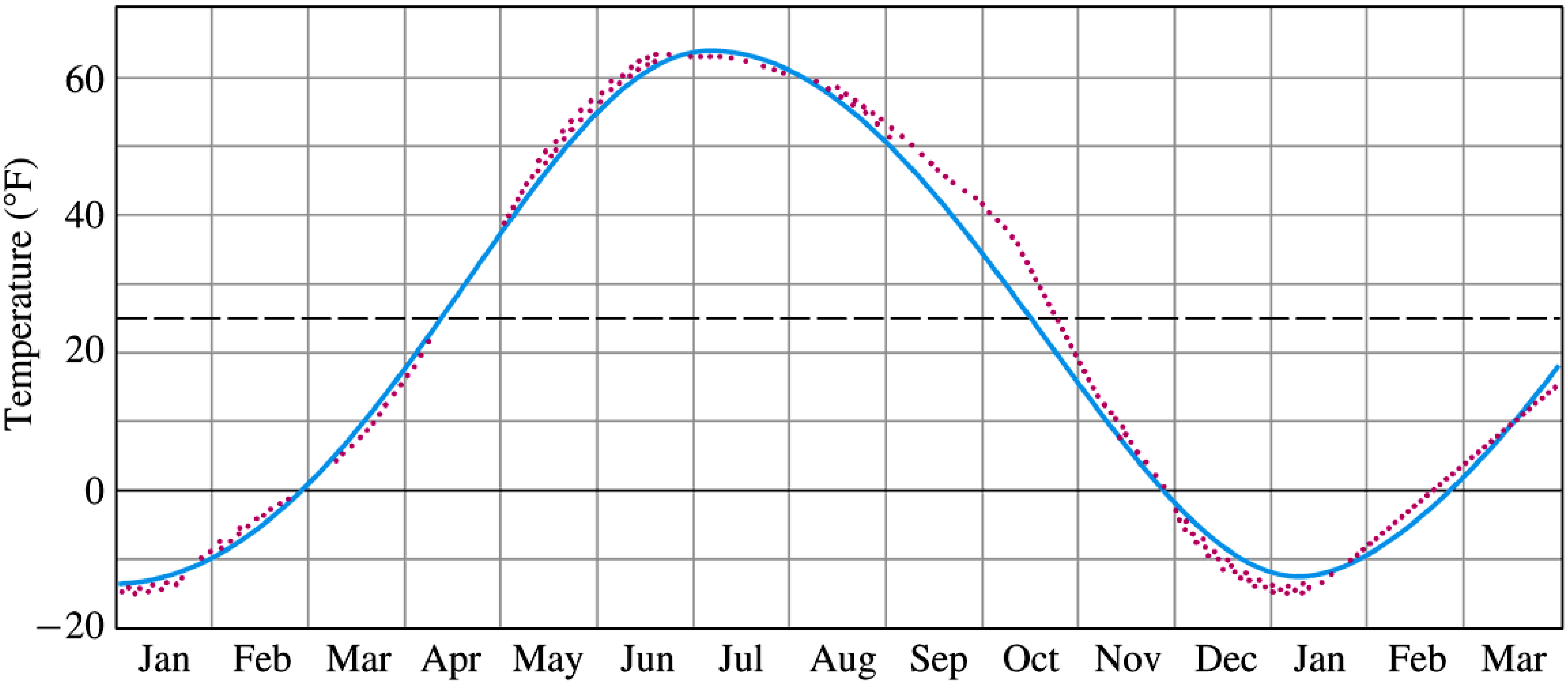


FIGURE 1.77 Normal mean air temperatures for Fairbanks, Alaska, plotted as data points (red). The approximating sine function (blue) is

$$f(x) = 37 \sin\left[(2\pi/365)(x - 101)\right] + 25.$$

1.7

Graphing with Calculators and Computers

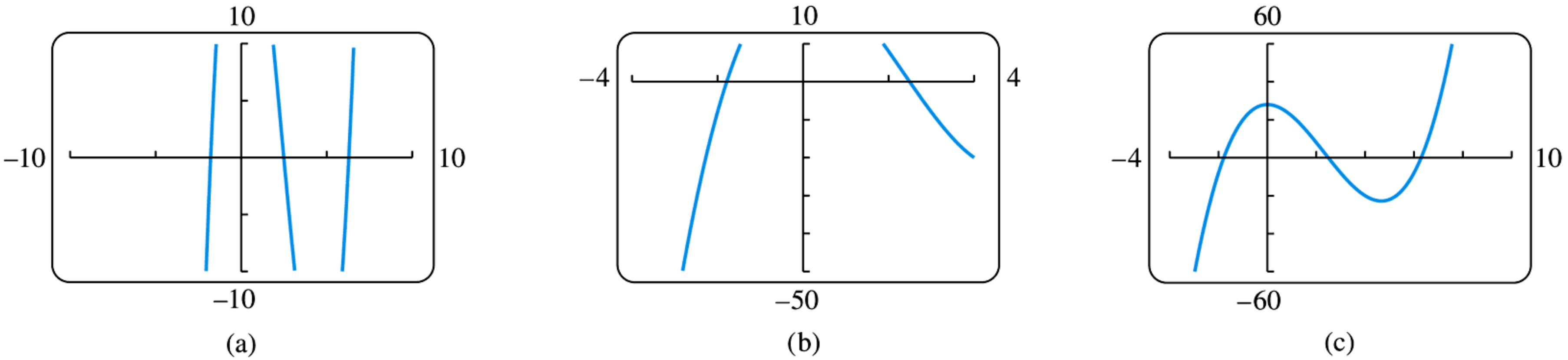


FIGURE 1.78 The graph of $f(x) = x^3 - 7x^2 + 28$ in different viewing windows (Example 1).

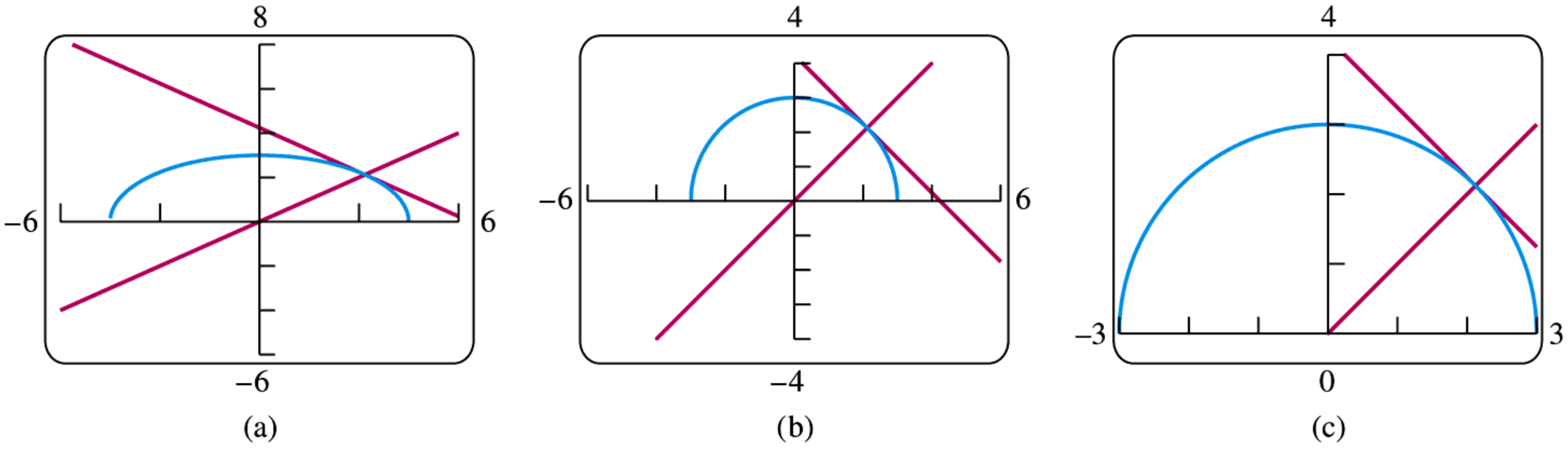


FIGURE 1.79 Graphs of the perpendicular lines $y = x$ and $y = -x + 3\sqrt{2}$, and the semicircle $y = \sqrt{9 - x^2}$, in (a) a nonsquare window, and (b) and (c) square windows (Example 2).

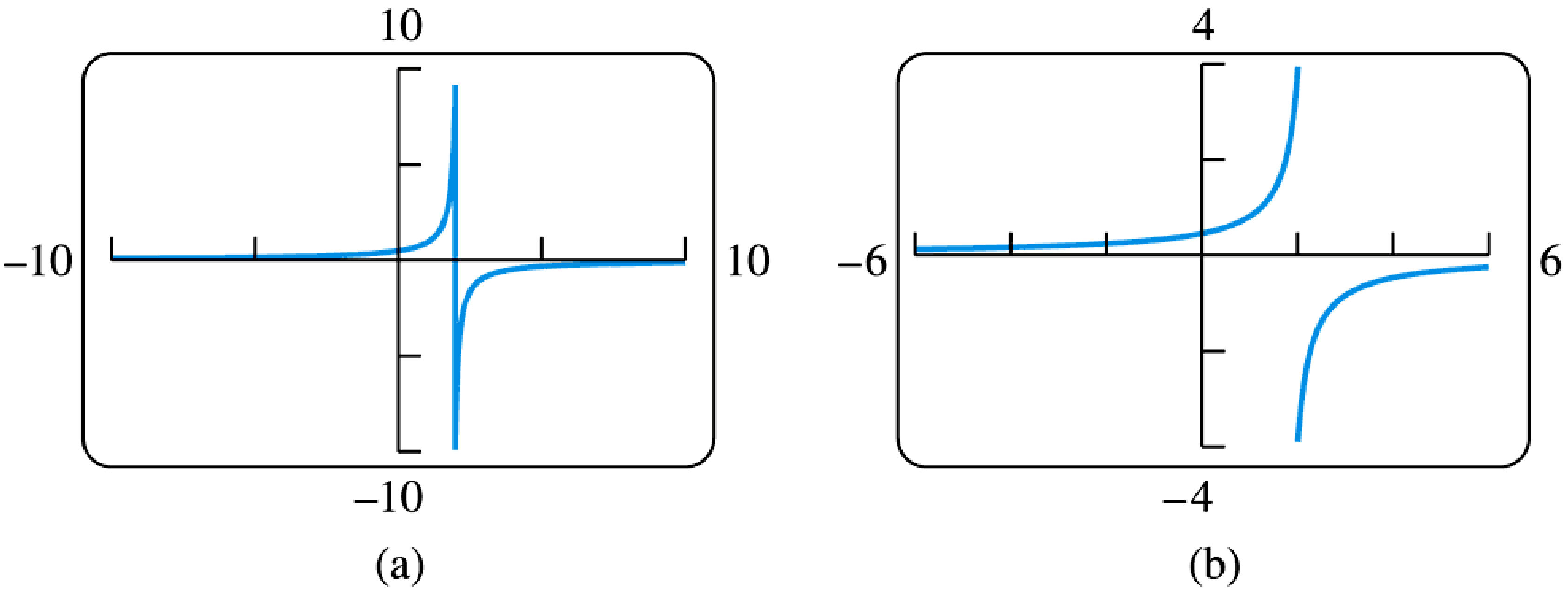


FIGURE 1.80 Graphs of the function $y = \dfrac{1}{2 - x}$ (Example 3).

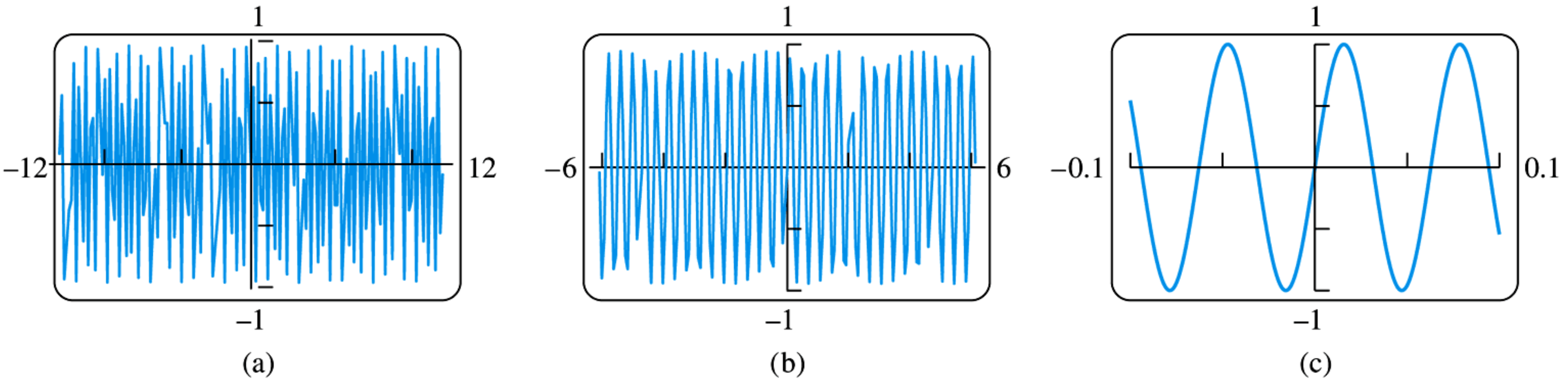


FIGURE 1.81 Graphs of the function $y = \sin 100x$ in three viewing windows. Because the period is $2\pi/100 \approx 0.063$, the smaller window in (c) best displays the true aspects of this rapidly oscillating function (Example 4).

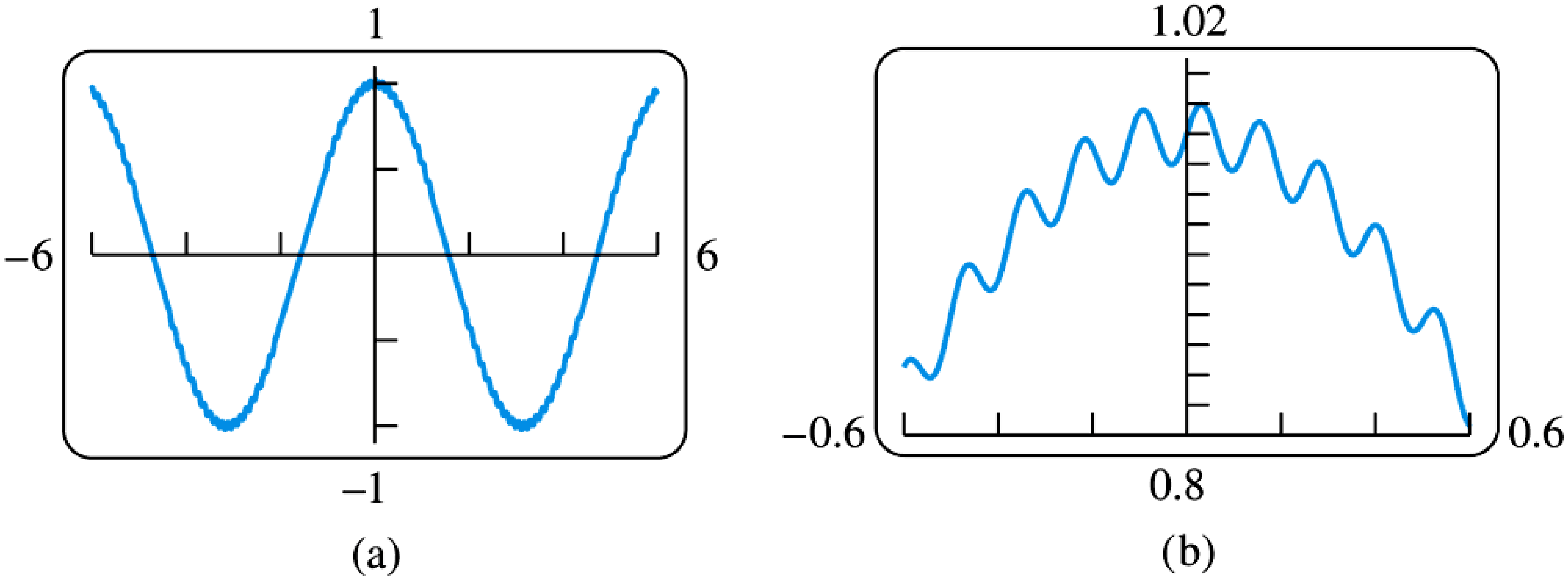


FIGURE 1.82 In (b) we see a close-up view of the function $y = \cos x + \frac{1}{50}\sin 50x$ graphed in (a). The term $\cos x$ clearly dominates the second term, $\frac{1}{50}\sin 50x$, which produces the rapid oscillations along the cosine curve (Example 5).

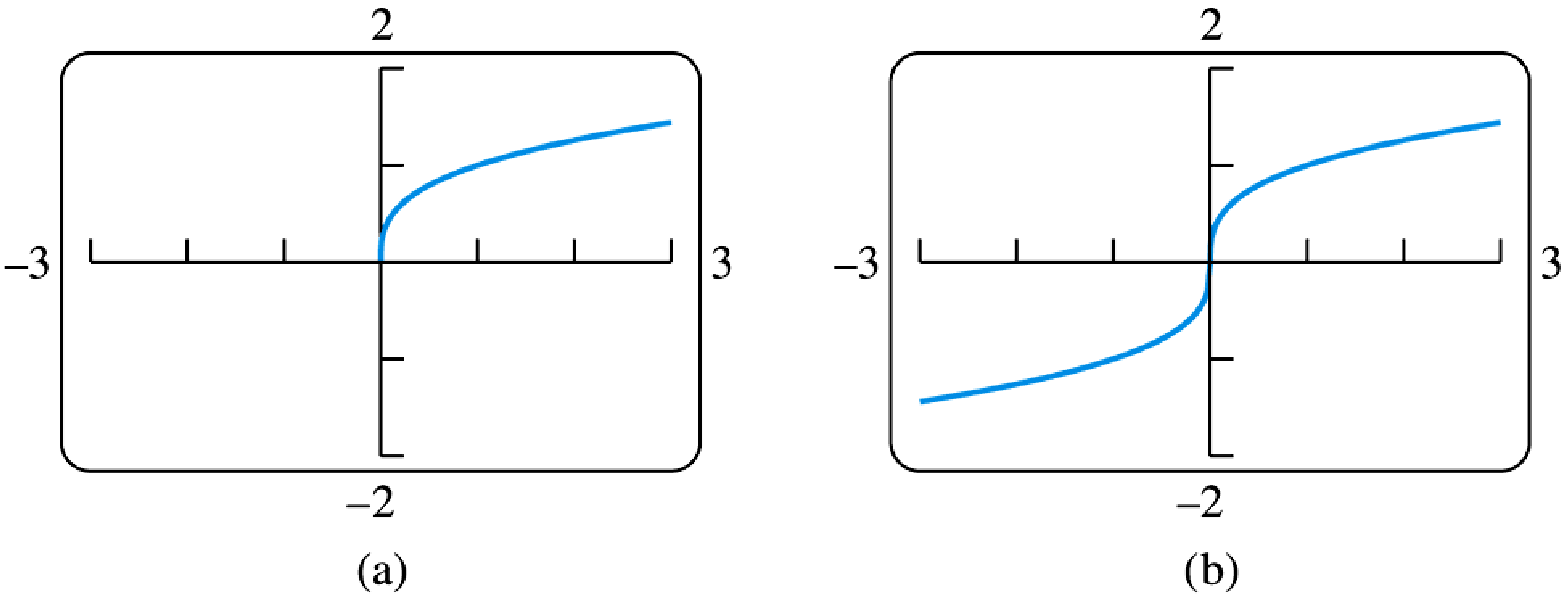


FIGURE 1.83 The graph of $y = x^{1/3}$ is missing the left branch in (a). In (b) we graph the function $f(x) = \frac{x}{|x|} \cdot |x|^{1/3}$ obtaining both branches. (See Example 6.)

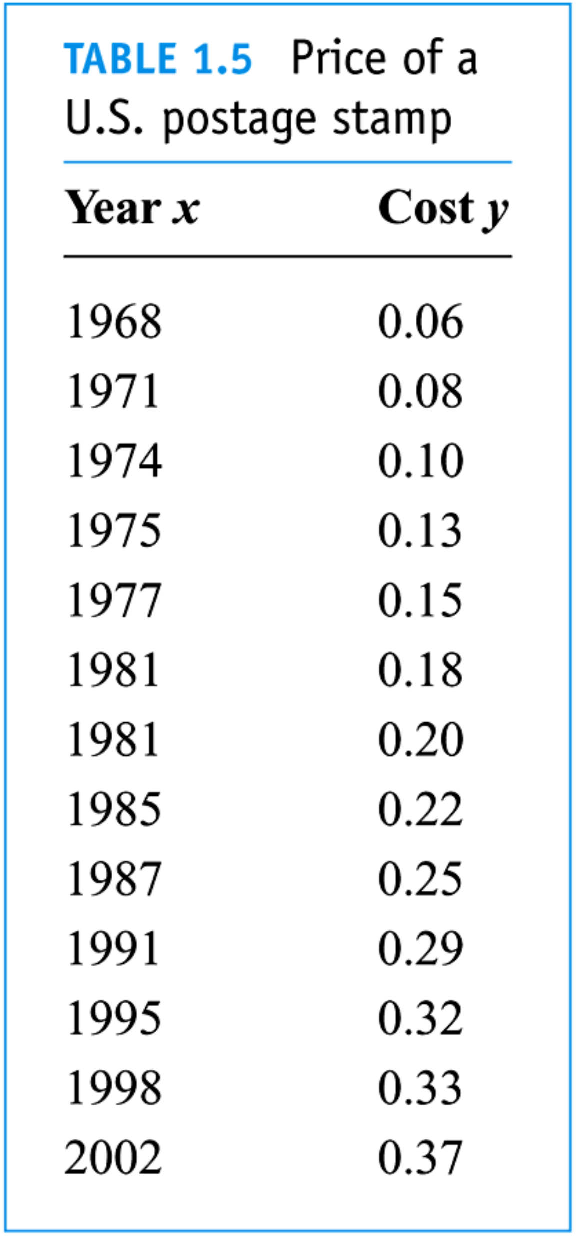


TABLE 1.5 Price of a U.S. postage stamp

Year x	Cost y
1968	0.06
1971	0.08
1974	0.10
1975	0.13
1977	0.15
1981	0.18
1981	0.20
1985	0.22
1987	0.25
1991	0.29
1995	0.32
1998	0.33
2002	0.37

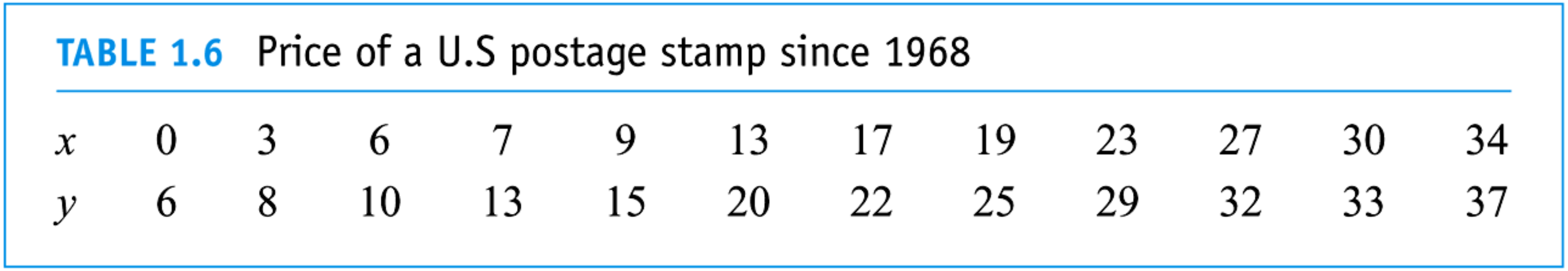
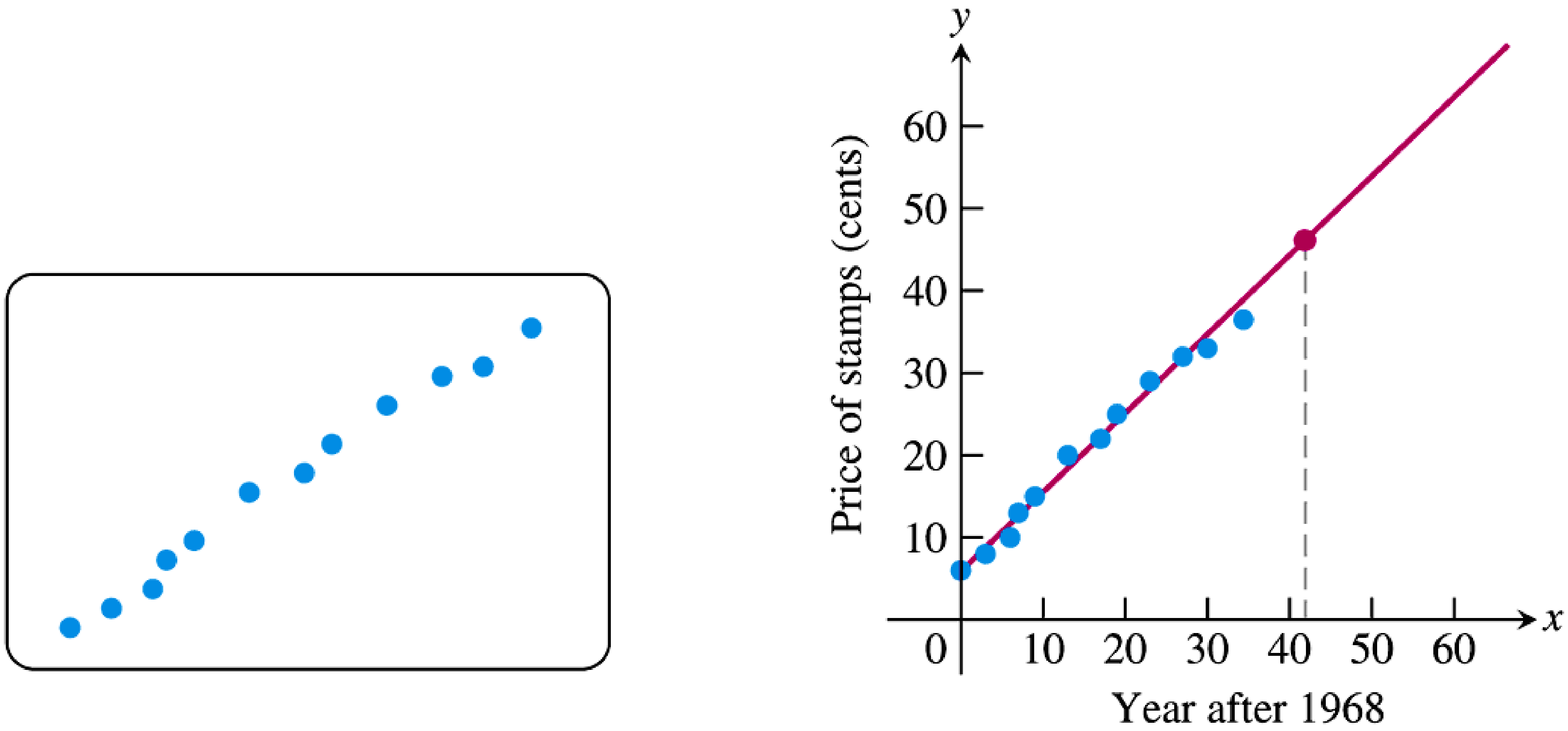


TABLE 1.6 Price of a U.S postage stamp since 1968

x	0	3	6	7	9	13	17	19	23	27	30	34
y	6	8	10	13	15	20	22	25	29	32	33	37



(a) (b)

FIGURE 1.84 (a) Scatterplot of (*x*, *y*) data in Table 1.6. (b) Using the regression line to estimate the price of a stamp in 2010. (Example 7).

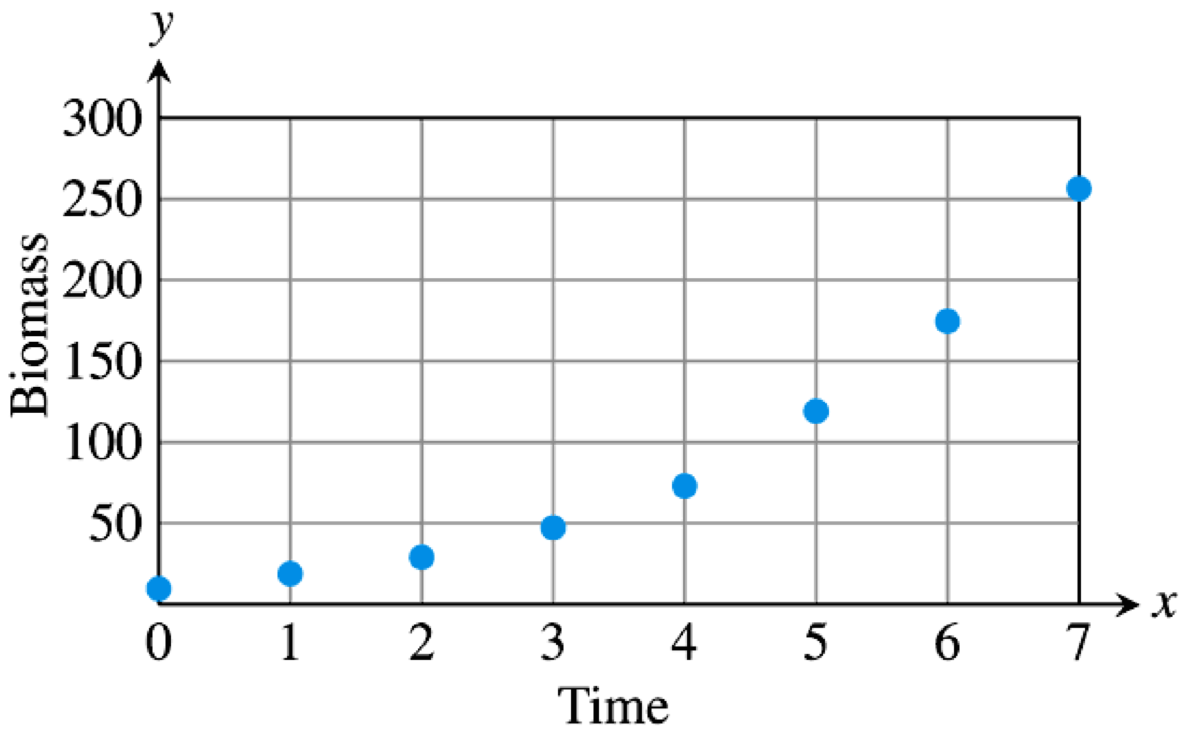


FIGURE 1.85 Biomass of a yeast culture versus elapsed time (Example 8).
(Data from R. Pearl, “The Growth of Population,” *Quart. Rev. Biol.*, Vol. 2 (1927), pp. 532–548.)

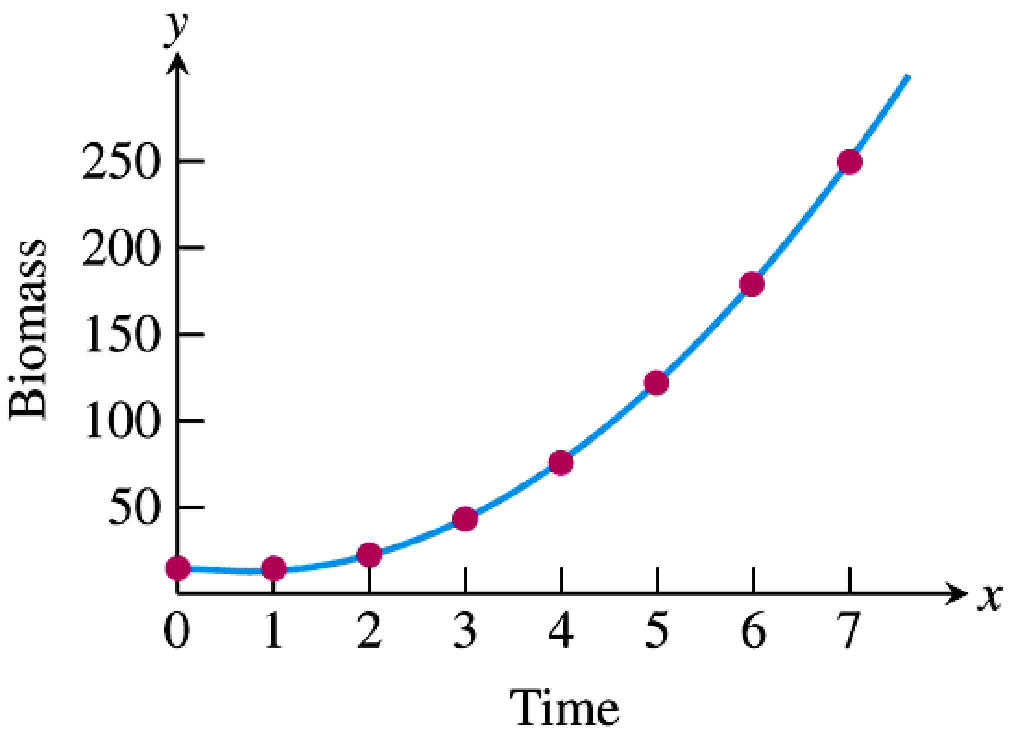


FIGURE 1.86 Fitting a quadratic to Pearl's data gives the equation $y = 6.10x^2 - 9.28x + 16.43$ and the prediction $y(17) = 1622.65$ (Example 8).

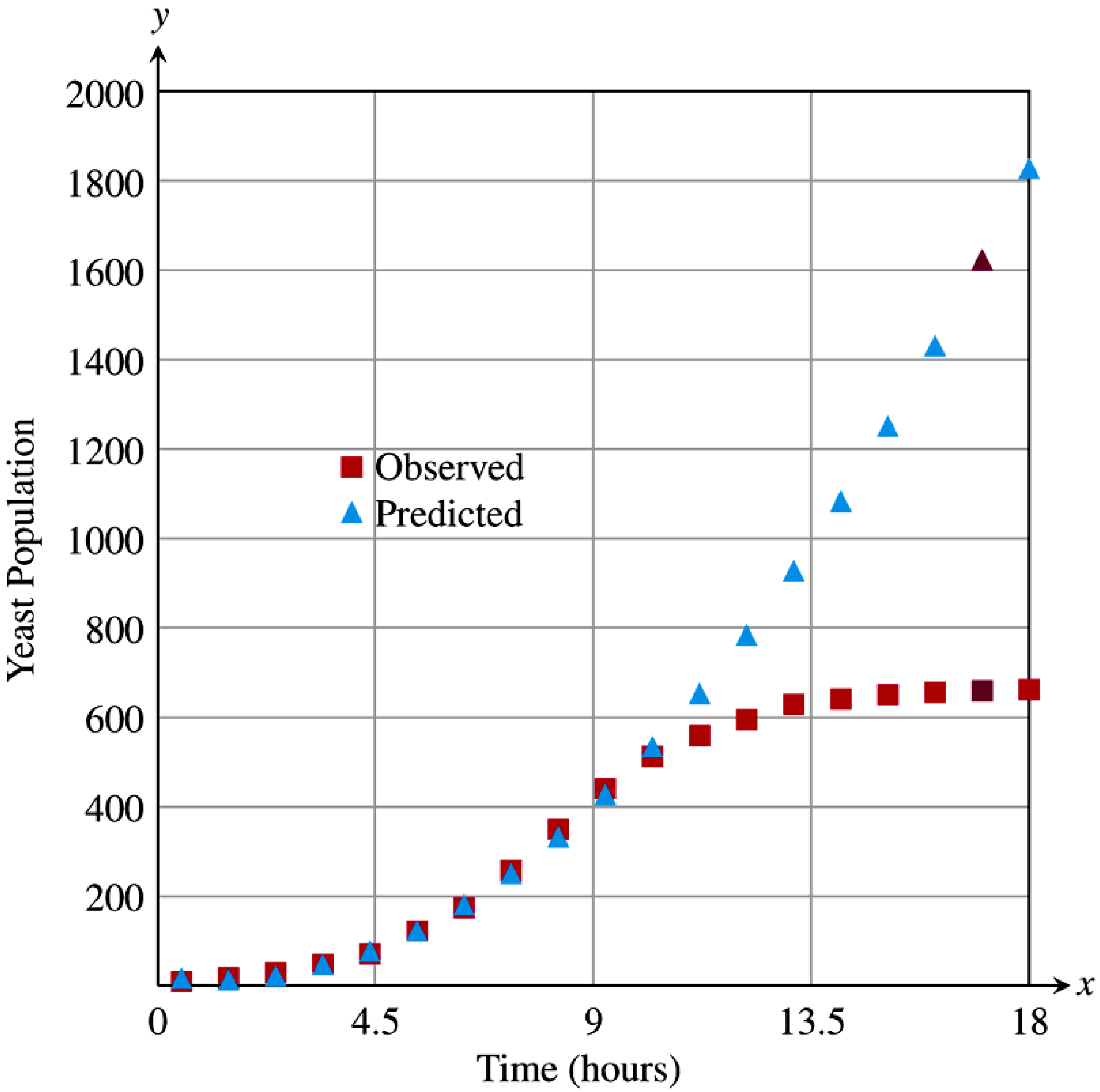
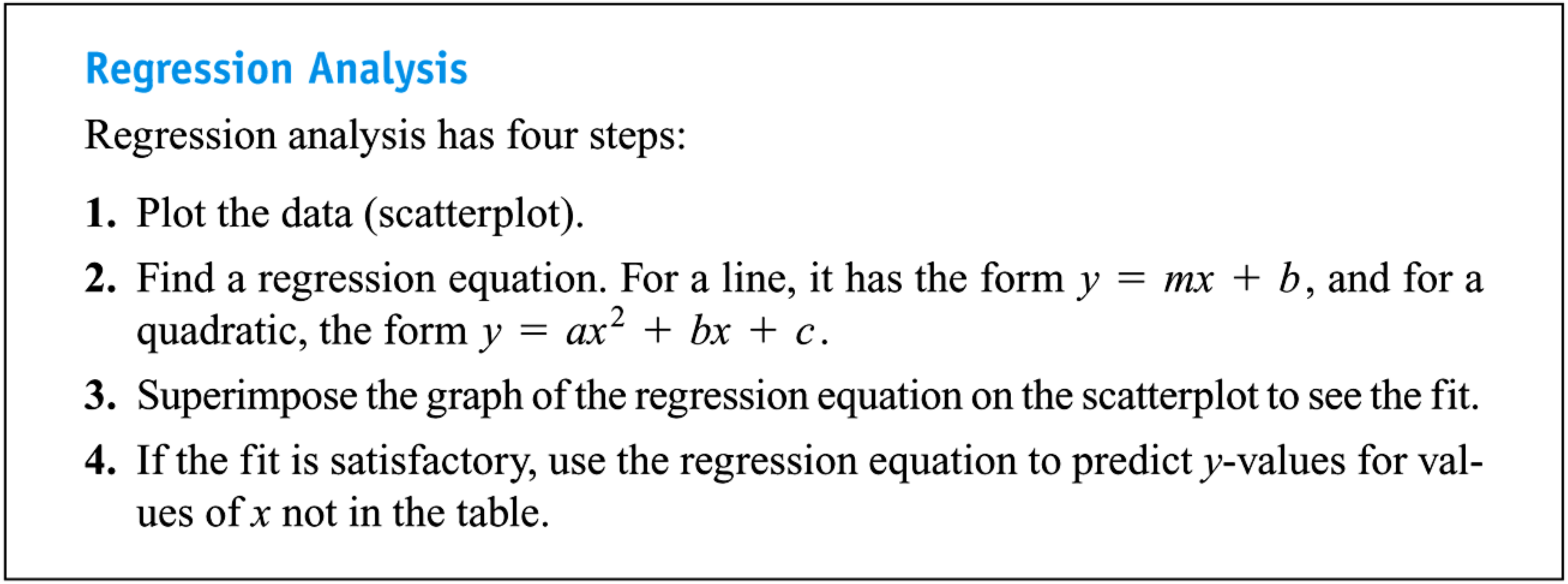


FIGURE 1.87 The rest of Pearl's data (Example 8).



Regression Analysis

Regression analysis has four steps:

1. Plot the data (scatterplot).
2. Find a regression equation. For a line, it has the form $y = mx + b$, and for a quadratic, the form $y = ax^2 + bx + c$.
3. Superimpose the graph of the regression equation on the scatterplot to see the fit.
4. If the fit is satisfactory, use the regression equation to predict y-values for values of x not in the table.

Workshop Solutions to Section 2.6

1) The inverse of the function
$f = \{(0,3), (-2,1), (3,4), (5,-2), (1,7)\}$ is
$f^{-1} = \{(3,0), (1,-2), (4,3), (-2,5), (7,1)\}$

2) Find the inverse of the function $f(x) = 2x + 3$.

Solution:

Let $y = 2x + 3$

$$2x = y - 3$$
$$x = \frac{y-3}{2}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \frac{x-3}{2}$$
$$\therefore\ f^{-1}(x) = \frac{x-3}{2}$$

3) Find the inverse of the function $f(x) = 3 - 2x$.

Solution:

Let $y = 3 - 2x$

$$2x = 3 - y$$
$$x = \frac{3-y}{2}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \frac{3-x}{2}$$
$$\therefore\ f^{-1}(x) = \frac{3-x}{2}$$

4) Find the inverse of the function $f(x) = 3 - \frac{x}{2}$.

Solution:

Let $y = 3 - \frac{x}{2}$

$$2y = 6 - x$$
$$x = 6 - 2y$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = 6 - 2x$$
$$\therefore\ f^{-1}(x) = 6 - 2x$$

5) Find the inverse of the function $f(x) = \sqrt{2x-3}$.

Solution:

Let $y = \sqrt{2x-3}$ by squaring both sides

$$y^2 = 2x - 3$$
$$2x = y^2 + 3$$
$$x = \frac{y^2+3}{2}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \frac{x^2+3}{2}$$
$$\therefore\ f^{-1}(x) = \frac{x^2+3}{2}$$

6) Find the inverse of the function $f(x) = \sqrt[3]{3-2x}$.

Solution:

Let $y = \sqrt[3]{3-2x}$ by cubing both sides

$$y^3 = 3 - 2x$$
$$2x = 3 - y^3$$
$$x = \frac{3-y^3}{2}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \frac{3-x^3}{2}$$
$$\therefore\ f^{-1}(x) = \frac{3-x^3}{2}$$

7) Find the inverse of the function
$$f(x) = (2x+3)^2, x \in [0, \infty).$$

Solution:

Let $y = (2x+3)^2$

Take the square root for both sides

$$\sqrt{y} = 2x + 3$$
$$2x = \sqrt{y} - 3$$
$$x = \frac{\sqrt{y}-3}{2}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \frac{\sqrt{x}-3}{2}$$
$$\therefore\ f^{-1}(x) = \frac{\sqrt{x}-3}{2}$$

8) Find the inverse of the function $f(x) = -(x-3)^3$.

Solution:

Let $y = -(x-3)^3$

$$-y = (x-3)^3$$

Take the cubic root for both sides

$$\sqrt[3]{-y} = x - 3$$
$$x = \sqrt[3]{-y} + 3$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \sqrt[3]{-x} + 3$$
$$\therefore\ f^{-1}(x) = \sqrt[3]{-x} + 3$$

9) Find the inverse of the function $f(x) = \frac{x}{x-3}$.

Solution:

Let $y = \frac{x}{x-3}$

$$y(x-3) = x$$
$$xy - 3y = x$$
$$xy - x = 3y$$
$$x(y-1) = 3y$$
$$x = \frac{3y}{y-1}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \frac{3x}{x-1}$$
$$\therefore\ f^{-1}(x) = \frac{3x}{x-1}$$

10) Find the inverse of the function $f(x) = \frac{x-3}{x}$.

Solution:

Let $y = \frac{x-3}{x}$

$$xy = x - 3$$
$$xy - x = -3$$
$$x(y-1) = -3$$
$$x = \frac{-3}{y-1} = -\frac{3}{y-1} = \frac{3}{-(y-1)} = \frac{3}{1-y}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \frac{3}{1-x}$$
$$\therefore\ f^{-1}(x) = \frac{3}{1-x}$$

11) Find the inverse of the function $f(x) = \frac{x+2}{x-3}$.

Solution:

Let $y = \frac{x+2}{x-3}$

$$y(x-3) = x+2$$
$$xy - 3y = x + 2$$
$$xy - x = 3y + 2$$
$$x(y-1) = 3y + 2$$
$$x = \frac{3y+2}{y-1}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \frac{3x+2}{x-1}$$

$$\therefore\ f^{-1}(x) = \frac{3x+2}{x-1}$$

12) Find the inverse of the function $f(x) = \sqrt{x} + 5$.

Solution:

Let $y = \sqrt{x} + 5$

$\sqrt{x} = y - 5$ by squaring both sides

$$x = (y-5)^2$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = (x-5)^2$$

$$\therefore\ f^{-1}(x) = (x-5)^2$$

13) Find the inverse of the function $f(x) = \sqrt[3]{x^5}$.

Solution:

Let $y = \sqrt[3]{x^5}$

$$y = x^{\frac{5}{3}}$$

$$y^{\frac{3}{5}} = (x^{\frac{5}{3}})^{\frac{3}{5}}$$

$$x = \sqrt[5]{y^3}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \sqrt[5]{x^3}$$

$$\therefore\ f^{-1}(x) = \sqrt[5]{x^3}$$

14) Find the inverse of the function $f(x) = 2x^3 - 5$.

Solution:

Let $y = 2x^3 - 5$

$$2x^3 = y + 5$$

$x^3 = \frac{y+5}{2}$ take the cubic root for both sides

$$x = \sqrt[3]{\frac{y+5}{2}}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \sqrt[3]{\frac{x+5}{2}}$$

$$\therefore\ f^{-1}(x) = \sqrt[3]{\frac{x+5}{2}}$$

15) Find the inverse of the function $f(x) = \sqrt[3]{\frac{x+2}{5}}$.

Solution:

Let $y = \sqrt[3]{\frac{x+2}{5}}$ by cubing both sides

$$y^3 = \frac{x+2}{5}$$
$$5y^3 = x + 2$$
$$x = 5y^3 - 2$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = 5x^3 - 2$$

$$\therefore\ f^{-1}(x) = 5x^3 - 2$$

16) Evaluate

$$2^{log_2(5x+3)}.$$

Solution:

$$2^{log_2(5x+3)} = 5x + 3$$

17) Evaluate

$$log_2 2^{(5x+3)}.$$

Solution:

$$log_2 2^{(5x+3)} = 5x + 3$$

18) $log_2 64 - log_2 32 + log_2 2 = log_2 \frac{64\times 2}{32}$

$$= log_2 4 = log_2 2^2$$
$$= 2 log_2 2$$
$$= 2 \times 1 = 2$$

OR

$$log_2 64 - log_2 32 + log_2 2 = log_2 2^6 - log_2 2^5 + log_2 2$$
$$= 6 - 5 + 1 = 2$$

19) $log_3 27 - log_3 81 + 5\, log_3 3 = log_3 \frac{27\times 3^5}{81}$

$$= log_3 81 = log_3 3^4$$
$$= 4 log_3 3$$
$$= 4 \times 1 = 4$$

OR

$$log_3 27 - log_3 81 + 5\, log_3 = log_3 3^3 - log_3 3^4 + 5 \times 1$$
$$= 3 - 4 + 5 = 4$$

20) $log_3 54 - log_3 2 = log_3 \frac{54}{2}$

$$= log_3 27 = log_3 3^3 = 3$$

21) If $log_2(6 + 2x) = 1$, then $x =$

Solution:

$$log_2(6 + 2x) = 1$$
$$2^{log_2(6+2x)} = 2^1$$
$$6 + 2x = 2$$
$$2x = 2 - 6 = -4$$
$$x = -2$$

22) If $ln(x+3) = 5$, then $x =$

Solution:

$$ln(x+3) = 5$$
$$e^{ln(x+3)} = e^5$$
$$x + 3 = e^5$$
$$x = e^5 - 3$$

23) If $ln(x) = 5$, then $x =$

Solution:

$$ln(x) = 5$$
$$e^{ln(x)} = e^5$$
$$x = e^5$$

24) If $e^{(2x-3)} = 5$, then $x =$

Solution:

$$e^{(2x-3)} = 5$$
$$lne^{(2x-3)} = ln5$$
$$2x - 3 = ln5$$
$$2x = ln5 + 3$$
$$x = \frac{ln5+3}{2}$$

25) $log_3 2 = \frac{ln2}{ln3}$

26) $\log\ 25 + \log\ 4 = \log(25 \times 4)$

$$= \log 100 = \log 10^2$$
$$= 2$$

27) $log_3 18 - log_3 6 = log_3 \frac{18}{6}$

$$= log_3 3$$
$$= 1$$

28) $log_2 6 - log_2 15 + log_2 20 = log_2 \frac{6\times 20}{15}$

$$= log_2 8 = log_2 2^3$$
$$= 3$$

29) $e^{3ln2} = e^{ln2^3} = 2^3 = 8$

30) If $3^{2-x} = 6$, then $x =$

Solution:

$$3^{2-x} = 6$$
$$log_3 3^{2-x} = log_3 6$$
$$2 - x = log_3 6$$
$$x = 2 - log_3 6 = 2 - log_3(3 \times 2)$$
$$= 2 - (log_3 3 + log_3 2) = 2 - (1 + log_3 2)$$
$$= 2 - 1 - log_3 2$$
$$= 1 - log_3 2$$

31) Find the inverse of the function $f(x) = 5 + lnx$.

Solution:

Let $y = 5 + lnx$

$$lnx = y - 5$$
$$e^{lnx} = e^{y-5}$$
$$x = e^{y-5}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = e^{x-5}$$
$$\therefore\ f^{-1}(x) = e^{x-5}$$

32) Find the domain of the function

$$f(x) = \sin^{-1}(3x+5).$$

Solution:

We know that the domain of $\sin^{-1}(x)$ is $[-1,1]$. So,

$$-1 \le 3x + 5 \le 1$$
$$-6 \le 3x \le -4$$
$$-2 \le x \le -\frac{4}{3}$$
$$\therefore\ D_f = \left[-2, -\frac{4}{3}\right]$$

33) Find the domain of the function

$$f(x) = \cos^{-1}(3x-5).$$

Solution:

We know that the domain of $\cos^{-1}(x)$ is $[-1,1]$. So,

$$-1 \le 3x - 5 \le 1$$
$$4 \le 3x \le 6$$
$$\frac{4}{3} \le x \le 2$$
$$\therefore\ D_f = \left[\frac{4}{3}, 2\right]$$

34) Find the domain of the function

$$f(x) = 2\sin^{-1}(x) + 1.$$

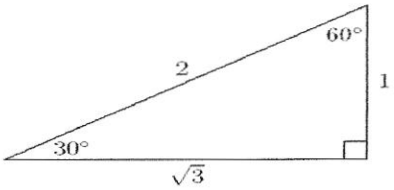
Solution:

We know that the domain of $\sin^{-1}(x)$ is $[-1,1]$. So,

$$\therefore\ D_f = [-1,1]$$

Before proceeding to the questions 35-55, we should be aware of the following well-known right triangles:

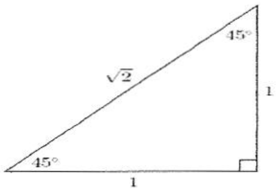
$30^o - 60^o$ Right Triangle



We know that $30^o = \frac{\pi}{6}$ and $60^o = \frac{\pi}{3}$, so

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \qquad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$
$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \qquad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$
$$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \qquad \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$
$$\cot\left(\frac{\pi}{6}\right) = \sqrt{3} \qquad \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$
$$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \qquad \sec\left(\frac{\pi}{3}\right) = 2$$
$$\csc\left(\frac{\pi}{6}\right) = 2 \qquad \csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$$

$30^o - 60^o$ Right Triangle



We know that $45^o = \frac{\pi}{4}$, so

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$
$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$
$$\tan\left(\frac{\pi}{4}\right) = 1$$
$$\cot\left(\frac{\pi}{4}\right) = 1$$
$$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$
$$\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$$

35) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

Solution:

Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\sin\theta = \frac{\sqrt{3}}{2}$

Use the $30^o - 60^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{3}$$

36) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

Solution:

Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\sin\theta = \frac{\sqrt{3}}{2}$

Use the $30^o - 60^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{3}$$

37) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$

Solution:

Let $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\tan\theta = \frac{1}{\sqrt{3}}$

Use the $30^o - 60^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{6}$$

38) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) =$

Solution:

Let $\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

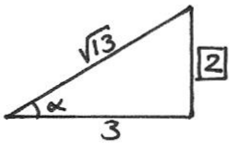
$\sin\theta = \frac{1}{\sqrt{2}}$

Use the $45^o - 45^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{4}$$

39) If $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$, then $\tan\alpha =$

Solution:



$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$

$$\cos\alpha = \frac{3}{\sqrt{13}} = \frac{adj}{hyp}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{\left(\sqrt{13}\right)^2 - 3^2} = \sqrt{13-9} = \sqrt{4} = 2$$

$$\therefore \tan\alpha = \frac{opp}{adj} = \frac{2}{3}$$

40) If $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$, then $\csc\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$

$$\cos\alpha = \frac{3}{\sqrt{13}} = \frac{adj}{hyp}$$

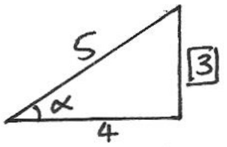
Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{\left(\sqrt{13}\right)^2 - 3^2} = \sqrt{13-9} = \sqrt{4} = 2$$

$$\therefore \csc\alpha = \frac{1}{\sin\alpha} = \frac{hyp}{opp} = \frac{\sqrt{13}}{2}$$

41) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\csc\alpha =$

Solution:



$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \dfrac{4}{5} = \dfrac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \csc\alpha = \frac{1}{\sin\alpha} = \frac{hyp}{opp} = \frac{5}{3}$$

42) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\cot\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \dfrac{4}{5} = \dfrac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \cot\alpha = \frac{1}{\tan\alpha} = \frac{adj}{opp} = \frac{4}{3}$$

43) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\tan\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \dfrac{4}{5} = \dfrac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \tan\alpha = \frac{1}{\cot\alpha} = \frac{opp}{adj} = \frac{3}{4}$$

44) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\sin\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \dfrac{4}{5} = \dfrac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \sin\alpha = \frac{opp}{hyp} = \frac{3}{5}$$

45) $\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right) =$

Solution:

Let $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \sin(\alpha) = \frac{opp}{hyp} = \frac{3}{5}$$

46) $\tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right) =$

Solution:

Let $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

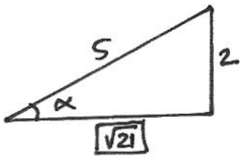
Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \tan(\alpha) = \frac{opp}{adj} = \frac{3}{4}$$

47) $\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) =$

Solution:



Let $\alpha = \sin^{-1}\left(\frac{2}{5}\right)$

$\sin\alpha = \frac{2}{5} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{5^2 - 2^2} = \sqrt{25-4} = \sqrt{21}$

$$\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin(2\alpha)$$

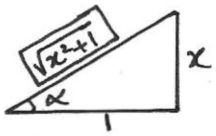
Now, use the identity $\sin(2x) = 2\sin x.\cos x$. Thus,

$$\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$

$$= 2 \times \frac{2}{5} \times \frac{\sqrt{21}}{5} = \frac{4\sqrt{21}}{25}$$

48) $\cos(\tan^{-1}x) =$

Solution:



Let $\alpha = \tan^{-1}x$

$\tan\alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}$

$$\cos(\tan^{-1}x) = \cos(\alpha) = \frac{adj}{hyp} = \frac{1}{\sqrt{x^2+1}}$$

49) $\sin(\tan^{-1}x) =$

Solution:

Let $\alpha = \tan^{-1}x$

$\tan\alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}$

$$\sin(\tan^{-1}x) = \sin(\alpha) = \frac{opp}{hyp} = \frac{x}{\sqrt{x^2+1}}$$

50) $\csc(\tan^{-1}x) =$

Solution:

Let $\alpha = \tan^{-1}x$

$\tan\alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}$

$$\csc(\tan^{-1}x) = \csc(\alpha) = \frac{1}{\sin\alpha} = \frac{hyp}{opp} = \frac{\sqrt{x^2+1}}{x}$$

51) $\sec(\tan^{-1}x) =$

Solution:

Let $\alpha = \tan^{-1}x$

$\tan\alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

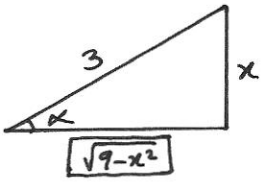
|hypotenuse|$= \sqrt{x^2+1^2} = \sqrt{x^2+1}$

$$\sec(\tan^{-1}x) = \sec(\alpha) = \frac{1}{\cos\alpha} = \frac{hyp}{adj} = \frac{\sqrt{x^2+1}}{1} = \sqrt{x^2+1}$$

52) $\sec\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$



$\sin\alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

|adjacent|$= \sqrt{3^2-x^2} = \sqrt{9-x^2}$

$$\sec\left(\sin^{-1}\frac{x}{3}\right) = \sec(\alpha) = \frac{1}{\cos\alpha} = \frac{hyp}{adj} = \frac{3}{\sqrt{9-x^2}}$$

53) $\cot\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$

$\sin\alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

|adjacent|$= \sqrt{3^2-x^2} = \sqrt{9-x^2}$

$$\cot\left(\sin^{-1}\frac{x}{3}\right) = \cot(\alpha) = \frac{1}{\tan\alpha} = \frac{adj}{opp} = \frac{\sqrt{9-x^2}}{x}$$

54) $\tan\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$

$\sin\alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

|adjacent|$= \sqrt{3^2-x^2} = \sqrt{9-x^2}$

$$\tan\left(\sin^{-1}\frac{x}{3}\right) = \tan(\alpha) = \frac{1}{\cot\alpha} = \frac{opp}{adj} = \frac{x}{\sqrt{9-x^2}}$$

55) $\cos\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$

$\sin\alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

|adjacent|$= \sqrt{3^2-x^2} = \sqrt{9-x^2}$

$$\cos\left(\sin^{-1}\frac{x}{3}\right) = \cos(\alpha) = \frac{adj}{hyp} = \frac{\sqrt{9-x^2}}{3}$$

Workshop Solutions to Section 2.5

How to find the domain and range of the exponential function $f(x) = a^x$?

1- If $f(x) = c.a^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (\pm k, \infty)$$

2- If $f(x) = -c.a^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (-\infty, \pm k)$$

3- If $f(x) = c.e^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (\pm k, \infty)$$

4- If $f(x) = -c.e^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (-\infty, \pm k)$$

1) Find the domain of the function $f(x) = 4^x$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	2) Find the range of the function $f(x) = 4^x$. Solution: From Step (1) above, we deduce that $R_f = (0, \infty)$
3) Find the domain of the function $f(x) = 4^x - 3$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	4) Find the range of the function $f(x) = 4^x - 3$. Solution: From Step (1) above, we deduce that $R_f = (-3, \infty)$
5) Find the domain of the function $f(x) = 5 - 3^x$. Solution: From Step (2) above, we deduce that $D_f = \mathbb{R}$	6) Find the range of the function $f(x) = 5 - 3^x$. Solution: From Step (2) above, we deduce that $R_f = (-\infty, 5)$
7) Find the domain of the function $f(x) = 3^{-x} + 1$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	8) Find the range of the function $f(x) = 3^{-x} + 1$. Solution: From Step (1) above, we deduce that $R_f = (1, \infty)$
9) Find the domain of the function $f(x) = e^x$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	10) Find the range of the function $f(x) = e^x$. Solution: From Step (3) above, we deduce that $R_f = (0, \infty)$
11) Find the domain of the function $f(x) = e^x - 3$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	12) Find the range of the function $f(x) = e^x - 3$. Solution: From Step (3) above, we deduce that $R_f = (-3, \infty)$
13) Find the domain of the function $f(x) = e^x + 1$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	14) Find the domain of the function $f(x) = \frac{1}{1-e^x}$. Solution: $f(x)$ is defined when $1 - e^x \neq 0$ $\Leftrightarrow \ e^x \neq 1 \ \Leftrightarrow \ \ln e^x \neq \ln 1$ $\Leftrightarrow \ x \neq 0$ $\therefore \ D_f = \mathbb{R} \setminus \{0\}$

15) Find the domain of the function $f(x) = \frac{1}{1+e^{x}}$.

Solution:

$f(x)$ is defined when $1 + e^{x} \neq 0$.

But there is no value of x makes $1 + e^{x} = 0$. Therefore,

$$D_f = \mathbb{R}$$

16) Find the domain of the function $f(x) = \sqrt{1 + 3^{x}}$.

Solution:

$f(x)$ is defined when $1 + 3^{x} \geq 0$.

But $1 + 3^{x} > 0$ always. Therefore,

$$D_f = \mathbb{R}$$

17) If $4^{(x+1)} = 8$, then $x =$

Solution:

$$4^{(x+1)} = 8$$
$$(2^{2})^{(x+1)} = 2^{3}$$
$$2^{2(x+1)} = 2^{3}$$
$$2(x+1) = 3$$
$$2x + 2 = 3$$
$$2x = 3 - 2 = 1$$
$$\therefore \quad x = \frac{1}{2}$$

18) If $4^{(x-1)} = 8$, then $x =$

Solution:

$$4^{(x-1)} = 8$$
$$(2^{2})^{(x-1)} = 2^{3}$$
$$2^{2(x-1)} = 2^{3}$$
$$2(x-1) = 3$$
$$2x - 2 = 3$$
$$2x = 3 + 2 = 5$$
$$\therefore \quad x = \frac{5}{2}$$

19) If $9^{(x+1)} = 27$, then $x =$

Solution:

$$9^{(x+1)} = 27$$
$$(3^{2})^{(x+1)} = 3^{3}$$
$$3^{2(x+1)} = 3^{3}$$
$$2(x+1) = 3$$
$$2x + 2 = 3$$
$$2x = 3 - 2 = 1$$
$$\therefore \quad x = \frac{1}{2}$$

20) If $9^{(x-1)} = 27$, then $x =$

Solution:

$$9^{(x-1)} = 27$$
$$(3^{2})^{(x-1)} = 3^{3}$$
$$3^{2(x-1)} = 3^{3}$$
$$2(x-1) = 3$$
$$2x - 2 = 3$$
$$2x = 3 + 2 = 5$$
$$\therefore \quad x = \frac{5}{2}$$

21) If $5^{2(x-1)} = 125$, then $x =$

Solution:

$$5^{2(x-1)} = 125$$
$$5^{2(x-1)} = 5^{3}$$
$$2(x-1) = 3$$
$$2x - 2 = 3$$
$$2x = 3 + 2 = 5$$
$$\therefore \quad x = \frac{5}{2}$$

22) If $5^{2(x+1)} = 125$, then $x =$

Solution:

$$5^{2(x+1)} = 125$$
$$5^{2(x+1)} = 5^{3}$$
$$2(x+1) = 3$$
$$2x + 2 = 3$$
$$2x = 3 - 2 = 1$$
$$\therefore \quad x = \frac{1}{2}$$

Workshop Solutions to Sections 2.1 and 2.2

1) Find the domain of the function $f(x) = 9 - x^2$.

Solution:

Since $f(x)$ is a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

Note: The domain of any polynomial is $\mathbb{R}$.

2) Find the range of the function $f(x) = 9 - x^2$.

Solution:

$$R_f = (-\infty, 9]$$

3) Find the domain of the function $f(x) = 6 - 2x$.

Solution:

Since $f(x)$ is a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

4) Find the range of the function $f(x) = 6 - 2x$.

Solution:

Since $f(x)$ is a polynomial of degree one ($i.e.$ is of an odd degree), then

$$R_f = \mathbb{R} = (-\infty, \infty)$$

5) Find the domain of the function $f(x) = x^2 - 2x - 3$.

Solution:

Since $f(x)$ is a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

6) Find the domain of the function $f(x) = 1 + 2x^3 - x^5$.

Solution:

Since $f(x)$ is a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

7) Find the domain of the function $f(x) = 5$.

Solution:

Since $f(x)$ is a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

8) Find the range of the function $f(x) = 5$.

Solution:

$$R_f = \{5\}$$

9) Find the domain of the function $f(x) = |x - 1|$.

Solution:

Since $f(x)$ is an absolute value of a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

Note: The domain of an absolute value of any polynomial is $\mathbb{R}$.

10) Find the domain of the function $f(x) = |x + 5|$.

Solution:

Since $f(x)$ is an absolute value of a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

11) Find the domain of the function $f(x) = |x|$.

Solution:

Since $f(x)$ is an absolute value of a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

12) Find the range of the function $f(x) = |x|$.

Solution:

$$R_f = [0, \infty)$$

Note: The range of an absolute value of any polynomial is always $[0, \infty)$.

13) Find the domain of the function $f(x) = |3x - 6|$.

Solution:

Since $f(x)$ is an absolute value of a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

14) Find the domain of the function $f(x) = |9 - 3x|$.

Solution:

Since $f(x)$ is an absolute value of a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

15) Find the domain of the function

$$f(x) = \frac{x + 2}{x - 3}$$

Solution:

$f(x)$ is defined when $x - 3 \neq 0 \implies x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$$

16) Find the domain of the function

$$f(x) = \frac{x - 2}{x + 3}$$

Solution:

$f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So,

$$D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$$

17) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-9}$$

Solution:

$f(x)$ is defined when $x^2 - 9 \neq 0 \Longrightarrow x^2 \neq 9 \Longrightarrow x \neq \pm 3$. So,

$$D_f = \mathbb{R} \setminus \{-3,3\} = (-\infty,-3)\mathrm{U}(-3,3)\mathrm{U}(3,\infty)$$

18) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-5x+6}$$

Solution:

$f(x)$ is defined when $x^2 - 5x + 6 \neq 0$

$\Longrightarrow (x-2)(x-3) \neq 0 \Longrightarrow x \neq 2$ or $x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{2,3\} = (-\infty,2)\mathrm{U}(2,3)\mathrm{U}(3,\infty)$$

19) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-x-6}$$

Solution:

$f(x)$ is defined when $x^2 - x - 6 \neq 0$

$\Longrightarrow (x+2)(x-3) \neq 0 \Longrightarrow x \neq -2$ or $x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{-2,3\} = (-\infty,-2)\mathrm{U}(-2,3)\mathrm{U}(3,\infty)$$

20) Find the domain of the function

$$f(x) = \frac{x+2}{x^2+9}$$

Solution:

$f(x)$ is defined when $x^2 + 9 \neq 0$ but for any value x the denominator $x^2 + 9$ cannot be 0. So,

$$D_f = \mathbb{R} = (-\infty,\infty)$$

21) Find the domain of the function

$$f(x) = \sqrt[3]{x-3}$$

Solution:

$$D_f = \mathbb{R} = (-\infty,\infty)$$

Note: The domain of an odd root of any polynomial is $\mathbb{R}$.

22) Find the domain of the function

$$f(x) = \sqrt{x-3}$$

Solution:

$f(x)$ is defined when $x - 3 \geq 0 \Longrightarrow x \geq 3$ because $f(x)$ is an even root. So,

$$D_f = [3,\infty)$$

23) Find the domain of the function

$$f(x) = \sqrt{3-x}$$

Solution:

$f(x)$ is defined when $3 - x \geq 0 \Longrightarrow -x \geq -3 \Longrightarrow x \leq 3$ because $f(x)$ is an even root. So,

$$D_f = (-\infty,3]$$

24) Find the domain of the function

$$f(x) = \sqrt{x+3}$$

Solution:

$f(x)$ is defined when $x + 3 \geq 0 \Longrightarrow x \geq -3$ because $f(x)$ is an even root. So,

$$D_f = [-3,\infty)$$

25) Find the domain of the function

$$f(x) = \sqrt{-x}$$

Solution:

$f(x)$ is defined when $-x \geq 0 \Longrightarrow x \leq 0$ because $f(x)$ is an even root. So,

$$D_f = (-\infty,0]$$

26) Find the range of the function

$$f(x) = \sqrt{-x}$$

Solution:

$$R_f = [0,\infty)$$

Note: The range of an even root is always ≥ 0 .

27) Find the domain of the function

$$f(x) = \sqrt{9-x^2}$$

Solution:

$f(x)$ is defined when $9 - x^2 \geq 0 \Longrightarrow -x^2 \geq -9 \Longrightarrow$

$x^2 \leq 9 \Longrightarrow \sqrt{x^2} \leq \sqrt{9} \Longrightarrow |x| \leq 3 \Longrightarrow -3 \leq x \leq 3$. So,

$$D_f = [-3,3]$$

28) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{x-3}}$$

Solution:

$f(x)$ is defined when $x - 3 > 0 \Longrightarrow x > 3$. So,

$$D_f = (3,\infty)$$

29) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{9-x^2}}$$

Solution:

$f(x)$ is defined when $9 - x^2 > 0 \Longrightarrow -x^2 > -9$

$\Longrightarrow x^2 < 9 \Longrightarrow \sqrt{x^2} < \sqrt{9} \Longrightarrow |x| < 3 \Longrightarrow -3 < x < 3$. So,

$$D_f = (-3,3)$$

30) Find the domain of the function

$$f(x) = \sqrt{x^2-9}$$

Solution:

$f(x)$ is defined when $x^2 - 9 \geq 0 \Longrightarrow x^2 \geq 9$

$\Longrightarrow \sqrt{x^2} \geq \sqrt{9} \Longrightarrow |x| \geq 3 \Longrightarrow x \geq 3$ or $x \leq -3$. So,

$$D_f = (-\infty,-3] \cup [3,\infty)$$

31) Find the range of the function

$$f(x) = \sqrt{x^2 - 9}$$

Solution:

$$R_f = [0, \infty)$$

32) Find the domain of the function

$$f(x) = \frac{x + 2}{\sqrt{x^2 - 9}}$$

Solution:

$f(x)$ is defined when $x^2 - 9 > 0 \implies x^2 > 9$

$\implies \sqrt{x^2} > \sqrt{9} \implies |x| > 3 \implies x > 3$ or $x < -3$.

So,

$$D_f = (-\infty, -3) \cup (3, \infty)$$

33) Find the domain of the function

$$f(x) = \sqrt{9 + x^2}$$

Solution:

$f(x)$ is defined when $9 + x^2 \geq 0$ but it is always true for any value x . So,

$$D_f = \mathbb{R}$$

34) Find the domain of the function

$$f(x) = \sqrt[4]{x^2 - 25}$$

Solution:

$f(x)$ is defined when $x^2 - 25 \geq 0 \implies x^2 \geq 25$

$\implies \sqrt{x^2} \geq \sqrt{25} \implies |x| \geq 5 \implies x \geq 5$ or $x \leq -5$.

So,

$$D_f = (-\infty, -5] \cup [5, \infty)$$

35) Find the domain of the function

$$f(x) = \sqrt[6]{16 - x^2}$$

Solution:

$f(x)$ is defined when $16 - x^2 \geq 0 \implies -x^2 \geq -16 \implies$ $x^2 \leq 16 \implies \sqrt{x^2} \leq \sqrt{16} \implies |x| \leq 4 \implies -4 \leq x \leq 4$.

So,

$$D_f = [-4,4]$$

36) Find the range of the function

$$f(x) = \sqrt{16 - x^2}$$

Solution:

We know that $f(x)$ is defined when $16 - x^2 \geq 0$

$\implies -x^2 \geq -16 \implies x^2 \leq 16 \implies \sqrt{x^2} \leq \sqrt{16}$

$\implies |x| \leq 4 \implies -4 \leq x \leq 4$. So,

$$D_f = [-4,4]$$

Using D_f we find the outputs vary from 0 to 4 . Hence,

$$R_f = [0,4]$$

37) Find the domain of the function

$$f(x) = \frac{x + |x|}{x}$$

Solution:

$f(x)$ is defined when $x \neq 0$. So,

$$D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$$

38) Find the domain of the function

$$f(x) = \begin{cases} -\dfrac{1}{x}, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Solution:

It is clear from the definition of the function $f(x)$ that

$$D_f = \mathbb{R} = (-\infty, \infty)$$

39) Find the domain of the function

$$f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$$

Solution:

$f(x)$ is defined when

1- $x \geq 0 \quad \implies \quad D_{\sqrt{x}} = [0, \infty)$

2- $x^2 + 1 > 0$ but this is always true for all x

$\implies D_{\sqrt{x^2+1}} = \mathbb{R}$.

Hence,

$$D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2+1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$$

40) Find the domain of the function

$$f(x) = \sqrt{x - 1} + \sqrt{x + 3}$$

Solution:

$f(x)$ is defined when

1- $x - 1 \geq 0 \implies x \geq 1 \implies D_{\sqrt{x-1}} = [1, \infty)$

2- $x + 3 \geq 0 \implies x \geq -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$

Hence,

$$D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$$

41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function.

42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function.

43) The function $f(x) = -3x^2 + 7$ is a quadratic function.

44) The function $f(x) = 2x + 3$ is a linear function.

45) The function $f(x) = x^7$ is a power function.

46) The function $f(x) = \frac{2x+3}{x^2-1}$ is a rational function.

47) The function $f(x) = \frac{x-3}{x+2}$ is a rational function and we can say it is an algebraic function as well.

48) The function $f(x) = \sin x$ is a trigonometric function.

49) The function $f(x) = e^x$ is a natural exponential function.

50) The function $f(x) = 3^x$ is a general exponential function.

51) The function $f(x) = x^2 + \sqrt{x-2}$ is an algebraic function.

52) The function $f(x) = -3$ is a constant function.

53) The function $f(x) = \log_3 x$ is a general logarithmic function.

54) The function $f(x) = \ln x$ is a natural logarithmic function.

55) The function $f(x) = 3x^4 + x^2 + 1$ is

Solution:

$$f(-x) = 3(-x)^4 + (-x)^2 + 1 = 3x^4 + x^2 + 1 = f(x)$$

Hence,

$f(x)$ is an even function.

56) The function $f(x) = 9 - x^2$ is

Solution:

$$f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x)$$

Hence,

$f(x)$ is an even function.

57) The function $f(x) = x^5 - x$ is

Solution:

$$\begin{aligned} f(-x) = (-x)^5 - (-x) &= -x^5 + x \\ &= -(x^5 - x) = -f(x) \end{aligned}$$

Hence,

$f(x)$ is an odd function.

58) The function $f(x) = 2 - \sqrt[5]{x}$ is

Solution:

$$\begin{aligned} f(-x) = 2 - \sqrt[5]{(-x)} = 2 - \sqrt[5]{-x} &= 2 + \sqrt[5]{x} \\ &= -\left(-2 - \sqrt[5]{x}\right) \end{aligned}$$

Hence,

$f(x)$ is neither even nor odd.

59) The function $f(x) = 3x + \frac{2}{\sqrt{x^2+9}}$ is

Solution:

$$\begin{aligned} f(-x) = 3(-x) + \frac{2}{\sqrt{(-x)^2+9}} &= -3x + \frac{2}{\sqrt{x^2+9}} \\ &= -\left(3x - \frac{2}{\sqrt{x^2+9}}\right) \end{aligned}$$

Hence,

$f(x)$ is neither even nor odd.

60) The function $f(x) = \frac{3}{\sqrt{x^2+9}}$ is

Solution:

$$f(-x) = \frac{3}{\sqrt{(-x)^2+9}} = \frac{3}{\sqrt{x^2+9}} = f(x)$$

Hence,

$f(x)$ is an even function.

61) The function $f(x) = \sqrt{4 + x^2}$ is

Solution:

$$f(-x) = \sqrt{4 + (-x)^2} = \sqrt{4 + x^2} = f(x)$$

Hence,

$f(x)$ is an even function.

62) The function $f(x) = 3$ is

Solution:

Since the graph of the constant function 3 is symmetric about the $y - axis$, then

$f(x)$ is an even function.

63) The function $f(x) = \frac{9-x^2}{x-2}$ is

Solution:

$$\begin{aligned} f(-x) = \frac{9 - (-x)^2}{(-x) - 2} &= \frac{9 - x^2}{-x - 2} \\ &= -\left(\frac{9 - x^2}{x + 2}\right) \end{aligned}$$

Hence,

$f(x)$ is neither even nor odd.

64) The function $f(x) = \frac{x^2-4}{x^2+1}$ is

Solution:

$$f(-x) = \frac{(-x)^2 - 4}{(-x)^2 + 1} = \frac{x^2 - 4}{x^2 + 1} = f(x)$$

Hence,

$f(x)$ is an even function.

65) The function $f(x) = 3|x|$ is

Solution:

$$f(-x) = 3|(-x)| = 3|x| = f(x)$$

Hence,

$f(x)$ is an even function.

66) The function $f(x) = x^{-2}$ is

Solution:

$$f(x) = x^{-2} = \frac{1}{x^2}$$

$$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$$

Hence, $f(x)$ is an even function.

67) The function $f(x) = x^3 - 2x + 5$ is Solution: $f(-x) = (-x)^3 - 2(-x) + 5 \quad = -x^3 + 2x + 5$ $= -(x^3 - 2x - 5)$ Hence, $f(x)$ is neither even nor odd.	68) The function $f(x) = \sqrt[3]{x^5} - x^3 + x$ is Solution: $f(-x) = \sqrt[3]{(-x)^5} - (-x)^3 + (-x) \quad = -\sqrt[3]{x^5} + x^3 - x$ $= -\left(\sqrt[3]{x^5} - x^3 + x\right) = -f(x)$ Hence, $f(x)$ is an odd function.
69) The function $f(x) = 7$ is Solution: Since the graph of the constant function 7 is symmetric about the $y - axis$, then $f(x)$ is an even function.	70) The function $f(x) = \frac{x^3-4}{x^3+1}$ is Solution: $f(-x) = \frac{(-x)^3 - 4}{(-x)^3 + 1} = \frac{-x^3 - 4}{-x^3 + 1} = -\frac{x^3 + 4}{-x^3 + 1}$ Hence, $f(x)$ is neither even nor odd.
71) The function $f(x) = \frac{x^2-1}{x^3+3}$ is Solution: $f(-x) = \frac{(-x)^2 - 1}{(-x)^3 + 3} = \frac{x^2 - 1}{-x^3 + 3} = -\frac{x^2 - 1}{x^3 - 3}$ Hence, $f(x)$ is neither even nor odd.	72) The function $f(x) = x^6 - 4x^2 + 1$ is Solution: $f(-x) = (-x)^6 - 4(-x)^2 + 1 = x^6 - 4x^2 + 1 = f(x)$ Hence, $f(x)$ is an even function.
73) The function $f(x) = x^2$ is increasing on $(0, \infty)$.	74) The function $f(x) = x^2$ is decreasing on $(-\infty, 0)$.
75) The function $f(x) = x^3$ is increasing on $(-\infty, \infty)$.	76) The function $f(x) = x^3$ is not decreasing at all.
77) The function $f(x) = \sqrt{x}$ is increasing on $(0, \infty)$.	78) The function $f(x) = \sqrt{x}$ is not decreasing at all.
79) The function $f(x) = \frac{1}{x}$ is not increasing at all.	80) The function $f(x) = \frac{1}{x}$ is decreasing on $(-\infty, \infty)$.

Workshop Solutions to Sections 2.3 and 2.4

1) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f+g)(x) =$ Solution: $(f+g)(x) = x^2 + \sqrt{4-x}$	2) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f+g} =$ Solution: $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \;\Leftrightarrow\; x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{f+g} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
3) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f-g)(x) =$ Solution: $(f-g)(x) = x^2 - \sqrt{4-x}$	4) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f-g} =$ Solution: $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \;\Leftrightarrow\; x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{f-g} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
5) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(fg)(x) =$ Solution: $(fg)(x) = x^2\sqrt{4-x}$	6) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{fg} =$ Solution: $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \;\Leftrightarrow\; x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{fg} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
7) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(f \circ g)(x) =$ Solution: $(f \circ g)(x) = f(g(x))$ $= f(\sqrt{4-x}) = (\sqrt{4-x})^2 = 4 - x$	8) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{f \circ g} =$ Solution: $(f \circ g)(x) = f(g(x))$ $= f(\sqrt{4-x}) = (\sqrt{4-x})^2 = 4 - x$ $D_g = (-\infty, 4]$ $D_{f(g(x))} = \mathbb{R}$ $D_{f \circ g} = D_g \cap D_{f(g(x))} = (-\infty, 4] \cap \mathbb{R} = (-\infty, 4]$
9) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $(g \circ f)(x) =$ Solution: $(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{4-x^2}$	10) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{g \circ f} =$ Solution: $(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{4-x^2}$ $D_f = \mathbb{R}$ $D_{g(f(x))} = [-2,2]$ $D_{g \circ f} = D_f \cap D_{g(f(x))} = \mathbb{R} \cap [-2,2] = [-2,2]$
11) If $f(x) = x^2$, then $(f \circ f)(x) =$ Solution: $(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$	12) If $f(x) = x^2$, then $D_{f \circ f} =$ Solution: $(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$ $D_f = \mathbb{R}$ $D_{f(f(x))} = \mathbb{R}$ $D_{f \circ f} = D_f \cap D_{f(f(x))} = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

13) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $\left(\frac{f}{g}\right)(x) =$ Solution: $\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x}}$	14) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{\frac{f}{g}} =$ Solution: $\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x}}$ $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \quad \Leftrightarrow \quad x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{\frac{f}{g}} = \{x \in D_f \cap D_g \mid g(x) \neq 0\}$ $= \mathbb{R} \cap (-\infty, 4) = (-\infty, 4)$
15) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $\left(\frac{g}{f}\right)(x) =$ Solution: $\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$	16) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{\frac{g}{f}} =$ Solution: $\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$ $D_f = \mathbb{R}$ $g(x)$ is defined when $4 - x \geq 0 \quad \Leftrightarrow \quad x \leq 4$. Thus, $D_g = (-\infty, 4]$ $D_{\frac{g}{f}} = \{x \in D_f \cap D_g \mid f(x) \neq 0\}$ $= \mathbb{R} \setminus \{0\} \cap (-\infty, 4] = (-\infty, 0) \cup (0, 4]$
17) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f + g)(x) =$ Solution: $(f + g)(x) = (9 - x^2) + (10) = 9 - x^2 + 10$ $= 19 - x^2$	18) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f - g)(x) =$ Solution: $(f - g)(x) = (9 - x^2) - (10) = 9 - x^2 - 10$ $= -x^2 - 1$
19) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g - f)(x) =$ Solution: $(g - f)(x) = (10) - (9 - x^2) = 10 - 9 + x^2$ $= 1 + x^2$	20) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(fg)(x) =$ Solution: $(fg)(x) = (9 - x^2)(10) = 90 - 10x^2$
21) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f \circ g)(x) =$ Solution: $(f \circ g)(x) = f(g(x)) = f(10)$ $= 9 - 10^2 = 9 - 100 = -91$	22) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g \circ f)(x) =$ Solution: $(g \circ f)(x) = g(f(x)) = g(9 - x^2) = 10$
23) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f \circ f)(x) =$ Solution: $(f \circ f)(x) = f(f(x)) = f(9 - x^2)$ $= 9 - (9 - x^2)^2$	24) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g \circ g)(x) =$ Solution: $(g \circ g)(x) = g(g(x)) = g(10) = 10$
25) If $f(x) = 9 - x^2$, $g(x) = \sin x$ and $h(x) = 3x + 2$, then $(f \circ g \circ h)(x) =$ Solution: $(f \circ g \circ h)(x) = f\Big(g\big(h(x)\big)\Big)$ $= f\big(g(3x + 2)\big)$ $= f(\sin(3x + 2))$ $= 9 - (\sin(3x + 2))^2$ $= 9 - sin^2(3x + 2)$	26) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then $(f + g)(x) =$ Solution: $(f + g)(x) = \sqrt{25 + x^2} + x^3$

27) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then $(f - g)(x) =$

Solution:

$$(f - g)(x) = \sqrt{25 + x^2 - x^3}$$

28) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then $(fg)(x) =$

Solution:

$$(fg)(x) = x^3\sqrt{25 + x^2}$$

29) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then $\left(\frac{f}{g}\right)(x) =$

Solution:

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{25 + x^2}}{x^3}$$

30) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then $(f \circ g)(x) =$

Solution:

$$(f \circ g)(x) = f(g(x)) = f(x^3) = \sqrt{25 + (x^3)^2}$$
$$= \sqrt{25 + x^6}$$

31) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then $(g \circ f)(x) =$

Solution:

$$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{25 + x^2}\right) = \left(\sqrt{25 + x^2}\right)^3$$
$$= \sqrt{(25 + x^2)^3}$$

32) If $f(x) = \sqrt{x}$ and $g(x) = x - 2$, then $(f \circ g)(x) =$

Solution:

$$(f \circ g)(x) = f(g(x)) = f(x - 2) = \sqrt{x - 2}$$

33) If $f(x) = \sqrt{x}$ and $g(x) = x - 2$, then $(g \circ f)(x) =$

Solution:

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{x} - 2$$

34) If $f(x) = \sqrt{x}$ and $g(x) = x - 2$, then $(g \circ g)(x) =$

Solution:

$$(g \circ g)(x) = g(g(x)) = g(x - 2) = (x - 2) - 2$$
$$= x - 2 - 2 = x - 4$$

35) If $f(x) = \sqrt{x}$ and $g(x) = x - 2$, then $(fg)(x) =$

Solution:

$$(fg)(x) = (\sqrt{x})(x - 2) = (x - 2)\sqrt{x}$$

36) If $f(x) = \sin 5x$ and $g(x) = x^2 + 3$, then $(f \circ g)(x) =$

Solution:

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 3\ ) = \sin 5(x^2 + 3\ )$$

37) If $f(x) = \sin 5x$ and $g(x) = x^2 + 3$, then $(g \circ f)(x) =$

Solution:

$$(g \circ f)(x) = g(f(x)) = g(\sin 5x\ ) = (\sin 5x)^2 + 3$$
$$= sin^2 5x + 3$$

38) If $f(x) = \sin 5x$ and $g(x) = x^2 + 3$, then $(fg)(x) =$

Solution:

$$(fg)(x) = (\sin 5x)(x^2 + 3) = (x^2 + 3)\sin 5x$$

39) If $f(x) = \sqrt{x}$ and $g(x) = \cos x$, then $(g \circ f)(x) =$

Solution:

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \cos\sqrt{x}$$

40) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1 - x^2$, then $(f \circ g)(x) =$

Solution:

$$(f \circ g)(x) = f(g(x)) = f(1 - x^2\ ) = (1 - x^2\ ) + \frac{1}{1 - x^2}$$

41) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1 - x^2$, then $(g \circ f)(x) =$

Solution:

$$(g \circ f)(x) = g(f(x)) = g\left(x + \frac{1}{x}\right) = 1 - \left(x + \frac{1}{x}\right)^2$$

42) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1 - x^2$, then $(fg)(x) =$

Solution:

$$(fg)(x) = \left(x + \frac{1}{x}\right)(1 - x^2)$$

43) If the graph of the function $f(x) = x^2$ is shifted a distance 2 units upwards , then the new graph represented the graph of the function is

Solution:

$$x^2 + 2$$

44) If the graph of the function $f(x) = x^2$ is shifted a distance 2 units downwards , then the new graph represented the graph of the function is

Solution:

$$x^2 - 2$$

45) If the graph of the function $f(x) = x^2$ is shifted a distance 2 units to the right , then the new graph represented the graph of the function is

Solution:

$$(x - 2)^2 = x^2 - 4x + 4$$

46) If the graph of the function $f(x) = x^2$ is shifted a distance 2 units to the left , then the new graph represented the graph of the function is

Solution:

$$(x + 2)^2 = x^2 + 4x + 4$$

47) If the graph of the function $f(x) = \cos x$ is stretched vertically by a factor of 2 , then the new graph represented the graph of the function is

Solution:

$$2\cos x$$

48) If the graph of the function $f(x) = \cos x$ is compressed vertically by a factor of $\frac{1}{2}$, then the new graph represented the graph of the function is

Solution:

$$\frac{1}{2}\cos x$$

49) If the graph of the function $f(x) = \cos x$ is compressed horizontally by a factor of 2 , then the new graph represented the graph of the function is

Solution:

$$\cos 2x$$

50) If the graph of the function $f(x) = \cos x$ is stretched horizontally by a factor of $\frac{1}{2}$, then the new graph represented the graph of the function is

Solution:

$$\cos\frac{x}{2}$$

51) The graph of the function $f(x) = \sqrt{x}$ is reflected about the $x - axis$ if

Solution:

$$f(x) = -\sqrt{x}$$

52) The graph of the function $f(x) = \sqrt{x}$ is reflected about the $y - axis$ if

Solution:

$$f(x) = \sqrt{-x}$$

53) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units upwards , then the new graph represented the graph of the function is

Solution:

$$e^x + 2$$

54) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units downwards , then the new graph represented the graph of the function is

Solution:

$$e^x - 2$$

55) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units to the right , then the new graph represented the graph of the function is

Solution:

$$e^{x-2}$$

56) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units to the left , then the new graph represented the graph of the function is

Solution:

$$e^{x+2}$$

57) $\frac{2\pi}{3}$ rad $= \frac{2\pi}{3} \times \frac{180^\circ}{\pi} = 120^\circ$

58) $\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

59) $\frac{7\pi}{6}$ rad $= \frac{7\pi}{6} \times \frac{180^\circ}{\pi} = 210^\circ$

60) $\frac{3\pi}{2}$ rad $= \frac{3\pi}{2} \times \frac{180^\circ}{\pi} = 270^\circ$

61) $120^\circ = 120 \times \frac{\pi}{180^\circ} = \frac{2\pi}{3}$ rad

62) $270^\circ = 270 \times \frac{\pi}{180^\circ} = \frac{3\pi}{2}$ rad

63) $\frac{5\pi}{12}$ rad $= \frac{5\pi}{12} \times \frac{180^\circ}{\pi} = 75^\circ$

64) $\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$ (Repeated)

65) $150^\circ = 150 \times \frac{\pi}{180^\circ} = \frac{5\pi}{6}$ rad

66) $210^\circ = 210 \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$ rad

67) $\frac{1}{\sec x} = \cos x$

68) $\frac{1}{\csc x} = \sin x$

69) $\frac{1}{\cot x} = \tan x$

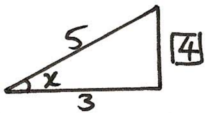
70) $\frac{\sin x}{\cos x} = \tan x$

71) $\frac{\cos x}{\sin x} = \cot x$

72) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\cot x =$

Solution:

$$\cos x = \frac{3}{5} = \frac{adj}{hyp}$$



Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

$$\therefore \cot x = \frac{1}{\tan x} = \frac{adj}{opp} = \frac{3}{4}$$

73) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\tan x =$

Solution:

$$\cos x = \frac{3}{5} = \frac{adj}{hyp}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

$$\therefore \tan x = \frac{1}{\cot x} = \frac{opp}{adj} = \frac{4}{3}$$

74) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\sin x =$

Solution:

$$\cos x = \frac{3}{5} = \frac{adj}{hyp}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$

$$\therefore \ \sin x = \frac{opp}{hyp} = \frac{4}{5}$$

75) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\csc x =$

Solution:

$$\cos x = \frac{3}{5} = \frac{adj}{hyp}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$

$$\therefore \ \csc x = \frac{1}{\sin x} = \frac{hyp}{opp} = \frac{5}{4}$$

76) $\sin\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

So, we deduce now that $\sin\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$\sin\left(\frac{5\pi}{6}\right) = \sin(150^\circ) = \sin(180^\circ - 30^\circ) = \sin(30^\circ) =$
$\sin\pi 6{=}12$

77) $\cos\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

So, we deduce now that $\cos\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$$\cos\left(\frac{5\pi}{6}\right) = \cos(150^\circ) = \cos(180^\circ - 30^\circ)$$
$$= -\cos(30^\circ) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

78) $\tan\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

So, we deduce now that $\tan\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$$\tan\left(\frac{5\pi}{6}\right) = \tan(150^\circ) = \tan(180^\circ - 30^\circ)$$
$$= -\tan(30^\circ) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

79) $\cot\left(\frac{5\pi}{6}\right) =$

Solution:

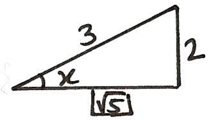
$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

So, we deduce now that $\cot\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$$\cot\left(\frac{5\pi}{6}\right) = \cot(150^\circ) = \cot(180^\circ - 30^\circ)$$
$$= -\cot(30^\circ) = -\cot\left(\frac{\pi}{6}\right) = -\sqrt{3}$$

80) If $\sin x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$, then $\sec x =$

Solution:



$$\sin x = \frac{2}{3} = \frac{opp}{hyp}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$

$$\therefore \ \sec x = \frac{1}{\cos x} = \frac{hyp}{adj} = \frac{3}{\sqrt{5}}$$

81) If $\sin x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$, then $\csc x =$

Solution:

$$\sin x = \frac{2}{3} = \frac{opp}{hyp}$$

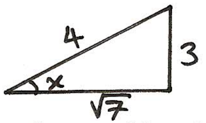
Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$

$$\therefore \ \csc x = \frac{1}{\sin x} = \frac{hyp}{opp} = \frac{3}{2}$$

82) If $\sin x = \frac{3}{4}$ and $0 < x < \frac{\pi}{2}$, then $\cos x =$

Solution:



$$\sin x = \frac{3}{4} = \frac{opp}{hyp}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{4^2 - 3^2} = \sqrt{16-9} = \sqrt{7}$

$$\therefore \ \cos x = \frac{adj}{hyp} = \frac{\sqrt{7}}{4}$$

83) If $\sin x = \frac{3}{4}$ and $0 < x < \frac{\pi}{2}$, then $\cot x =$

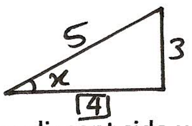
Solution:

$$\sin x = \frac{3}{4} = \frac{opp}{hyp}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{4^2 - 3^2} = \sqrt{16-9} = \sqrt{7}$

$$\therefore \ \cot x = \frac{1}{\tan x} = \frac{adj}{opp} = \frac{\sqrt{7}}{3}$$

84) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cos x =$ Solution: $\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$  Now, we should find the length of the adjacent side using the Pythagorean Theorem, so $\lvert\text{adjacent}\rvert = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ $\therefore \cos x = \frac{adj}{hyp} = \frac{4}{5}$	85) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\sec x =$ Solution: $\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$ Now, we should find the length of the adjacent side using the Pythagorean Theorem, so $\lvert\text{adjacent}\rvert = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ $\therefore \sec x = \frac{1}{\cos x} = \frac{hyp}{adj} = \frac{5}{4}$
86) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cot x =$ Solution: $\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$ Now, we should find the length of the adjacent side using the Pythagorean Theorem, so $\lvert\text{adjacent}\rvert = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ $\therefore \cot x = \frac{1}{\tan x} = \frac{adj}{opp} = -\frac{4}{3}$	87) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\tan x =$ Solution: $\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$ Now, we should find the length of the adjacent side using the Pythagorean Theorem, so $\lvert\text{adjacent}\rvert = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ $\therefore \tan x = \frac{1}{\cot x} = \frac{opp}{adj} = -\frac{3}{4}$
88) If $f(x) = \sin x$, then $D_f = \mathbb{R}$	89) If $f(x) = \cos x$, then $D_f = \mathbb{R}$
88) If $f(x) = \sin x$, then $R_f = [-1,1]$	88) If $f(x) = \sin x$, then $R_f = [-1,1]$

1.5 Exponential Function.

Notes

6

- التركيز على المفاهيم الأساسية.
- شرح أبواب المنهج حسب الخطة.
- أمثلة توضيحية وتدريبات.
- نماذج اختبارات.

السعدي

رياضيات ١١٠
Math. 110

جمال السعدي
استاذ الرياضيات والإحصاء للمرحلة الجامعية
0566664790

1.5

Exponential Functions

In general,

Exponential function : $F(x) = a^x \longrightarrow a > 0$

where a is positive constant.

Notes :

- $a^n = a \cdot a \cdot \cdots\cdots\cdot a$
  $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

- $a^{-n} = \frac{1}{a^n} \longrightarrow 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

- $a^{n/m} = \sqrt[m]{a^n} = (\sqrt[m]{a})^n$
  $2^{3/5} = \sqrt[5]{2^3} = (\sqrt[5]{2})^3$

- $a^0 = 1$

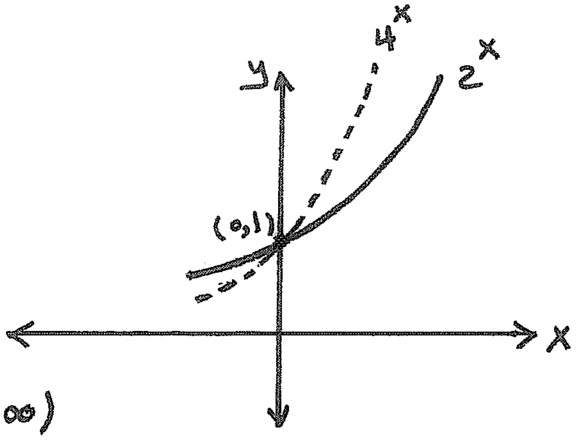
أى عدد أس zero بواحد

بشرط أن الأساس $a \neq 0$

$2^0 = 1$ ، $(\frac{-2}{3})^0 = 1$ ، $e^0 = 1$

$$F(x) = a^x$$

(1) $a > 1$



* Domain $F(x) = (-\infty, \infty)$

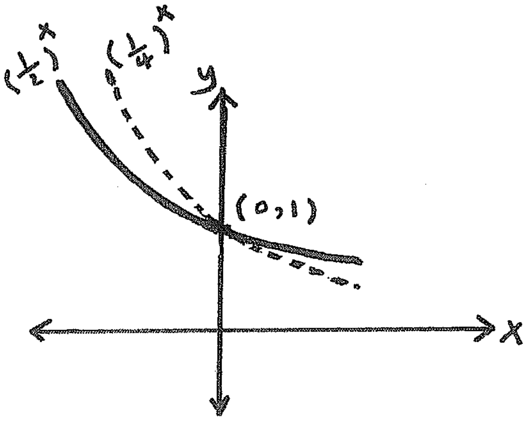
المجال من محور x

* Range $F(x) = (0, \infty)$

المدى من محور y

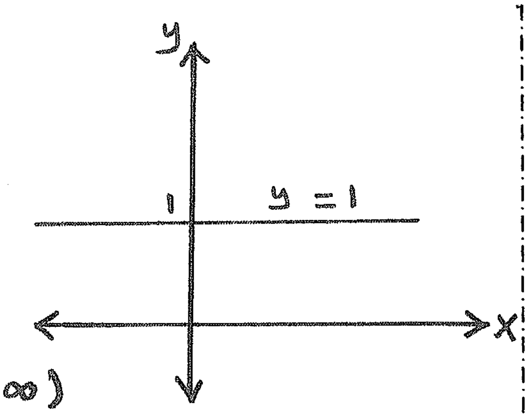
* $F(x)$ is increasing on $(-\infty, \infty)$

(2) $0 < a < 1$



* Domain $F(x) = (-\infty, \infty)$
* Range $F(x) = (0, \infty)$
* $F(x)$ is decreasing on $(-\infty, \infty)$

(3) $a = 1$



* Domain $F(x) = (-\infty, \infty)$

* Range $F(x) = \{1\}$

* $F(x)$ is constant on $(-\infty, \infty)$

Laws of exponents :

(1) $a^x \cdot a^y = a^{x+y}$ عند الضرب نجمع الأسس بشرط تساوي الأساسات

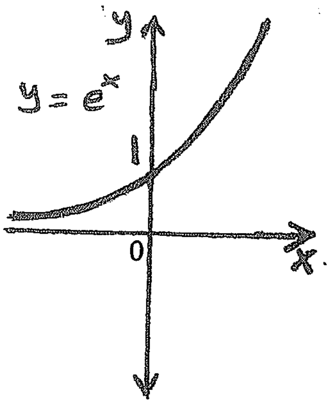
(2) $\frac{a^x}{a^y} = a^{x-y}$ 〃 القسمة نطرح 〃 〃 〃 〃

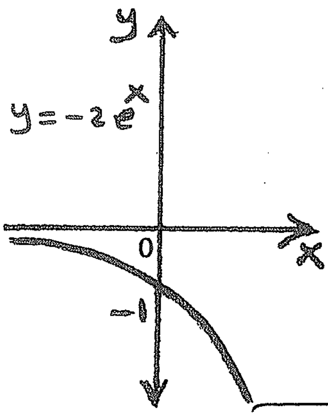
(3) $(a^x)^y = a^{x \cdot y}$ ضرب الأسس الداخلي في الخارجي

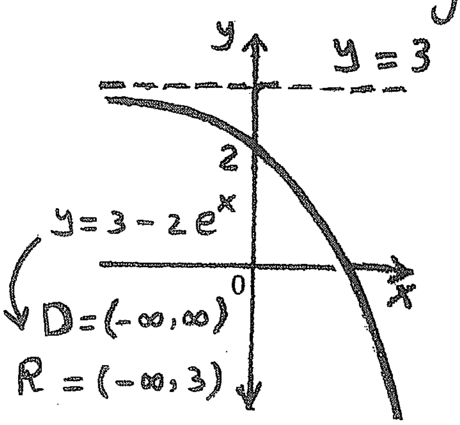
(4) $(ab)^x = a^x \cdot b^x$ توزيع الأسس الخارجي على حاصل الضرب

(5) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$ 〃 〃 〃 〃 خارج القسمة 〃

$\overset{\text{أرسم}}{\text{Sketch}}$ the graph of the function $\boxed{y = 3 - 2e^x}$ and determine the domain and range







$D = (-\infty, \infty)$

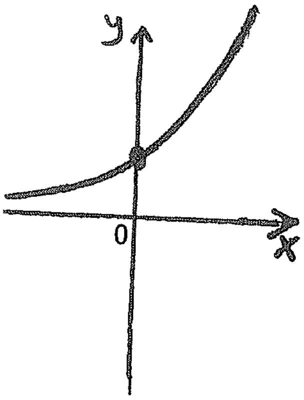
$R = (-\infty, 3)$

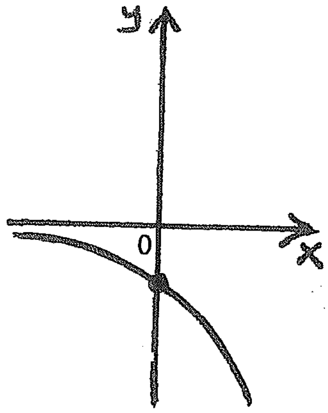
- If : $y = 3^x$

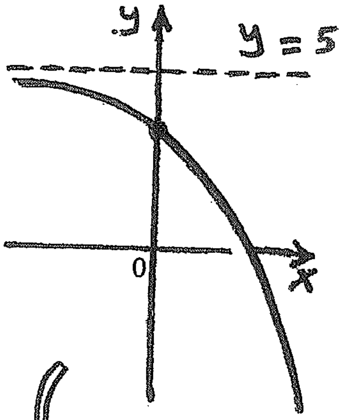
Find the new function , domain and range where $y = 3^x$ reflect about x – axis and shifted $\underline{\underline{5}}$ units upword .

{ solution }

$y = 3^x$







Domain = $(-\infty, \infty)$

Range = $(-\infty, 5)$

$y = 3^x$

reflect about x – axis

$y = -3^x$

shifted $\underline{\underline{5}}$ units upward

$y - 5 = -3^x$

$y = 5 - 3^x$

سؤال للشطار فقط

Find the range

For : $y = 6 - 3^x$?

بدون الرسم .

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Compare the exponential $f(x) = 2^x$ and the power function $g(x) = x^2$ which function grows more quickly when x is large ?

{Solution}

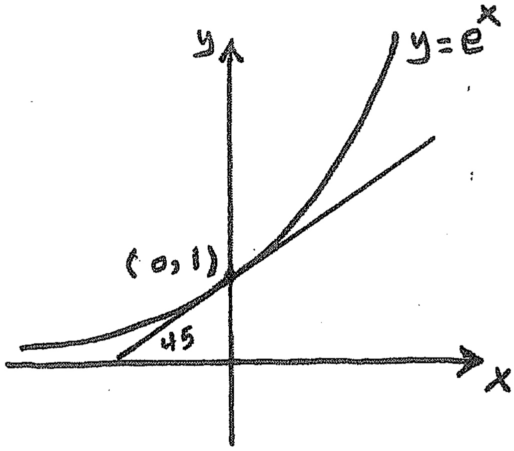
x	$f(x) = 2^x$	$g(x) = x^2$
1	$f(1) = 2^1 = 2$	$g(1) = 1^2 = 1$
2	$f(2) = 2^2 = 4$	$g(2) = 2^2 = 4$
⋮	⋮	⋮
5	$f(5) = 2^5 = 32$	$g(5) = 5^2 = 25$
⋮	⋮	⋮
10	$f(10) = 2^{10} = \underline{\underline{1024}}$	$g(10) = 10^2 = \underline{\underline{100}}$

$$f(x) = 2^x > g(x) = x^2$$

$\therefore$ $f(x) = 2^x$ is grows quickly more than the power function $g(x) = x^2$

The Number $e \approx 2.71828$

- The natural exponential function



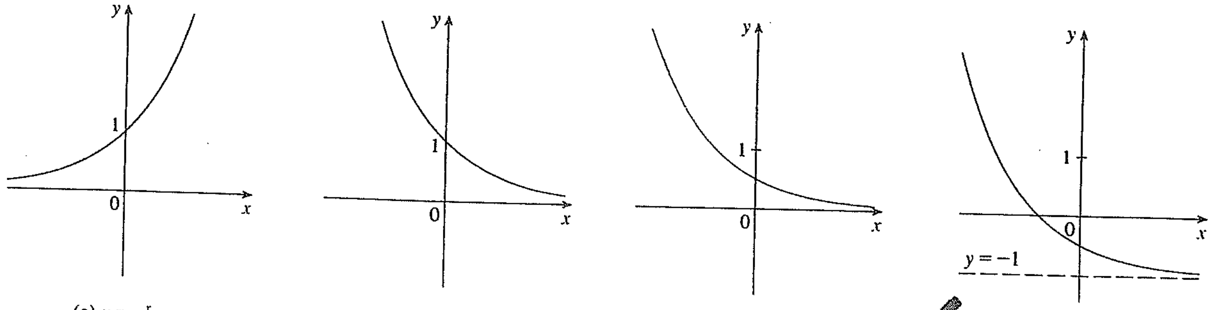
$*$ Domain $= (-\infty, \infty)$
$*$ Range $= (0, \infty)$
$*$ $y = e^x$ is increasing
$*$ slope at $(0,1)$ is $m = 1$
$m = \tan 45 = 1$

- Example :

Graph the function: $y = \frac{1}{2}e^{-x} - 1$

and find the domain and range.

{Solution}



(a) $y = e^x$ (b) $y = e^{-x}$ (c) $y = \frac{1}{2}e^{-x}$ (d) $y = \frac{1}{2}e^{-x} - 1$

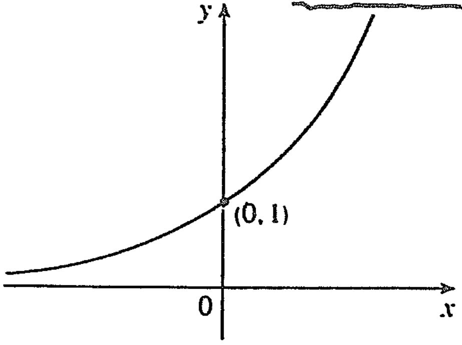
$*$ Domain $= (-\infty, \infty)$
$*$ Range $= (-1, \infty)$

Graph the given Function:

① $y = 8^x$

$a = 8 > 1$

ملحوظه ١:
اذا كان الأساس
$a > 1$
يكون المنحنى صاعد
اى الداله تزايديه



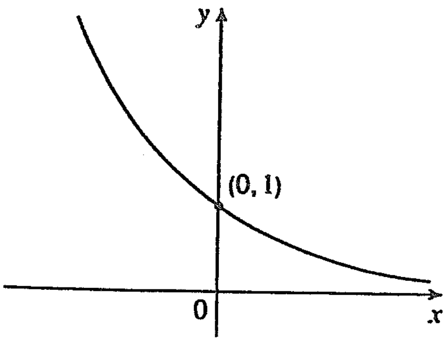
② $y = 8^{-x}$

$\hookrightarrow y = \left(\frac{1}{8}\right)^x$

ملحوظه:
اذا كان الأساس
$0 < a < 1$
يكون المنحنى نازل
اى الداله تناقصيه

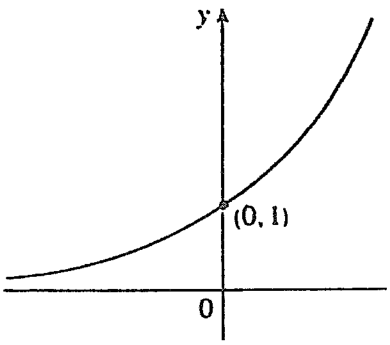
لتغير اشاره
الأس
من سالب إلى
موجب
نقلب الأساس

$0 < a = \frac{1}{8} < 1$



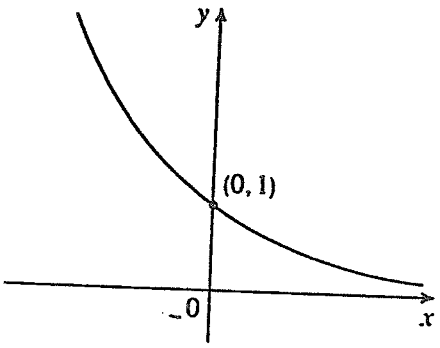
③ $y = e^x$

$0 < a = e \approx 2.7 < 1$



④ $y = e^{-x} = \left(\frac{1}{e}\right)^x$

$0 < a = \frac{1}{e} < 1$

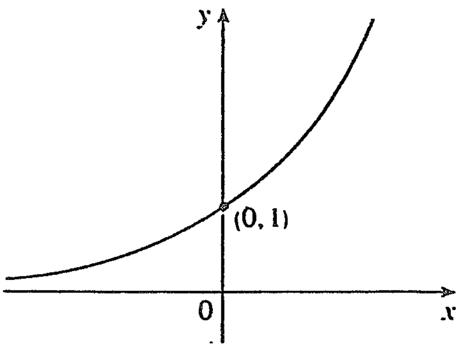


⑤ $y = \left(\frac{2}{3}\right)^{-x}$

$\hookrightarrow y = \left(\frac{3}{2}\right)^x$

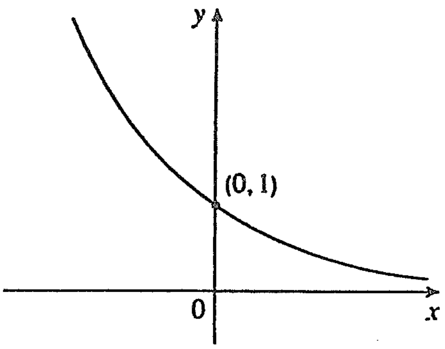
$a = \frac{3}{2} > 1$

لتغيير اشاره
الأس من
سالب لموجب
نقلب الأساس



⑥ $y = 0.3^x$

$0 < a = 0.3 < 1$



8

Starting with the graph of $y = e^x$, write the equation of the graph that results from

(a) shifting 2 units downward
(b) shifting 2 units to the right
(c) reflecting about the x-axis
(d) reflecting about the y-axis
(e) reflecting about the x-axis and then about the y-axis

Solution

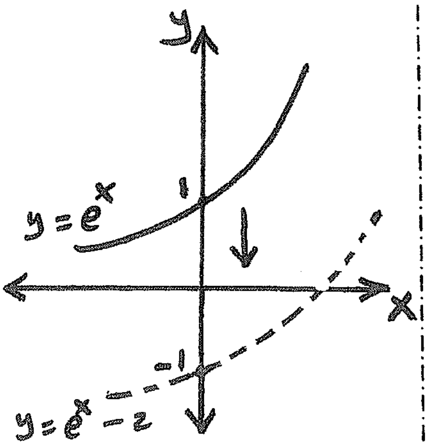
$y = e^x$

(a) shifting $\underline{\underline{2}}$ units downward

$\boxed{y+2}$ الحركه على محور y بمقدار 2 لأسفل

$y + 2 = e^x$

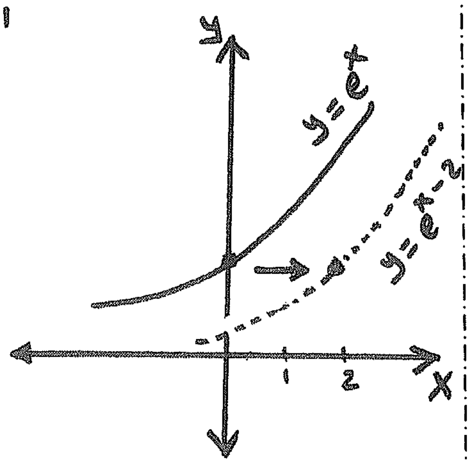
$y = e^x - 2$



(b) shifting $\underline{\underline{2}}$ units to the right

$\boxed{x-2}$ الحركه على محور X بمقدار 2 يمين

$y = e^{x-2}$

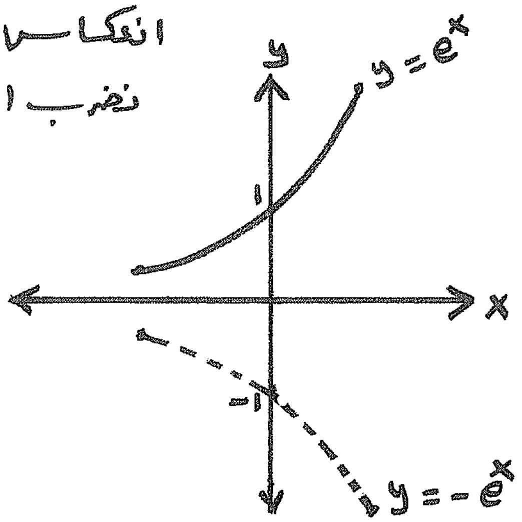


(c) reflecting about the x - axis

انعكاس حول محور x

نضرب الدالة الأصلية في سالب

$y = -e^{x}$

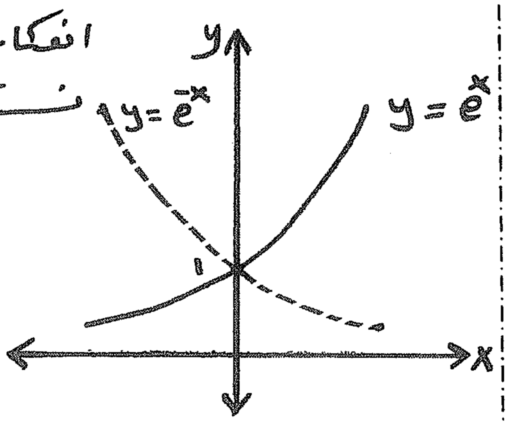


(d) reflecting about the y - axis

انعكاس حول محور y

نستبدل x بـ -x في الدالة الأصلية

$y = e^{-x}$



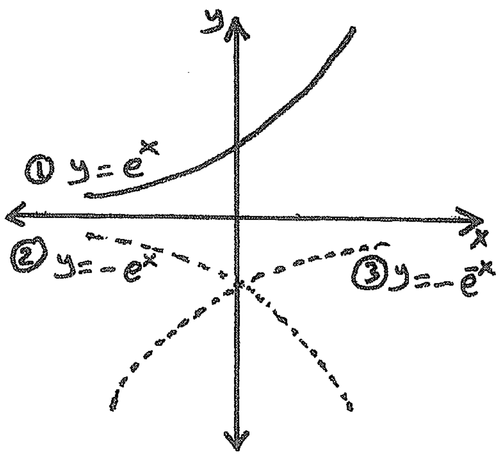
(e) reflecting about the x - axis and then about the y - axis

انعكاس حول محور x ثم انعكاس حول محور y

نضرب الدالة الأصلية في سالب

ونستبدل x بـ -x

$y = -e^{-x}$



* نلاحظ أن المنحنى ① انعكس حول محور x اعطى المنحنى ②
المنحنى ② انعكس حول محور y اعطى المنحنى ③
*الناتج # ها

Starting with the graph of $y = e^x$, find the equation of the graph that results from

(a) reflecting about the line $y = 4$

(b) reflecting about the line $x = 2$

solution

(a) reflecting about the line $y = 4$

$$\begin{tikzcd} y = e^{x} \arrow[d, bend right, "\text{انعكاس حول محور } X"'] \\ y = -e^{x} \arrow[d, bend right, "\text{ازاحه 8 وحدات لأعلى}"'] \\ y - 8 = -e^{x} \end{tikzcd}$$

$$\Longrightarrow \underline{y = -e^{x} + 8}$$

line $y = 4$

هو مستقيم يوازى محور X

* نوجد انعكاس الداله حول محور X

وذلك بضرب الداله المعطاه فى سالب

* ثم ازاحه لأعلى (لأنه y موجبه)

بمقدار ضعف المسافه اى 8 فى اتجاه y^{+}

(b) reflecting about the line $x = 2$

$$\begin{tikzcd} y = e^{x} \arrow[d, bend right, "\text{انعكاس حول محور } y"'] \\ y = e^{-x} \arrow[d, bend right, "\text{ازاحه 4 وحدات يمين}"'] \\ y = e^{-(x-4)} \end{tikzcd}$$

$$\Longrightarrow \underline{y = e^{-x+4}}$$

line $X = 2$

هو مستقيم يوازى محور y

* نوجد انعكاس الداله حول محور y

وذلك باستبدال X بـ $-x$

* ثم ازاحه لليمين (لأنه X موجبه)

بمقدار ضعف المسافه اى 4 فى اتجاه X^{+}

Find The Domain ?

① $F(x) = \dfrac{1}{1+e^{x}}$

∴ المقام ليس له أصفار

$\hookrightarrow Df(x) = R = (-\infty, \infty)$

* أصفار المقام

$1 + e^{x} = 0$

$e^{x} = -1$

discard

مرفوض (مستبعد)

where $e^{x} \neq -1$

∴ المقام ليس له أصفار

---

② $F(x) = \dfrac{1}{1-e^{x}}$

* أصفار المقام

$1 - e^{x} = 0$

$e^{x} = 1$

$\cancel{e}^{x} = \cancel{e}^{0}$

$x = 0$

$Df(x) = R - \{0\}$

$= (-\infty, 0) \cup (0, \infty)$

③ $g(t) = \sin(e^{t})$

داله اسيه + داله مثلثيه

$\overset{e^{t}}{\downarrow}$ مجالها R ∩ $\overset{\sin}{\downarrow}$ مجالها R

$= R = (-\infty, \infty)$

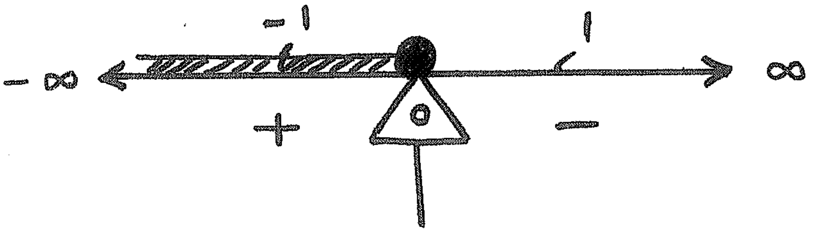
$$\therefore D\,g(t) = R = (-\infty, \infty)$$

---

④ $g(t) = \sqrt{1 - 2^{t}}$

المجال هو الفترات الموجبه لما تحت الجذر

* لدراسه اشاره ما تحت الجذر بالتعويض ب ١ ، -١



$1 - 2^{t} = 0$

$2^{t} = 1$

$\cancel{2}^{t} = \cancel{2}^{0}$

$t = 0$

$$\therefore D\,g(x) = (-\infty, 0]$$

---

⑤ $g(x) = \cos(e^{\sqrt{x}})$

* داله $\sqrt{x}$ (جذريه) مجالها $[0, \infty)$

* داله e (أسيه) مجالها R

* داله cos (مثلثيه) مجالها R

* المجال المشترك للدوال السابقه هو أصغر مجال وهو $[0, \infty)$ يمثل مجال الداله $g(x)$

$$\therefore D\,g(x) = [0, \infty)$$

IF : $F(x) = 5^x$

show that : $\dfrac{F(x+h) - F(x)}{h} = 5^x\left(\dfrac{5^h - 1}{h}\right)$

{Solution}

استبدال x بـ x+h في الداله الأصليه.

الداله الأصليه كما هي

$$L.H.S. = \frac{F(x+h) - f(x)}{h}$$

$$= \frac{5^{x+h} - 5^x}{h}$$

$$= \frac{5^x \cdot 5^h - 5^x}{h}$$

{عامل مشترك 5^x}

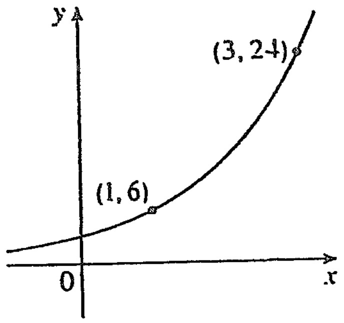
$$= 5^x\left(\frac{5^h - 1}{h}\right) = R.H.S.$$

Find the exponential function

$F(x) = ca^x$ where graph is given

Ⓐ $F(x) = 3\left(\frac{1}{2}\right)^x$ Ⓑ $F(x) = 2\left(\frac{1}{3}\right)^x$

Ⓒ $F(x) = 2(3)^x$ Ⓓ $F(x) = 3(2)^x$



{Solution}

$F(x) = ca^x$ معادله المنحنى

∵ المنحنى يمر بالنقطه $(1, 6)$ ∴ تحقق معادله المنحنى

$\therefore F(1) = ca^1$

$\boxed{6 = ca} \longrightarrow$ ①

∵ المنحنى يمر بالنقطه $(3, 24)$ ∴ تحقق معادله المنحنى

$\therefore F(3) = ca^3$

$\boxed{24 = ca^3} \longrightarrow$ ②

بقسمه المعادله ② على المعادله ①

$$\Longrightarrow \frac{24}{6} = \frac{ca^3}{ca}$$

$4 = a^2 \xRightarrow[\sqrt{}]{by} a = \pm 2$ (∵ a لابد تكون موجبه ∴ نستبعد السالب)

$\therefore \boxed{a = 2} \longrightarrow$ بالتعويض فى المعادله الأولى لتحديد c

$\Rightarrow 6 = (c) \cdot 2 \xrightarrow{\div 2} \boxed{c = 3}$

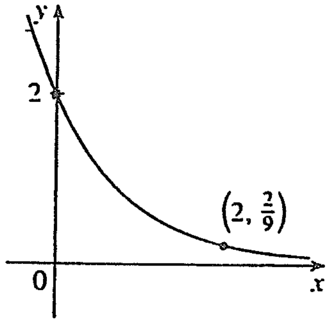
∴ معادله المنحنى المطلوب هى

$\boxed{F(x) = 3(2)^x} \longrightarrow$

∵ الاختبار اختيارات
ممكن الحل بطريقه اسهل وهى
نعوض بالنقطتين فى كل الاختيارات
الاختيار الذى تحققه النقطتين
هو الصواب

السعدي

Find the exponential function $\boxed{F(x) = c\,a^x}$ whose graph is given.



Ⓐ $c = -2$, $a = -\frac{1}{3}$ Ⓑ $c = 2$, $a = -\frac{2}{9}$

Ⓒ $c = 2$, $a = \frac{1}{3}$ Ⓓ $c = 2$, $a = 1$

<u>لاحظ أن</u> : a لا تكون سالبة ولا تكون ١

حيث أن شكل الرسم يدل على أن $0 < a < 1$ ∴ الاختيار الصحيح هو Ⓒ

{solution}

- ∵ المنحنى يمر بالنقطة (0, 2) ∴ تحقق معادلة المنحنى ⇐ $F(0) = c\,a^0$

$$2 = c\,(1)$$

$$2 = c \Longrightarrow \boxed{c = 2}$$

- ∵ المنحنى يمر بالنقطة ($2, \frac{2}{9}$) ∴ تحقق معادلة المنحنى ⇐ $\therefore F(2) = c\,a^2$

$$\frac{2}{9} = 2\,a^2 \xrightarrow[\frac{1}{2}\ \text{ف}]{\text{بالضرب}} \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{9} = \frac{1}{\cancel{2}} \cdot \cancel{2}\,a^2$$

$$\longrightarrow \frac{1}{9} = a^2 \quad \text{by } \sqrt{\ }$$

$$\longrightarrow a = \pm\frac{1}{3}$$

$$\longrightarrow \boxed{a = \frac{1}{3}}$$

السالب مرفوض لأن $0 < a < 1$ حسب شكل الرسم

∴ معادلة المنحنى هي

$$F(x) = 2\left(\frac{1}{3}\right)^x$$

كل التمنيات الطيبة والدعوات الصادقة للجميع بالتوفيق... جمال السعدي

Notes

- التركيز على المفاهيم الاساسية.
- شرح أبواب المنهج حسب الخطة.
- امثلة توضيحية وتدريبات.
- نماذج اختبارات.

السعدي

7

رياضيات ١١٠
Math.110

جمال السعدي

استاذ الرياضيات والإحصاء للمرحلة الجامعية

0566664790

1.6

Inverse functions and logarithms
الدوال العكسيه واللوغاريتمات

Definition :

f is called One - to - One

if it never takes on the same value twice

$$f(x_1) \neq f(x_2) \quad \text{where} \quad x_1 \neq x_2$$

تكون الداله واحد لواحد اذا كانت العناصر المختلفه لها صور مختلفه

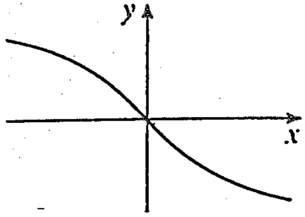
- **Horizontal line test** (اختبار الخط الأفقي)

F is one - to - one
if and only if (اذا وفقط اذا)
no horizontal line intersects
its graph more than once.

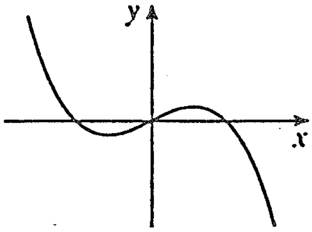
* يقال أن الداله واحد لواحد اذا تأكد عدم وجود خط أفقياً يقطع منحنى الداله في أكثر من نقطه.

Example : Determine (حدد) whether (ما إذا كانت) it is one-to-one:

5.



6.



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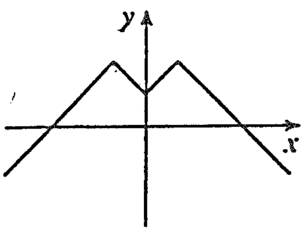
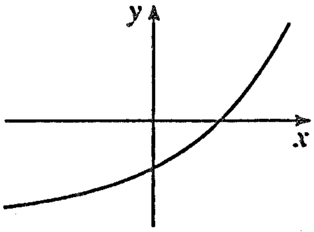


8.



Solution

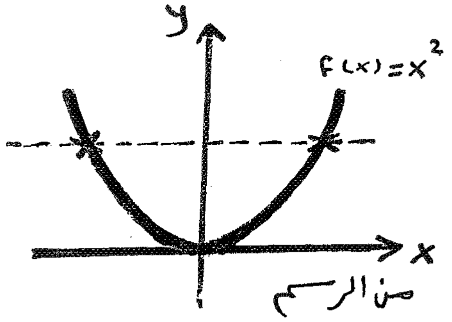
* 5. and 8. are one-to-one.
* 6. and 7. are not one-to-one.

Example :

① Is the function : $f(x) = x^2$ one-to-one

Solution

$$\begin{array}{ccc} -2 & \neq & 2 \\ \swarrow & & \searrow \\ F(-2) = (-2)^2 = \underline{\underline{4}} & & F(2) = (2)^2 = \underline{\underline{4}} \end{array}$$



عنصران مختلفان لهما نفس الصوره

$\therefore$ f(x) is <u>not</u> one-to-one

من الرسم

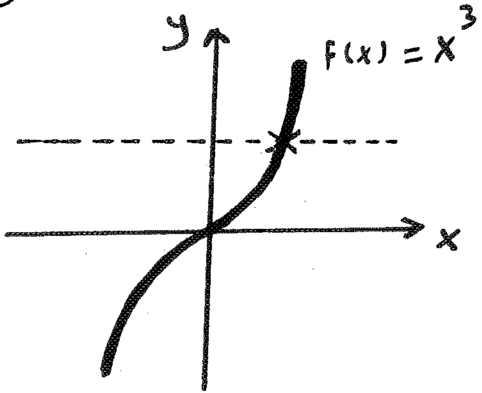
الخط الأفقي قطع منحنى الداله في أكثر من نقطه

$\therefore$ f(x) is not one-to-one

---

② Is the function : $f(x) = x^3$ one-to-one

Solution



let : $x_1 \neq x_2$ بالتكعيب

$x_1^3 \neq x_2^3$

$f(x_1) \neq f(x_2)$

$\therefore$ f(x) is one-to-one

ملحوظه : في حاله كثيره الحدود : اذا كان أعلى أس زوجيا ← **Not** one-to-one

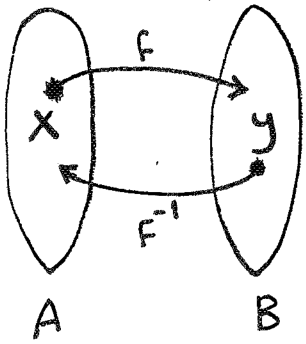
اذا كان أعلى أس فردي ← One-to-one

If : F is one - to - one with domain A and range B

then F^{-1} is inverse function 〃 〃 B 〃 〃 A

$F : A \longrightarrow B$

$F^{-1} : B \longrightarrow A$



Note that :

$$\boxed{\begin{array}{l} * \ \overset{\text{مجال}}{\text{Domain}}\ f = \overset{\text{مدى}}{\text{Range}}\ F^{-1} \\ * \ \overset{\text{مدى}}{\text{Range}}\ F = \overset{\text{مجال}}{\text{Domain}}\ f^{-1} \end{array}}$$

Note that:

$$F^{-1}(x) \neq \frac{1}{f(x)} \quad \text{but} \quad [f(x)]^{-1} = \frac{1}{F(x)}$$

$$\boxed{\text{If: } F(x) = y \iff F^{-1}(y) = x}$$

Example :

If : $F(1) = 5$, $F(3) = 7$ and $F(8) = -10$

Find : $F^{-1}(5)$, $F^{-1}(7)$ and $F^{-1}(-10)$

Solution

$\because F(1) = 5$
$\Rightarrow F^{-1}(5) = 1$

$\because F(3) = 7$
$\Rightarrow F^{-1}(7) = 3$

$\because F(8) = -10$
$\Rightarrow F^{-1}(-10) = 8$

$$(F^{-1} \circ F)(x) = F^{-1}(F(x)) = x$$

$$(F \circ F^{-1})(x) = F(F^{-1}(x)) = x$$

Example :

$$(F^{-1} \circ f)(2) = F^{-1}(F(2)) = 2$$

$$(F \circ F^{-1})(3) = F(F^{-1}(3)) = 3$$

Exercises

⑮ page 70 → If: F is one-to-one

such that $F(2)=9$

what is $F^{-1}(9)$?

$\because$ F is one-to-one

then : $F(2)=9 \Longrightarrow F^{-1}(9) = 2$

---

⑯ page 70 → let $F(x) = 3 + x^2 + \tan\left(\frac{\pi x}{2}\right)$

where $-1 < x < 1$

(a) Find $F^{-1}(3)$

لإيجاد $F^{-1}(3)$ نعوض في الدالة F بأرقام
حتى نحصل على العدد 3

$F(0) = 3 + 0^2 + \tan(0)$

$= 3 + 0 + 0 = 3$

$\therefore F(0) = 3 \Longrightarrow F^{-1}(3) = 0$

(b) Find $F(F^{-1}(5))$

$\cancel{F}(\cancel{F^{-1}}(5)) = 5$

⑰ page 70 → If: $g(x) = 3 + x + e^{x}$

Find $g^{-1}(4)$?

$g(x)$ is one-to-one

لأن أي عنصران مختلفان لهما صورتان مختلفتان .

* جرب التعويض بأرقام من داله $g(x)$ حتى تحصل على العدد 4

$g(0) = 3 + 0 + e^{0} = 3 + 1 = 4$

$\therefore g(0) = 4 \Longrightarrow g^{-1}(4) = 0$

---

⑱ The graph of f is given. ← الشكل المعطى

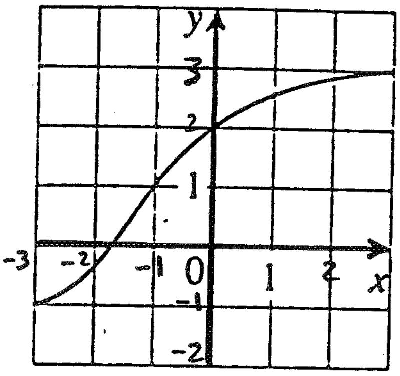
(a) Why is f one-to-one?

(b) What are the domain and range of f^{-1}?

(c) What is the value of $f^{-1}(2)$?

(d) Estimate the value of $f^{-1}(0)$.

خمن



(a) $F(x)$ is one-to-one because any horizontal line intersect the curve in one point.

(b) * Domain F^{-1} = Range $F = [-1, 3]$

* Range F^{-1} = Domain $F = [-3, 3]$

(c) $F^{-1}(2)$? $\longrightarrow$ $y = 2$ على المنحنى
يناظرها $x = 0$

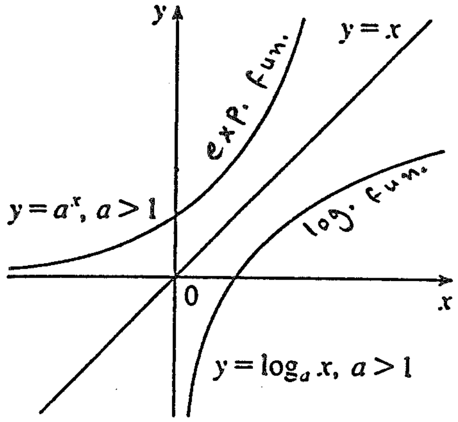
$\therefore F^{-1}(2) = 0$

(d) $F^{-1}(0)$? $\longrightarrow$ $y = 0$ على المنحنى هي نقطة تقاطع المنحنى مع محور X
يناظرها $X \simeq -1.7$

$\therefore F^{-1}(0) \approx -1.7$

الداله اللوغاريتميه

Logarithm function



- $F(x) = a^x$ الداله الأسيه
  Exponential function

* Domain $= (-\infty, \infty)$
* Range $= (0, \infty)$
and its one-to-one

- Inverse function: $F^{-1}(x) = \log_a x$
  Logarithm function

* Domain $= (0, \infty)$
* Range $= (-\infty, \infty)$
and its One-to-one

If: $\log_a x = y \Longleftrightarrow x = a^y$

where $a > 0$ and $a \neq 1$

Laws of logarithms

قوانين اللوغاريتمات.

① $\log_a (x\,y) = \log_a x + \log_a y$
الضرب يحول جمع

② $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
القسمه تحول طرح

③ $\log_a x^n = n \log_a x$

④ $\log_a a^x = x$

⑤ $a^{\log_a x} = x$

⑥ $\log_a a = 1$

⑦ $\log_a 1 = 0$

الطبيعيه اللوغاريتمات
Natural logarithms

$$\log_e x = \ln x$$

نفس الخواص السابقه لل log تنطبقه على ln

- $\ln x = y \iff x = e^y$
- $\ln e^x = x$ where $x \in R$ اي أن x ممكن تكونه موجبه أو سالبه
- $e^{\ln x} = x$ where $x > 0$ اي أن x موجب فقط

•• $\ln e = 1$ •• $\ln 1 = 0$

- $\log_a x = \dfrac{\ln x}{\ln a} \longrightarrow$ كتابه log بدلاله ln

Example: Solve the equation?

① $e^{2x-1} = 9$

بأخذ ln للطرفين

$\cancel{\ln e}^{2x-1} = \ln 9$

$2x - 1 = \ln 9$

$2x = \ln 9 + 1$ (بالضرب في $\frac{1}{2}$)

$x = \frac{1}{2}(\ln 9 + 1)$

• ملحوظه هامه •

دائماً

* للتخلص من e نأخذ ln للطرفين

* " " ln نأخذ e للطرفين

مع ملاحظه أن e ، ln

يلغوا بعضهم دائماً .

② $2\ln x = 1$

نرفع العدد 2 للأس

$\ln x^2 = 1$ بأخذ e للطرفين

$\cancel{e}^{\cancel{\ln} x^2} = e^1$

$x^2 = e$ by √

$x = \pm\sqrt{e}$

④ $e^{2x+3} - 7 = 0$

$e^{2x+3} = 7$ بأخذ ln للطرفين

$\cancel{\ln e}^{2x+3} = \ln 7$

$2x + 3 = \ln 7$

$2x = \ln 7 - 3$ بالضرب في $\frac{1}{2}$

$x = \frac{1}{2}(\ln 7 - 3)$

⑥ $2^{x-5} = 3$ بأخذ ln للطرفين

$\ln 2^{(x-5)} = \ln 3$

$(x-5)\ln 2 = \ln 3$

بقسمة الطرفين على ln 2

$x - 5 = \frac{\ln 3}{\ln 2}$

$x = \frac{\ln 3}{\ln 2} + 5$

$x = \log_2 3 + 5$

③ $e^{-x} = 5$

بأخذ ln للطرفين

$\cancel{\ln e}^{-x} = \ln 5$

$-x = \ln 5$ بالضرب في سالب

$x = -\ln 5$

⑤ $\ln(5-2x) = -3$

بأخذ e للطرفين

$\cancel{e}^{\cancel{\ln}(5-2x)} = e^{-3}$

$5 - 2x = e^{-3}$

$-2x = e^{-3} - 5$

بالضرب في $-\frac{1}{2}$

$\Rightarrow x = -\frac{1}{2}(e^{-3} - 5)$

⑦ $e^{ax} = c\,e^{bx}$ بأخذ ln للطرفين

$\ln e^{ax} = \ln(c\,e^{bx})$

$\cancel{\ln e}^{ax} = \ln c + \cancel{\ln e^{bx}}$

$ax = \ln c + bx$

الأكسات في طرف واحد

$ax - bx = \ln c$

$x(a-b) = \ln c$

$x = \frac{\ln c}{(a-b)}$

Solve the equation :

① $\ln(\ln x) = 1$ * بأخذ $\underline{\underline{e}}$ للطرفين

$\cancel{e}^{\cancel{\ln}(\ln x)} = e^1$

$\ln x = e$ * بأخذ $\underline{\underline{e}}$ للطرفين

$\cancel{e}^{\cancel{\ln} x} = e^e \Rightarrow x = e^e$

---

② $e^{e^x} = 2$ * بأخذ $\underline{\underline{\ln}}$ للطرفين

$\cancel{\ln e}^{e^x} = \ln 2$

$e^x = \ln 2$ * بأخذ $\underline{\underline{\ln}}$ للطرفين

$\cancel{\ln e}^{x} = \ln(\ln 2)$

$x = \ln(\ln 2)$

Note that

- $\log 1 = 0$, $\log 10 = 1$, $\log 100 = 2$, $\log 1000 = 3$, ...

$\log 1000000 = 6$ * الناتج هو عدد الأصفار الموجود بجوار الواحد

- $\ln e = 1$, $\ln 1 = 0$

حل المتباينة

Solve the inequality for x :

① $e^x < 10$ بأخذ ln للطرفين

$\cancel{\ln e}^{x} < \ln 10$

$x < \ln 10$

② $\ln x > -1$ بأخذ e للطرفين

$\cancel{e}^{\ln x} > e^{-1}$

$x > \frac{1}{e}$

③ $2 < \ln x < 9$

بأخذ e لكل الحدود

$e^2 < \cancel{e}^{\ln x} < e^9$

$e^2 < x < e^9$

④ $e^{2-3x} > 4$

بأخذ ln للطرفين

$\cancel{\ln e}^{2-3x} > \ln 4$

$2 - 3x > \ln 4$

$-3x > \ln 4 - 2$

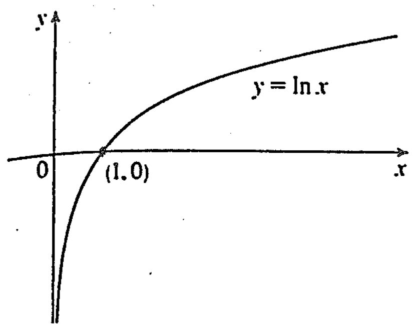
بالضرب في $-\frac{1}{3}$

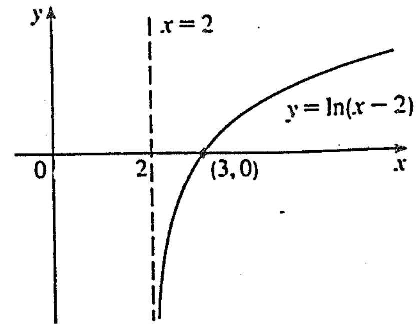
$x < -\frac{1}{3}(\ln 4 - 2)$

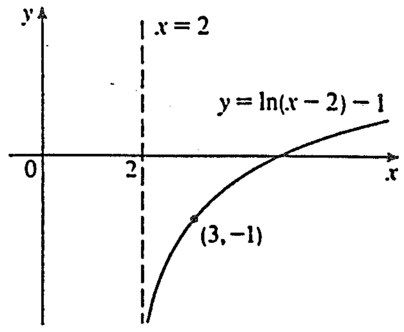
ارسم

Sketch the graph of the function :

$$y = \ln(x-2) - 1$$







Find the exact value of each expression :

① $\log_5 125$

$= \cancel{\log_5 5}^{3}$

$= 3$

② $\log_3 \frac{1}{27}$

$= \cancel{\log_3 3}^{-3}$

$= -3$

③ $\ln\left(\frac{1}{e}\right)$

$= \cancel{\ln e}^{-1}$

$= -1$

④ $\log_{10} \sqrt{10}$

$= \cancel{\log_{10} 10}^{\frac{1}{2}}$

$= \frac{1}{2}$

⑤ $\log_2 6 - \log_2 15 + \log_2 20$

$= \log_2 \left(\frac{{}^{2}\cancel{6} \times {}^{4}\cancel{20}}{\cancel{15}\,\cancel{5}\,1} \right)$

اللوغاريتمات الموجبة تحول إلى
حاصل ضرب من البسط
والسالبة من المقام

$= \log_2 8$

$= \cancel{\log_2 2}^{3} \quad = 3$

⑥ $\log_3 100 - \log_3 18 - \log_3 50$

$= \log_3 \left(\frac{100}{18 \times 50} \right)$

$= \log_3 \left(\frac{1}{9} \right)$

$= \cancel{\log_3 3}^{-2} \quad = -2$

⑦ $e^{-2\ln 5}$

$= \cancel{e}^{\cancel{\ln} 5^{-2}}$

$= 5^{-2} \quad = \frac{1}{25}$

⑧ $\ln(\cancel{\ln e}^{10})$

$= \cancel{\ln (e}^{10})$

$= 10$

⑨ $e^{2\ln 3}$

$= \cancel{e}^{\cancel{\ln} 3^{2}}$

$= 3^{2} \quad = 9$

⑩ $\log 25 + \log 4$

$= \log (25 \times 4)$

$= \log 100 \quad = \cancel{\log_{10} 10}^{2} \quad = 2$

ملحوظة :
اذا لم يذكر الأساس
فهو دائماً 10

Express the given quantity as a single logarithm:

عبر عن الكمية المعطاه كالوغاريتم واحد .

① $\ln 5 + 5 \ln 3$

$= \ln 5 + \ln 3^5$

$= \ln 5 + \ln 243 \quad = \ln (5 \times 243) \quad = \ln 1215$

② $\ln (a+b) + \ln (a-b) - 2 \ln c$

$= \ln (a+b) + \ln (a-b) - \ln c^2$

$= \ln \left(\frac{(a+b) \cdot (a-b)}{c^2} \right) \quad = \ln \left(\frac{a^2 - b^2}{c^2} \right)$

③ $\ln (1 + x^2) + \frac{1}{2} \ln x - \ln \sin x$

$= \ln (1 + x^2) + \ln x^{1/2} - \ln \sin x$

$= \ln \left(\frac{(1 + x^2) \cdot \sqrt{x}}{\sin x} \right)$

How to Find the Inverse Function

كيفيه ايجاد الداله العكسيه $F^{-1}(x)$

لداله $f(x)$ one-to-one

① نستبدل $f(x)$ ب y

② ايجاد x بدلاله y (أى x كداله فى y)

③ نستبدل x ب $F^{-1}(x)$

نستبدل y ب x

Example :

Find the inverse function of $F(x) = x^3 + 2$

Solution

$$y = x^3 + 2$$

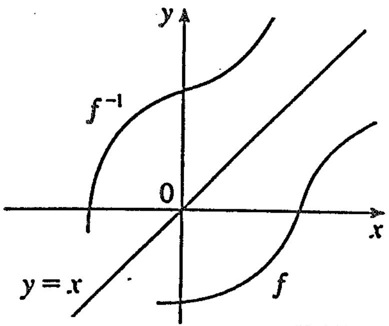
$$x^3 = y - 2 \quad (\text{by } \sqrt[3]{\ })$$

$$x = \sqrt[3]{y-2}$$

$$F^{-1}(x) = \sqrt[3]{x-2}$$

$F^{-1}(x)$ is obtained by reflecting $F(x)$ about the line $y = x$

انعكاس



Find the formula (صيغة) for the inverse (الدالة العكسية) of the function :

① $F(x) = \sqrt{10-3x}$

$y = \sqrt{10-3x}$ بالتربيع

$y^2 = 10 - 3x$

$3x = 10 - y^2$ بالضرب في $\frac{1}{3}$

$x = \frac{1}{3}(10 - y^2)$

$F^{-1}(x) = \frac{1}{3}(10 - x^2)$

② $F(x) = e^{x^3}$

$y = e^{x^3}$ بأخذ ln للطرفين

$\ln y = \cancel{\ln e}^{x^3}$

$\ln y = x^3$ by $\sqrt[3]{\ }$

$\sqrt[3]{x^3} = \sqrt[3]{\ln y}$

$x = \sqrt[3]{\ln y}$

$F^{-1}(x) = \sqrt[3]{\ln x}$

③ $y = \frac{e^x}{1+2e^x}$

$e^x = y + 2ye^x$

$e^x - 2ye^x = y$

$e^x(1-2y) = y$

$e^x = \frac{y}{1-2y}$ بأخذ ln للطرفين

$\cancel{\ln e}^x = \ln\left(\frac{y}{1-2y}\right)$

$x = \ln\left(\frac{y}{1-2y}\right)$

$F^{-1}(x) = \ln\left(\frac{x}{1-2x}\right)$

④ $F(x) = \frac{4x-1}{2x+3}$

$y = \frac{4x-1}{2x+3}$

$4x - 1 = 2xy + 3y$

الأكسات في طرف واحد

$4x - 2xy = 3y + 1$

$x(4-2y) = 3y+1$

$x = \frac{3y+1}{4-2y}$

$F^{-1}(x) = \frac{3x+1}{4-2x}$

If : $F(x) = \sqrt{3 - e^{2x}}$

① Find the domain of $F(x)$?

∵ الجذر تربيعي ∴ مجال الدالة F هو

ماتحت الجذر ≥ 0

$3 - e^{2x} \geq 0 \Rightarrow -e^{2x} \geq -3 \Rightarrow e^{2x} \leq 3$ بأخذ ln للطرفين

$\cancel{\ln e}^{2x} \leq \ln 3 \Rightarrow 2x \leq \ln 3 \Rightarrow x \leq \frac{1}{2} \ln 3$

$\Rightarrow x \leq \ln 3^{\frac{1}{2}} \Rightarrow x \leq \ln\sqrt{3}$

$\therefore$ Domain $f(x) = (-\infty, \ln\sqrt{3}]$

---

② Find $F^{-1}(x)$ and Range of $F^{-1}(x)$?

$y = \sqrt{3 - e^{2x}}$ بالتربيع $\Rightarrow y^2 = 3 - e^{2x}$

$\Rightarrow e^{2x} = 3 - y^2$ بأخذ ln للطرفين $\Rightarrow \cancel{\ln e}^{2x} = \ln(3 - y^2)$

$\Rightarrow 2x = \ln(3 - y^2) \Rightarrow x = \frac{1}{2} \ln(3 - y^2)$

$\Rightarrow x = \ln(3 - y^2)^{\frac{1}{2}} \Rightarrow x = \ln\sqrt{3 - y^2}$

$\therefore F^{-1}(x) = \ln\sqrt{3 - x^2}$

- Range $F^{-1}(x)$ = Domain $F(x) = (-\infty, \ln\sqrt{3}]$ ← ① من رقم

If: $f(x) = \ln(2 + \ln x)$

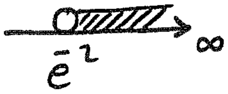
① Find the domain of $f(x)$?

To find Domain of $f(x) = \ln(2 + \ln x)$

put : $2 + \ln x > 0$

$\Rightarrow \ln x > -2$ بأخذ $\underline{\underline{e}}$ للطرفين

$\Rightarrow \cancel{e}^{\cancel{\ln} x} > e^{-2} \Rightarrow x > e^{-2}$



$\therefore$ Domain $f(x) = (e^{-2}, \infty)$

② Find $f^{-1}(x)$ and Range of $F^{-1}(x)$?

$y = \ln(2 + \ln x)$ بأخذ $\underline{\underline{e}}$ للطرفين

$e^{y} = e^{\ln(2 + \ln x)}$

$e^{y} = 2 + \ln x$

$\ln x = e^{y} - 2$ بأخذ $\underline{\underline{e}}$ للطرفين

$\cancel{e}^{\cancel{\ln} x} = e^{(e^{y} - 2)} \Rightarrow x = e^{(e^{y} - 2)} \Rightarrow F^{-1}(x) = e^{(e^{x} - 2)}$

- Range $f^{-1}(x)$ = Domain $f(x) = (e^{-2}, \infty)$ ← ①من رقم

العكسية المثلثية الدوال

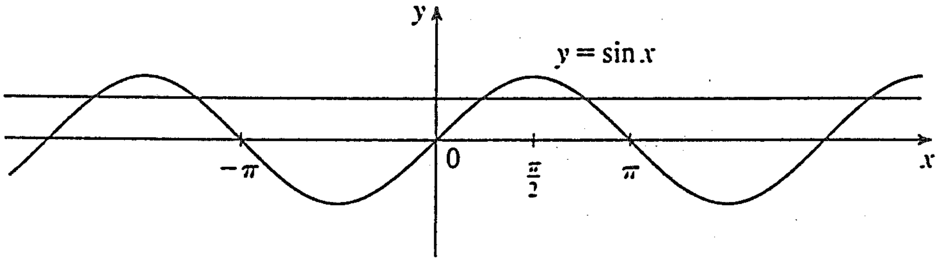
Inverse trigonometric functions :

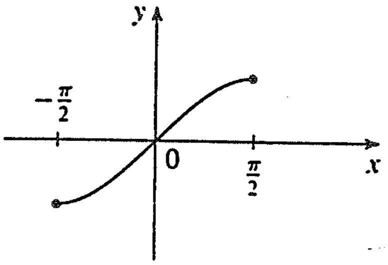
$y = \sin x$ is not one-to-one
(by the horizontal line test)

but: $y = \sin x$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$ is one-to-one

يرمز لها

and is denoted by $\underline{\underline{\sin^{-1}}}$ or $\underline{\underline{\text{arcsin}}}$



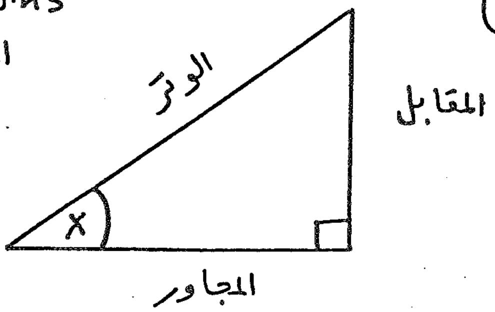


Note that :

- $\sin^{-1} x = y \iff x = \sin y$ where $y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- $\sin^{-1}(\sin x) = x$ where $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- $\sin(\sin^{-1} x) = x$ where $x \in [-1, 1]$
- $\cos^{-1}(\cos x) = x$ where $x \in [0, \pi]$
- $\cos(\cos^{-1} x) = x$ where $x \in [-1, 1]$

Remark
تذكر أن

Trigonometric functions
الدوال المثلثية



الدوال الأساسية

$\sin x = \dfrac{\text{المقابل}}{\text{الوتر}}$

$\cos x = \dfrac{\text{المجاور}}{\text{الوتر}}$

$\tan x = \dfrac{\text{المقابل}}{\text{المجاور}} = \dfrac{\sin x}{\cos x}$

المقلوبات

$\csc x = \dfrac{1}{\sin x}$

$\sec x = \dfrac{1}{\cos x}$

$\cot x = \dfrac{1}{\tan x}$

ملاحظات

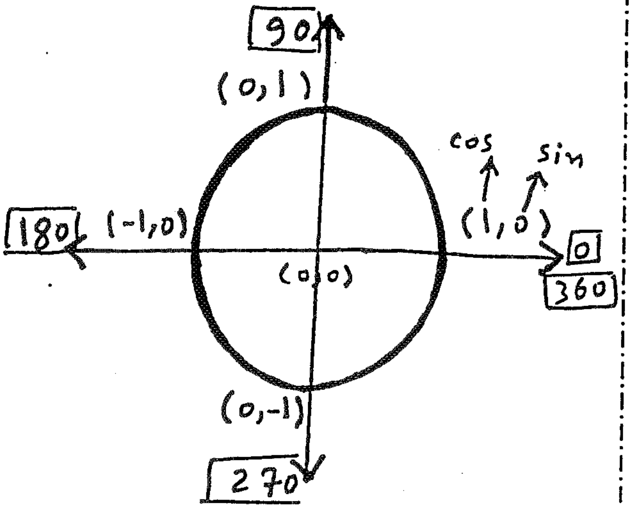
١ = مقلوبها . الدالة →

$\sin x \cdot \csc x = 1$

$\cos x \cdot \sec x = 1$

$\tan x \cdot \cot x = 1$

الدالة / الزاوية	0	$\pi/2$ 90	π 180	$3\pi/2$ 270	2π 360
Sin	0	1	0	-1	0
cos	1	0	-1	0	1
tan	0	undef.	0	undef.	0



الدالة / الزاوية	$\pi/6$ 30	$\pi/4$ 45	$\pi/3$ 60
Sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

$* \cos 2x = \cos^2 x - \sin^2 x$

$* \sin 2x = 2 \sin x \cos x$

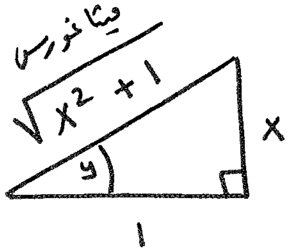
Simplify the expression :

① $\cos(\tan^{-1}x)$?

put : $y = \tan^{-1}x$

$\Rightarrow \quad \tan y = x$

$\tan y = \frac{x \rightarrow \text{المقابل}}{1 \rightarrow \text{المجاور}}$



$\therefore \cos(\tan^{-1}x)$

$= \cos(y)$

$= \frac{\text{المجاور}}{\text{الوتر}}$

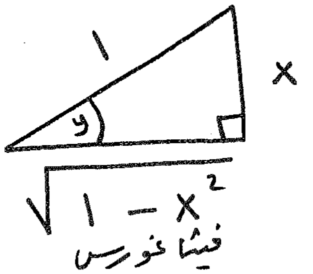
$= \frac{1}{\sqrt{x^2+1}}$

② $\tan(\sin^{-1}x)$

put : $y = \sin^{-1}x$

$\Rightarrow \quad \sin y = x$

$\sin y = \frac{x \rightarrow \text{المقابل}}{1 \rightarrow \text{الوتر}}$



$\therefore \tan(\sin^{-1}x)$

$= \tan(y)$

$= \frac{\text{المقابل}}{\text{المجاور}}$

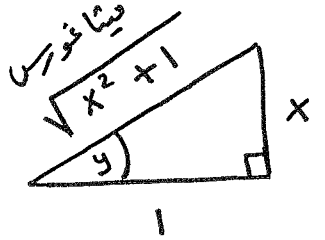
$= \frac{x}{\sqrt{1-x^2}}$

③ $\cos(2\tan^{-1}x)$

put : $y = \tan^{-1}x$

$\Rightarrow \quad \tan y = x$

$\tan y = \frac{x \rightarrow \text{المقابل}}{1 \rightarrow \text{المجاور}}$



$\therefore \cos(2\tan^{-1}x)$

$= \cos(2y)$ قانون

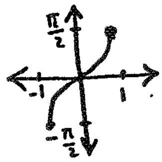
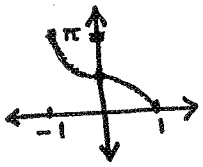
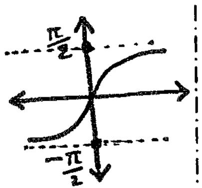
$= \cos^2 y - \sin^2 y$

$= \left(\frac{1}{\sqrt{x^2+1}}\right)^2 - \left(\frac{x}{\sqrt{x^2+1}}\right)^2$

$= \frac{1}{x^2+1} - \frac{x^2}{x^2+1}$

$= \frac{1-x^2}{x^2+1}$

Domain and Range for inverse trigonometric functions.

y	Domain $= x$	Range $= y$
$\sin^{-1} x = y$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$ 
$\cos^{-1} x = y$	$[-1, 1]$	$[0, \pi]$ 
$\tan^{-1} x = y$	$x \in R$ $= (-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$ 
$\cot^{-1} x = y$	$x \in R$ $= (-\infty, \infty)$	$(0, \pi)$
$\sec^{-1} x = y$	$\lvert x \rvert \geqslant 1$	$(0, \frac{\pi}{2}] \cup (\pi, \frac{3\pi}{2}]$
$\csc^{-1} x = y$	$\lvert x \rvert \geqslant 1$	$[0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$

Note that: هام

- IF: $\sin^{-1} a = b$
  $\Rightarrow \sin^{-1} -a = -b$
- IF: $\tan^{-1} a = b$
  $\Rightarrow \tan^{-1} -a = -b$
- IF: $\cos^{-1} a = b$
  $\Rightarrow \cos^{-1} -a = 180 - b$

Find the exact value of each expression:

① $\sin^{-1}(\sqrt{3}/2)$

$= 60^\circ$

$= \frac{\pi}{3} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

② $\cos^{-1}(-1)$

$= 180^\circ$

$= \pi \quad \in [0, \pi]$

③ $\arctan(1)$

$= \tan^{-1}(1) = 45^\circ$

$= \frac{\pi}{4} \quad \in \quad (-\frac{\pi}{2}, \frac{\pi}{2})$

④ $\tan(\arctan 10)$

$= \cancel{\tan}(\cancel{\tan^{-1}} 10)$

$= 10$

⑤ $\cancel{\sin^{-1}}(\cancel{\sin}(\frac{7\pi}{3}))$

$= \frac{7\pi}{3}$

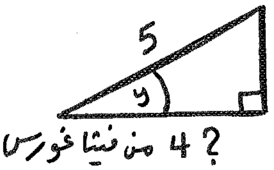
⑥ $\tan^{-1}(-1) = -45^\circ$

$= -\frac{\pi}{4} \in (-\frac{\pi}{2}, \frac{\pi}{2})$

⑦ $\sin(2\sin^{-1}(3/5))$

put: $y = \sin^{-1}\frac{3}{5}$

$\Rightarrow \sin y = \frac{3}{5}$ 3 → المقابل
5 → الوتر



$\therefore \sin(2\sin^{-1}(3/5))$

$= \sin(2y)$

$= 2\sin y \cos y$

$= 2\cdot(\frac{3}{5})\cdot(\frac{4}{5})$

$= \frac{24}{25}$

⑧ $\cot^{-1}(-\sqrt{3})$

$= 180 - 30$

$= 150 = \frac{5\pi}{6} \in (0, \pi)$

⑨ $\sin^{-1}(\frac{1}{2}) = 30^\circ$

$= \frac{\pi}{6} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

⑩ $\sin^{-1}(-\frac{1}{2}) = -30^\circ$

$= -\frac{\pi}{6} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

⑪ $\cot^{-1}(\sqrt{3})$

$= 30 = \frac{\pi}{6} \in (0, \pi)$

⑫ $\cos^{-1}(\frac{1}{\sqrt{2}}) = 45^\circ$

$= \frac{\pi}{4} \in [0, \pi]$

⑬ $\sec^{-1}(2)$

$= \cos^{-1}(\frac{1}{2}) =$

$= 60 = \frac{\pi}{3} \in [0, \frac{\pi}{2})$

⑭ $\cos^{-1}(-\frac{1}{\sqrt{2}}) = 180 - 45$

$= 135 = \frac{3\pi}{4} \in [0, \pi]$

Exercises :

الداله المعطاه بجدول

A function is given by a table of values,

صيغه وصف بالكلام

a formula , or a verbal description.

Determine whether it is one-to-one.

①

X	1	2	3	4	5	6
F(X)	1.5	2.0	3.6	5.3	2.8	2.0

$\Rightarrow$ Not one-to-one

* لوجود عنصران مختلفان 2, 6

لهما نفس الصوره 2.0

②

X	1	2	3	4	5	6
F(X)	1	2	4	8	16	32

$\Rightarrow$ One-to-one

* لأنه كل العناصر X مختلفه

لهما صور F(x) مختلفه .

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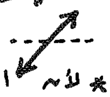
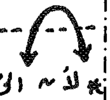
③ $F(x) = \frac{1}{2}(x+5)$



One-to-one

* لأنه الخط الأفقى يقطع الداله فى نقطه واحده

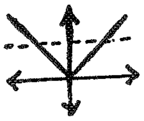
④ $F(x) = 1 + 4x - x^2$



Not one-to-one

* لأنه الخط الأفقى يقطع منحنى الداله فى أكثر من نقطه

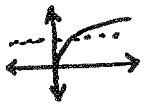
⑤ $g(x) = |x|$



Not one-to-one

* لأنه الخط الأفقى يقطع الداله فى أكثر من نقطه .

⑥ $g(x) = \sqrt{x}$



One-to-one

* لأنه الخط الأفقى يقطع الداله فى نقطه واحده .

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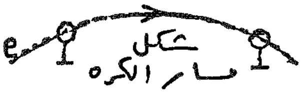
⑦ F(t) is the height of a football t seconds after kickoff. $\Rightarrow$ Not one-to-one



شكل مسار الكره

الخط الأفقى يقطع مسار الكره فى أكثر من نقطه .

⑧ F(t) is your height at age t . $\Rightarrow$ Not one-to-one

t: عمرك يتغير من 20 سنه إلى 30 سنه

F(t): 180 cm وطولك ثابت ولكن

السعدى

كل التمنيات بالنجاح والتوفيق