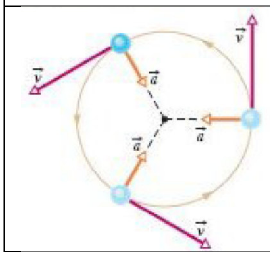
CHAPTER(4) Motion in Two and Three Dimensions

	1D	2/3 D
1-Position	In x-axis ➔ x In y-axis ➔ y In z-axis ➔ z	*Position vector:* $(\vec{r})$ $\vec{r} = x\,i + y\,j + z\,k$
2-Displacement	$\Delta x = x_2 - x_1$ $\Delta y = y_2 - y_1$ $\Delta z = z_2 - z_1$	$\Delta r = r_2 - r_1$ $\Delta r = \Delta x\,i + \Delta y\,j + \Delta z\,k$
3-Velocity: **-Average Velocity**	$\upsilon_{avg} = \frac{\Delta x}{\Delta t}$	*Velocity Vector* $\upsilon_{avg} = \frac{\Delta r}{\Delta t} = \frac{r_2 - r_1}{\Delta t}$ $= \frac{\Delta x}{\Delta t} i + \frac{\Delta y}{\Delta t} j + \frac{\Delta z}{\Delta t} k$
- Velocity	$\upsilon = \frac{dx}{dt}$	$v = \frac{dr}{dt} = \frac{dx}{dt} i + \frac{dy}{dt} j + \frac{dz}{dt} k$ $v = \frac{dr}{dt} = v_x i + v_y j + v_z k$ The direction of v of a particle is always tangent to the particle's path at the particle's position.
4- Acceleration Average acceleration:	$a_{avg} = \frac{\Delta \upsilon}{\Delta t} = \frac{\Delta x}{(\Delta t)^2}$	$a_{avg} = \frac{\Delta \upsilon}{\Delta t} = \frac{r_2 - r_1}{(\Delta t)^2} = \frac{\Delta x}{\Delta t^2} i + \frac{\Delta y}{\Delta t^2} j + \frac{\Delta z}{\Delta t^2} k$
Acceleration	$a = \frac{d\upsilon}{dt} = \frac{d^2 x}{dt^2}$	$a = \frac{dv}{dt} = \frac{dv_x}{dt} i + \frac{dv_y}{dt} j + \frac{dv_z}{dt} k$ $a = \frac{d^2 r}{dt^2} = \frac{d^2 x}{dt^2} i + \frac{d^2 y}{dt^2} j + \frac{d^2 z}{dt^2} k$
Magnitude & Driection	$V =$ (±) direction (no) magnitude	$\lvert V \rvert = V = \sqrt{V_x{}^2 + V_y{}^2}$ $\theta = \tan^{-1} \frac{V_y}{V_x}$
6-Uniform circular Motion 	//	$V =$ (dir.) (mag.) variable (tangent) Const. (speed=|v|) $a =$ (dir.) (mag.) variable (inward) Const. $(a = \frac{v^2}{r})$ Period= $T = \frac{Circumference(distance)}{speed} = \frac{2\pi r}{\lvert v \rvert}$

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Projectiles المقذوفات

1D

Const. acceleration

الحركة الأفقية بتسارع ثابت

Horizontal motion(x-axis)

1- $v = v_0 + a\,t$

2- $x - x_0 = v_0 t + \frac{1}{2} a t^2$

3- $v^2 = v_0^2 + 2\,a(x - x_0)$

4- $x - x_0 = \frac{1}{2}\ (v + v_0)\ t$

5- $x - x_0 = vt - \frac{1}{2} a t^2$

Free Falling

الحركة العمودية بتسارع ثابت

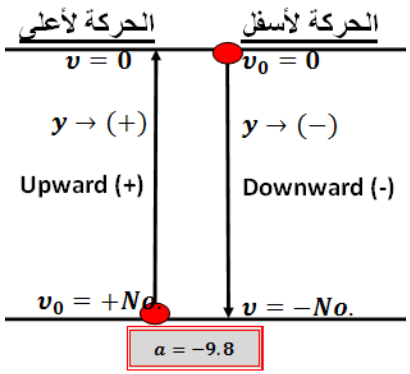
Vertical motion(y-axis)

1- $v = v_0 - g\,t$

2- $y - y_0 = v_0 t - \frac{1}{2} g t^2$

3- $v^2 = v_0^2 - 2\,g(y - y_0)$

4- $y - y_0 = \frac{1}{2}\ (v + v_0)\ t$



2D

Projectile motion

$v_0 = v_{0x} i + v_{0y} j$ $\quad a = a_x i + a_y j$

Horizontal motion

$a_x = 0,\ v_{0x} = v_0 cos\theta$

1- $v_x = v_{0x}$

2- $x = v_{0x} t$

Horizontal range (R)=

$$R = \frac{v_0^2}{g} \sin 2\theta$$

Maximum range

$\theta = 45^0$ ➔ $R_{max} = \frac{v_0^2}{g}$

Vertical motion

$a_y = -g,\ v_{0y} = v_0 \sin\theta$

1- $v_y = v_{0y} - g\,t$

2- $y = v_{0y} t - \frac{1}{2} g t^2$

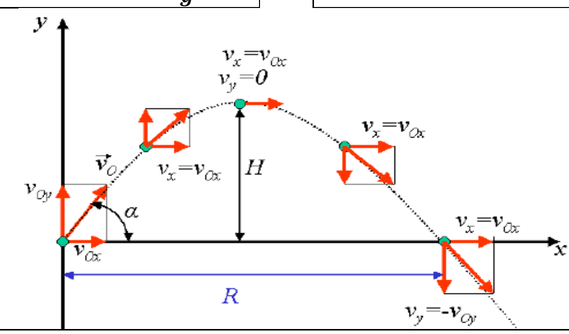
3- $v_y^2 = v_{0y}^2 - 2\,gy$

Vertical height (H)=

$H = \frac{(v_0 \sin\theta)^2}{2g}$, where $v_y = 0$

Maximum height

$\theta = 90^0$ ➔ $H_{max} = \frac{v_0^2}{2g}$



Problems:

1- A particle goes from $x = -2\,\text{m}$, $y = 3\,\text{m}$, $z = 1\,\text{m}$ to $x = 3\,\text{m}$, $y = -1\,\text{m}$, $z = 4\,\text{m}$. Its displacement is:

A. $(1\,\text{m})\,\hat{\text{i}} + (2\,\text{m})\,\hat{\text{j}} + (5\,\text{m})\,\hat{\text{k}}$
B. $(5\,\text{m})\,\hat{\text{i}} - (4\,\text{m})\,\hat{\text{j}} + (3\,\text{m})\,\hat{\text{k}}$
C. $-(5\,\text{m})\,\hat{\text{i}} + (4\,\text{m})\,\hat{\text{j}} - (3\,\text{m})\,\hat{\text{k}}$
D. $-(1\,\text{m})\,\hat{\text{i}} - (2\,\text{m})\,\hat{\text{j}} - (5\,\text{m})\,\hat{\text{k}}$
E. $-(5\,\text{m})\,\hat{\text{i}} - (2\,\text{m})\,\hat{\text{j}} + (3\,\text{m})\,\hat{\text{k}}$

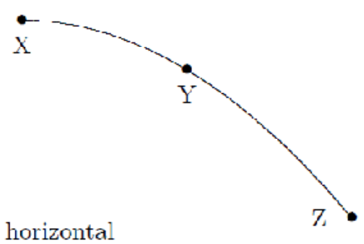
ans: B

2- A stone thrown from the top of a tall building follows a path that is:

A. circular
B. made of two straight line segments
C. hyperbolic
D. parabolic
E. a straight line

ans: D

3- A stone is thrown horizontally and follows the path XYZ shown. The direction of the acceleration of the stone at point Y is:



A. ↓
B. →
C. ↘
D. ↙
E. ↗

ans: A

4- A large cannon is fired from ground level over level ground at an angle of 30° above the horizontal. The muzzle speed is $980\,\text{m/s}$. Neglecting air resistance, the projectile will travel what horizontal distance before striking the ground?

A. 4.3 km
B. 8.5 km
C. 43 km
D. 85 km
E. 170 km

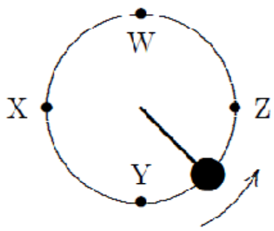
ans: D

5- A projectile is fired from ground level over level ground with an initial velocity that has a vertical component of $20\,\text{m/s}$ and a horizontal component of $30\,\text{m/s}$. Using $g = 10\,\text{m/s}^2$, the distance from launching to landing points is:

A. 40 m
B. 60 m
C. 80 m
D. 120 m
E. 180 m

ans: D

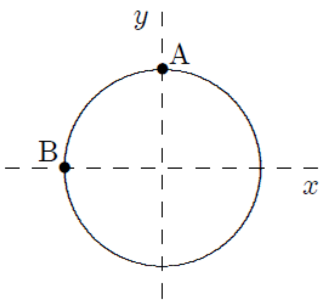
6- An object, tied to a string, moves in a circle at constant speed on a horizontal surface as shown. The direction of the displacement of this object, as it travels from W to X is:



A. ←
B. ↓
C. ↑
D. ↗
E. ↙

ans: E

7- A toy racing car moves with constant speed around the circle shown below. When it is at point A its coordinates are $x = 0$, $y = 3\,\mathrm{m}$ and its velocity is $(6\,\mathrm{m/s})\,\hat{\mathrm{i}}$. When it is at point B its velocity and acceleration are:



A. $-(6\,\mathrm{m/s})\,\hat{\mathrm{j}}$ and $(12\,\mathrm{m/s^2})\,\hat{\mathrm{i}}$, respectively
B. $(6\,\mathrm{m/s})\,\hat{\mathrm{i}}$ and $-(12\,\mathrm{m/s^2})\,\hat{\mathrm{i}}$, respectively
C. $(6\,\mathrm{m/s})\,\hat{\mathrm{j}}$ and $(12\,\mathrm{m/s^2})\,\hat{\mathrm{i}}$, respectively
D. $(6\,\mathrm{m/s})\,\hat{\mathrm{i}}$ and $(2\,\mathrm{m/s^2})\,\hat{\mathrm{j}}$, respectively
E. $(6\,\mathrm{m/s})\,\hat{\mathrm{j}}$ and 0, respectively

ans: C

8- An object is moving on a circular path of radius π meters at a constant speed of $4.0\,\mathrm{m/s}$. The time required for one revolution is:

A. $2/\pi^2\,\mathrm{s}$
B. $\pi^2/2\,\mathrm{s}$
C. $\pi/2\,\mathrm{s}$
D. $\pi^2/4$
E. $2/\pi\,\mathrm{s}$

ans: B

9- A particle moves at constant speed in a circular path. The instantaneous velocity and instantaneous acceleration vectors are:

A. both tangent to the circular path
B. both perpendicular to the circular path
C. perpendicular to each other
D. opposite to each other
E. none of the above

ans: C

10 A car rounds a 20-m radius curve at $10\,\mathrm{m/s}$. The magnitude of its acceleration is:

A. 0
B. $0.20\,\mathrm{m/s^2}$
C. $5.0\,\mathrm{m/s^2}$
D. $40\,\mathrm{m/s^2}$
E. $400\,\mathrm{m/s^2}$

ans: C