3.2 : Equations of Lines and Linear Models

معادلات الخط المستقيم .

(x, y) , (x_1, y_1)

slope (الميل) $\leftarrow$ $m = \dfrac{y - y_1}{x - x_1}$

$(x_1, y_1), (x_2, y_2) \Longrightarrow$ slope : $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point – slope Form:-

let m slope , point (x_1, y_1)

$$\boxed{y - y_1 = m(x - x_1)}$$

معادلة الخط المستقيم بدلالة نقطة وميل .

Example 1. p(99) :~

Write an equation of Line through $(-4, 1)$, having slope -3

أوجد معادلة الخط المستقيم المار بالنقطة $(-4,1)$ وميله -3 .

$x_1 = -4$, $y_1 = 1$, $m = -3$

$y - y_1 = m(x - x_1)$

$y - 1 = -3(x - (-4))$

$y - 1 = -3(x + 4)$

$y - 1 = -3x - 12$ $\Longrightarrow$ $\boxed{y = -3x - 11}$

HW 1 P.(99)

Write an equation of the line through $(-3,2)$ and $(2,-4)$. Write the result in standard form

$$Ax+By=C$$

اكتبي معادلة الخط المستقيم المار بالنقطتين .

* هنا في السؤال : الميل غير موجود .
أولاً نوجد الميل :

Find Slope , $(\overset{x_1}{-3},\overset{y_1}{2})$, $(\overset{x_2}{2},\overset{y_2}{-4})$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-2}{2-(-3)}=\frac{-6}{5}$$

ثانياً : نعوض في معادلة الخط المستقيم $y-y_1=m(x-x_1)$ بأحد أي نقطة من النقاط .

Let the point $(-3,2) \Rightarrow x_1=-3,\ y_1=2$

$$m=-\frac{6}{5}$$

$$y-y_1=m(x-x_1)$$

$$y-2=-\frac{6}{5}(x-(-3))$$

$$y-2=-\frac{6}{5}(x+3)$$

$$y-2=-\frac{6}{5}x-\frac{18}{5}$$

$$y=-\frac{6}{5}x-\frac{18}{5}+2$$

$$y=-\frac{6}{5}x-\frac{18}{5}+\frac{10}{5}$$

$$y=-\frac{6}{5}x-\frac{8}{5}$$

ملاحظة : عند التعويض في المعادلة المستقيم يكون عندنا حرية اختيار أحدى النقطتين

لو اخترنا النقطة الثانية $(2,-4)$ نطلع بنفس الناتج

Standard form: $y+\frac{6}{5}x=-\frac{8}{5}$

$$5y+6x=-8$$

slope - Intercept Form:

$y = mx + b$ → b: y-intercept (الجزء الذي يقطع محور y)

m: slope (الميل)

Example 2 p.(100) :- Find the slope and y-intercept of the Line with equation

$$4x + 5y = -10$$

(أوجدي الميل والجزء الذي يقطع محور y)

$$5y = -10 - 4x$$

$$y = -\frac{4}{5}x - \frac{10}{5}$$

$$y = \left(-\frac{4}{5}\right)x + (-2)$$

$-\frac{4}{5}$: slope, -2: y-intercept

∴ slope $m = -\frac{4}{5}$, y-intercept -2

HW2 p.(100) :- Write an equation of the Line through (1,1) and (2,4). Then graph (ارسمي) the Line using the slope intercept form.

① Find the slope: $(x_1, y_1), (x_2, y_2)$ $(1,1), (2,4)$ (نوجد الميل) (أوجدي معادلة المستقيم المار بالنقطتين)

$$m = \frac{y_2 - y_1}{x_2 - x} = \frac{4-1}{2-1} = \frac{3}{1} = 3$$

②

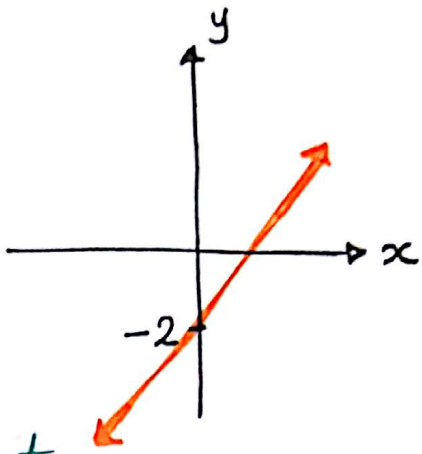
$$y - y_1 = m(x - x_1) \quad \text{at point } (1,1)$$

$$y - 1 = 3(x - 1)$$

$$y - 1 = 3x - 3$$

$$y = 3x - 3 + 1$$

$$y = \underbrace{3}_{\text{slope}}x \underbrace{-2}_{y\text{-intercept}}$$

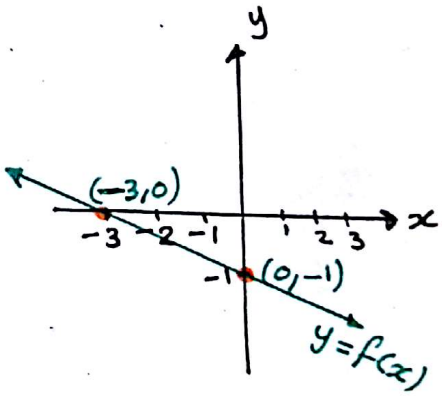


Example 3 p. (100) :~ use the graph استخدمي الرسم

ⓐ Find the slope, y-intercept, x-intercept

ⓑ Write the equation that defines f.

ⓐ We have the points $\overset{x_1\ y_1}{(-3,0)}$, $\overset{x_2\ y_2}{(0,-1)}$



$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 0}{0 - (-3)}$$

$$= \frac{-1}{3}$$

الجزء الذي يقطع محور y : y-intercept -1

الجزء الذي يقطع محور x : x-intercept -3

ⓑ $m = -\frac{1}{3}$, point $(-3,0)$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{1}{3}(x - (-3))$$

$$y = -\frac{1}{3}(x + 3) = -\frac{1}{3}x - 1$$

$$\boxed{f(x) = -\frac{1}{3}x - 1}$$

General Form: الصيغة العامة

$$Ax + By = C$$

$$By = -Ax + C$$

$$y = \left(\frac{-A}{B}\right)x + \left(\frac{C}{B}\right)$$

$\frac{C}{B}$ → y-intercept

$\frac{-A}{B}$ → slope

$$Ax = -By + C$$

$$x = \frac{-B}{A}y + \left(\frac{C}{A}\right)$$

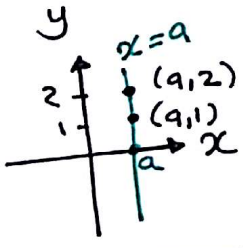
$\frac{C}{A}$ → x-intercept

Equations of Vertical and Horizontal Line

Vertical Line الخط الرأسي

(a,b)

$x = a$



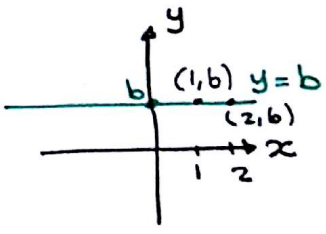
slope: undefined

$m = \frac{2-1}{a-a} = \frac{1}{0}$ undefined

Horizontal Line الخط الأفقي

(a,b)

$y = b$



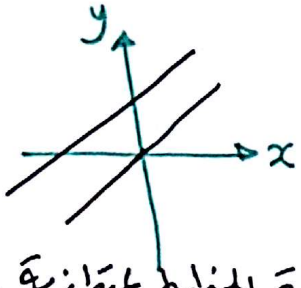
Slope = 0

$m = \frac{b-b}{2-1} = \frac{0}{1} = 0$

Parallel and Perpendicular Lines

Parallel Lines //

خطوط متوازية



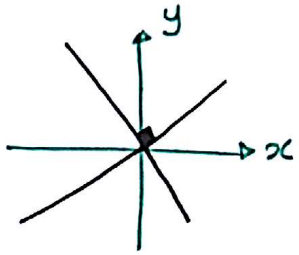
→ slope of parallel line - equal

∴ ميزة الخطوط المتوازية - لها نفس الميل

Same slope.

Perpendicular Lines ⊥

خطوط متعامدة



ميزة الخطوط المتعامدة :

حاصل ضرب ميلهم = -1

$m_1 m_2 = -1$

HW3 · p. (102) :~

Write the equation of line that passes through the point (3, 5)

أوجدي معادلة الخط المستقيم المار بالنقطة (3,5).

ⓐ Parallel to the line $2x + 5y = 4$

- المتوازي للخط ← الخطوط المتوازية لها نفس الميل.

① Find the slope. نوجد الميل

$2x + 5y = 4 \Longrightarrow 5y = -2x + 4$

$y = -\frac{2}{5}x + \frac{4}{5}$

∴ Slope $m = -\frac{2}{5}$

② Find the equation of line , $(3,5)$ x_1, y_1

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{2}{5}(x - 3)$$

$$y - 5 = -\frac{2}{5}x + \frac{6}{5}$$

$$y = -\frac{2}{5}x + \frac{6}{5} + 5$$

$$y = -\frac{2}{5}x + \frac{6+25}{5}$$

$$\boxed{y = -\frac{2}{5}x + \frac{31}{5}}$$

Standard form: $\boxed{5y + 2x = 31}$

(b) perpendicular to the line $2x + 5y = 4$

العمودي على المستقيم

من الفقرة a وجدنا أن $m_1 = -\frac{2}{5}$

الحل ← في المستقيمات المتعامدة حاصل ضرب ميلهم $= -1$

$$m_1 \quad (-\frac{2}{5})(\frac{5}{2}) = -1 \quad m_2$$

$\therefore m_2 = \frac{5}{2}$ ← slope

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{5}{2}(x - 3)$$

$$y - 5 = \frac{5}{2}x - \frac{15}{2}$$

$$y = \frac{5}{2}x - \frac{15}{2} + 5$$

$$y = \frac{5}{2}x \; \frac{-15+10}{2}$$

$$\boxed{y = \frac{5}{2}x - \frac{5}{2}}$$

Standard form : $\boxed{2y - 5x = -5}$

Exercises 3.2 P. (103)

Write an equation for the line.

(7) through $(-1, 3)$ and $(3, 4)$

① Find the slope : $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{3-(-1)} = \frac{1}{4}$

② Find the equation : $y - y_1 = m(x - x_1)$

نختار نقطة من النقاط $(-1, 3)$

$$y - 3 = \frac{1}{4}(x - (-1))$$

$$y - 3 = \frac{1}{4}(x + 1)$$

$$y - 3 = \frac{1}{4}x + \frac{1}{4}$$

$$y = \frac{1}{4}x + \frac{1}{4} + 3$$

$$\boxed{y = \frac{1}{4}x + \frac{13}{4}}$$

Standard form:

$$\boxed{4y - x = 13}$$

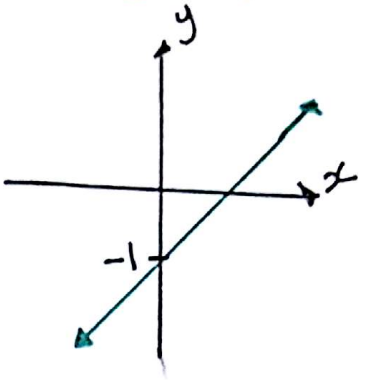
Give the slope and y-intercept and graph it

أوجد الميل ، الجزء الذي يقطع المحور y ، ارسم .

(15) $y = 3x - 1$

3 → Slope

-1 → y-intercept



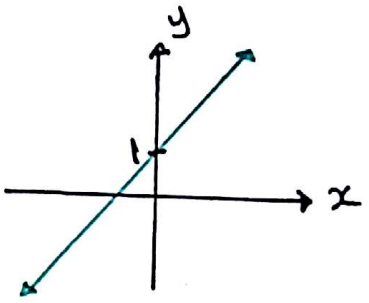
Slope : $m = 3$, y-intercept -1

(19) $y - \frac{3}{2}x - 1 = 0$

$y = \frac{3}{2}x + 1$

$\frac{3}{2}$ → Slope

+1 → y-intercept



Slope : $m = \frac{3}{2}$, y-intercept 1

(26) Write an equation of line through $(-5, 6)$,

perpendicular to $x = -2$ خط رأسي

متعامد

المستقيم المتعامد للخط الرأسي هو الخط الأفقي .

المستقيم العامودي على المستقيم $x = -2$ هيا بالنقطة $(-5, 6)$ هو خط افقي $\boxed{y = 6}$