مدونة المناهج السعودية

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الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

Exercises

Show that the given function is continuous at the given number

$f(x) = x^2 - 3x^2 + x - 2 \ , \ a = 2$

$D(f) = \mathbb{R}$

$\because \ 2 \in D(f)$

$\therefore$ f is continuous at $x = 2$.

$f(x) = \sqrt{x+1} - 3x \ , \ a = 3$

- $f(3) = \sqrt{3+1} - 3(3) = -7$
- $\lim\limits_{x \to 3} f(x) = \lim\limits_{x \to 3} \sqrt{x+1} - 3 = -7$
- $\lim\limits_{x \to 3} f(x) = f(3)$

$\therefore$ f is continuous at $a = 3$

$$f(x) = \begin{cases} \frac{\sin 3x}{x}, x \neq 0 \\ \frac{1}{3}, x = 0 \end{cases}$$

1)_ $f(0) = \frac{1}{3}$

2)_ $\lim\limits_{x \to 0} \frac{\sin 3x}{x} = \frac{3}{1} = 3$

3)_ $\lim\limits_{x \to 0} f(x) \neq f(0)$

$\therefore$ f is discontinuous at $x = 0$

$$F(x) = \begin{cases} \dfrac{x^2-2x-8}{x-4}, & x<4 \\ 2x-2, & x \geq 4 \end{cases}$$

1)- $F(4) = 2(4) - 2 = 6$

2)- • $\lim\limits_{x \to 4^-} F(x) = \lim\limits_{x \to 4^-} \dfrac{x^2-2x-8}{x-4}$

$$= \lim_{x \to 4^-} \frac{(x-4)(x+2)}{x-4}$$

$$= \lim_{x \to 4^-} x+2 = 4+2 = 6$$

• $\lim\limits_{x \to 4^+} F(x) = \lim\limits_{x \to 4^+} 2x-2 = 2(4) - 2 = 6$

$\Rightarrow \lim\limits_{x \to 4} f(x) = 6$

3) $\lim\limits_{x \to 4} F(x) = F(4)$

$\therefore$ F is continuous at $x=1$.

Show that the given function is continuous at the given interval

$f(x) = \sqrt[4]{x-3} + x \qquad [3, \infty)$

$g(x) = \sqrt[4]{x-3}$

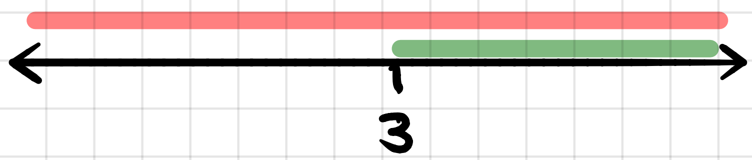
$h(x) = x$

g is continuous iff $x-3 \geq 0$

$\Rightarrow x \geq 3$

$h(x)$ is continuous on R

$\therefore D(g) = [3, \infty)$

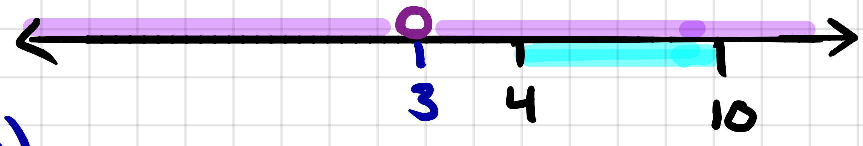


F is continuous on $[3, \infty)$

$$F(x) = \frac{1}{x-3} \qquad [4,10]$$

F is continuous iff $x-3 \neq 0$

$x \neq 3$



$\therefore D(F) = \mathbb{R} - \{3\} = (-\infty, 3) \cup (3, \infty)$

$\because [4,10] \subset D(F) \Rightarrow F$ is continuous on $[4,10]$.

Find all the numbers for which the given function is discontinuous

$$F(x) = \frac{x-1}{x^3 - x^2 + 4x - 4}$$

F is continuous iff $x^3 - x^2 + 4x - 4 \neq 0$

$\Rightarrow x^2(x-1) + 4(x-1) \neq 0$

$\Rightarrow (x-1)(x^2+4) \neq 0$

$\Rightarrow x-1 \neq 0 \Rightarrow x \neq 1$

F is continuous on $\mathbb{R} - \{1\} = (-\infty, 1) \cup (1, \infty)$

F is discontinuous at $x = 1$

$F(x) = [\![x-1]\!]$

F is discontinuous at $n \in \mathbb{Z}$

F is discontinuous from the left. for $n \in \mathbb{Z}$

F continuous from the right. for $n \in \mathbb{Z}$.

$$F(x) = \frac{x+1}{\sqrt{2x-1} - 3}$$

F is continuous iff

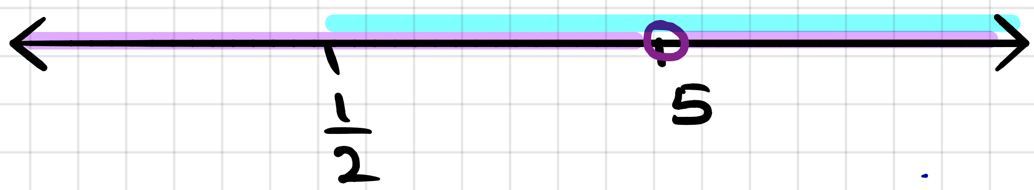
$\sqrt{2x-1} - 3 \neq 0$ and $2x - 1 \geqslant 0$

$\sqrt{2x-1} \neq 3$ $\qquad 2x \geqslant 1$

$2x - 1 \neq 9$ $\qquad x \geqslant \frac{1}{2}$

$2x \neq 10$

$x \neq 5$



$\therefore$ F is continuous on $[\frac{1}{2}, 5) \cup (5, \infty)$

F is discontinuous on $(-\infty, \frac{1}{2}) \cup \{5\}$

$$F(x) = \begin{cases} \dfrac{\sin(3x-6)}{x-2}, & x \neq 2 \\ 3, & x = 2 \end{cases}$$

1)- $F(2) = 3$

2)- $\lim\limits_{x \to 2} F(x) = \lim\limits_{x \to 2} \dfrac{\sin(3x-6)}{x-2}$

$= \lim\limits_{x \to 2} \dfrac{\sin 3(x-2)}{x-2}$

$= \lim\limits_{x \to 2} \dfrac{3 \sin(x-2)}{x-2} = 3$

3)- $\lim\limits_{x \to 2} F(x) = F(2) = 0$

F is continuous at $x = 2$

$$q(x) = \frac{x-2}{\sqrt{x^2-4}}$$

$q(x)$ is continuous iff

$$x^2 - 4 > 0$$

$$x^2 > 4$$

$$\sqrt{x^2} > \sqrt{4}$$

$$|x| > 2$$

$\Rightarrow \quad x > 2 \quad$ or $\quad x < 2$

$\therefore$ f is continuous on $(-\infty, -2) \cup (2, \infty)$

f is discontinuous on $[-2, 2]$.

$$f(x) = \frac{x+1}{x|x^2-1|}$$

$$x|x^2-1| \neq 0$$

$\Rightarrow \quad x \neq 0 \quad$ or $\quad |x^2-1| \neq 0$

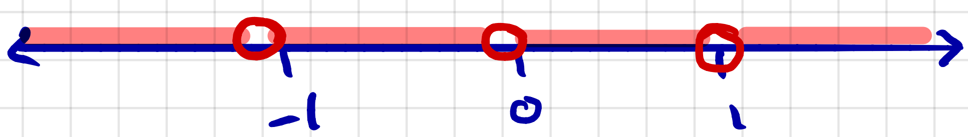
$$x^2 - 1 \neq 0$$

$$x^2 \neq 1$$

$$x \neq \pm 1$$

f is continuous on $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$

f is discontinuos on $-1, 0, 1$



Find all numbers for which the given function is continuous

$$f(x) = \sqrt[12]{3x-6} + \frac{1}{x-7}$$

$$g(x) = \sqrt[12]{3x-6}$$

$$h(x) = \frac{1}{x-7}$$

g is continuous iff $3x-6 \geq 0$

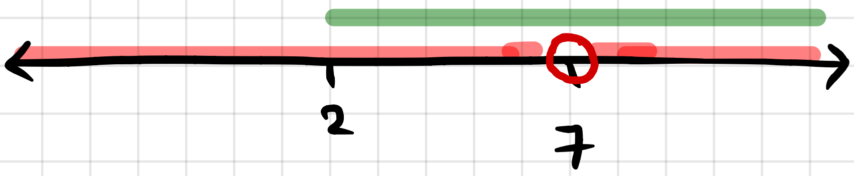
$$3x \geq 6$$

$$x \geq 2$$

h is continuous iff

$$x-7 \neq 0$$

$$\Rightarrow \quad x \neq 7$$



$\therefore$ f is continuous on

$[2,7) \cup (7,\infty)$

$$f(x) = \frac{3x-4}{(x^2-5)(x^2-4x+3)}$$

f is continuous iff

$$(x^2-5)(x^2-4x+3) \neq 0$$

$$x^2-5 \neq 0$$

$$x^2 \neq 5$$

$$\sqrt{x^2} \neq \sqrt{5}$$

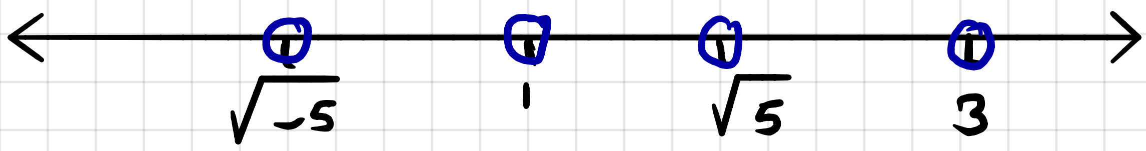
$$x \neq \pm\sqrt{5}$$

$$x^2-4x+3 \neq 0$$

$$(x-1)(x-3) \neq 0$$

$$\Rightarrow \quad x \neq 1 \;,\; x \neq 3$$

$D = \mathbb{R} - \{\pm\sqrt{5}, 1\}$



$\therefore$ f is continuous on

$(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, 1) \cup (1, \sqrt{5}) \cup (\sqrt{5}, 3) \cup (3, \infty)$

Find the constant and Such that the function is continuous on the real entire real line

$$f(x) = \begin{cases} \dfrac{\sin(\alpha x)}{x} & , x \neq 0 \\ 4 & , x = 0 \end{cases}$$

1)_ $f(0) = 4$

2)_ $\lim\limits_{x \to 0} \dfrac{\sin(\alpha x)}{x} = \alpha$

f is continuous iff $\lim\limits_{x \to 0} f(x) = f(0)$

$\Rightarrow \quad \alpha = 4$

$$F(x) = \begin{cases} 3x+1 & , x \geqslant 2 \\ \alpha x^2 - 1 & , x < 2 \end{cases}$$

F is continuous iff

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

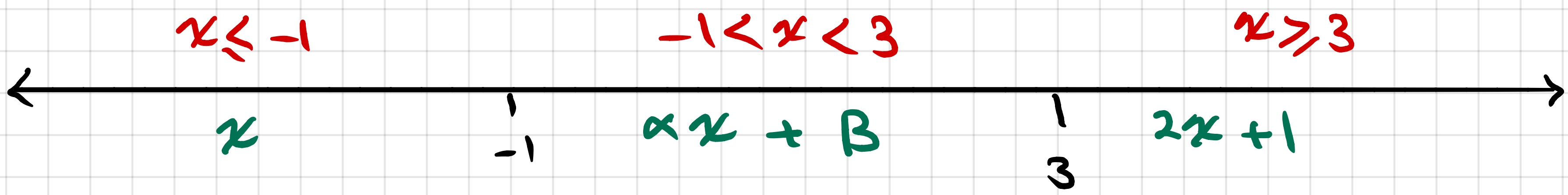
$$\lim_{x \to 2^-} \alpha x^2 - 1 = \lim_{x \to 2^+} 3x+1$$

$$\alpha (2)^2 - 1 = 3(2) + 1$$

$$4\alpha - 1 = 7$$

$$\Rightarrow 4\alpha = 8 \Rightarrow \alpha = 2$$

$$f(x)=\begin{cases} x & , x \leq -1 \\ \alpha x+\beta & , -1<x<3 \\ 2x+1 & , x \geq 3 \end{cases}$$



At $x=-1$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} x = -1$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \alpha x+\beta = -\alpha+\beta$$

$$-\alpha+\beta=-1 \longrightarrow 1$$

At $x=3$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \alpha x+\beta = 3\alpha+\beta$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2x+1 = 7$$

$$3\alpha+\beta=7 \longrightarrow 2$$

$-\alpha+\beta=-1 \quad (\times 3)$

$3\alpha+\beta=7$

$\Rightarrow$

$$\begin{aligned} -\cancel{3\alpha}+3\beta &= -3 \\ \cancel{3\alpha}+\beta &= 7 \\ \hline 4\beta &= 4 \Rightarrow \beta=1 \end{aligned}$$

To find α:

$-\alpha+1=-1 \Rightarrow -\alpha=-2$

$\Rightarrow \alpha=2$