KING SAUD UNIVERSITY
DEANSHIP OF THE FIRST YEAR COMMON
BASIC SCIENCES DEPARTMENT

MATH 101 | HW # 1 / SECONDSEMESTER 1441 | Date: 06/02/2020

Question 1 — 2 Marks for each

A. Classify the following numbers into rational and irrationals.

$$\left\{\sqrt[3]{-8},\ \frac{3.17}{6},\ \left(\sqrt{2}\right)^8,\ \left(\sqrt{2}+\sqrt{8}\right)^2,\ \pi+1.\overline{2},\ \sqrt{5+\sqrt{16}},\ \cot 30^\circ,\ |1-\pi|\right\}$$

$\sqrt[3]{-8} = -2 \in Q$

$\frac{3.17}{6} = 0.528\overline{3} \in Q$

$(\sqrt{2})^8 = 16 \in Q$

$(\sqrt{2}+\sqrt{8})^2 = 18 \in Q$

$\pi + 1.\overline{2} \in I$

$\sqrt{5+\sqrt{16}} = 3 \in Q$

$\cot 30 = \sqrt{3} \in I$

$|1-\pi| \in I$

B. Solve the following inequalities and write the solution in interval notation.

1. $4+3(2x-1) \geq x+2$
2. $x+3 \leq 4x+1 < x+7$
3. $\frac{x^2}{x-1} \leq 0$
4. $(4-x)^2+7 \geq 11$
5. $1 \leq |x-2| < 6$
6. $x^2-10x+25 \leq 0$

$\boxed{1}$ $4+3(2x-1) \geq x+2$

$4+6x-3 \geq x+2$

$6x+1 \geq x+2$

$6x - x \geq 2-1$

$5x \geq 1$

$x \geq \frac{1}{5}$

$\frac{1}{5}$ ∞

S.S $[\frac{1}{5}, \infty)$

$\boxed{2}$ $x+3 \leq 4x+1 < x+7$

$-x \quad -x \quad -x$

$3 \leq 3x+1 < 7$

$-1 \quad -1 \quad -1$

$\frac{2}{3} \leq \frac{3x}{3} < \frac{6}{3}$

$\frac{2}{3} \leq x < 2$

$\frac{2}{3}$ 2

SS: $[\frac{2}{3}, 2)$

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Q1 : B :

3)
$$\frac{x^2}{x-1} < 0$$

$x^2 \geq 0$

$x-1=0 \rightarrow x=1$

$- \quad - \quad \oplus \quad + \quad +$

$-\infty \qquad 1$

SS: $(-\infty, 1)$

4)
$(4-x)^2+7 \geq 11$

$16-8x+x^2+7-11 \geq 0$

$x^2-8x+12 \geq 0$

$x^2-8x+12=0$

$(x-2)(x-6)=0$

$x-2=0 \rightarrow x=2$

$x-6=0 \rightarrow x=6$

$+ \quad + \quad | \quad - \quad - \quad | \quad + \quad +$

$-\infty \qquad 2 \qquad 6 \qquad \infty$

SS: $(-\infty, 2] \cup [6, \infty)$

5)
$1 \leq |x-2| < 6$

$|x-2| < 6$ and $|x-2| \geq 1$

$|x-2| < 6$

$-6 < x-2 < 6$

$+2 \qquad +2 \qquad +2$

$-4 < x < 8$

$|x-2| \geq 1$

$x-2 \geq 1$ or $x-2 \leq -1$

$+2 \quad +2 \qquad +2 \quad +2$

$x \geq 3 \qquad x \leq 1$

$-4 \qquad 1 \qquad 3 \qquad 8$

SS: $(-4, 1] \cup [3, 8)$

6)
$x^2-10x+25 \leq 0$

$(x-5)^2 \leq 0$

$|x-5| \leq 0$

$|x-5| = 0$

$x-5=0 \rightarrow x=5$

SS: $\{5\}$

S.S = $\{5\}$

Question 2

2 Marks for each

Find the domain of the following functions

1. $f(x) = 2x^3 + 5x - 1$
2. $f(x) = \sqrt{3 - |x - 2|}$
3. $f(x) = \dfrac{2}{\csc(4x)}$
4. $f(x) = \sqrt[3]{\dfrac{2x}{2x - 8}}$

1) f is poly →

$D_f : \mathbb{R}$

2) $f(x) = \sqrt{3 - |x-2|}$

$3 - |x-2| \geq 0$

$3 \geq |x-2|$

$|x-2| \leq 3$

$-3 \leq x - 2 \leq 3$

$+2 \quad +2 \quad +2$

$-1 \leq x \leq 5$

$D_f : [-1, 5]$

3) $f(x) = \dfrac{2}{\csc(4x)}$

$\csc(4x)$ not defined

when: $4x = n\pi$

$x = \dfrac{n\pi}{4} \quad n \in \mathbb{Z}$

$D_f : \mathbb{R} - \left\{ \dfrac{n\pi}{4} \right\}$

4) $f(x) = \sqrt[3]{\dfrac{2x}{2x-8}}$

$2x - 8 = 0 \rightarrow 2x = 8$

$\rightarrow x = 4$

$D_f = \mathbb{R} - \{4\}$

Question 3

2 Marks **one** for each

Determine whether the functions are the same or not.

1. $f(x) = \dfrac{1}{1+\tan^2 x}$, $g(x) = \cos^2 x$.
2. $f(x) = \dfrac{x^3+x}{x^2+1}$, $g(x) = x$.

1) $f(x) = \dfrac{1}{1+\tan^2 x}$

$\tan x$ not defined

when: $x = \dfrac{\pi}{2} + n\pi$
$n \in \mathbb{Z}$

$D_f : \mathbb{R} - \left\{ \dfrac{\pi}{2} + n\pi \right\}, n \in \mathbb{Z}$

$g(x) = \cos^2 x$

$D_g = \mathbb{R}$

$D_f \neq D_g$

f and g are not the same

2) $f(x) = \dfrac{x^3+x}{x^2+1}$

$x^2+1 \neq 0 \longrightarrow$

$D_f : \mathbb{R}$

$D_g : \mathbb{R}$

1) $D_f = D_g$

2) $f(x) = \dfrac{x^3+x}{x^2+1} =$

$\dfrac{x\cancel{(x^2+1)}}{\cancel{x^2+1}} = x = g(x)$

$f(x) = g(x)$

f and g are the same

Question 4

2 Marks for each

Let $f(x) = \dfrac{1}{x^2 - 4}$, $\quad g(x) = \sqrt{2x-1}$.

1. Find $\dfrac{f}{g}$ and its domain.
2. Find $\left(f \circ g\right)(x)$ and $\left(f \circ g\right)(5)$.

1) $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{1}{x^2-4}}{\sqrt{2x-1}}$

$D_{\frac{f}{g}} = D_f \cap D_g - \{g(x)=0\}$

D_f : $x^2 - 4 = 0 \rightarrow x^2 = 4 \rightarrow x = \mp 2$

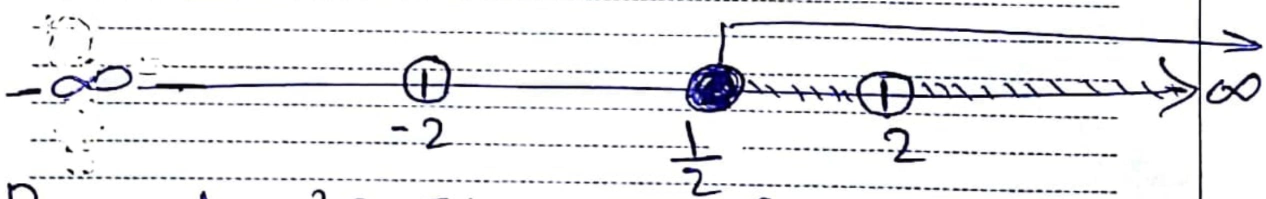
$D_f = R - \{\mp 2\}$

D_g : $2x - 1 \geq 0 \rightarrow 2x \geq 1 \rightarrow x \geq \frac{1}{2}$

$D_g = [\frac{1}{2}, \infty)$

$g(x) = 0 \rightarrow \sqrt{2x-1} = 0 \longrightarrow$

$2x - 1 = 0 \rightarrow 2x = 1 \rightarrow x = \frac{1}{2}$



$D_{\frac{f}{g}} = R - \{\mp 2\} \cap [\frac{1}{2}, \infty) - \{\frac{1}{2}\}$

$= (\frac{1}{2}, 2) \cup (2, \infty)$

Q4 : $\boxed{2}$ Find $f \circ g(x)$ and $f \circ g(5)$

$$f(x) = \frac{1}{x^2-4} \quad ; \quad g(x) = \sqrt{2x-1}$$

$$f \circ g(x) = f\big(g(x)\big) =$$

$$= f\left(\sqrt{2x-1}\right) =$$

$$f \circ g(x) = \frac{1}{\left(\sqrt{2x-1}\right)^2 - 4}$$

$$f \circ g(5) = \frac{1}{\left(\sqrt{2(5)-1}\right)^2 - 4} = \frac{1}{5}$$

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Question 3 2 Marks for each

Determine whether the following functions are one to one or not.

1. $f(x) = x^2 - 2x, \quad x \geq -4$

2. $f(x) = \dfrac{2x+1}{x-3}$

1) Let $x_1, x_2 \in D_f$ and $x_1 \geq -4$, $x_2 \geq -4$

$$f(x_1) = f(x_2)$$

$$x_1^2 - 2x_1 = x_2^2 - 2x_2$$

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$$x_1^2 - 2x_1 + 1 = x_2^2 - 2x_2 + 1$$

بالجذر

$$(x_1 - 1)^2 = (x_2 - 1)^2$$

$$|x_1 - 1| = |x_2 - 1|$$

$$x_1 - 1 = \mp (x_2 - 1)$$

f is not one to one

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Q 5 : $f(x) = \dfrac{2x+1}{x-3}$; D_f : $R - \{3\}$

Let $x_1, x_2 \in D_f$ and

$$f(x_1) = f(x_2)$$

$$\frac{2x_1+1}{x_1-3} = \frac{2x_2+1}{x_2-3}$$

$$(2x_1+1)(x_2-3) = (2x_2+1)(x_1-3)$$

$$\cancel{2x_1x_2} - 6x_1 + x_2 \cancel{-3} = \cancel{2x_1x_2} - 6x_2 + x_1 \cancel{-3}$$

$$-6x_1 + x_2 = x_1 - 6x_2$$

$$-6x_1 - x_1 = -6x_2 - x_2$$

$$\frac{-7x_1}{-7} = \frac{-7x_2}{-7}$$

$$x_1 = x_2$$

f is one to one

Question 6

2 Marks for each

Given that $f(x) = \frac{-x}{x+2}$ is one-to-one function.

1. Find f^{-1}.
2. Find R_f.

$f(x) = \frac{-x}{x+2}$; $D_f : \mathbb{R} - \{-2\}$

$y = \frac{-x}{x+2} \rightarrow x = \frac{-y}{y+2}$

$x(y+2) = -y$

$xy + 2x = -y$

$xy + y = -2x$

$y(x+1) = -2x$

$y = \frac{-2x}{x+1}$

$f^{-1}(x) = \frac{-2}{x+1}$

$D_{f^{-1}} : \mathbb{R} - \{-1\}$

$R_f = D_{f^{-1}} = \mathbb{R} - \{-1\}$

Question 7 2 Marks for each

Without using calculator find the values of:

1. $\cos\left(-\frac{5}{6}\pi\right)$
2. $\tan(15^\circ)$
3. $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$
4. $\cos\left[\sin^{-1}\left(\frac{-2}{3}\right)+\sin^{-1}\left(\frac{1}{3}\right)\right]$
5. $\sin 13^\circ \cos 17^\circ + \cos 13^\circ \sin 17^\circ$

1) $\cos\left(-\frac{5}{6}\pi\right) = \cos\left(5\frac{\pi}{6}\right)$

$= \cos\left(\pi - \frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = \frac{-\sqrt{3}}{2}$

2) $\tan(15) = \tan(45 - 30)$

$= \frac{\tan 45 - \tan 30}{1 + \tan 45 . \tan 30} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$

$= \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} = 2 - \sqrt{3}$

3) $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$

$\sin\left(\frac{4\pi}{3}\right) = \sin\left(\pi + \frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = \frac{-\sqrt{3}}{2}$

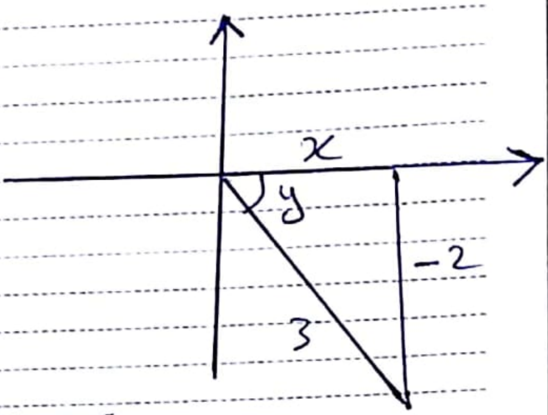
$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

Q 7:

H) $\cos\left[\sin^{-1}\left(\frac{-2}{3}\right)+\sin^{-1}\left(\frac{1}{3}\right)\right]$

Let $\sin^{-1}\left(-\frac{2}{3}\right)=y \quad -\frac{\pi}{2}\le y\le\frac{\pi}{2}$

$\sin y=\frac{-2}{3}$

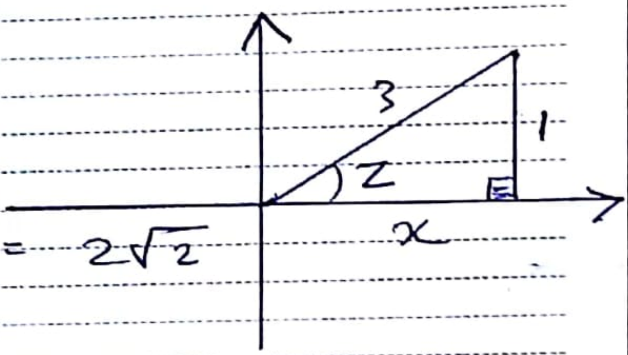


$x=\sqrt{3^2-(-2)^2}=\sqrt{5}$

$\cos y=\frac{\sqrt{5}}{3}$

Let $\sin^{-1}\left(\frac{1}{3}\right)=Z \quad \frac{-\pi}{2}\le Z\le\frac{\pi}{2}$

$\sin Z=\frac{1}{3}$



$x=\sqrt{3^2-1^2}=\sqrt{8}=2\sqrt{2}$

$\cos Z=\frac{2\sqrt{2}}{3}$

$\cos(y+Z)=\cos y\cdot\cos Z-\sin y\cdot\sin Z$

$=\frac{\sqrt{5}}{3}\times\frac{2\sqrt{2}}{3}-\frac{-2}{3}\times\frac{1}{3}=$

$=\frac{2\sqrt{10}}{9}+\frac{2}{9}=\frac{2\sqrt{10}+2}{9}$

Q7: 51

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin \overset{x}{13} \cos \overset{y}{17} + \cos 13 \sin 17$

$\sin(13+17) = \sin 30 = \frac{1}{2}$

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Question 8

2 Marks for each

1. Prove the identity

$$\frac{\cos\left(\frac{\pi}{2}-x\right)}{1-\cos^2 x} = \csc x$$

2. Solve the trigonometric equation:

$$\sin x + \sin(2x) = 0, \quad x \in [0, 2\pi]$$

1) $\dfrac{\cos\left(\frac{\pi}{2}-x\right)}{1-\cos^2 x} = \dfrac{\cancel{\cos\frac{\pi}{2}\cos x} + \sin\frac{\pi}{2}\sin x}{\sin^2 x}$

$= \dfrac{\sin x}{\sin^2 x} = \dfrac{1}{\sin x} = \csc x$

2) $\sin x + 2\sin x \cos x = 0$

$\sin x\,[1 + 2\cos x] = 0$

$\sin x = 0$	$1 + 2\cos x = 0$
$\sin x = \sin 0$	$2\cos x = -1$
$x = 0$	$\cos x = \frac{-1}{2}$
$x = \pi$	$\cos x = \cos\frac{2\pi}{3}$
$x = 2\pi$	$x = \frac{2\pi}{3}$
	$x = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$

$S.S = \left\{0, \pi, 2\pi, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$