Instructions : (33 points). Solve each of the following problems and choose the correct answer.

(1) The domain of the function $f(x) = |3x - 6|$ is
- **(a)** $\mathbb{R} - \{2\}$
- **(b)** $[2, \infty)$
- **(c)** $\mathbb{R} - \{-2\}$
- **(d)** $\mathbb{R}$ *

(2) The domain of the function $f(x) = \dfrac{x+2}{x^2 + x - 6}$ is
- **(a)** $\mathbb{R} - \{-2, 3\}$
- **(b)** $\mathbb{R} - \{-2, -3\}$
- **(c)** $\mathbb{R} - \{2, -3\}$ *
- **(d)** $\mathbb{R} - \{2, 3\}$

(3) The domain of the function $f(x) = \sqrt{4 - x^2}$ is
- **(a)** $(-2, 2)$
- **(b)** $[-2, 2]$ *
- **(c)** $(-\infty, -2] \cup [2, \infty)$
- **(d)** $(2, \infty)$

(4) The range of the function $f(x) = \sqrt{25 + x^2}$ is
- **(a)** $(-\infty, 5]$
- **(b)** $(-\infty, 5)$
- **(c)** $(5, \infty)$
- **(d)** $[5, \infty)$ *

(5) The range of the function $f(x) = 9 - x^2$ is
- **(a)** $(-\infty, 9]$ *
- **(b)** $[9, \infty)$
- **(c)** $(-\infty, -9]$
- **(d)** $[-9, \infty)$

(6) The function $f(x) = 10 - x^3$ is even.
- **(a)** True
- **(b)** False *

(7) The function $f(x) = x^{\frac{2}{3}} + x^2$ is
- **(a)** Algebraic function *
- **(b)** Power function
- **(c)** Polynomial function
- **(d)** Exponential function

(8) If $h(x) = |\cos x|$, $f(x) = \cos x$, $g(x) = |x|$, then
- **(a)** $h = f \circ g$
- **(b)** $h = g \circ f$ *
- **(c)** $h = f.g$
- **(d)** $h = f \circ f$

(9) The function $f(x) = \dfrac{7 - x^2}{x^3 + 3x}$ is symmetric about the origin.
- **(a)** True *
- **(b)** False

(10) The function $f(x) = (x-1)^2$ is

- **(a)** increasing on $(1,\infty)$ *
- **(b)** increasing on $(-\infty,1)$
- **(c)** decreasing on $(1,\infty)$
- **(d)** decreasing on $(-1,\infty)$

(11) The degree measure of $\theta = \frac{7\pi}{6}$ is

- **(a)** 100°
- **(b)** 120°
- **(c)** 210° *
- **(d)** 75°

(12) The radian measure of $\theta = 150^\circ$ is

- **(a)** $\frac{5\pi}{6}$ *
- **(b)** $\frac{10\pi}{3}$
- **(c)** $\frac{10\pi}{9}$
- **(d)** $\frac{4\pi}{3}$

(13) If $f(x) = x^2$ and $g(x) = \sqrt{2+x}$, then $(f \circ g)(x) =$

- **(a)** $2 + x^2$
- **(b)** $\sqrt{2+x^2}$
- **(c)** $(2+x)^2$
- **(d)** $2 + x$ *

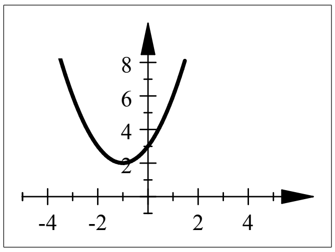
(14) If $f(x) = x$ and $g(x) = 3x^2 + x$, then $(\frac{f}{g})(x) =$

- **(a)** $\frac{x}{3x^2-1}$
- **(b)** $\frac{1}{3x+1}$ *
- **(c)** $\frac{1}{3x-1}$
- **(d)** $\frac{x}{3x^2+1}$

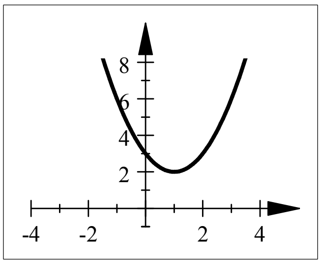
(15) If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, then the domain of $(f+g)(x)$ is

- **(a)** $(-\infty,2]$
- **(b)** $[0,2]$ *
- **(c)** $[0,\infty)$
- **(d)** $(0,2)$

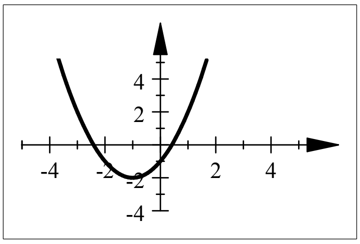
(16) The graph of the function $f(x) = (x+1)^2 + 2$ is



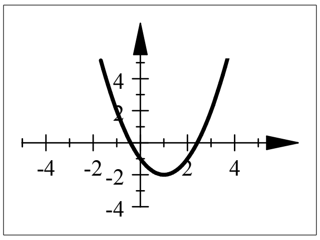
(a)*



(b)



(c)



(d)

(17) The graph of $g(x) = |x - 4|$ is a shifting of the graph of $f(x) = |x|$

- **(a)** 4 units to the left
- **(b)** 4 units to the right *
- **(c)** 4 units downward
- **(d)** 4 units upward

(18) If the graph of $f(x) = 3^x$ is reflected about the $y-$ *axis*, then the equation of the new function is

- **(a)** $(\frac{1}{3})^{-x}$
- **(b)** $(-3)^x$
- **(c)** $(\frac{1}{3})^x$ *
- **(d)** $-(3^x)$

(19) If $\cos x = \frac{3}{2}$, $\sin x = \frac{1}{2}$, then $\sin(2x) =$

- **(a)** $\frac{3}{2}$ *
- **(b)** 2
- **(c)** 4
- **(d)** $\frac{3}{4}$

(20) The function $f(x) = \left(\frac{1}{2}\right)^x$ is increasing on $\mathbb{R}$.

- **(a)** True
- **(b)** False *

(21) If $\sin\theta = \frac{3}{4}$ and $0 < \theta < \frac{\pi}{2}$, then $\cos\theta =$

- **(a)** $\frac{-3}{\sqrt{7}}$
- **(b)** $-\frac{\sqrt{7}}{4}$
- **(c)** $\frac{3}{\sqrt{7}}$
- **(d)** $\frac{\sqrt{7}}{4}$ *

(22) If $\theta = \frac{-\pi}{3}$, then $\sin\theta =$

- **(a)** $\frac{1}{2}$
- **(b)** $\frac{\sqrt{3}}{2}$

(c) $\frac{-\sqrt{3}}{2}$ *

(d) $\frac{-1}{2}$

(23) The range of the function $f(x) = \sin x$ is

(a) $\mathbb{R}$

(b) $(-1, 1)$

(c) $\mathbb{R} - (-1, 1)$

(d) $[-1, 1]$ *

(24) The function $f(x) = \cot x$ is

(a) even

(b) odd *

(c) even and odd

(d) neither even nor odd

(25) If a is a positive number and x, y are real numbers, then $(a^x)^y =$

(a) a^{x+y}

(b) $a^{x.y}$ *

(c) $a^x . a^y$

(d) $a^{x/y}$

(26) The range of the function $y = 2^x + 1$ is

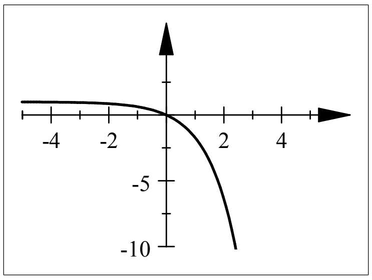
(a) $(1, \infty)$ *

(b) $[1, \infty)$

(c) $(-\infty, 1)$

(d) $(-\infty, 1]$

(27) The following graph represents the function $f\ (x) =$



(a) $-e^x - 1$

(b) $e^{-x} + 1$

(c) $e^{-x} - 1$

(d) $1 - e^x$ *

(28) The domain of the function $f(x) = \frac{1}{1 - e^{2x}}$ is

(a) $\mathbb{R} - \{0\}$ *

(b) $\mathbb{R} - \{1\}$

(c) $\mathbb{R} - \{0, 1\}$

(d) $\mathbb{R}$

(29) If $f(x) = 3x + 2$, then $f^{-1}(x) =$

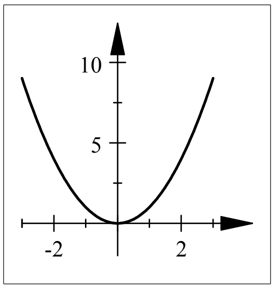
(a) $\frac{x-3}{2}$

(b) $\frac{x+3}{2}$

(**c**) $\frac{x-2}{3}$ *

(**d**) $\frac{x+2}{3}$

(**30**) The following graph represents one - to - one function



1. (**a**) true

(**b**) false *

(**31**) The range of the function $f(x) = \sqrt{x}$ is

(**a**) $\mathbb{R}$

(**b**) $\mathbb{R} - \{0\}$

(**c**) $[0, \infty)$ *

(**d**) $(0, \infty)$

(**32**) One of the following identities is true

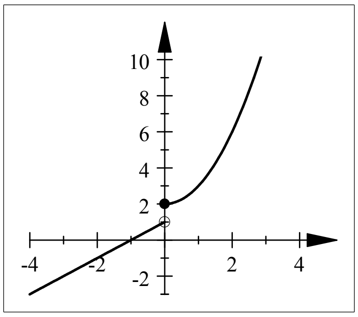
(**a**) $\cos(2x) = \cos^2 x - \sin^2 x$ *

(**b**) $\cos(2x) = \cos^2 x + \sin^2 x$

(**c**) $\cos(2x) = \cos^2(2x) - \sin^2(2x)$

(**d**) $\cos(2x) = 2\sin x.\cos x$

(**33**) The following graph



represents the function :

(**a**) $f(x) = \begin{cases} x^2 + 2 & \text{if } x > 0 \\ x + 1 & \text{if } x \le 0 \end{cases}$

(**b**) $f(x) = \begin{cases} x^2 + 2 & \text{if } x \ge 0 \\ x + 1 & \text{if } x < 0 \end{cases}$ *

(**c**) $f(x) = \begin{cases} x^2 + 2 & \text{if } x < 0 \\ x + 1 & \text{if } x \ge 0 \end{cases}$

(**d**) $f(x) = \begin{cases} x^2 + 2 & \text{if } x \le 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

Form B : **Instructions**: **(33 points). Solve each of the following problems and choose the correct answer** :

1. $\log_2 8 - \log_2 4 =$

(**a**) 1 $*$
(**b**) 2
(**c**) 0
(**d**) -1

2. If $\ln(2x-9) = 0$, then $x =$

(**a**) $\frac{9}{2}$
(**b**) 4
(**c**) -5
(**d**) 5 $*$

3. $\sin\left(\cos^{-1}\frac{3}{x}\right) =$

(**a**) $\frac{x}{\sqrt{x^2-9}}$
(**b**) $\frac{x}{\sqrt{9-x^2}}$
(**c**) $\frac{\sqrt{x^2-9}}{x}$ $*$
(**d**) $\frac{\sqrt{9-x^2}}{x}$

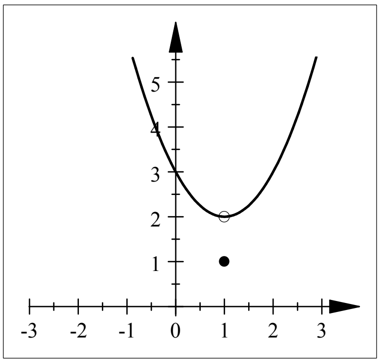
4. The domain of the function $f(x) = \cos^{-1}(3x+4)$ is

(**a**) $\left[-\frac{5}{3},-1\right]$ $*$
(**b**) $\left(-\frac{5}{3},-1\right)$
(**c**) $\left[1,\frac{5}{3}\right]$
(**d**) $\left(1,\frac{5}{3}\right)$

5. The exact value of the expression $e^{-2\ln 3}$ is

(**a**) -6
(**b**) 9
(**c**) $\frac{1}{9}$ $*$
(**d**) $\frac{1}{6}$

6. If $f(x)$ is the function whose graph is shown ,



then $\lim_{x \to 1} f(x) =$

- (**a**) 3
- (**b**) 1
- (**c**) 2 $*$
- (**d**) Does not exist.

7. $\lim_{x \to -3} \frac{x^2 + 2x - 3}{x + 3} =$

- (**a**) –4 *
- (**b**) -1
- (**c**) -2
- (**d**) 4

8. If $\lim_{x \to 3} \frac{f(x) - 2}{x^2} = 2$, then $\lim_{x \to 3} f(x) =$

- (**a**) 0
- (**b**) 20 $*$
- (**c**) 16
- (**d**) 4

9. $\lim_{x \to 3} \frac{(x-1)^2 - 4}{x - 3} =$

- (**a**) ∞
- (**b**) 2
- (**c**) -4
- (**d**) 4 *

10. If $5(x-1) \le f(x) \le x^3 + x^2 - 2$, then $\lim\limits_{x \to -3} f(x) =$

- (**a**) 34
- (**b**) 20
- (**c**) –20 *
- (**d**) Does not exist.

11. $\lim\limits_{x \to 0} \dfrac{\sqrt{25+x} - 5}{x} =$

- (**a**) $\frac{1}{10}$ *
- (**b**) $\frac{1}{25}$
- (**c**) 0
- (**d**) ∞

12. $\lim\limits_{x \to \frac{\pi}{2}} x \sin x =$

- (**a**) $\frac{\pi}{2}$ *
- (**b**) 0
- (**c**) $-\frac{\pi}{2}$
- (**d**) Does not exist.

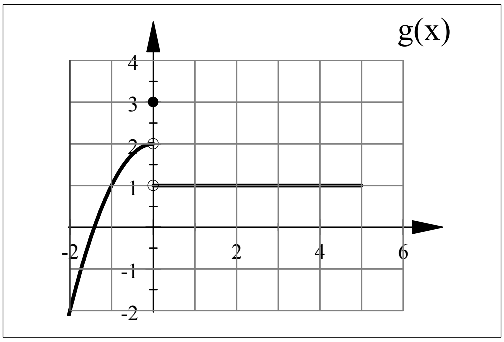
13. $\lim\limits_{x \to 2} \dfrac{x-2}{x^3 - 8} =$

- (**a**) 2
- (**b**) $\frac{1}{12}$ *
- (**c**) $\frac{1}{4}$
- (**d**) Does not exist.

14. $\lim\limits_{x \to -4} \dfrac{\frac{1}{4} + \frac{1}{x}}{4 + x} =$

- (**a**) $\frac{1}{16}$
- (**b**) $-\frac{1}{16}$ *
- (**c**) 16
- (**d**) −16

15. If $g(x)$ is the function whose graph is shown,



then $\lim\limits_{x \to 0^+} g(x) =$

- (**a**) 3
- (**b**) 2
- (**c**) 1 *
- (**d**) Does not exist.

16. If $f(x) = \begin{cases} -3x+1 & \text{if } x > 1 \\ x+2 & \text{if } x < 1 \end{cases}$, then $\lim\limits_{x \to 1^-} f(x) =$

- (**a**) −2
- (**b**) 3 *
- (**c**) 2
- (**d**) Does not exist.

17. If $f(x) = \dfrac{x^2-4}{|x-2|}$, then $\lim\limits_{x \to 2^-} f(x) =$

- (**a**) 16
- (**b**) 4
- (**c**) –4 *
- (**d**) Does not exist.

18. $\lim\limits_{x \to 4} \dfrac{\sin(x-4)}{2x-8} =$

- (**a**) 0
- (**b**) 1
- (**c**) $-\dfrac{1}{2}$
- (**d**) $\dfrac{1}{2}$ *

19. $\lim\limits_{\theta \to 0} \dfrac{\tan\theta}{2\theta + \sin\theta} =$

- (**a**) 0
- (**b**) 1
- (**c**) $\dfrac{1}{3}$ *
- (**d**) Does not exist.

20. $\lim\limits_{x \to \infty} \dfrac{x-4}{x^2 - x - 12} =$

(**a**) 0 *
(**b**) $\frac{1}{3}$
(**c**) 4
(**d**) ∞

21. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2 - 2}}{3x - 5} =$

(**a**) $\frac{1}{3}$
(**b**) $-\frac{1}{3}$ *
(**c**) ∞
(**d**) $-\infty$.

22. The horizontal asymptote of $f(x) = \dfrac{7x^3 - 5x^2 - 3}{8x^3 + x}$ is

(**a**) $y = \frac{7}{8}$ *
(**b**) $x = \frac{7}{8}$
(**c**) $y = -\frac{7}{8}$
(**d**) $x = -\frac{7}{8}$

23. The function $f(x) = \dfrac{-3x^4 - 4x^2 + 35}{x^3 - 8}$ does not have a horizontal asymptote.

(**a**) True *
(**b**) False

24. $\lim\limits_{x \to -\infty} (5x^2 + 2x + 7) =$

(**a**) $-\infty$
(**b**) 5
(**c**) 7
(**d**) ∞ *

25. $\lim\limits_{x \to -\infty} \cos(\dfrac{1}{2x + \pi}) =$

(**a**) 1 *
(**b**) $\frac{\pi}{2}$
(**c**) 0
(**d**) Does not exist.

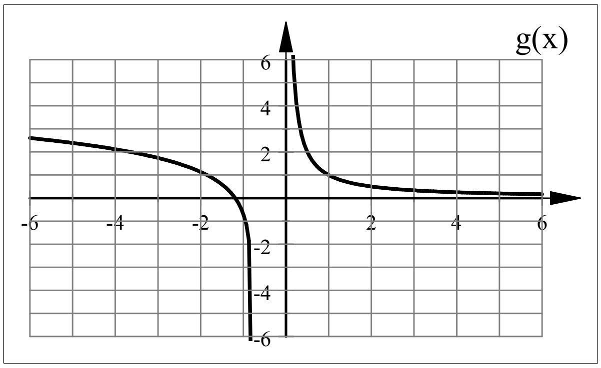
26. $\lim\limits_{x \to 3^-} \dfrac{2x}{x - 3} =$

(**a**) ∞
(**b**) $-\infty$ *
(**c**) 2
(**d**) 6

27. The vertical asymptote(s) of $f(x) = \dfrac{x-3}{x^2+x-12}$ is (are)

(**a**) $y = 3$, $y = -4$
(**b**) $x = 3$, $x = -4$
(**c**) $y = -4$
(**d**) $x = -4$ *

28. The horizontal asymptote(s) of the following function is (are)



(**a**) $y = 0$, $y = 3$ *
(**b**) $y = -1$, $y = 0$
(**c**) $x = 0$, $x = 3$
(**d**) $x = 0$

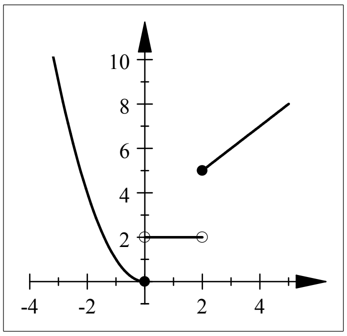
29. The function $f(x) = \begin{cases} x-1 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$ is continuous at $a = 0$

(**a**) Ttue
(**b**) False *

30. The function $f(x) = \begin{cases} k^2x - 3x & \text{if } x \geq 2 \\ 6x & \text{if } x < 2 \end{cases}$ is continuous on $\mathbb{R}$ if

(**a**) $k = \pm 3$ *
(**b**) $k = \pm 9$
(**c**) $k = 9$
(**d**) $k = -9$.

31. If $f(x)$ is the function whose graph is shown below ,



then $f(x)$ is

(**a**) continuous from the right at $x = 0$
(**b**) discontinuous from the right at $x = 0$ *
(**c**) continuous from the left at $x = 2$
(**d**) discontinuous from the right at $x = 2$

32. The function $f(x) = \tan x$ is discontinuous at $x =$
(**a**) $(2n+1)\pi$, $n \in Z$
(**b**) $(2n+1)\frac{\pi}{2}$, $n \in Z$ *
(**c**) $n\pi$, $n \in Z$
(**d**) $\frac{n\pi}{2}$, $n \in Z$

33. The function $f(x) = \frac{\sqrt{4 - x^2}}{x - 2}$ is continuous on
(**a**) $[-2,2]$
(**b**) $[-2,2)$ *
(**c**) $(-\infty,-2] \cup (2,\infty)$
(**d**) $(-\infty,-2) \cup (2,\infty)$.

Form C. **Instructions**: **(44 points). Solve each of the following problems and choose the correct answer** :

(**1**) The range of the function $f(x) = \frac{x+2}{|x+2|}$ is

- (**a**) $[0, \infty)$
- (**b**) $\{-1, 1\}$ *
- (**c**) $\mathbb{R}$
- (**d**) $\mathbb{R} - \{-2\}$.

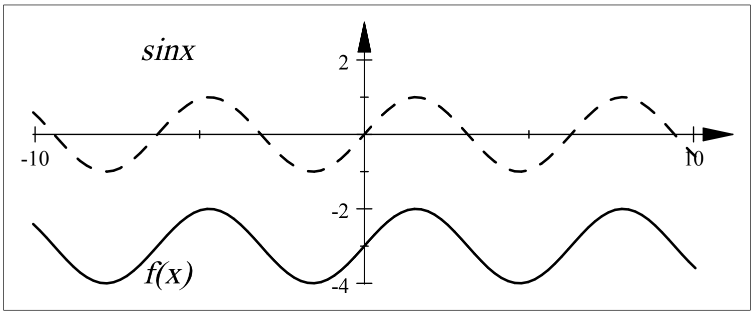
(**2**) The function $f(x)$ is an even if $f(-x) = -f(x)$ for every $x \in D_f$

- (**a**) True
- (**b**) False. *

(**3**) $\cos(\frac{5\pi}{2} + 2\pi) = \cos\frac{5\pi}{2}$

- (**a**) True *
- (**b**) False.

(**4**) The following figure shows the graph of $y = \sin x$ shifted to a new position.



An equation for the new function is

- (**a**) $f(x) = \sin(x-3)$
- (**b**) $f(x) = \sin x + 3$
- (**c**) $f(x) = \sin(x+3)$
- (**d**) $f(x) = \sin x - 3$. *

(**5**) The domain of the function $f(x) = \frac{1}{1+e^x}$ is

- (**a**) $(0, \infty)$
- (**b**) $\mathbb{R} - \{-1\}$
- (**c**) $\mathbb{R}$ *
- (**d**) $\mathbb{R} - \{0\}$.

(**6**) If $f(x) = 2 + e^x$, then $f^{-1}(x) =$

- (**a**) $\ln(x-2)$ *
- (**b**) $\ln x - 2$
- (**c**) $\ln(x+2)$
- (**d**) $\ln x + 2$.

(**7**) $\sec^{-1}(\frac{2}{\sqrt{3}}) = \frac{\pi}{6}$

(**a**) True *
(**b**) False.

(**8**) If $e^{2x+3} = 1$, then $x =$

(**a**) $\frac{2}{3}$
(**b**) $\frac{3}{2}$
(**c**) $-\frac{2}{3}$
(**d**) $-\frac{3}{2}$ *

(**9**) $\lim\limits_{x \to 0^-} \frac{3x + |x|}{x} =$

(**a**) 1
(**b**) 4
(**c**) 2 *
(**d**) Does not exist.

(**10**) $\lim\limits_{x \to -4} \frac{e^c}{9} =$

(**a**) $\frac{e^c}{9}$ *
(**b**) $\frac{e^{-4}}{9}$
(**c**) $-\frac{4}{9}$
(**d**) 0

(**11**) If $\lim\limits_{x \to a} f(x) = \frac{2}{5}$ and $\lim\limits_{x \to a} g(x) = \frac{4}{7}$, then $\lim\limits_{x \to a} \frac{f(x)}{g(x)} =$

(**a**) $\frac{10}{7}$
(**b**) $\frac{7}{10}$ *
(**c**) $\frac{35}{8}$
(**d**) $\frac{8}{35}$

(**12**) $\lim\limits_{x \to 1^+} \frac{x+2}{x-1} = -\infty$

(**a**) True
(**b**) False. *

(**13**) $\lim\limits_{x \to 0} \frac{\sin 3x}{\sin 7x} =$

(a) $\frac{3}{7}$ *
(b) $\frac{7}{3}$
(c) 1
(d) Does not exist.

(14) The horizontal asymptote(s) of the function $f(x) = \dfrac{\sqrt{4x^2 - 3x}}{x - 2}$ is (are)

(a) $x = 2$
(b) $y = -1$
(c) $y = 1$
(d) $y = 2$, $y = -2$. *

(15) $\lim\limits_{x \to \infty} (1 - e^x) =$

(a) 0
(b) ∞
(c) $-\infty$ *
(d) -1

(16) The vertical asymptote(s) of the curve $y = \dfrac{x - 3}{x^2 - 9}$ is (are)

(a) $y = -3$
(b) $x = 3$, $x = -3$
(c) $x = 3$
(d) $x = -3$ *

(17) The function $f(x) = \begin{cases} \dfrac{x^2 + 2x}{x + 2} & if \;\; x \neq -2 \\ 1 & if \;\; x = -2 \end{cases}$ is continuous on

(a) $\mathbb{R} - \{-2\}$ *
(b) $\mathbb{R} - \{2\}$
(c) $\mathbb{R} - \{1\}$
(d) $\mathbb{R}$.

(18) The function $f(x) = \dfrac{3x^2 + 5}{x^2 + 4x + 4}$ is continuous on

(a) $\mathbb{R}$
(b) $\mathbb{R} - \{-2\}$ *
(c) $\mathbb{R} - \{2\}$
(d) $\mathbb{R} - \{2, -2\}$

(19) If $f(x) = \tan x$, then $f'(x) =$

- **(a)** $\lim\limits_{h \to 0} \dfrac{\tan x - \tan(x+h)}{h}$
- **(b)** $\lim\limits_{h \to 0} \dfrac{\tan(x-h) + \tan x}{h}$
- **(c)** $\lim\limits_{h \to 0} \dfrac{\tan(x+h) - \tan x}{h}$ *
- **(d)** $\lim\limits_{h \to 0} \dfrac{\tan(x+h) + \tan x}{h}$

(20) If $f(x) = \sqrt{x+4}$, then $f(x)$ is differentiable at $x = -4$

- **(a)** True
- **(b)** False. *

(21) The equation for the tangent line to the curve $y = f(x)$, $f(-2) = 2$, $f'(-2) = -4$

- **(a)** $y = -4x - 6$ *
- **(b)** $y = -4x - 10$
- **(c)** $y = 4x + 6$
- **(d)** $y = 4x + 10$.

(22) $\frac{d}{dx}\cos(\pi/6) =$

- **(a)** 0 *
- **(b)** $\sqrt{3}/2$
- **(c)** $\sin(\pi/6)$
- **(d)** $-\sin(\pi/6)$

(23) The slope of the tangent line to the curve $f(x) = \sqrt{x}(1+x^2)$ at the point $(1,0)$ is

- **(a)** 2
- **(b)** 3 *
- **(c)** −3
- **(d)** 5

(24) If $y = 5x^5 + 3x^3 - 7x^2 + 2$, then $y^{(6)} =$

- **(a)** 0 *
- **(b)** 30
- **(c)** 1
- **(d)** 5

(25) If $f(x) = 3ax^2 + 3x$ and $f''(x) = -12$, then $a =$

- **(a)** $-\frac{1}{2}$
- **(b)** $\frac{1}{2}$
- **(c)** -2 *
- **(d)** 2

(26) If $f(2) = 4$, $f'(2) = 3$, $g(2) = 2$, $g'(2) = 1$, then $\frac{d}{dx}\left(\frac{g}{f}\right)(2) =$

(**a**) $\frac{1}{8}$
(**b**) $-\frac{1}{2}$
(**c**) $\frac{1}{2}$
(**d**) $-\frac{1}{8}$ *

(**27**) $\frac{d}{dx}\left(\frac{4^x}{\sin x}\right) =$

(**a**) $\dfrac{4^x(\sin x - \cos x)}{\sin^2 x}$
(**b**) $\dfrac{4^x(\ln 4 \sin x - \cos x)}{\sin^2 x}$ *
(**c**) $\dfrac{4^x(\cos x - \ln 4 \sin x)}{\sin^2 x}$
(**d**) $\dfrac{4^x(\cos x - \sin x)}{\sin^2 x}$

(**28**) The 15^{th} derivative of $\sin x$ is

(**a**) $\sin x$
(**b**) $-\sin x$
(**c**) $\cos x$
(**d**) $-\cos x$ *

(**29**) The equation of the tangent line to the curve $f(x) = -\sin x + \cos x$ at the point $(0,1)$ is

(**a**) $y = x - 1$
(**b**) $y = -x - 1$
(**c**) $y = 1 - x$ *
(**d**) $y = x + 1$

(**30**) If $y = -e^{\tan x}$, then $y' =$

(**a**) $-\tan x\, e^{\sec^2 x}$
(**b**) $\tan x\, e^{\sec^2 x}$
(**c**) $\sec^2 x\, e^{\tan x}$
(**d**) $-\sec^2 x\, e^{\tan x}$ *

(**31**) If $y = (x + \cot x)^5$, then $y' =$
(**a**) $5(x + \cot x)^4(1 + \csc^2 x)$
(**b**) $5(x + \cot x)^4(1 - \csc^2 x)$ *
(**c**) $-5(x + \cot x)^4(1 - \csc^2 x)$
(**d**) $-5(x + \cot x)^4(1 + \csc^2 x)$

(**32**) If $x^2y^3 = 5$, then $y' =$
(**a**) $-\frac{3x}{2y}$

(b) $\frac{3x}{2y}$

(c) $-\frac{2y}{3x}$ *

(d) $\frac{2y}{3x}$

(33) $\frac{d}{dx}\left(\cos^{-1} x^2\right) = \frac{-2}{\sqrt{1-x^4}}$

(a) True

(b) False *

(34) If $y = (x^3 + 2x^2)^{3/2}$, then $y' =$

(a) $\frac{3}{2}(x^3 + 2x^2)^{1/2}$

(b) $\frac{3}{2}(x^3 + 2x^2)^{1/2}(3x^2 + 4x)$ *

(c) $\frac{3}{2(x^3 + 2x^2)^{1/2}}$

(d) $\frac{3(3x^2 + 4x)}{2(x^3 + 2x^2)^{1/2}}$.

(35) If $f(x) = \ln(\cos x^3)$, then $f'(x) =$

(a) $3x^2 \tan x^3$

(b) $-3x^2 \cot x^3$

(c) $-3x^2 \tan x^3$ *

(d) $3x^2 \cot x^3$.

(36) If $y = x^{\cos x}$, then $y' =$

(a) $\frac{\cos x}{x} - \sin x \ln x$

(b) $x^{\cos x}\left(\frac{\cos x}{x} - \sin x \ln x\right)$ *

(c) $\cos x \,(x^{\cos x - 1})$

(d) $-x^{\cos x} \sin x \ln x$.

(37) The critical numbers of the function $f(x) = x^3 - 3x^2 - 24x$ are

(a) $2, 4$

(b) $-2, -4$

(c) $-2, 4$ *

(d) $2, -4$

(38) The absolute extreme of the function $f(x) = x^2 - 2x - 5$ on $[0,3]$ are

	Absolute minimum	Absolute maximum
(a)	$f(3)$	$f(0)$
(b)	$f(0)$	$f(1)$
(c)	$f(0)$	$f(3)$
(d)	$f(1)$	$f(3)$ *

(39) The value(s) of c that satisfies Rolle's theorem for the function $f(x) = 2x^3 - 18x$ on $[0,3]$ is(are)

- **(a)** $\sqrt{3}$ *
- **(b)** $-\sqrt{3}$
- **(c)** $\pm\sqrt{3}$
- **(d)** 3

(40) The function $f(x) = x^3 - 3x$ is decreasing on

- **(a)** $(-\infty,-1)$
- **(b)** $(-1,\infty)$
- **(c)** $(-\infty,-1) \cup (1,\infty)$
- **(d)** $(-1,1)$ *

(41) If $f''(x) > 0$ for $1 < x < 3$, then the graph of $f(x)$ is concave down on $(1,3)$

- **(a)** True
- **(b)** False *

(42) The inflection point of the function $f(x) = x^3 - 12x + 12$ is

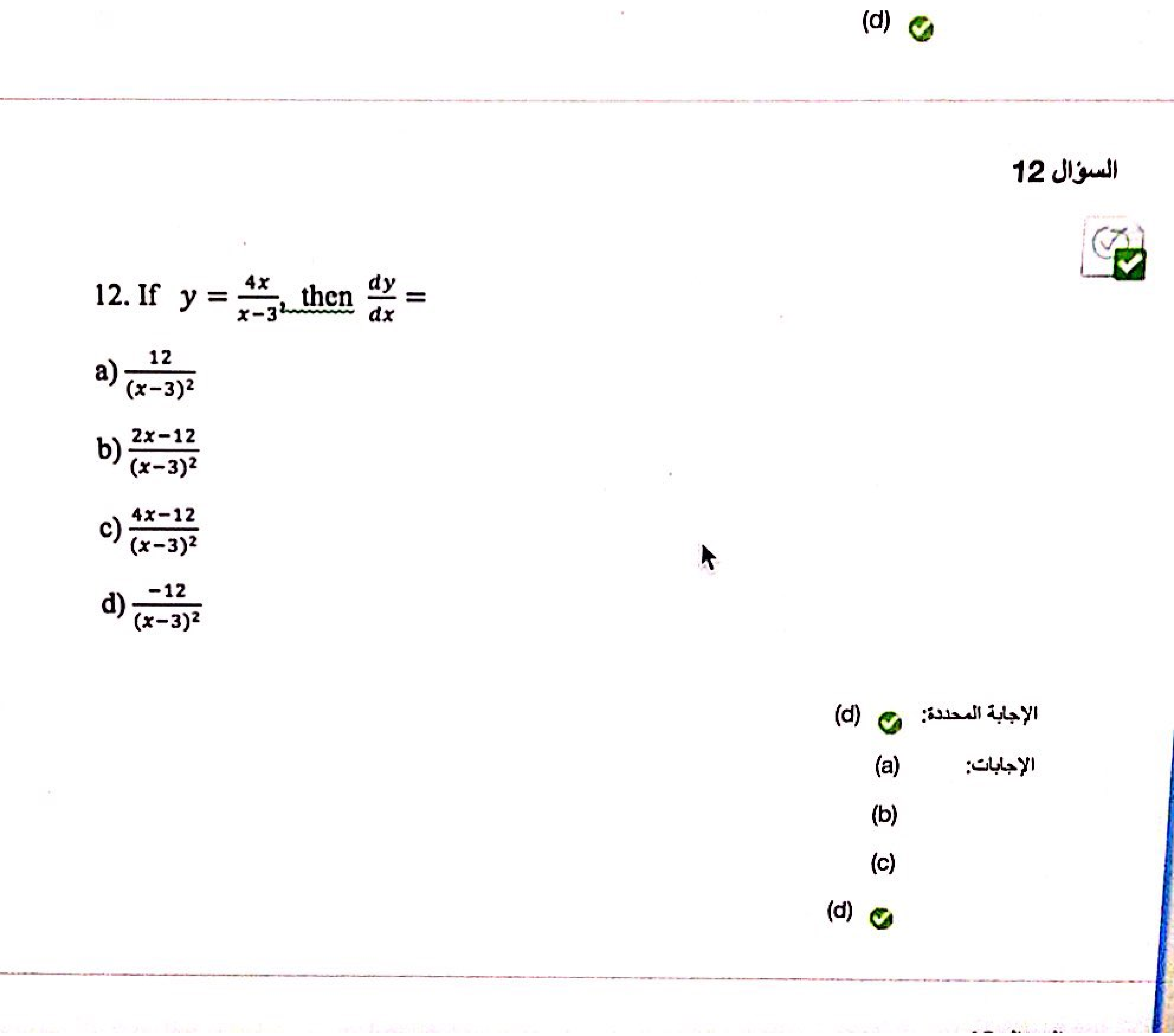
- **(a)** $(2,-4)$
- **(b)** $(-2,28)$
- **(c)** $(0,12)$ *
- **(d)** f does not have an inflection point.

(43) $\lim\limits_{x \to -\infty} \dfrac{e^{-x}+2}{x^2+1} =$

- **(a)** $-\infty$
- **(b)** ∞ *
- **(c)** 0
- **(d)** 2

(44) $\lim\limits_{x \to 0} \dfrac{1-\cos x}{5x^2} =$

- **(a)** $\dfrac{1}{10}$ *
- **(b)** $-\dfrac{1}{10}$
- **(c)** 0
- **(d)** 1



(d) ✓

السؤال 12

12. If $y = \frac{4x}{x-3}$, then $\frac{dy}{dx} =$

a) $\frac{12}{(x-3)^2}$

b) $\frac{2x-12}{(x-3)^2}$

c) $\frac{4x-12}{(x-3)^2}$

d) $\frac{-12}{(x-3)^2}$

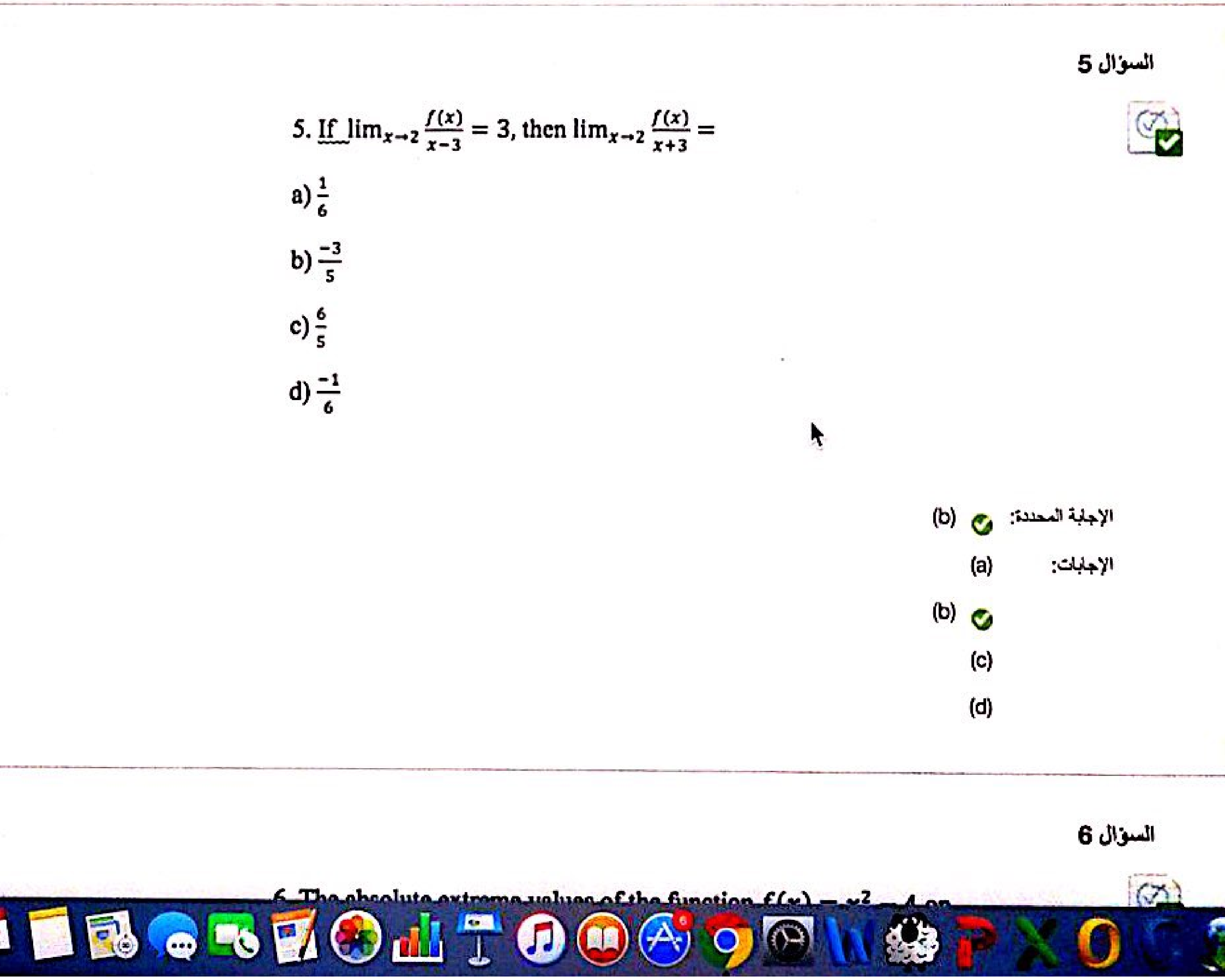
الإجابة المحددة: ✓ (d)

الإجابات: (a)

(b)

(c)

✓ (d)



السؤال 5

5. If $\lim_{x\to2}\frac{f(x)}{x-3} = 3$, then $\lim_{x\to2}\frac{f(x)}{x+3} =$

a) $\frac{1}{6}$

b) $\frac{-3}{5}$

c) $\frac{6}{5}$

d) $\frac{-1}{6}$

الإجابة المحددة: ✓ (b)

الإجابات: (a)

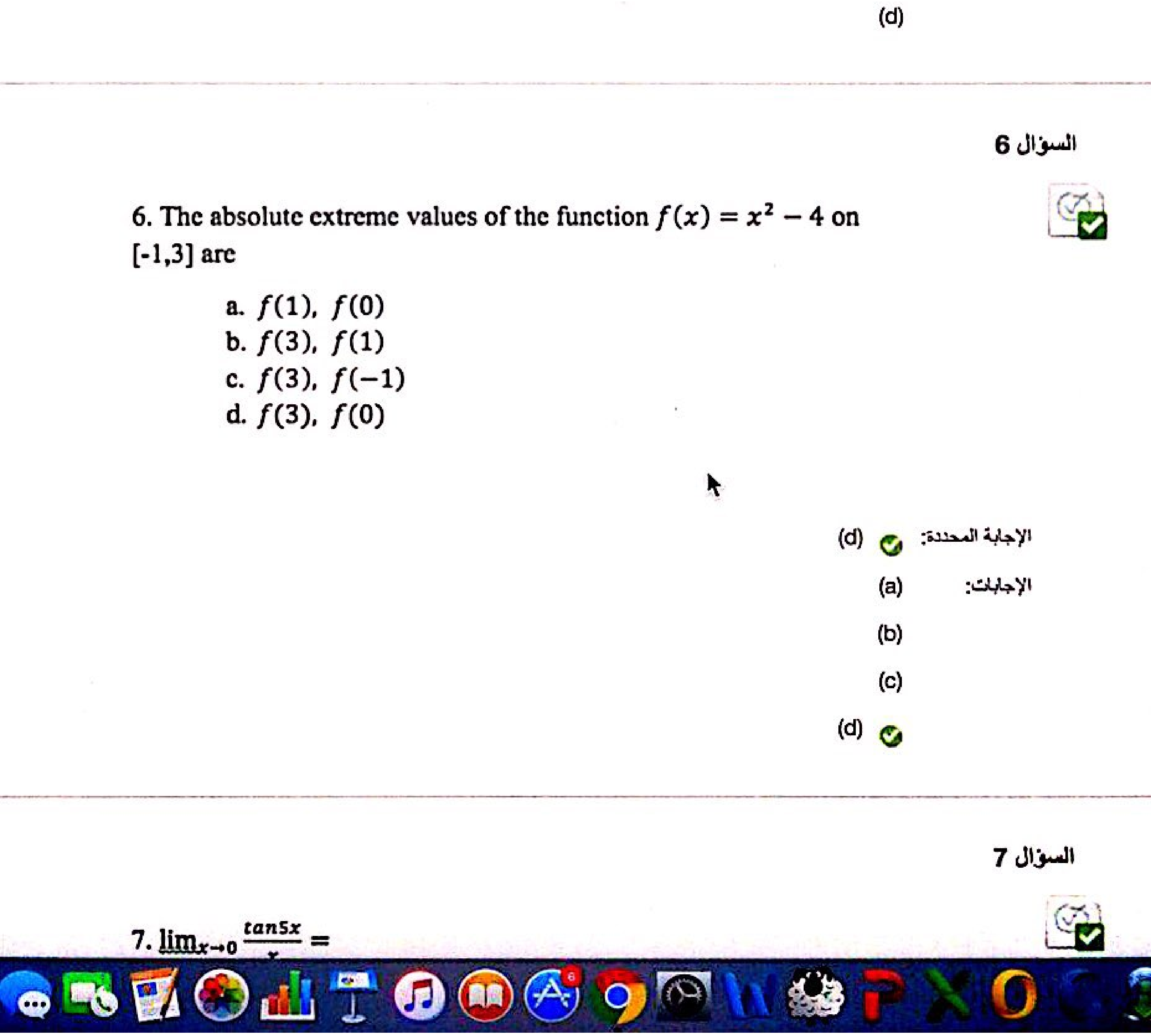
✓ (b)

(c)

(d)

السؤال 6

6. The absolute extreme values of the function $f(x) = x^2 - 4$ on



(d)

السؤال 6

6. The absolute extreme values of the function $f(x) = x^2 - 4$ on [-1,3] are

a. $f(1),\ f(0)$
b. $f(3),\ f(1)$
c. $f(3),\ f(-1)$
d. $f(3),\ f(0)$

الإجابة المحددة: ✓ (d)

الإجابات: (a)

(b)

(c)

✓ (d)

السؤال 7

7. $\lim_{x\to 0} \frac{tan5x}{x} =$

(c)

(d)

السؤال 10

10.If $2^{5-x} = 4$,then $x =$

a) -7

b) -3

c) 3

d) 7

الإجابة المحددة: ✓ (c)

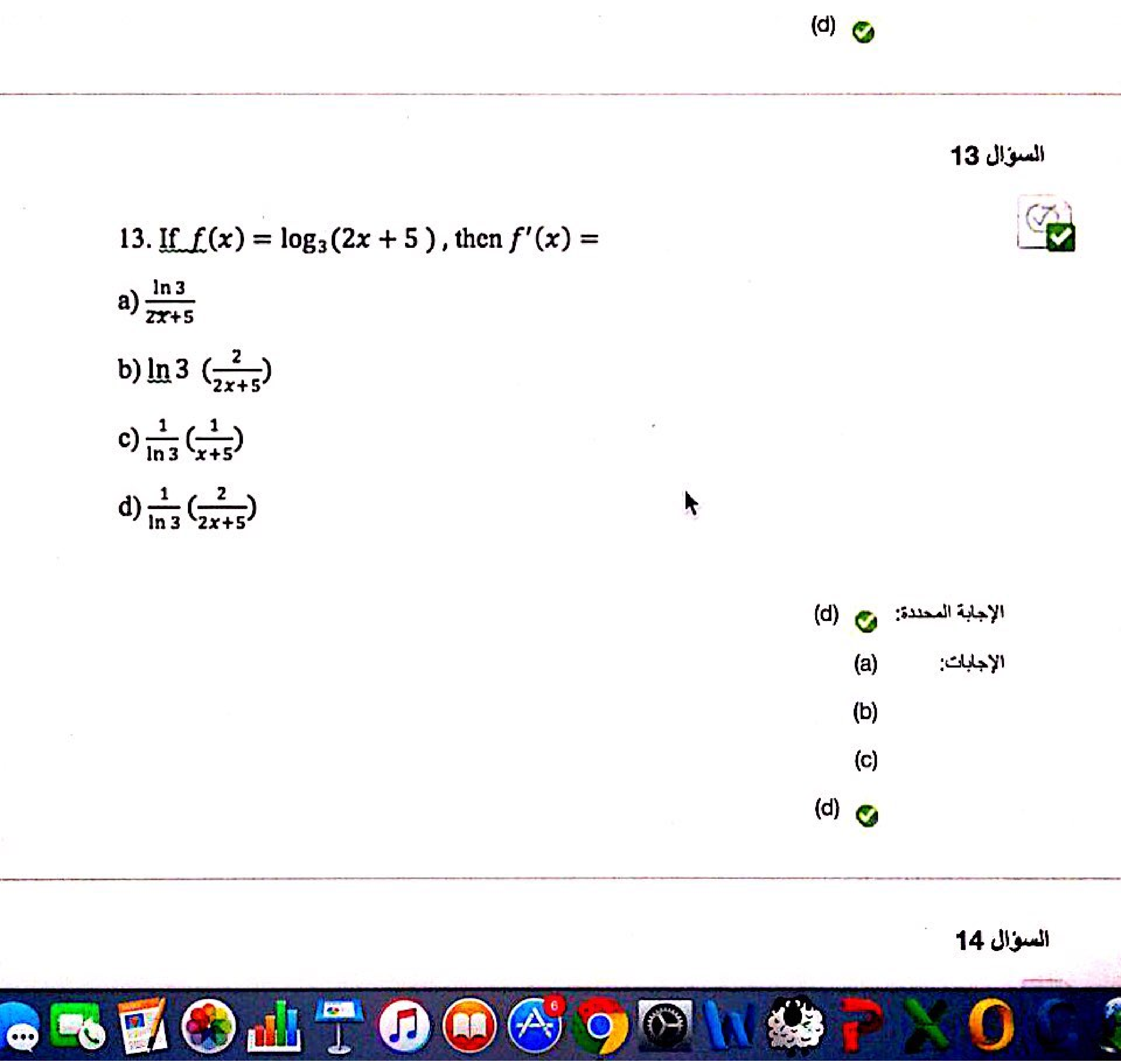
الإجابات: (a)

(b)

✓ (c)

(d)

السؤال 11



(d) ✓

السؤال 13

☑

13. If $f(x) = \log_3(2x + 5)$, then $f'(x) =$

a) $\frac{\ln 3}{2x+5}$

b) $\ln 3\ (\frac{2}{2x+5})$

c) $\frac{1}{\ln 3}(\frac{1}{x+5})$

d) $\frac{1}{\ln 3}(\frac{2}{2x+5})$

الإجابة المحددة: ✓ (d)

الإجابات: (a)

(b)

(c)

(d) ✓

السؤال 14

(d) ✓

السؤال 14

14. The function $f(x) = \frac{x+1}{x^2-x-6}$ is discontinuous at $x =$

a) -2,3

b) -3,2

c) 2,3

d) -3,-2

الإجابة المحددة: ✓ (a)

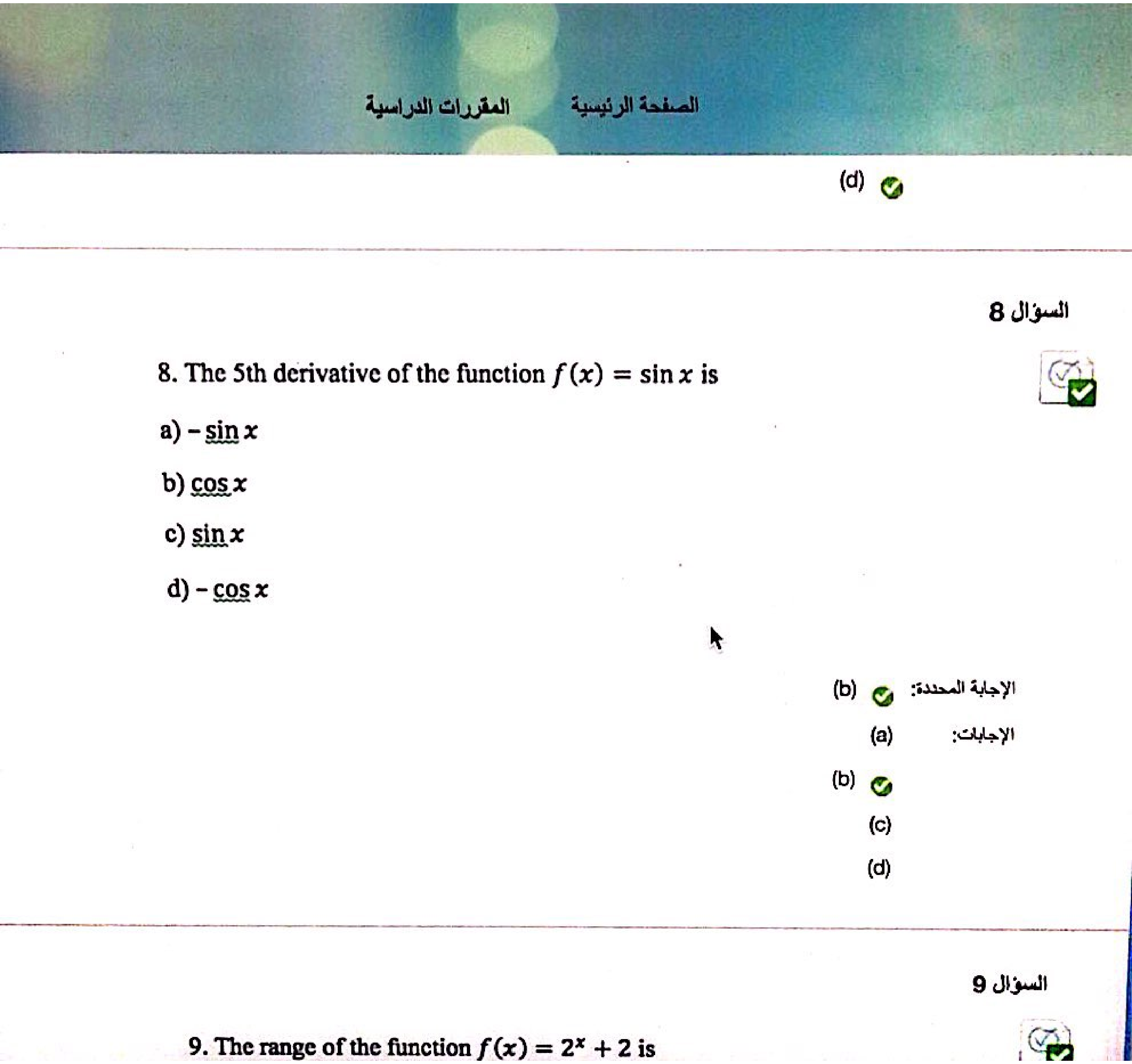
الإجابات: ✓ (a)

(b)

(c)

(d)

السؤال 15



✓ (d)

السؤال 8

8. The 5th derivative of the function $f(x) = \sin x$ is

a) $-\sin x$

b) $\cos x$

c) $\sin x$

d) $-\cos x$

الإجابة المحددة: ✓ (b)

الإجابات: (a)

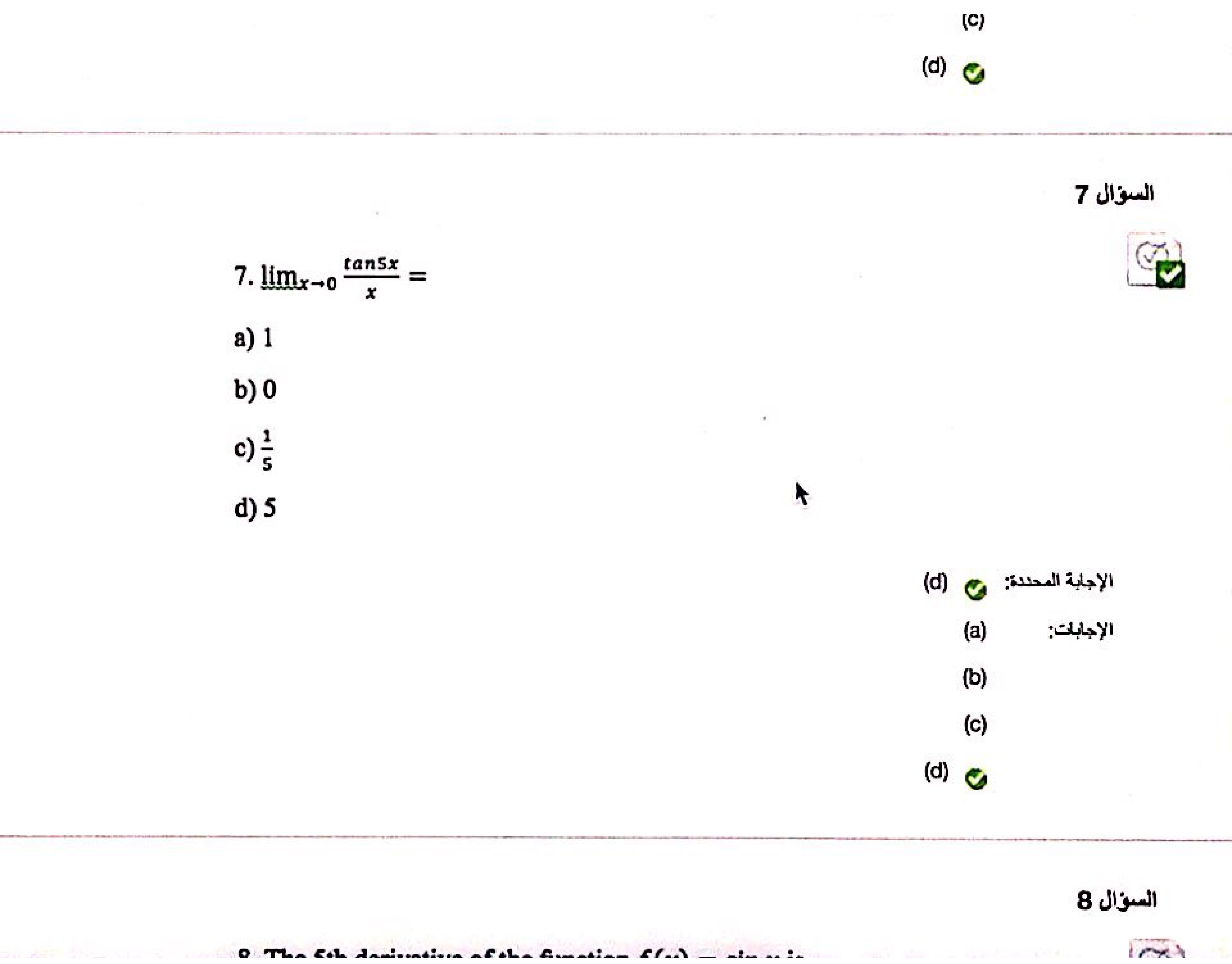
✓ (b)

(c)

(d)

السؤال 9

9. The range of the function $f(x) = 2^x + 2$ is



(c)

✓ (d)

السؤال 7

7. $\lim_{x\to 0}\frac{\tan 5x}{x} =$

a) 1

b) 0

c) $\frac{1}{5}$

d) 5

الإجابة المحددة: ✓ (d)

الإجابات: (a)

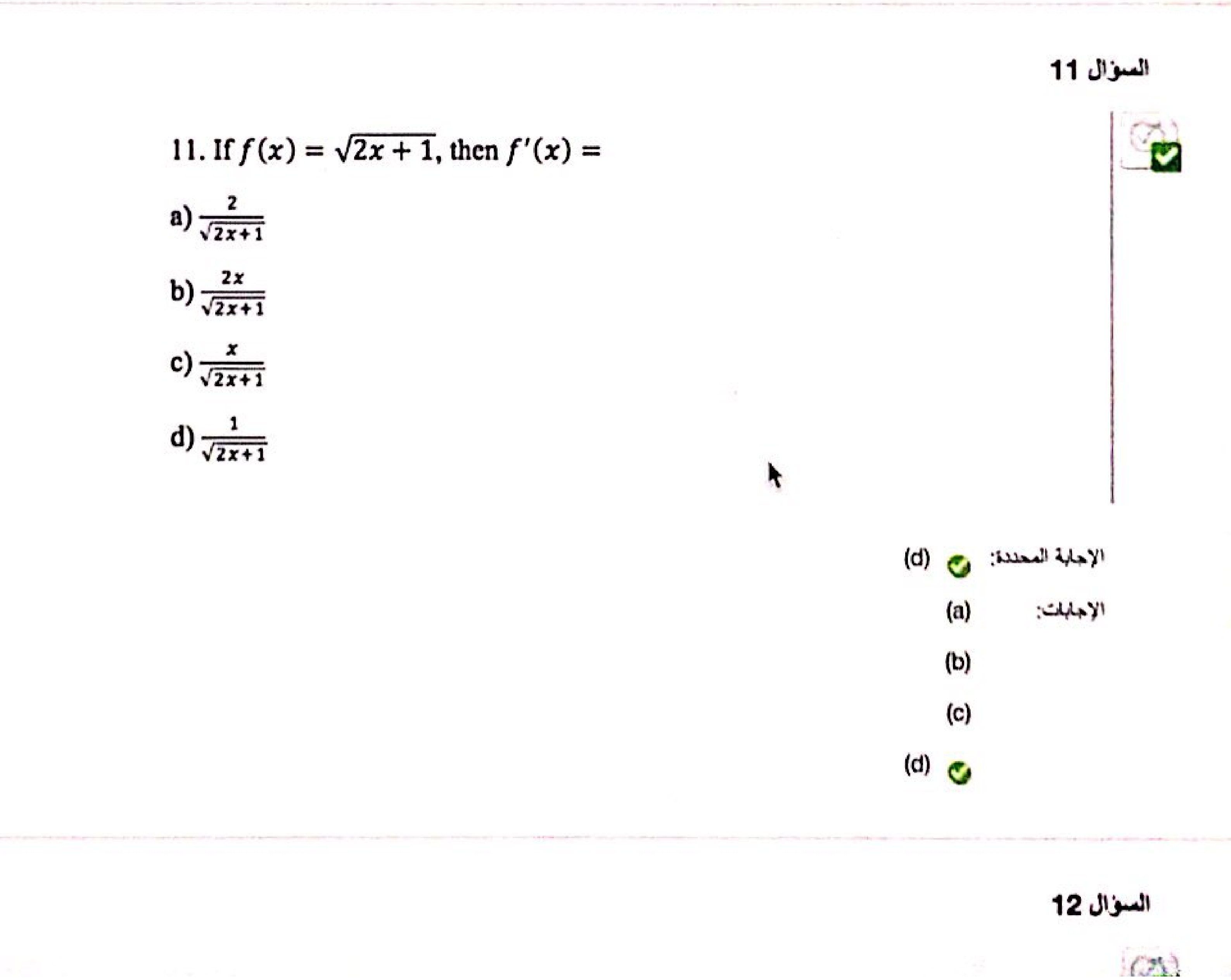
(b)

(c)

✓ (d)

السؤال 8

8. The 5th derivative of the function $f(x) = \sin x$ is



السؤال 11

11. If $f(x) = \sqrt{2x+1}$, then $f'(x) =$

a) $\frac{2}{\sqrt{2x+1}}$

b) $\frac{2x}{\sqrt{2x+1}}$

c) $\frac{x}{\sqrt{2x+1}}$

d) $\frac{1}{\sqrt{2x+1}}$

الإجابة المحددة: ✓ (d)

الإجابات: (a)

(b)

(c)

✓ (d)

السؤال 12

السؤال 15

15. The equation of the tangent line to the curve $y = x^3 e^x + 4x$ at the point (0,1) is

a) $y = 4x - 1$

b) $y = 4x + 4$

c) $y = 4x$

d) $y = 4x + 1$

الإجابة المحددة: ✓ (d)

الإجابات: (a)

(b)

(c)

✓ (d)

(c)

(d) ✓

السؤال 16

☒

16. The value of c that satisfies the conditions of the Mean Value Theorem for

$f(x) = x^2 + 3x - 1$ on $[-1,3]$ is $c =$

a) $\frac{1}{2}$

b) $\frac{-1}{2}$

c) -1

d) 1

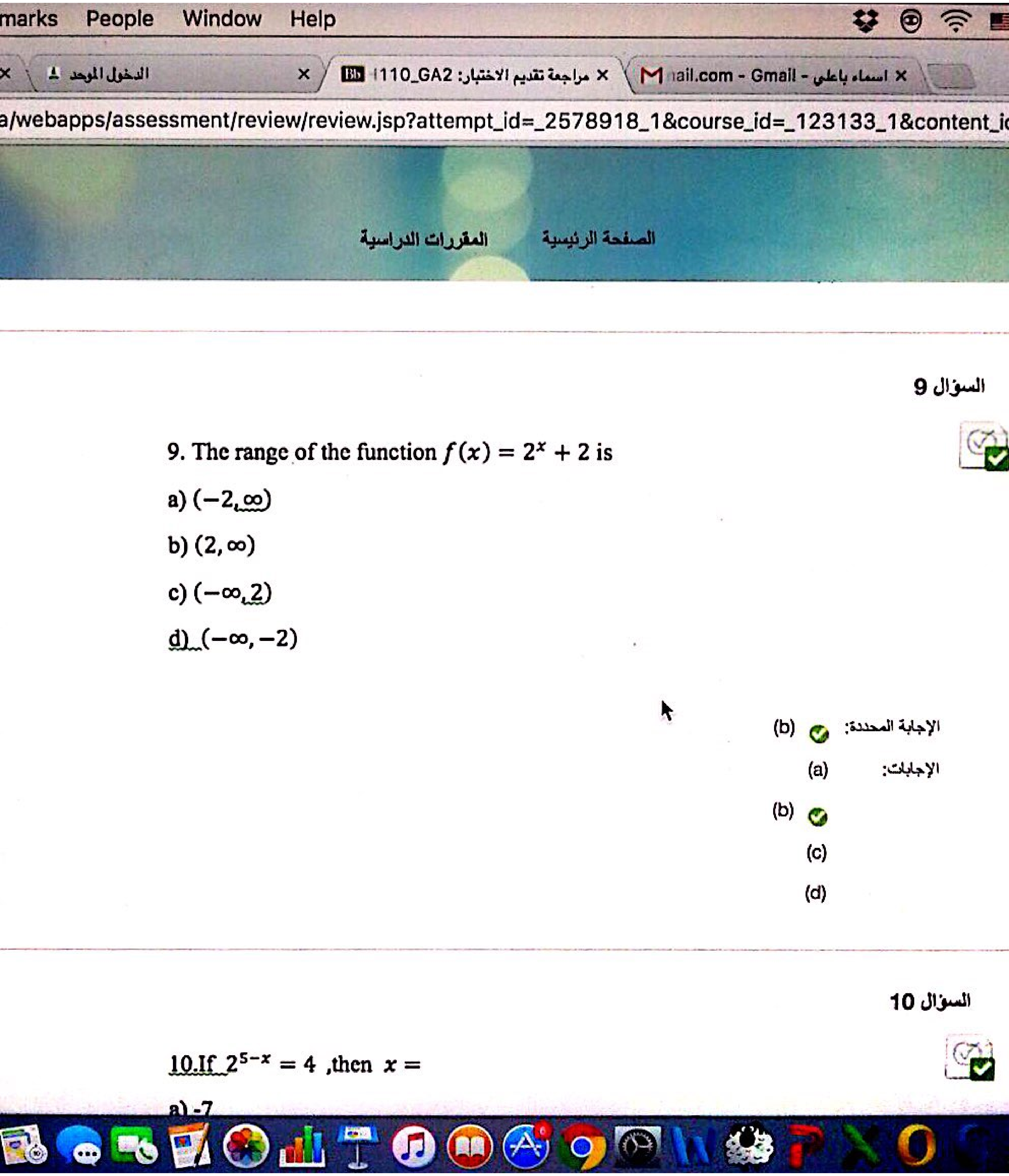
الإجابة المحددة: ✗ [لم يتم إعطاء إجابة]

الإجابات: (a)

(b)

(c)

(d) ✓



السؤال 9

9. The range of the function $f(x) = 2^x + 2$ is

a) $(-2, \infty)$

b) $(2, \infty)$

c) $(-\infty, 2)$

d) $(-\infty, -2)$

الإجابة المحددة: ✓ (b)

الإجابات: (a)

✓ (b)

(c)

(d)

السؤال 10

10.If $2^{5-x} = 4$,then $x =$

a) -7

(d)

السؤال 20

20. If $1 - 4x^2 \le f(x) \le \ln x - 3x$, then $\lim_{x\to 1} f(x) =$

a) 0

b) -5

c) -3

d) 1

الإجابة المحددة: ✓ (c)

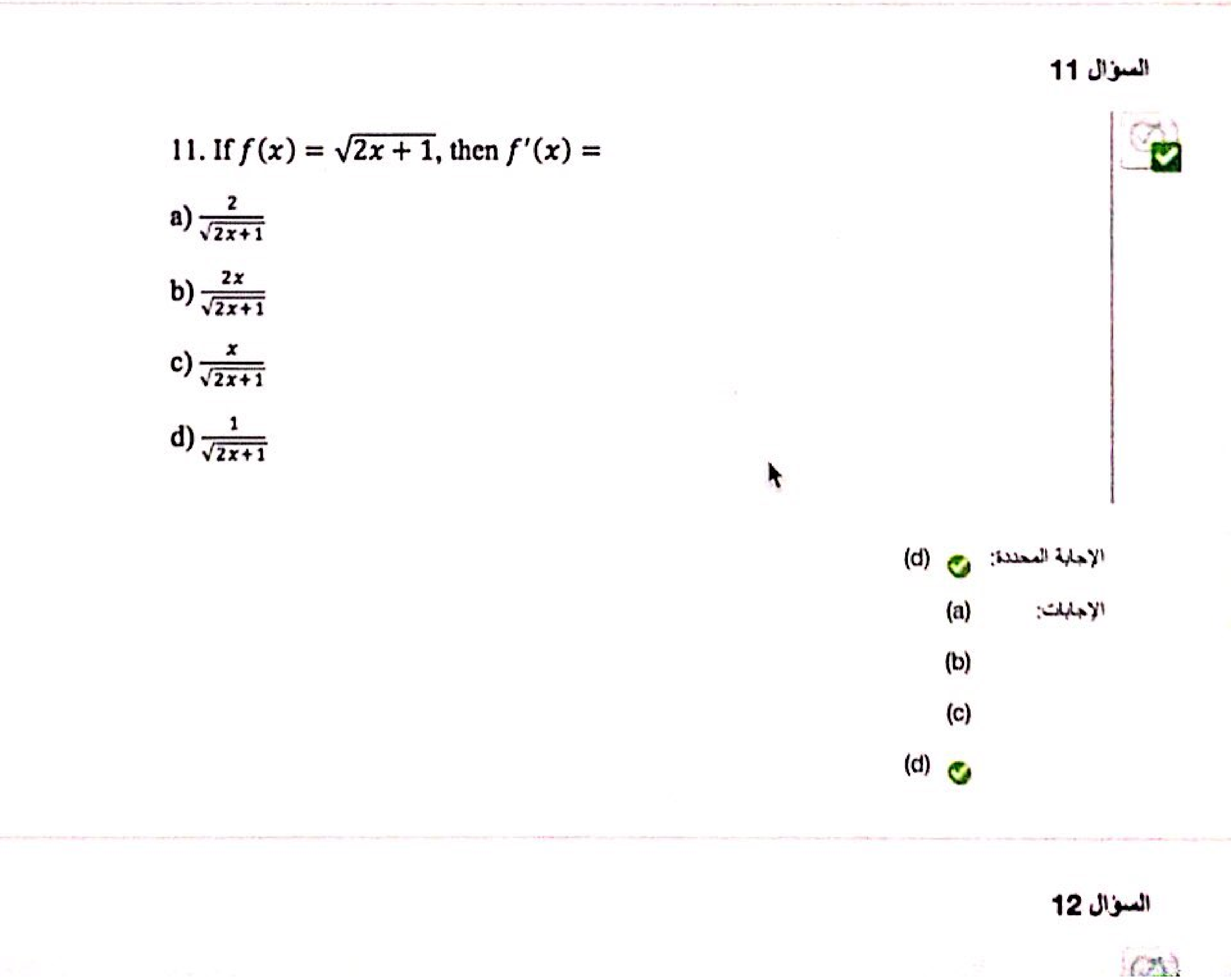
الإجابات: (a)

(b)

✓ (c)

(d)

السؤال 21



السؤال 11

11. If $f(x) = \sqrt{2x+1}$, then $f'(x) =$

a) $\frac{2}{\sqrt{2x+1}}$

b) $\frac{2x}{\sqrt{2x+1}}$

c) $\frac{x}{\sqrt{2x+1}}$

d) $\frac{1}{\sqrt{2x+1}}$

الإجابة المحددة: ✓ (d)

الإجابات: (a)

(b)

(c)

✓ (d)

السؤال 12

السؤال 15

15. The equation of the tangent line to the curve $y = x^3 e^x + 4x$ at the point (0,1) is

a) $y = 4x - 1$

b) $y = 4x + 4$

c) $y = 4x$

d) $y = 4x + 1$

الإجابة المحددة: ✓ (d)

الإجابات: (a)

(b)

(c)

✓ (d)

(c)

(d) ✓

السؤال 16

16. The value of c that satisfies the conditions of the Mean Value Theorem for

$$f(x) = x^2 + 3x - 1 \text{ on } [-1,3] \text{ is } c =$$

a) $\frac{1}{2}$

b) $\frac{-1}{2}$

c) -1

d) 1

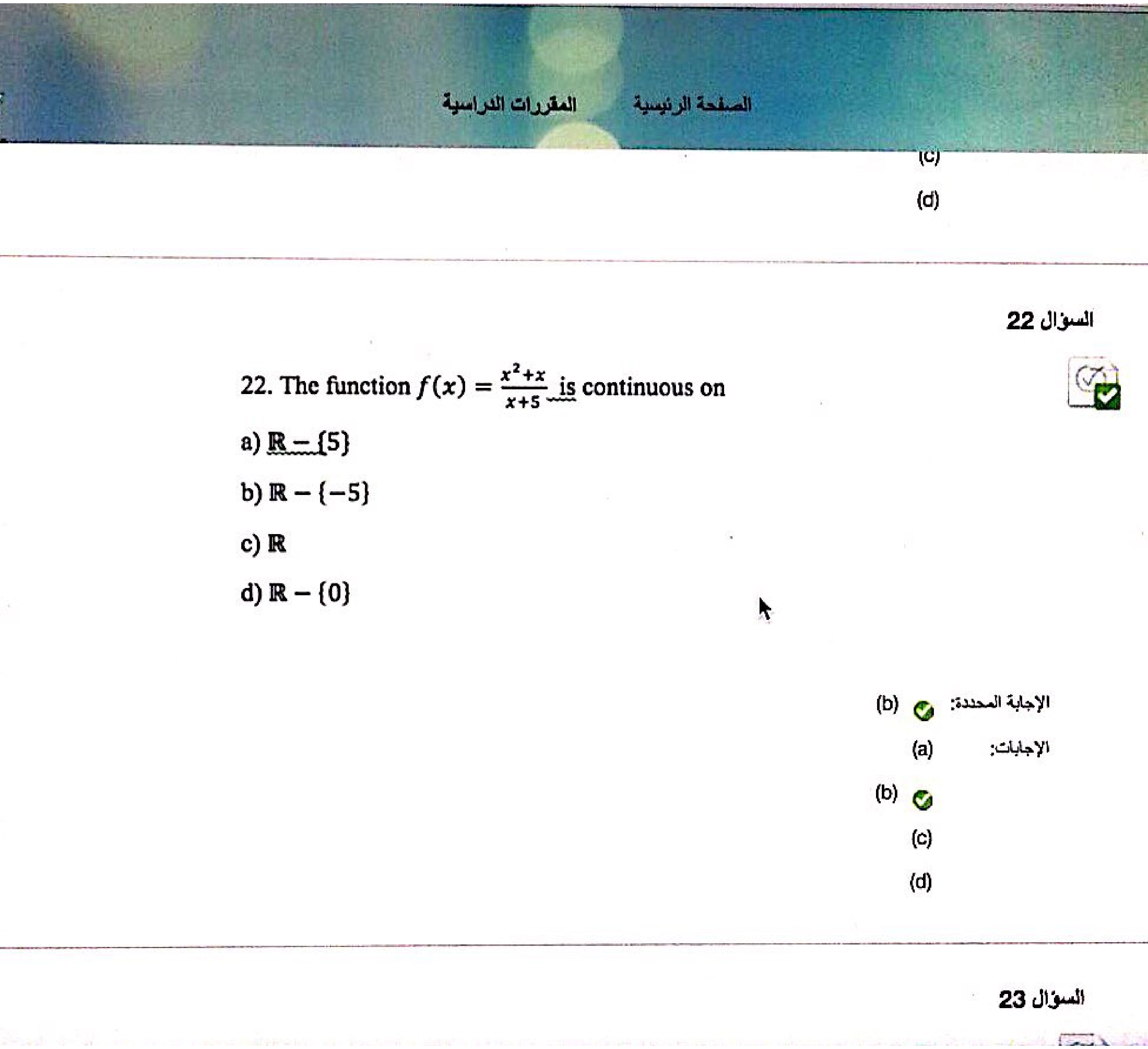
الإجابة المحددة: ✗ [لم يتم إعطاء إجابة]

الإجابات: (a)

(b)

(c)

(d) ✓



(c)

(d)

السؤال 22

22. The function $f(x) = \frac{x^2+x}{x+5}$ is continuous on

a) $\mathbb{R} - \{5\}$

b) $\mathbb{R} - \{-5\}$

c) $\mathbb{R}$

d) $\mathbb{R} - \{0\}$

الإجابة المحددة: ✓ (b)

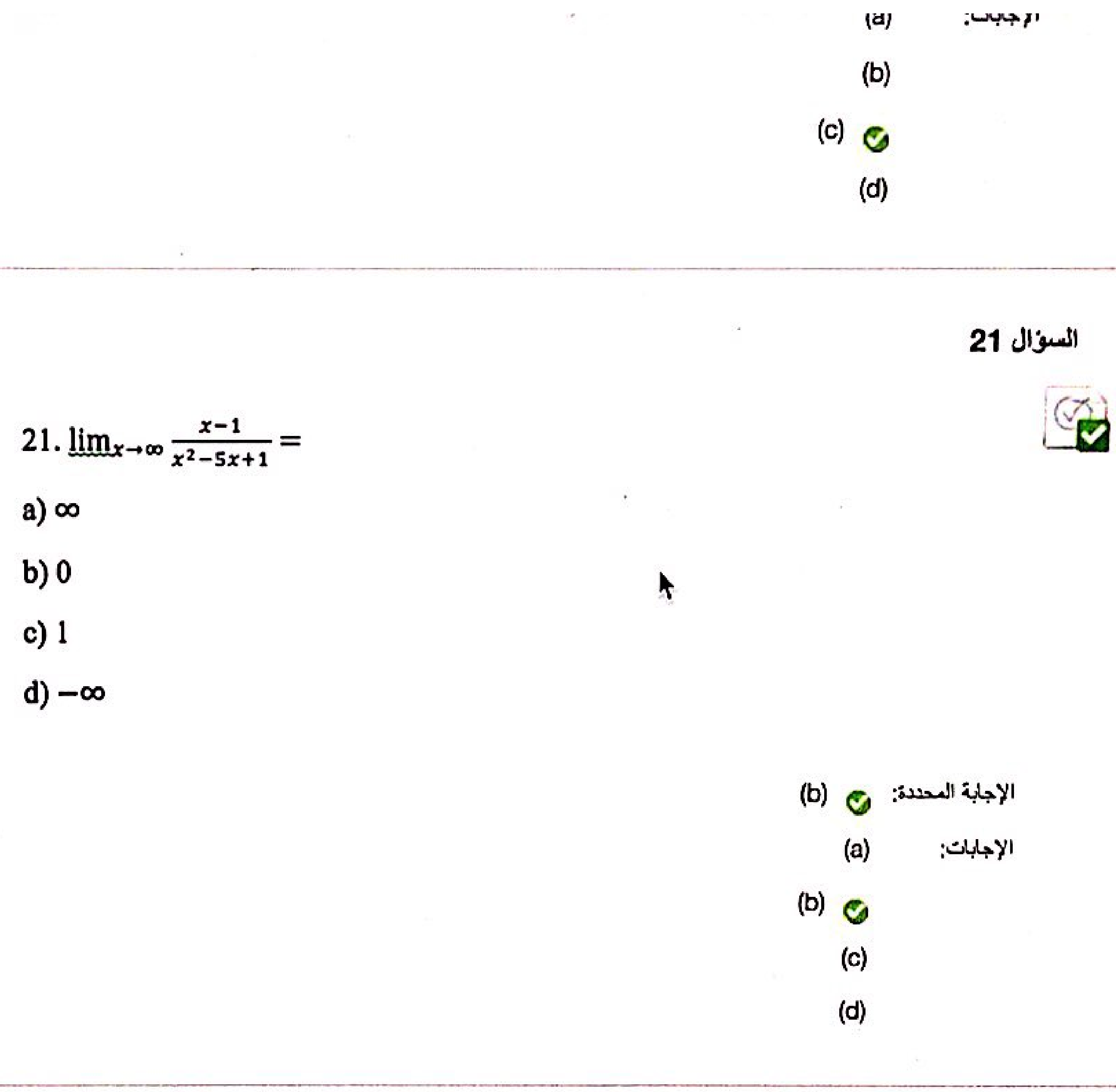
الإجابات: (a)

✓ (b)

(c)

(d)

السؤال 23



الإجابات: (a)

(b)

✓ (c)

(d)

السؤال 21

21. $\lim_{x\to\infty} \frac{x-1}{x^2-5x+1} =$

a) ∞

b) 0

c) 1

d) $-\infty$

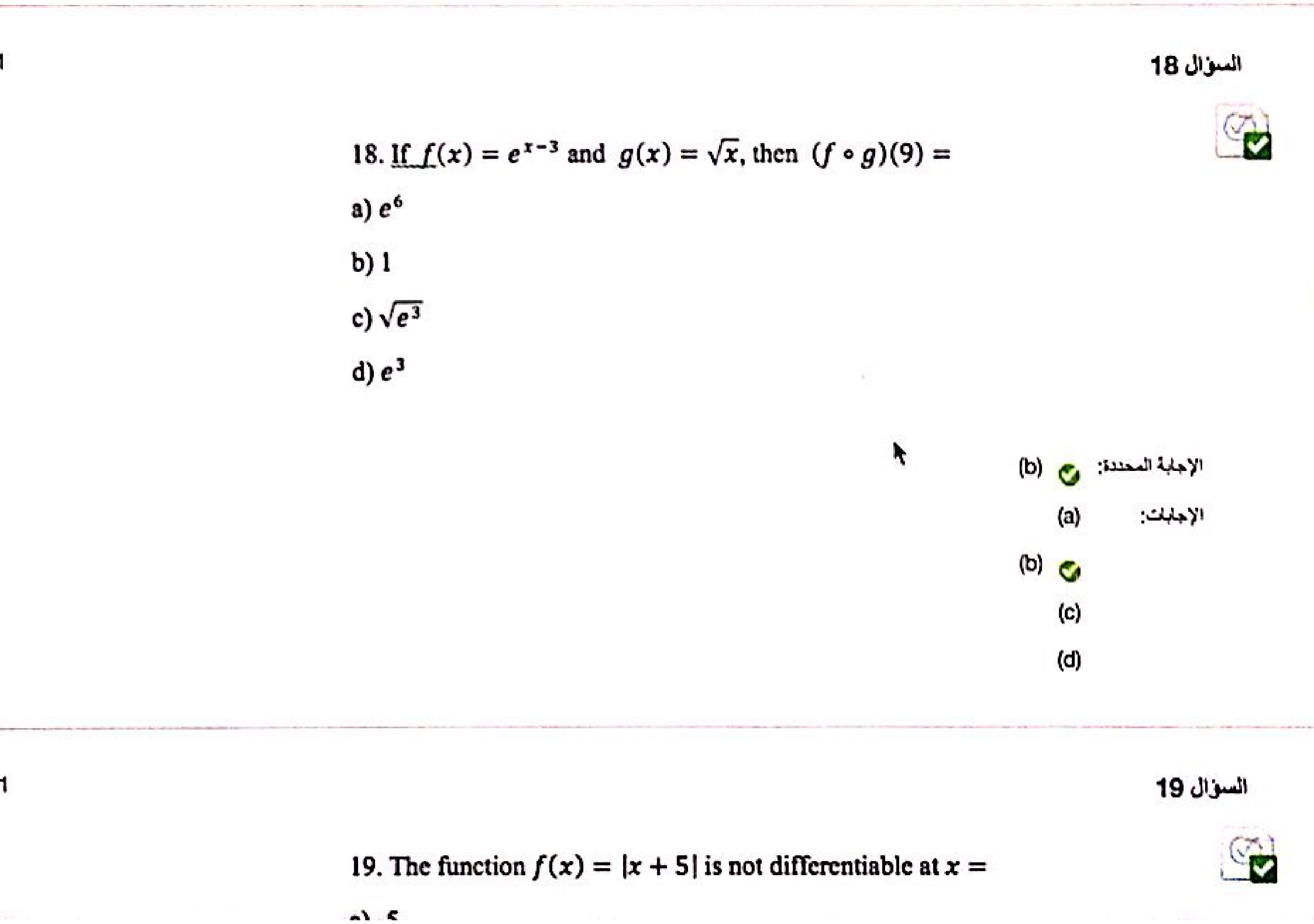
الإجابة المحددة: ✓ (b)

الإجابات: (a)

✓ (b)

(c)

(d)



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السؤال 18

18. If $f(x) = e^{x-3}$ and $g(x) = \sqrt{x}$, then $(f \circ g)(9) =$

a) e^6

b) 1

c) $\sqrt{e^3}$

d) e^3

الإجابة المحددة: ✓ (b)

الإجابات: (a)

✓ (b)

(c)

(d)

1

السؤال 19

19. The function $f(x) = |x + 5|$ is not differentiable at $x =$

a) -5

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(c)

(d)

السؤال 19

19. The function $f(x) = |x + 5|$ is not differentiable at $x =$

a) -5

b) 5

c) 25

d) 0

الإجابة المحددة: ✓ (a)

الإجابات: ✓ (a)

(b)

(c)

(d)

السؤال 20

(d)

السؤال 23

23. If $y = fg, f(2) = 3, \ g(2) = -4, \ f'(2) = 3, \ g'(2) = 2$,then $y'(2) =$

a) 6

b) -6

c) -18

d) 18

الإجابة المحددة: ✓ (b)

الإجابات: (a)

✓ (b)

(c)

(d)

السؤال 24

(b) ✓

(c)

(d)

السؤال 24

✓

24. $\frac{d}{dx}(-3x^2 \cot x) =$

a) $6x(\csc^2 x - \cot x)$

b) $-3x(x \csc^2 x + 2 \cot x)$

c) $3x(x \csc^2 x - 2\cot x)$

d) $3x(2 \cot x - x \csc^2 x)$

الإجابة المحددة: ✓ (c)

الإجابات: (a)

(b)

(c) ✓

(d)

(c) ✓

(d)

السؤال 25

25. $\lim_{x\to -5^+} \sqrt{25 - x^2} =$

a) 5

b) does not exist

c) 0

d) -5

الإجابة المحددة: ✓ (c)

الإجابات: (a)

(b)

✓ (c)

(d)

السؤال 26

السؤال 17

☑

17. If $f(x) = 3x^2 - e^2$, then $f'(x) =$

a) $6x - 2e$

b) $6x - 1$

c) $6x$

d) $6x - e^2$

الإجابة المحددة: ✓ (c)

الإجابات: (a)

(b)

✓ (c)

(d)

السؤال 18

☑

18. If $f(x) = e^{x-3}$ and $g(x) = \sqrt{x}$, then $(f \circ g)(9) =$

Section (1.6) :—

if $x_1 \neq x_2 \Longrightarrow f(x_1) \neq f(x_2)$

or $f(x_1) = f(x_1) \Longrightarrow x_1 = x_2$

① One-to-one :
- * Def ① بأي التعريف ✓ أو X
- * By the graph does the function one-to-one ??

أي يعطيك رسم ويسأل هل الدالة one-to-one؟
- * أمثلة على one-to-one في الكتاب

مثال ① و ⓒ

② Inverse function :
- * كل داله ومعكوسها متماثله حول المستقيم $y = x$
- * Cancellation law $f(f^{-1}(x)) = x$

$$f^{-1}(f(x)) = x$$

Example : If $f(x) = \sqrt{x} + 11$ ~~and~~ then $(f^{-1} \circ f)(x) = \ldots\ldots$

ⓐ $\frac{1}{x}$ $\boxed{ⓑ\ x}$ ⓒ $\sqrt{x}+11$ ⓓ $\frac{1}{\sqrt{x}+11}$

* Find the inverse function :

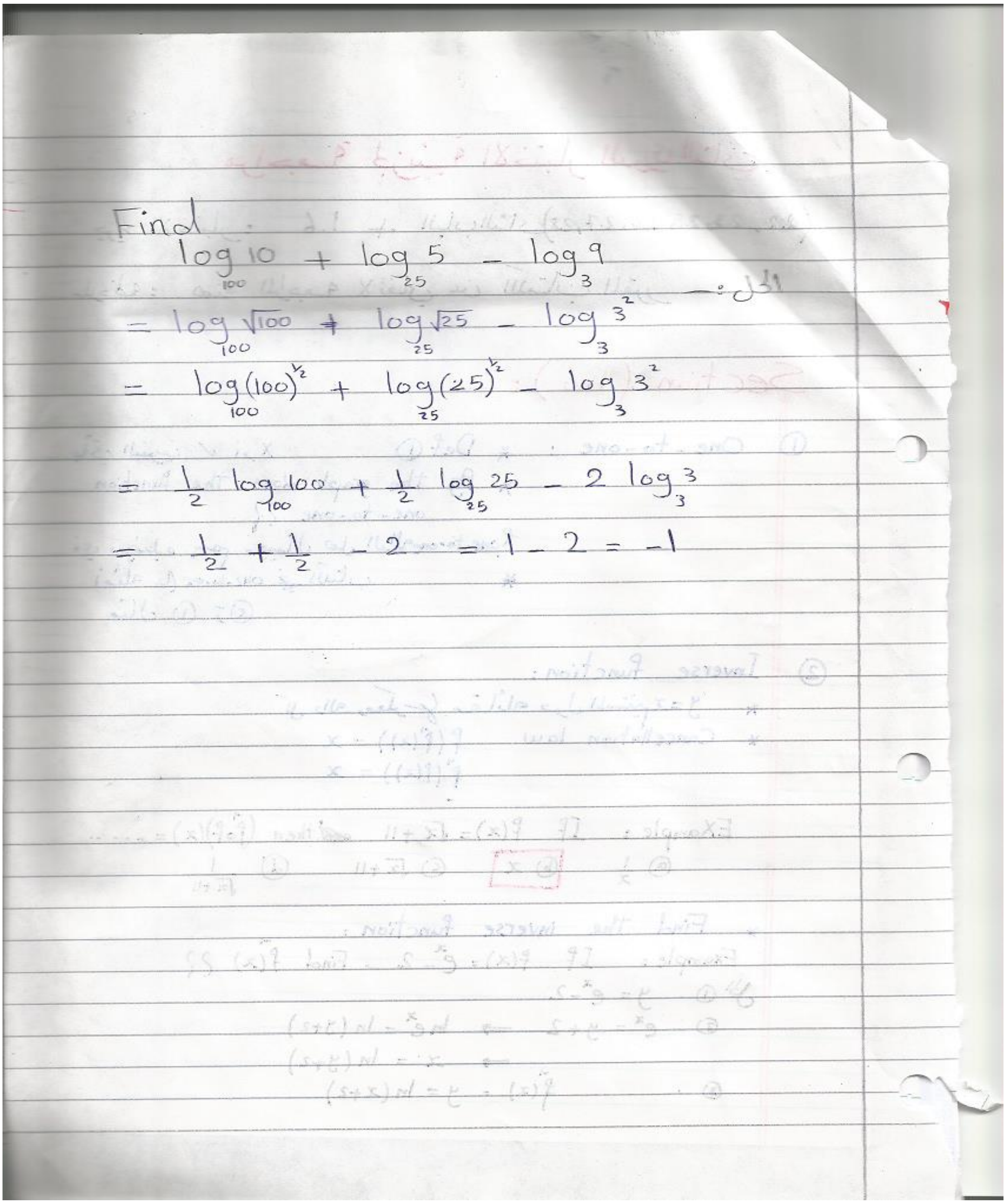
Example : If $f(x) = e^{x} - 2$. Find $f^{-1}(x)$??

الحل ① $y = e^{x} - 2$

② $e^{x} = y + 2 \Longrightarrow \ln e^{x} = \ln(y+2)$

$\Longrightarrow x = \ln(y+2)$

③ $f^{-1}(x) = y = \ln(x+2)$



Find

$\log_{100} 10 + \log_{25} 5 - \log_{3} 9$

الحل :-

$= \log_{100} \sqrt{100} + \log_{25} \sqrt{25} - \log_{3} 3^{2}$

$= \log_{100} (100)^{\frac{1}{2}} + \log_{25} (25)^{\frac{1}{2}} - \log_{3} 3^{2}$

$= \frac{1}{2} \log_{100} 100 + \frac{1}{2} \log_{25} 25 - 2 \log_{3} 3$

$= \frac{1}{2} + \frac{1}{2} - 2 = 1 - 2 = -1$

③ logarithmic function :-

* Definition $\quad y = \log_a x \iff x = a^y$ $\Big\}$ (T or $\underline{F}$)

* قوانين اللوغاريتم

* Find ?? Example : $\log_5 20 - \log_5 4 + \log_5 25$

$$= \log_5 \frac{(20)(25)}{4}$$

$$= \log_5 5^3 = 3 \log_5 5 = 3$$

* Find x ??

Example $\longrightarrow$ ① $e^{5-3x} = 10$

$$\ln e^{5-3x} = \ln 10$$

$$5 - 3x = \ln 10$$

$$5 - \ln 10 = 3x$$

$$x = \frac{5 - \ln 10}{3} = \frac{1}{3}(5 - \ln 10)$$

---

② $\ln x = 5$

$$e^{\ln x} = e^5$$

$$\boxed{x = e^5}$$

④ Inverse trigonometric function :-

* Domain and range

* Find $\longrightarrow$ ① $\sin^{-1} \frac{1}{2}$

الحل :- Let $\theta = \sin^{-1} \frac{1}{2}$

$\longrightarrow \sin\theta = \frac{1}{2} \Longrightarrow \theta = \pi/6 = 30^\circ$

---

② $\sin\left(\cot^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

— الحل : —

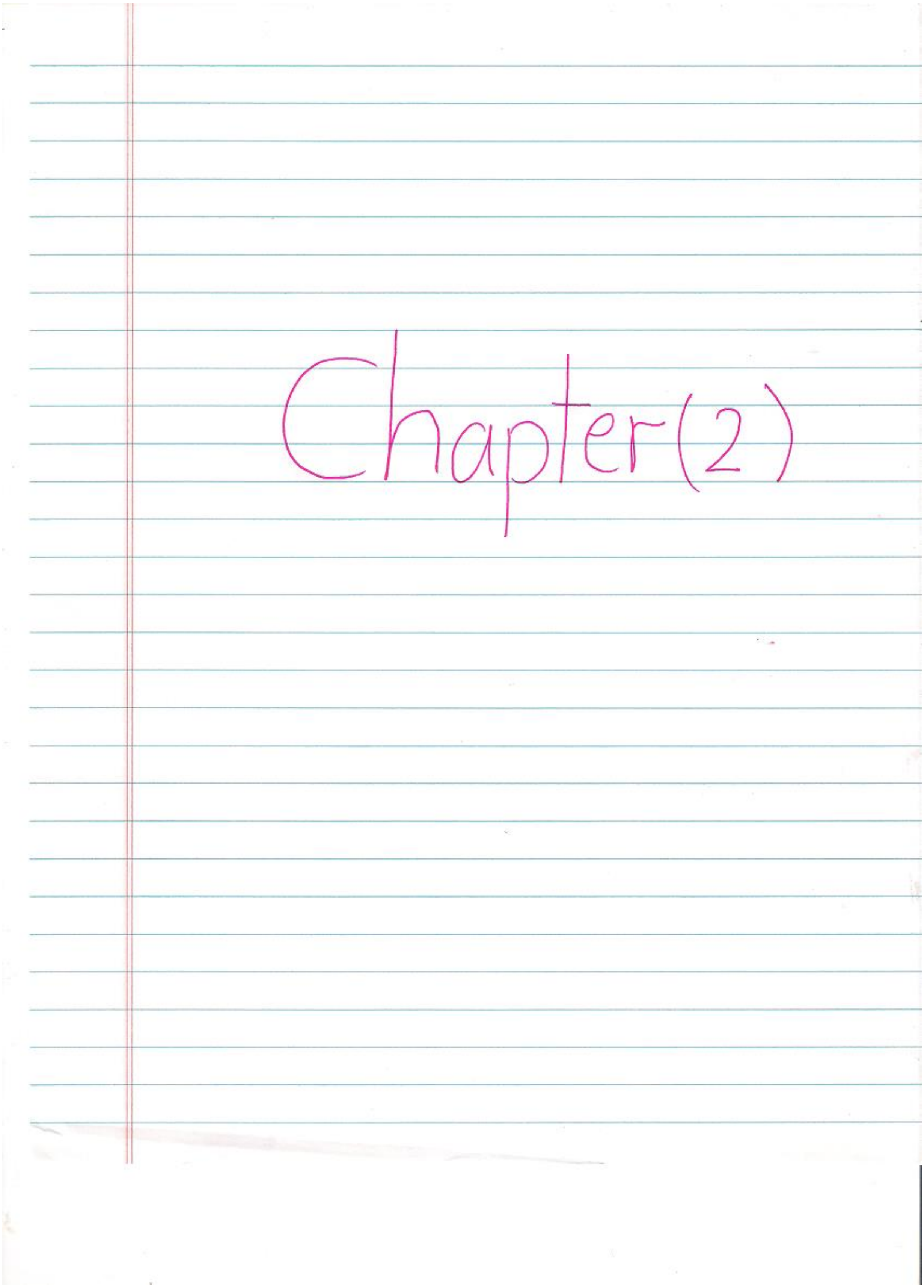
Let $\theta = \cot^{-1}\frac{\sqrt{3}}{2} \longrightarrow \cot\theta = \frac{\sqrt{3}}{2} = \frac{\cos\theta}{\sin\theta} = \frac{\text{المجاور}}{\text{المقابل}}$

$\therefore \sin\left(\cot^{-1}\frac{\sqrt{3}}{2}\right) = \sin\theta = \frac{\text{المقابل}}{\text{الوتر}}$

$\therefore (\text{الوتر})^2 = (\text{المقابل})^2 + (\text{المجاور})^2$

$= 4 + 3 = 7 \Longrightarrow \text{الوتر} = \sqrt{7}$

$$\boxed{\therefore \sin\theta = \frac{2}{\sqrt{7}}}$$



Chapter(2)

limit

$$\lim_{x\to a} f(x)$$

نبدأ بالتقويض المباشر
وفي حالة الحصول عل
أحد حالات عدم التقيين

$$\left(\frac{\infty}{\infty}, \frac{0}{0}, \infty-\infty, \infty+\infty, 0(\infty), 0(-\infty)\right)$$

أقوم بالاختصار أو
الضرب في المرافق أو ....

$$\lim_{x\to\pm\infty} f(x)$$

And horizontal Asymptote

$$\lim_{x\to a} f(x) = \pm\infty$$

And Vertical Asymptote $x = a$

If f(x) is rational function

$$\therefore f(x) = \frac{g(x)}{h(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_0}$$

① If $n = m \Rightarrow \lim_{x\to\pm\infty} \frac{g(x)}{h(x)} = \frac{a_n}{b_m}$

② If $m > n \Rightarrow \lim_{x\to\infty} \frac{g(x)}{h(x)} = \pm\infty$

$$\lim_{x\to-\infty} \frac{g(x)}{h(x)} = \pm\infty$$

③ If $m < n \Rightarrow \lim_{x\to\pm\infty} \frac{g(x)}{h(x)} = 0$

If f(x) is a polynomial function

$$\therefore f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$$

① $\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} (a_n x^n) = a_n \lim_{x\to\pm\infty} x^n = \infty$ } n even

② $\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} (a_n x^n) = a_n \lim_{x\to\pm\infty} x^n = \pm\infty$ } n odd

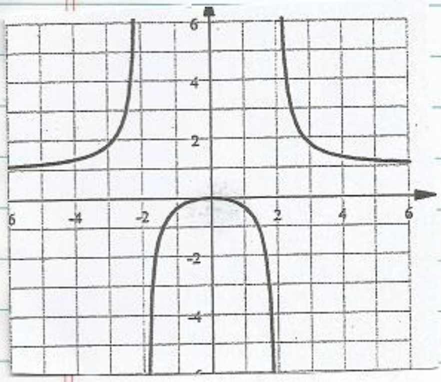
Section (2.2):

① Find the limit by the graph. ايجاد النهاية من الرسم

② Find one side limit by the graph.

③ Vertical asymptote

ايجاده من الرسم



∴ the vertical asymptote are $\boxed{x=2}$, $\boxed{x=-2}$

ايجاده بالحل

Example:-

$f(x) =$ [illegible] $\frac{3-x}{x^2-x-6}$

الحل:-

① الاختصار

$f(x) = \frac{3-x}{(x-3)(x+2)}$

$= \frac{-(x-3)}{\cancel{(x-3)}(x+2)} = \frac{-1}{x+2}$

② أصفار المقام

$x+2=0 \Longrightarrow \boxed{x=-2}$

∴ the vertical asymptote is $x=-2$

④ $\lim_{x\to(\frac{\pi}{2})^-} \tan x = +\infty$

$\lim_{x\to(\frac{\pi}{2})^+} \tan x = -\infty$

تحفظ

x ممكن أيضاً تأخذ القيم $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

Section (2.3) :—

① Compute the limit by direct substiution

اوجدي النهايه بالتعويض المباشر

Example: $\lim_{x\to -2}(x^3-2x+1) = -8+4+1 = -3$

② Example :—

* $\lim_{x\to 2}\frac{f(x)-8}{x-1} = 10$. Find $\lim_{x\to 2} f(x)$?

هذا المثال موجود في صفحه 108 تمرين (57)

الحل :—

$$\frac{\lim_{x\to 2} f(x) - \lim_{x\to 2} 8}{\lim_{x\to 2} x - \lim_{x\to 2} 1} = 10$$

$$\Longrightarrow \frac{\lim f(x) - 8}{2-1} = 10$$

$$\Longrightarrow \frac{\lim f(x) - 8}{1} = 10$$

$$\Longrightarrow \lim_{x\to 2} f(x) - 8 = 10 \Longrightarrow \boxed{\lim_{x\to 2} f(x) = 10+8 = 18}$$

---

* $\lim_{x\to 1}\frac{f(x)+3x}{x^2-5f(x)} = 1$. Find $\lim_{x\to 1} f(x)$??

الحل :—

$$\frac{\lim_{x\to 1} f(x) + \lim_{x\to 1} 3x}{\lim_{x\to 1} x^2 - 5\lim_{x\to 1} f(x)} = 1$$

$$\Longrightarrow \frac{\lim f(x) + 3}{1 - 5\lim f(x)} \xcancel{=} \frac{1}{1}$$

$$\Longrightarrow \lim_{x\to 1} f(x) + 3 = 1 - 5\lim_{x\to 1} f(x)$$

$$\Longrightarrow \lim f(x) + 5\lim f(x) = 1 - 3$$

$$\Longrightarrow 6\lim_{x\to 1} f(x) = -2 \Longrightarrow \boxed{\lim_{x\to 1} f(x) = \frac{-2}{6} = \frac{-1}{3}}$$

③ حساب النهايه عندما نحصل على $\frac{0}{0}$

Example:—

① $\lim\limits_{x\to 3} \frac{x-3}{x^3-27} = \frac{0}{0} \longrightarrow \lim\limits_{x\to 3} \frac{\cancel{x-3}}{(\cancel{x-3})(x^2+3x+9)} = \frac{1}{9+9+9} = \frac{1}{27}$

قاعدة لوبيتال $\downarrow$

$\lim\limits_{x\to 3} \frac{1}{3x^2} = \frac{1}{3(9)} = \frac{1}{27}$

---

② $\lim\limits_{x\to 0} \frac{\sqrt{x+25}\ -5}{x} = \frac{0}{0} \longrightarrow$ بالضرب في المرافق

قاعد لوبيتال $\downarrow$

$\lim\limits_{x\to 0} \frac{\frac{1}{2}(x+25)^{-\frac{1}{2}}(1)}{1} = \lim\limits_{x\to 0} \frac{1}{2\sqrt{x+25}} = \frac{1}{2(5)} = \frac{1}{10}$

---

③ $\lim\limits_{x\to 0} \frac{x^3-7x^2}{x^2} = \frac{0}{0} \longrightarrow \lim\limits_{x\to 0} \frac{\cancel{x^2}(x-7)}{\cancel{x^2}} = -7$

قاعدة لوبيتال $\downarrow$

$\lim\limits_{x\to 0} \frac{3x^2-14x}{2x} = \frac{0}{0}$

نعيد قاعدة لوبيتال مرة أخرى:

$\lim\limits_{x\to 0} \frac{6x-14}{2} = \frac{-14}{2} = -7$

④ limit of trigonometric function

By direct subsititution

Example :—

$$\lim_{x\to \pi/2}(\sin x+\cos x)$$

$$= \sin \pi/2 + \cos \pi/2$$

$$= 1+0=1$$

the limit with $\frac{0}{0}$

EX : $\lim_{x\to 0} \frac{\cos^2 x-1}{\cos x-1} = \frac{\cos 0-1}{\cos 0-1} = \frac{1-1}{1-1} = \frac{0}{0}$

$$\therefore \lim_{x\to 0} \frac{\cos^2 x-1}{\cos x-1} = \lim_{x\to 0} \frac{(\cos x-1)(\cos x+1)}{(\cos x-1)}$$

$$= \lim_{x\to 0}(\cos x+1) = 1+1=2$$

⑤ استخدام النظريه

$$\boxed{\begin{array}{l}\lim_{x\to 0} \frac{\sin x}{x} = 1 \\ \lim_{x\to 0} \frac{\tan x}{x} = 1\end{array}}$$

النظريه موجوده في الباب الثالث صفحه 192

وكذلك في صفحه 193 ايجاد النظريه $\lim_{\theta\to 0} \frac{\cos\theta-1}{\theta}$

والتمارين المتعلقه بها في صفحه 198

⑥ ايضاً استخدام

$$\boxed{\lim_{\substack{x\to\infty \\ x\to-\infty}} \frac{\sin x}{x} = 0}$$

Example : If $y=\frac{x}{\sin x+2x}+\frac{2x^2+x+1}{x^2+x-5}$ find the horizontal asymptote ?

الحل

$$\lim_{x\to\infty} \frac{x}{\sin x+2x} + \lim_{x\to\infty} \frac{2x^2+x+1}{x^2+x-5}$$

$$= \lim_{x\to\infty} \frac{1}{\frac{\sin x+2x}{x}} + 2$$

$$= \lim_{x\to\infty} \frac{1}{\frac{\sin x}{x}+2} + 2 = \frac{1}{0+2}+2 = \frac{1}{2}+2 = \frac{1}{2}+\frac{4}{2} = \frac{5}{2}$$

⑤ Sandwich theorem :—

Example :— If $\frac{x^2+9}{x-3} \leq f(x) \leq x-3$

find $\lim\limits_{x\to 0} f(x)$??

الحل :-

$\because \lim\limits_{x\to 0} x-3 = -3$, $\lim\limits_{x\to 0} \frac{x^2+9}{x-3} = \frac{9}{-3} = -3$

$\therefore \boxed{\lim\limits_{x\to 0} f(x) = -3}$

Example : find $\lim\limits_{x\to 0} x^2 \sin(\frac{1}{x})$??

الحل

$\because \quad -1 \leq \sin(\frac{1}{x}) \leq 1$

$-x^2 \leq x^2 \sin(\frac{1}{x}) \leq x^2$

$\because \lim\limits_{x\to 0} (-x^2) = 0$, $\lim\limits_{x\to 0} x^2 = 0$

$\boxed{\therefore \lim\limits_{x\to 0} x^2 \sin(\frac{1}{x}) = 0}$

⑥ Find the limit of piecewise function :

Example : If $f(x) = \begin{cases} 3x+2 & x<2 \\ x^2+1 & x>2 \end{cases}$, find $\lim\limits_{x\to 2} f(x)$??

الحل :-

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} (3x+2) = 3(2)+2 = 6+2 = 8$

⑥ find the limit of absolute value function :

Example :

Find $\lim_{x\to 7^-} \frac{|x-7|}{x-7}$

$$\frac{|x-7|}{x-7} = \begin{cases} \frac{x-7}{x-7} & x-7>0 \\ \frac{-(x-7)}{x-7} & x-7<0 \end{cases} = \begin{cases} 1 & x>7 \\ -1 & x<7 \end{cases}$$

$$\therefore \lim_{x\to 7^-} \frac{|x-7|}{x-7} = -1$$

Example :

find $\lim_{x\to 2^+} \frac{|x-2|}{x^2-4}$

$$\frac{|x-2|}{x^2-4} = \begin{cases} \frac{x-2}{x^2-4} & x-2>0 \\ \frac{-(x-2)}{x^2-4} & x-2<0 \end{cases} = \begin{cases} \frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)} = \frac{1}{x+2} & x>2 \\ \frac{-\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \frac{-1}{x+2} & x<2 \end{cases}$$

$$\therefore \lim_{x\to 2^+} \frac{|x-2|}{x^2-4} = \lim_{x\to 2^+} \frac{1}{x+2} = \frac{1}{4}$$

Section (2.5) :—

① Continuaty at point from the graph

ايجاد الاتصال من الرسم

② Continuaty at point by solution

ايجاد الاتصال بالحل :

Example: disc the continuaty of $f(x) = \begin{cases} 2-x & x \leq 2 \\ (x-2)^2 & x > 2 \end{cases}$

الحل :-

$\boxed{1}$ $f(2) = 2-(2) = 0$

$\boxed{2}$ $\lim\limits_{x\to 2} f(x) \longrightarrow \lim\limits_{x\to 2^-} 2-x = 0$

$\searrow \lim\limits_{x\to 2^+} (x-2)^2 = 0$

$\therefore \lim\limits_{x\to 2} f(x) = 0$

$\boxed{3}$ $\lim\limits_{x\to 2} f(x) = f(2)$

$\therefore$ $f(x)$ is Continuous function

③ Find the interval where the function is Continuous :

(ايجاد الفترة التي تكون عندها الداله متصله)

* For piecewise function :—

Example : $f(x) = \begin{cases} 5x+2 & x \geqslant 3 \\ 3x+4 & x < 3 \end{cases}$

الحل :-

$\boxed{1}$ $f(3) = 5(3)+2 = 17$

$\boxed{2}$ $\lim\limits_{x\to 3} f(x) \longrightarrow \lim\limits_{x\to 3^+} (5x+2) = 17$

$\searrow \lim\limits_{x\to 3^-} (3x+4) = 13$

$\boxed{3}$ $\lim\limits_{x\to 3^-} f(x) \neq f(3)$

$\therefore$ $f(x)$ is ~~not~~ disContinuous at $x = 3$

* By Domain :—

Example : D the continuous of $f(x) = \frac{x+1}{3-\ln x}$

الحل :-

$\because$ f(x) is rational function with logarithmic function

$\Longrightarrow$ f(x) is continuous on its domain

$D_f = \mathbb{R} - \{e^3\}$

أصفار المقام

$\downarrow$

$3 - \ln x = 0$

$\ln x = 3$

$e^{\ln x} = e^3$

$\boxed{x = e^3}$

---

④ Find where the function is discontinuous :—

Examples :-

[a] $f(x) = \begin{cases} 5x^2+1 & x \geq -2 \\ x^2+15 & x < -2 \end{cases}$

الحل :-

[1] $f(-2) = 5(4)+1 = 21$

[2] $\lim\limits_{x\to -2} f(x) \longrightarrow \lim\limits_{x\to -2^+} 5x^2+1 = 21$

$\searrow \lim\limits_{x\to -2^-} x^2+15 = 19$

$\therefore$ f(x) is discontinuous at $x=-2$

---

[b] $f(x) = \frac{x^2+5x-1}{x-4}$

الحل :-

[1] f(4) not defined

$\therefore$ f(x) is discontinuous at $\boxed{x=4}$

⑤ Intermediat Value Theorem نظرية القيمة المتوسطه

تأتى ✓ أو X

① $f(x)$ is Continuous on $[a,b]$

② $f(a) < N < f(b)$

③ $f(a) \neq f(b)$

$\Longrightarrow \exists\, c \in (a,b)$ s.t. $f(c) = N$

⑥ Find for what value of k the function is Continuous.

Example:—

$$f(x) = \begin{cases} kx^2 + 2x & , x < 2 \\ x^3 - kx & , x \geq 2 \end{cases}$$

الحل: —

① $f(2) = 2^3 - 2k = 8 - 2k$

② $\lim\limits_{x\to 2} f(x)$ $\longrightarrow$ $\lim\limits_{x\to 2^+} (x^3 - kx) = 8 - 2k$

$\searrow$ $\lim\limits_{x\to 2^-} (kx^2 + 2x) = 4k + 4$

③ $\lim\limits_{x\to 2} f(x) = f(2)$

$\Rightarrow 4k + 4 = 8 - 2k$

$\Rightarrow 4k + 2k + 4 - 8 = 0$

$\Rightarrow 6k - 4 = 0$

$\Rightarrow 6k = 4 \Longrightarrow \boxed{k = \frac{4}{6} = \frac{2}{3}}$

حل أسئلة

Blackbord

1) $\lim_{x\to 3^+} \dfrac{|x-3|}{(x+1)(x-3)} =$

[✓ a. $\frac{1}{4}$]

b. $-\frac{1}{4}$

c. 1

d. does not exist

$$\frac{|x-3|}{(x+1)(x-3)} = \begin{cases} \dfrac{(x-3)}{(x+1)(x-3)} & x-3>0 \text{ اذا} \\ \dfrac{-(x-3)}{(x+1)(x-3)} & x-3<0 \end{cases}$$

$$= \begin{cases} \dfrac{1}{x+1} & x>3 \\ \dfrac{-1}{x+1} & x<3 \end{cases}$$

$$\therefore \lim_{x\to 3^+} \frac{|x-3|}{(x+1)(x-3)} = \lim_{x\to 3^+} \frac{1}{x+1} = \frac{1}{3+1} = \frac{1}{4}$$

2) If the function $f(x) = \begin{cases} (k^2-1)x, & x \geq 1 \\ 3x, & x < 1 \end{cases}$ is continuous on $\mathbb{R}$, then

a. $k = \pm 1$

b. $k = \pm 4$

[✓ c. $k = \pm 2$]

d. $k = \pm 3$

① $f(1) = k^2 - 1$

② $\lim_{x\to 1} f(x)$ $\begin{cases} \nearrow ~~\lim_{x\to 1^+} (k^2-1)x = k^2-1~~ \\ \searrow \lim_{x\to 1^-} (3x) = 3 \end{cases}$

③ $k^2 - 1 = 3$

$k^2 = 4 \Longrightarrow \boxed{k = \pm 2}$

---

3) $\lim_{x\to 0} \dfrac{\sin 3x}{x} = \dfrac{3}{1} = 3$

a. 1

[✓ b. 3]

c. 0

d. does not exist

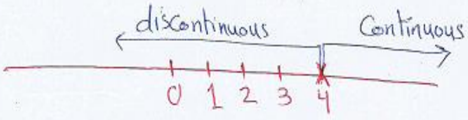
4) The function $f(x) = \sqrt{x-4}$ is continuous from the right at $x = 0$ because

a. $\lim_{x\to 4} f(x) = f(4)$

[✓ b. $\lim_{x\to 4^+} f(x) = f(4)$]

c. $\lim_{x\to 4^-} f(x) = f(4)$

d. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x)$



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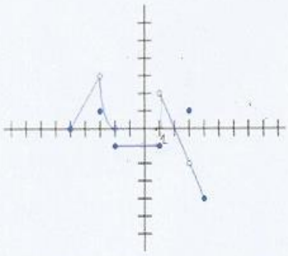
5) If $f(x)$ is a function whose graph is shown then $\lim_{x\to 1} f(x) =$

a. 2

b.-1

[✓ c. does not exist]

d. 0



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6) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)}{\cancel{\sqrt{x}-2}} = 2+2=4$

a. -4

b. -2

c. 2

[✓ d. 4]

---

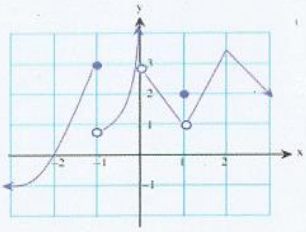
7) Consider the function $f(x)$ whose graph is shown, then $\lim_{x\to -1^-} f(x) =$

a. 1

b. does not exist

[✓ c. 3]

d. 0



$$\frac{|x-3|}{x} = \begin{cases} \frac{x-3}{x} & x>3 \\ \frac{-(x-3)}{x} & x<3 \end{cases}$$

8) $\lim_{x\to3}\frac{|x-3|}{x} =$

$$\lim_{x\to3^+}\frac{x-3}{x} = \frac{3-3}{3} = \frac{0}{3} = 0$$

$$\lim_{x\to3^-}\frac{-(x-3)}{x} = \frac{-(3-3)}{3} = \frac{0}{3} = 0$$

[✓ a. 0]
b. 1
c. -1
d. Does not exist

9) $\lim_{x\to0}\frac{(x+4)^2-16}{x} = \lim_{x\to0}\frac{x^2+8x+\cancel{16}-\cancel{16}}{x}$

$$= \lim_{x\to0}\frac{x(x+8)}{\cancel{x}} = 8$$

a. -8
[✓ b. 8]
c. does not exist

10) If $f(x) = \begin{cases} \sqrt{x+4}, & x \geq 0 \\ \frac{\tan 2x}{x}, & x < 0 \end{cases}$, then $\lim_{x\to0} f(x) =$

$$\lim_{x\to0^+}\sqrt{x+4} = \sqrt{4} = 2$$

$$\lim_{x\to0^-}\frac{\tan 2x}{x} = \frac{2}{1} = 2$$

[✓ a. 2]
b. 0
c. does not exist
d. -2

11) $\lim_{x\to\infty}\frac{2}{3x+1} = 0$ لأن درجة البسط < درجة المقام

a. $\frac{2}{3}$
[✓ b. 0]
c. ∞
d. $-\infty$

12) $\lim_{x\to\infty} \frac{2x^5+3x-1}{3x^5-x^2+1} = \frac{2}{3}$

a. ∞

b. $-\infty$

$\boxed{\text{c. } \frac{2}{3}}$

d. 0

درجة البسط = درجة المقام

13) $\lim_{x\to 2} \frac{x^3-2x^2}{x-2} = \frac{0}{0}$

a. 0

b. ∞

$\boxed{\text{c. } 4}$

d. $-\infty$

هناك طريقتان للحل :-

① قاعدة لوبيتال

$$\lim_{x\to 2} \frac{3x^2-4x}{1} = 12-8=4$$

② أخذ عامل مشترك

$$\lim_{x\to 2} \frac{x^2\cancel{(x-2)}}{\cancel{x-2}} = 4$$

14) $\lim_{x\to\infty} \frac{\sqrt{9x^2+3x}+2}{3x-1} = \frac{\sqrt{9}}{3} = \frac{3}{3} = 1$

a. 3

b. -3

$\boxed{\text{c. } 1}$

d. -1

درجة البسط = درجة المقام

15) $f(x) = \begin{cases} 2x+1, & x \geq 3 \\ x-5, & x < 3 \end{cases}$ is continuous on

a. $\mathbb{R} - \{-3\}$

b. $\mathbb{R}$

c. $\mathbb{R} - \{3, -3\}$

$\boxed{\text{d. } \mathbb{R} - \{3\}}$

① $f(3) = 6+1 = 7$

② $\lim_{x\to 3} f(x) \begin{cases} \lim_{x\to 3^+} (2x+1) = 7 \\ \lim_{x\to 3^-} x-5 = -2 \end{cases}$

$\therefore \lim_{x\to 3} f(x)$ does not exist

$\therefore f(x)$ is discontinuous at $x=3$

16) $\lim_{x\to 0} x\cos\left(x+\frac{1}{x}\right) = 0(\infty)$

a. ∞

[✓ b. 0]

c. 1

d. -1

$-1 \le \cos\left(x+\frac{1}{x}\right) \le 1$

$-x \le x\cos\left(x+\frac{1}{x}\right) \le x$

by Sandwich theorem:—

$\lim_{x\to 0}(-x) = 0 = \lim_{x\to 0} x = 0$

$\therefore \lim_{x\to 0} x\cos\left(x+\frac{1}{x}\right) = 0$

17) If $\lim_{x\to 2}\frac{f(x)+1}{x} = 3$, then $\lim_{x\to 2} f(x) =$

a. 3

b.4

[✓ c. 5]

d. 6

$$\frac{\lim_{x\to 2} f(x) + \lim_{x\to 2} 1}{\lim_{x\to 2} x} = 3$$

$$\frac{\lim f(x) + 1}{2} \neq 3$$

$\lim f(x) + 1 = 6$

$\boxed{\lim_{x\to 2} f(x) = 5}$

18) The function $f(x) = 3 + e^{x}$ is continuous on

a. $\mathbb{R} - \{0\}$

[✓ b. $\mathbb{R}$]

c. $[0, \infty)$

d. $(0, \infty)$

$f(x)$ is continuous on its domain $= \mathbb{R}$

✳ 19) If $\lim_{x\to1} f(x) = 9$, $\lim_{x\to1} g(x) = -4$, and $\lim_{x\to1} h(x) = -1$

Then $\lim_{x\to1}\left(\frac{\sqrt{f(x)}}{g(x)+(h(x))^2} \right)$ $= \frac{\sqrt{\lim_{x\to1} f(x)}}{\lim_{x\to1} g(x) + \left(\lim_{x\to1} h(x)\right)^2}$

a. +1

b. $-\frac{3}{5}$

[✓ c. -1]

d. $\frac{3}{5}$

$= \frac{\sqrt{9}}{-4+(-1)^2} = \frac{3}{-4+1} = \frac{3}{-3} = -1$

✳ 20) $\lim_{x\to0} \frac{\cos^2 x - 1}{\cos x - 1}$ $= \frac{0}{0}$

[✓ a. 2]

b. 0

c. -1

d. -2

[illegible]

$$\lim_{x\to0} \frac{\cancel{(\cos x - 1)}(\cos x + 1)}{\cancel{(\cos x - 1)}}$$

$$= \lim (\cos x + 1) = 1+1 = 2$$

هذه المراجعة لا تغني عن الكتاب المقرر

Appendix D :

① Convert :

$$\begin{tikzcd} \text{degree} \arrow[r, bend left, "\times \frac{\pi}{180}"] & \text{radian} \arrow[l, bend left, "\times \frac{180}{\pi}"] \end{tikzcd}$$

② Find the trigonometric function

EX : $\sin\theta = \frac{1}{2}$ then find the other five

③ period :–

$\sec x, \csc x, \sin x, \cos x \Longrightarrow 2\pi$

$\tan x, \cot x \Longrightarrow \pi$

④ Even or odd :–

even → Constant, $x^2, x^4, x^6, \ldots, |x|, \cos x, \sec x$

odd → $x, x^3, x^5, \ldots, \tan x, \cot x, \sin x, \csc x$

Laws :–

① even + even = even

② odd + odd = odd

③ even + odd = neither even nor odd

④ even × even = even

⑤ odd × odd = ~~odd~~ even

⑥ even × odd = odd

⑦ $\frac{\text{even}}{\text{odd}} = \frac{\text{odd}}{\text{even}} = \text{odd}$

⑧ $\frac{\text{even}}{\text{even}} = \frac{\text{odd}}{\text{odd}} = \text{even}$

⑤ Table (Choose the correct value)
page A27

⑥ Identities
in page A28, A29 (T or F)

6, 7, 8, 9, 10a, 10b, 11a, 11b
15a, 15b, 17a, 17b

⑦ The Domain and rang of trigonometric function.

موجوده في موقعي.

(1.1) Four ways to repersent a function

① Find the domain of :-

① polynomial	$\mathbb{R}$
② $\sqrt[3]{f(x)}$, $\sqrt[5]{f(x)}$	$\mathbb{R}$
$\sqrt{f(x)}$, $\sqrt[4]{f(x)}$	ما بداخل الجذر ≥ 0
$\sqrt{a^2-x^2}$	$[-a,a]$
$\sqrt{x^2-a^2}$	$(-\infty,-a] \cup [a,\infty)$
$\sqrt{x^2+a^2}$	$\mathbb{R}$
③ Rational	$\mathbb{R}$ – أصفار المقام
④ Absolute value	$\mathbb{R}$
⑤ piecewise	على حسب تعريف x المعطى في السؤال

Example: $f(x) = \begin{cases} x^2 & x < -5 \\ -x^2+7 & |x| < 5 \leftrightarrow -5 < x < 5 \\ 3x-1 & x > 5 \end{cases}$

The Domain is :-

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R}-\{-5\}$ $\boxed{ⓒ\ \mathbb{R}-\{\pm 5\}}$ ⓓ $R-\{5\}$

② Find the range :-

① Absolute value	$[0,\infty)$
② $x^2 \pm a$, $a-x^2$	أرسمي ,استنتجي المدى
③ $\sqrt{a^2-x^2}$	$[0,a]$
$\sqrt{x^2-a^2}$	$[0,\infty)$
$\sqrt{x^2+a}$	$[a,\infty)$
④ $\sqrt{x\pm a}$ or $\sqrt{a\pm x}$	بالرسم ,استنتاج المدى

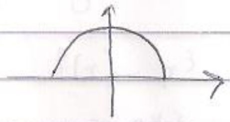
③ Choose the correct graph (f(x) is given)
<u>or</u> the correct function (the graph is given)

أي يعطيك الرسمه ,يطلب اختيار الداله الصحيحه
أو يعطيك الداله ,يطلب منك اختيار الرسمه الصحيحه

← ④ Function or not ??

Vertical line test استخدام

Ex :



function ?

[✓ a True]   b False

⑤ From the graph find: من الرسم أوجدي المجال أو المدى:
* Domain or * Range

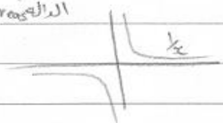
⑥ Increasing and decreasing :

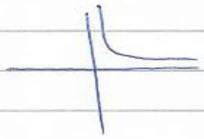
* definition (T or F) يسأل عن التعريف

* $\frac{1}{x}$ is increasing in :

ⓐ $(0, \infty)$ ⓑ $(-\infty, 0)$ ⓒ $\mathbb{R}$ [✓ ⓓ not exist]

من الرسم يتضح أن الدالة decreasing



* From the graph  the function

increasing on $(-\infty, 0)$ (T or [✓ F])

⑦ Even and Odd :-

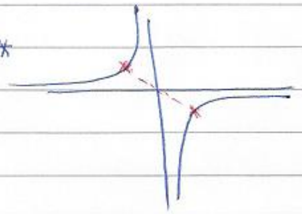
* definition (T or F)

* $\frac{x^2-1}{x^4-x^2}$ is [✓ ⓐ even] ⓑ odd ⓒ even and odd ⓓ neither odd nor even

* $\frac{x^3-x}{\cos x}$ symmetric about

[✓ ⓐ Origin] ⓑ x-axis ⓒ y-axis ⓓ $y=x$

*



From the graph the function is :

ⓐ even [✓ ⓑ odd] ⓒ even and odd

ⓓ neither even nor odd

الداله الفرديه متماثلة حول نقطة الأصل

الداله الزوجيه متماثله حول y-axis

(1.2) A catalog of essential functions :-

1] Classify : صنفي أ، اذكري نوع الداله

Ex ①: $\log_5 7$ is :-

$\boxed{\text{a] Constant}}$ b] logarithmic
c] Exponential d] [illegible] trigonometric

Ex ②: $f(x) = \_\_\_\_\_$ exponential

a] $\boxed{(7.5)^x}$ b] $(-7)^x$ c] x^7 d] x^t

2] Domain and Range for (power, polynomial, exponential, logarithmic, ...)

(1.3) New functions from old functions :-

① Find the domain :-

$(f \circ g)(x) \longrightarrow D_{f\circ g} \cap D_g$

$$\left.\begin{matrix} f+g \\ f-g \\ fg \end{matrix}\right\} \longrightarrow D_f \cap D_g$$

$\frac{f}{g} \longrightarrow D_f \cap D_g -$ اصفار المقام

② Evaluate or Find :

- * $f(x) \circ g(x)$
- * $f(x) \pm g(x)$
- * $f(x)\, g(x)$
- * $f(x) \circ g(x) \circ h(x)$
- * find $f(x)$, $g(x)$ from $f(x) \circ g(x)$

Ex ① $f(x) = 10$, $g(x) = x - 1$

*Find $f \circ g(x)$??

$$f(g(x)) = f(x-1) = 10$$

* Find $f \circ g(2)$??

$$f(g(2)) = 10$$

Ex ② $f(x) = e^{x-4}$ $g(x) = \sqrt{x}$

Find $f \circ g(16)$??

$$f \circ g(x) = f(g(x)) = f(\sqrt{x}) = e^{\sqrt{x} - 4}$$

$$\therefore f \circ g(16) = e^{\sqrt{16} - 4} = e^{4-4} = e^{0} = 1$$

③ Shifting and reflecting :-

* Find the domain and range

Ex ① the Domain of x^2+3

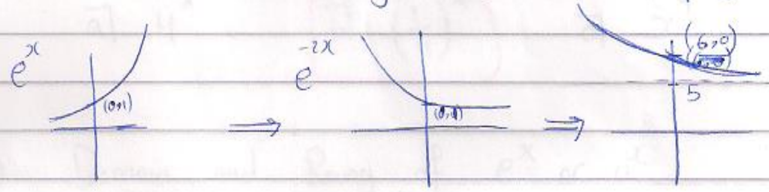
$D = \mathbb{R}$

$R = [3, \infty)$

* نعطيك الرسم ونطلب منك المعادلة

* نعطيك معادلة بعد حدوث الـ shift ونطلب الرسم الصحيح.

EX ② Find the range of $f(x) = 5 + e^{-2x}$



$\therefore$ range $= (5, \infty)$

EX ③ Find $f(x) = \pi + e^{-x}$

بنفس طريقة EX ② نجد أن

range $= (\pi, \infty)$

(1.5) Exponential Functions :

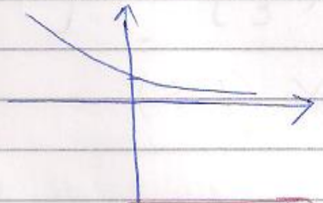
① Laws (T or F)

② increasing or decreasing (T or F)

$a^x \rightarrow 0<a<1 \rightarrow$ decreasing

$\searrow a>1 \rightarrow$ increasing

EX:



a] 4^x $\boxed{\text{b] } \left(\frac{1}{4}\right)^x}$ c] x^4

③ Domain and Rang of e^x or a^x
with shift or reflect

EX: $f(x) = \pi + e^{-x}$

$(2.3)^x$

EX: ① $\left(\frac{7}{3}\right)^x$ is increasing $\forall x \in \mathbb{R}$

[✓ a] T] b] F

② $f(x) = \ldots$ exponential function

[✓ a $(7.5)^x$] b -7^x c x^7 d x^t

③ $f(x) = \ldots$ exponential

[✓ a 4^{-x}] b $(-4)^x$ c x^4 d x^t

$$4^{-x} = \frac{1}{4^x} = \frac{1^x}{4^x} = \left(\frac{1}{4}\right)^x$$

EX :-

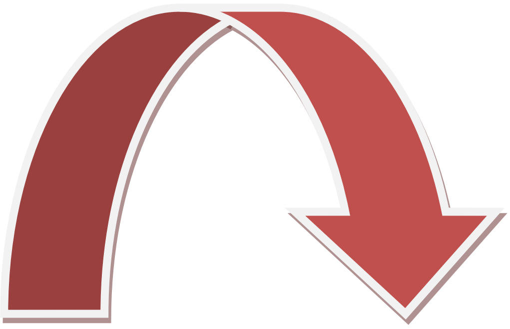
$f(x) = \frac{|x|}{3x}$ find the range?

$$\frac{|x|}{3x} = \begin{cases} \frac{x}{3x} = \frac{1}{3} & x \geq 0 \\ \frac{-x}{3x} = \frac{-1}{3} & x < 0 \end{cases} = \begin{cases} \frac{1}{3} & x \geq 0 \\ \frac{-1}{3} & x < 0 \end{cases}$$

$\therefore$ range $= \left\{ \frac{1}{3}, \frac{-1}{3} \right\}$ ✓

$\left(\frac{-1}{3}, \frac{1}{3}\right)$ ✗

نموذج اختبار



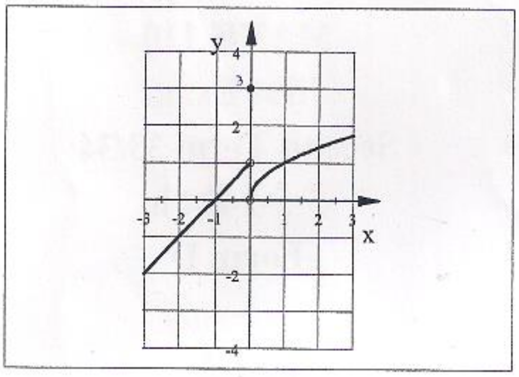
Form D

1. The range of the function $f(x) = \frac{|x|}{4x}$ is

$\frac{|x|}{4x} = \begin{cases} \frac{x}{4x}, & x \geq 0 \\ \frac{-x}{4x}, & x < 0 \end{cases} = \begin{cases} \frac{1}{4}, & x \geq 0 \\ -\frac{1}{4}, & x < 0 \end{cases}$

- (a) $\mathbb{R} - \{0\}$
- [✓ (b) $\{-\frac{1}{4}, \frac{1}{4}\}$]
- (c) $\{-4, 4\}$
- (d) $(-\frac{1}{4}, \frac{1}{4})$

2. The following graph represents the function $g(x) =$



- (a) $\begin{cases} x+1 & \text{if } x > 0 \\ \sqrt{x} & \text{if } x < 0 \\ 3 & \text{if } x = 0 \end{cases}$ ×
- [✓ (b) $\begin{cases} \sqrt{x} & \text{if } x > 0 \\ x+1 & \text{if } x < 0 \\ 3 & \text{if } x = 0 \end{cases}$] ✓
- (c) $\begin{cases} x^2 & \text{if } x > 0 \\ x+1 & \text{if } x < 0 \\ 3 & \text{if } x = 0 \end{cases}$ ×
- (d) $\begin{cases} x^2 & \text{if } x > 0 \\ x+1 & \text{if } x < 0 \\ 1 & \text{if } x = 0 \end{cases}$ ×

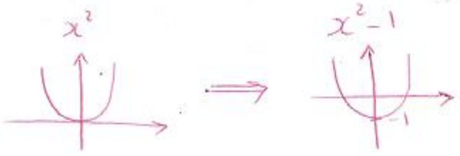
3. The function $f(x) = \left(\frac{7}{3}\right)^x$ is increasing for all $x \in \mathbb{R}$.

- [✓ (a) True]
- (b) False

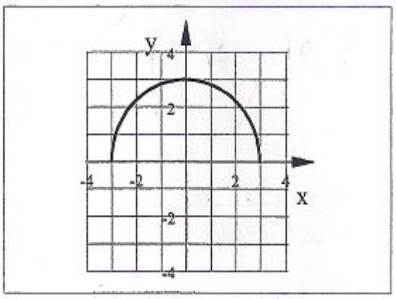
because $\frac{7}{3} = 2.3$ a<1 > 1 a>1

4. The range of the function $f(x) = x^2 - 1$ is

- (a) $[0, \infty)$
- (b) $(-\infty, \infty)$
- (c) $[1, \infty)$
- [✓ (d) $[-1, \infty)$]



5. The following figure represents a graph of a function.



بواسطة اختبار الخط الرأسي

[✓ (a) True]
(b) False

---

6. The domain of the function $f(x) = \dfrac{x^2+3x+\sqrt{5}}{x^2+3x-4}$ is

(a) $\mathbb{R}-\{1,4\}$
(b) $\mathbb{R}-\{-4,-1\}$
[✓ (c) $\mathbb{R}-\{-4,1\}$]
(d) $\mathbb{R}-\{-1,4\}$

$(x+4)(x-1)$

$x^2+3x-4=0$

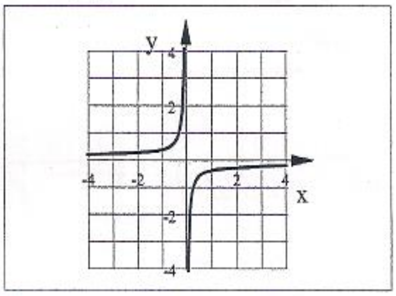
$(x+4)(x-1)=0$

$\boxed{x=-4}$, $\boxed{x=1}$

أصفار المقام

---

7. The following figure represents a graph of



لوجود تماثل حول نقطة الأصل

[✓ (a) an odd function]
(b) an even function
(c) neither odd nor even function
(d) an odd and even function

---

8. If $f(x)=\sqrt{x+2}$ and $g(x)=x^2+5$,then the domain of $(g\circ f)(x)$ is

(a) $[2,\infty)$
(b) $(-\infty,2]$
(c) $(-\infty,-2]$
[✓ (d) $[-2,\infty)$]

$g(f(x)) = g(\sqrt{x+2}) = (\sqrt{x+2})^2+5 = x+7$

$\therefore D = D_{g\circ f} \cap D_f = \mathbb{R} \cap [-2,\infty)$

$D_f = [-2,\infty)$ $[-2,\infty)$

---

9. If $f(x)=\sqrt{16-x^2}$ and $g(x)=\dfrac{x-1}{x+4}$,then the domain of $(f+g)(x)$ is

(a) $[-4,4)$
[✓ (b) $(-4,4]$]
(c) $(-4,4)$
(d) $[-4,4]$

$f(x)+g(x) = \sqrt{16-x^2} + \dfrac{x-1}{x+4}$ $(-4,4]$

$=$

$|x| \leq 4$

$D_f:$
$16-x^2 \geq 0$
$16 \geq x^2$
$\sqrt{x^2} \leq \sqrt{16}$
$|x| \leq 4$
$-4 \leq x \leq 4$

$D_{f+g} = D_f \cap D_g$
$= [-4,4] \cap \mathbb{R}-\{-4\}$
$= (-4,4]$

Form D

10. If $f(x) = \frac{1}{x}$, $g(x) = x^3$ and $h(x) = \sin x$, then $(f \circ g \circ h)(x) =$

- **(a)** $\frac{1}{\sin x^3}$
- **(b)** $\sec^3 x$
- **(c)** $\sin \frac{1}{x^3}$
- [✓ **(d)** $\csc^3 x$]

$f(g(h(x))) = f(g(\sin x))$

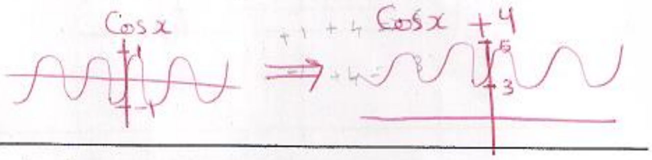
$f \sin^3 x = f(\sin^3 x)$

$\frac{1}{\sin^3 x} = \frac{1}{\sin^3 x} = \csc^3 x$

---

11. If the graph of the function $f(x) = \cos x$ is shifted 4 units upward, then the range of the new function is

- [✓ **(a)** $[3,5]$]
- **(b)** $[-5,-3]$
- **(c)** $[-4,4]$
- **(d)** $[-1,1]$



---

12. The graph of the function $f(x) = |x+5| - 2$ is obtained by shifting the graph of $f(x) = |x|$

- **(a)** 2 units downward and 5 units to the right.
- **(b)** 2 units upward and 5 units to the left.
- **(c)** 2 units upward and 5 units to the right.
- [✓ **(d)** 2 units downward and 5 units to the left.]

---

13. The domain of the function $f(x) = \frac{1}{\sqrt{x^2 - 25}}$ is

- **(a)** $[-5,5]$
- [✓ **(b)** $(-\infty,-5) \cup (5,\infty)$]
- **(c)** $(-\infty,-5] \cup [5,\infty)$
- **(d)** $(-5,5)$

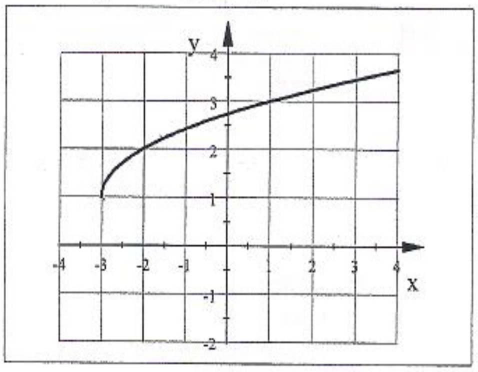
$D_f = D_{\text{البسط}} \cap D_{\text{المقام}} - \text{أصفار المقام}$

$= \mathbb{R} \cap [(-\infty,5] \cup [5,\infty)] - \{\pm 5\}$

$= (-\infty,5) \cup (5,\infty)$

---

14. The range of the function $f(x)$ whose graph is given is



- **(a)** $(1,\infty)$
- **(b)** $[-3,\infty)$
- **(c)** $(-3,\infty)$
- [✓ **(d)** $[1,\infty)$]

ب

15. If $\sin\theta = \frac{-\sqrt{3}}{2}$ and $\cos\theta = \frac{1}{2}$, then $\cos 2\theta =$

$\frac{1}{4} - \frac{3}{4} = \frac{-2}{4}$

- (a) $\frac{1}{2}$
- (b) 1
- [✓ (c) $-\frac{1}{2}$]
- (d) -1

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{1}{4} - \frac{3}{4} = \frac{-2}{4} = \frac{-1}{2}$$

16. The following function is an exponential function

- (a) $(-0.4)^x$
- [✓ (b) $(\frac{1}{3})^x$]
- (c) e^5
- (d) x^{-7}

17. If $\sec\theta = \frac{-3}{2}$, $\pi < \theta < \frac{3\pi}{2}$, then $\sin\theta =$

الربع الثالث

- [✓ (a) $\frac{-\sqrt{5}}{3}$]
- (b) $\frac{\sqrt{5}}{3}$
- (c) $\frac{-3}{\sqrt{5}}$
- (d) $\frac{3}{\sqrt{5}}$

3, -2

$\sqrt{9-4} = \sqrt{5}$

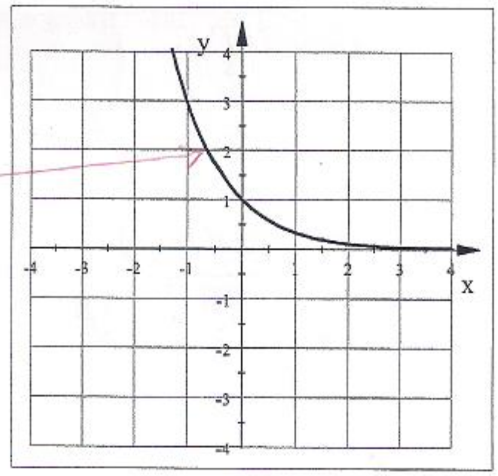
$$\sec\theta = \frac{-3}{2} \Rightarrow \cos\theta = \frac{-2}{3} = \frac{adj}{hyp}$$

$$opp^2 = 9 - 4 = 5$$

$$\therefore \boxed{opp = \sqrt{5}}$$

$$\therefore \sin\theta = \frac{opp}{hyp} = -\frac{\sqrt{5}}{3}$$

18. The following figure represents the graph of the function $f(x) =$



decreasing a^x الدالة

$\therefore 0 < a < 1$

- (a) 3^x
- (b) -3^{-x}
- [✓ (c) 3^{-x}] $\Longrightarrow 3^{-x} = \left(\frac{1}{3}\right)^x$
- (d) -3^x

$3^{-(-1)} = 3$

19. $\cos(\frac{11\pi}{5} + \pi) = \cos\frac{11\pi}{5}$

$= \pi(\frac{11}{5} + 1) = \pi(2 + \frac{1}{5} + 1)$

$= \pi(2 + \frac{6}{5})$

- (a) True
- [✓ (b) False]

$2\pi + \frac{6\pi}{5}$

Form D

20. The range of the function $f(x) = \sqrt{x^2+9}$ is

(a) $(3,\infty)$
(b) $(0,\infty)$
[✓ **(c)** $[3,\infty)$]
(d) $[0,\infty)$

---

21. The function $f(x) = \dfrac{x^{\frac{1}{3}}-2}{x^2+3}$ is

(a) a polynomial function
(b) a power function
(c) a rational function
[✓ **(d)** an algebraic function]

---

22. The range of the function $f(x) = \log_7 x$ is

[✓ **(a)** $(-\infty,\infty)$]
(b) $(0,\infty)$
(c) $[0,\infty)$
(d) $(1,\infty)$

---

23. The function $f(x) = \dfrac{1}{x^2}$ is

$\frac{1}{x^2} = x^{-2}$

[✓ **(a)** increasing on $(-\infty,0)$]
(b) increasing on $\mathbb{R}-\{0\}$
(c) increasing on $(0,\infty)$
(d) decreasing on $\mathbb{R}-\{0\}$

---

24. If $f(x) = 2x-6$ and $g(x) = \sqrt{4x^2+20}$, then $\left(\dfrac{f}{g}\right)(x) =$

(a) $\dfrac{x-3}{2\sqrt{x^2+5}}$
(b) $\dfrac{x-3}{\sqrt{x^2+20}}$
(c) $\dfrac{\sqrt{x^2+5}}{x-3}$
[✓ **(d)** $\dfrac{x-3}{\sqrt{x^2+5}}$]

$$\frac{f}{g} = \frac{2x-6}{\sqrt{4x^2+20}} = \frac{2(x-3)}{\sqrt{4(x^2+5)}}$$

$$= \frac{2(x-3)}{\sqrt{4}\sqrt{x^2+5}} = \frac{2(x-3)}{2\sqrt{x^2+5}} = \frac{x-3}{\sqrt{x^2+5}}$$

---

25. If $F(x) = \csc\sqrt{x}$, then $F = f\circ g$ where

(a) $f(x) = \sqrt{x},\ g(x) = \csc x$
[✓ **(b)** $f(x) = \csc x,\ g(x) = \sqrt{x}$]
(c) $f(x) = \sqrt{x},\ g(x) = \csc\sqrt{x}$
(d) $f(x) = \csc\sqrt{x},\ g(x) = \sqrt{x}$

$f\circ g(x) = f(g(x)) = f(\sqrt{x})$

---

26. The graph of the function $g(x) = -e^x+5$ is obtained from the graph of $g(x) = e^x$ by

[✓ **(a)** reflecting about the $x-$ *axis* then shifting 5 units upward]
(b) reflecting about the $x-$ *axis* then shifting 5 units downward
(c) reflecting about the $y-$ *axis* then shifting 5 units downward
(d) reflecting about the $y-$ *axis* then shifting 5 units upward

Form D

27. The range of the function $f(x) = -\sqrt{x} + 3$ is

- (a) $[3,\infty)$
- (b) $[-3,\infty)$
- [✓ (c) $(-\infty,3]$]
- (d) $(-\infty,-3]$

---

28. The domain of the function $f(x) = \sec x$ is

- (a) $\mathbb{R} - \{0, \pi, 2\pi, 3\pi, \ldots\}$
- (b) $\mathbb{R} - \{0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\}$
- (c) $\mathbb{R} - \{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\}$
- [✓ (d) $\mathbb{R} - \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots\}$]

---

29. The domain of the function $f(x) = \sqrt{1+4^x}$ is

- (a) $\mathbb{R} - \{1\}$
- [✓ (b) $\mathbb{R}$]
- (c) $\mathbb{R} - \{0\}$
- (d) $\mathbb{R} - \{-1\}$

لأنه لايوجد قيمة تجعل مابداخل الجذر ≤ 0

فكل قيم $\mathbb{R}$ تحقق أن مابداخل الجذر ≥ 0

---

30. The function $f(x) = \frac{x^2-1}{x^4-x^2}$ is

- (a) an odd function
- [✓ (b) an even function]
- (c) neither odd nor even function
- (d) an odd and even function

$\frac{\text{even}}{\text{even}} = \text{even}$

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31. The function $y = f(x)$ whose graph is shifted 5 units to the right is given by $y =$

- (a) $f(x+5)$
- (b) $f(x)+5$
- [✓ (c) $f(x-5)$]
- (d) $f(x)-5$

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32. $160° =$

- (a) $\frac{\pi}{2}$ *rad*
- (b) π *rad*
- (c) $\frac{7\pi}{9}$ *rad*
- [✓ (d) $\frac{8\pi}{9}$ *rad*]

$160 \times \frac{\pi}{180} = \frac{16}{18}\pi = \frac{8\pi}{9}$

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33. The graph of the function $f(x) = \frac{x^3-x}{\cos x}$ is symmetric about

- [✓ (a) the origin]
- (b) the line $x = y$
- (c) the $y-$ *axis*
- (d) the $x-$ *axis*

$\frac{\text{odd}}{\text{even}} = \text{odd} \Longrightarrow$ symmetric about the origin