BMT-222 (September 2018)

Chapter 11 Differentiation (Summary)

Terminology and notation

The process of finding the rate of change of given function is called differentiation. The function is said to be differentiated. If y is a function of the independent variable x, we say that y is differentiated with respect to (w.r.t.) x.

- There is a notation for writing down the derivative of a function. If the function is $y(x)$, we denote the derivative of y by $\frac{dy}{dx}$, pronounced 'dee y by dee x'.
- Another notation for the derivative is simply y', pronounced y dash.
- Similarly if the function is $y(t)$ we write the derivative as $\frac{dy}{dt}$ *or* $\dot{y}$, pronounced "y dot".

Block 1 **Exercises (Terminology and notation) (page 624)**

Problem # 1 If x is a function of the independent variable t, write down two ways in which the derivative can be written.

Solutions: (1) $\frac{dx}{dt}$ *or* $\dot{x}$

Problem # 2. If f is a function of x, write down two ways in which the derivative can be written.

Solutions: (2) $\frac{df}{dx}$ *or* f'

Block 2 **Exercises (Using table of derivatives) (page 629)**

Problem # 1 Find the derivative of the following functions:

(a) $9x$ (b) $4x$ (c) $6x^3$ (d) $-3x^2$ (e) $\ln 3t$

Problem # 2. Find $\frac{dz}{dt}$ when z is defined by

(a) $\frac{4}{t^3}$ (b) $\sqrt{t^3}$ (c) $5t^{-2}$ (d) $-\frac{3}{2}t^{\frac{3}{2}}$

Problem # 3. Find the derivative of each of the following functions:

(a) $\sin 5x$ (b) $\cos 4t$ (c) $\tan 3r$ (d) e^{2s} (e) $\frac{1}{e^{3t}}$

Problem # 4. Find the derivative of the following:

(a) $\cos\frac{2x}{3}$ (b) $\sin(-2x)$ (c) $\tan ® x$ (d) $e^{\frac{x}{2}}$

HW: Problem # 1 (a) and (b), Problem # 2 (c), (d), Problem # 3 (a) and (d), Problem # 4 (b) and (d)

(Problems solved in class # 1 (c), (d), and (e), # 2 (a), (b), # 3(b), (c) and (e) # 4(a), and (c))

Extending the table of derivatives

- The derivative of $f(x) \pm g(x)$ *is* $\dfrac{df}{dx} \pm \dfrac{dg}{dx}$
- The derivative of $k\, f(x)$ *is* $k\dfrac{df}{dx}$

Exercises (Extending the table of derivatives) (page 631)

Problem # 1 Find $\dfrac{dy}{dx}$ when y is defined by

(a) $4x^6 + 8x^3$ (b) $-3x^4 + 2x^{1.5}$ (c) $\dfrac{9}{x^2} + \dfrac{14}{x} - 3x$ (d) $\dfrac{3+2x}{4}$ (e) $(2+3x)^2$

Problem # 2. Find the derivative of each of the following functions:

(a) $z(t) = 5\sin t + \sin 5t$ (b) $h(v) = 3\cos 2v - 6\sin\dfrac{v}{2}$ (c) $m(n) = 4e^{2n} + \dfrac{2}{e^{2n}} + \dfrac{n^2}{2}$

(d) $H(t) = \dfrac{e^{3t}}{2} + 2\tan 2t$ (e) $S(r) = (r^2+1)^2 - 4e^{-2r}$

Problem # 3. Differentiate the following functions:

(a) $A(t) = (3+e^t)^2$ (b) $B(s) = \text{®} e^{2s} + \dfrac{1}{s} + 2\sin \text{®} s$ (c) $V(r) = \left(1 + \dfrac{1}{r}\right)^2 + (r+1)^2$

(d) $M(\text{¯}) = 6\sin 2\text{¯} - 2\cos\left(\dfrac{\text{¯}}{4}\right) + 2\text{¯}^2$ (e) $H(t) = 4\tan 3t + 3\sin 2t - 2\cos 4t$

HW: Problem # 1 (a) and (c), Problem # 2 (a), (c), (e) Problem # 3 (b) and (e)

(Problems solved in class # 1 (b), (d), (e), # 2 (b), (d), # 3(a), (c) and (d))

Exercises (Evaluating a derivative) (page 633)

Problem # 1 Calculate the derivative of

$$y = 3x^2 + e^x \text{ when } x = 0.5.$$

Problem # 2. Calculate the rate of change of

$$i(t) = \sin 2t + 3t \text{ when (a) } t = \frac{\text{®}}{3}, \text{ (b) } t = 0.6.$$

Problem # 3. Evaluate the rate of change of

$$H(t) = 5\sin t - 3\cos 2t \text{ when (a) } t = 0, \text{ (b), } t = 1.3.$$

HW: Problem # 2 (Problems solved in class # 1, # 3)

Higher derivatives

Notation

We know that the first derivative is denoted by $\dfrac{dy}{dx}$ or y'. The second derivative is calculated by differentiating the first derivative, that is

$$\text{Second derivative } = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

So, the second derivative is denoted by $\dfrac{d^2y}{dx^2}$. This often written more concisely as y''.

If $y = y(x)$

$$\textit{first derivative} = \frac{dy}{dx} \qquad \textit{second derivative} = \frac{d^2y}{dx^2} \qquad \textit{third derivative} = \frac{d^3y}{dx^3}$$

- Derivative with respect to **t** are often indicated using a dot notation, so $\dfrac{dx}{dt}$ can be written as $\dot{x}$.

Similarly, a second derivative with respect to **t** can be written as $\ddot{x}$, pronounced x double dot.

- We may need to evaluate higher derivatives at specific points. The second derivative of $y(x)$, evaluated

at, say, $x = 2$, is written as $\dfrac{d^2y}{dx^2}(2)$, or more simply as $y''(2)$.

Block 3 **Exercises (Higher derivatives) (page 637)**

Problem # 1 Find $\dfrac{d^2y}{dx^2}$ where $y(x)$ is defined by

(a) $3x^2 - e^{2x}$ (b) $\sin 3x + \cos x$ (c) $\sqrt{x}$ (d) $e^x + e^{-x}$ (e) $1 + x + x^2 + \ln x$

Problem # 2. Find $\dfrac{d^3y}{dx^3}$ where $y(x)$ is defined is given in question 1.

Problem # 3. Find $\ddot{y}(1)$ where $y(t)$ is given by

(a) $t(t^2+1)$ (b) $\sin(-2t)$ (c) $2e^t + e^{2t}$ (d) $\dfrac{1}{t}$ (e) $\cos\left(\dfrac{t}{2}\right)$

Problem # 4 Find $\ddot{y}(-1)$ of the functions given in question 3.

HW: Problem # 1 (a) and (e), Problem # 2 (a), (b), (e) Problem # 3 (b) and (e) Problem # 4 (b) and (e)

(Problems solved in class # 1 (b), (c), (d), # 2 (c), (d), # 3(a), (c) and (d), # 4(a), (c) and (d))

Exercises (end of block exercises) (page 637-638)

Problem # 1 Calculate y'' where y is given by

(a) $\cos 2t - \sin 2t$ (b) $e^{2x} - e^x$ (c) $2x^6 - 3x^7$ (d) $-x^3 + 3x^2$ (e) $9 - \dfrac{9}{x}$

Problem # 2. Find the fourth derivative of the following functions:

(a) e^{3t} (b) e^{kx}, where k is a constant (c) $\sin 2t$ (d) $\sin kt$ where k constant (d) $\cos kt$ where k constant

Problem # 3. Show that $y = e^x + 2x$ satisfies the equation $y'' - y' - y = -2 - 2x - e^x$

Problem # 4. Evaluate $y'''(0)$ where y is given by

(a) $\sin 3t + t^3$ (b) $2\cos t + \cos 2t$ (c) $e^{-x}(e^x + 1)$ (d) $3 - 3t^4$ (e) $\dfrac{e^{2x}+1}{e^x}$

Problem # 5. The function $y(x) = x^4 - 3x^3 + 3x^2 + 1$, Calculate the value of x where $y'' = 0$.

HW: Problem # 1 (c) and (e), Problem # 2 (b), (c), (e) Problem # 4 (b) and (e)

(Problems solved in class # 1 (a), (b), (d), # 2 (a), (d), # 3, # 4(a), (c) and (d), # 5)