24/12

Rules for inequalities (المتباينات)

1: $a<b \rightarrow a+c<b+c$

2: $a<b \rightarrow a-c<b-c$

3: $a<b$ and $c>0 \rightarrow ac<bc$

4: $a<b$ and $c<0 \rightarrow ac>bc$; $-a>-b$

5: $a>0 \rightarrow \frac{1}{a}>0$

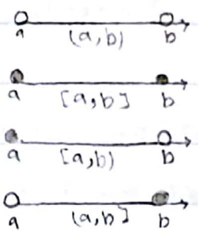
6: $0<a<b \rightarrow \frac{1}{b}<\frac{1}{a}$

الأعداد الطبيعية
natural numbers : 1, 2, 3, 4

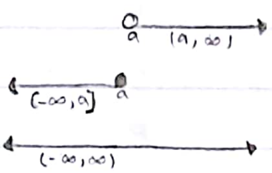
الأعداد الكلية
integers : $0, \pm1, \pm2, \pm3, \pm4$

الأعداد الحقيقية
rational numbers : $n \neq 0$ → (تتضمن) الكسور والأعداد الدورية والجذور

الفترات المحدودة
Finite intervals



الفترات الغير محدودة
infinite intervals



2: ⓐ $2x-1>x+3$

$2x-\cancel{1}+\cancel{1}>x+3+1$

$2x>x+4$ $-x$

$x>4$

$(4,\infty)$

2: ⓑ $-\frac{x}{3} \geq 2x-1$

$-3\left(-\frac{x}{3}\right) \leq -3(2x-1)$

$x \leq -6x+3$

$7x \leq 3$

$x \leq \frac{3}{7}$

$\left(-\infty, \frac{3}{7}\right]$

3: ⓐ $3 \leq 2x+1 \leq 5$

★ $3-1 \leq 2x+\cancel{1}-\cancel{1} \leq 5-1$

$\frac{2}{2} \leq \frac{2x}{2} \leq \frac{4}{2}$

$1 \leq x \leq 2$

$[1,2]$

★ (طريقة أخرى)

$3 \leq 2x+1$

$3-1 \leq 2x+1-1$

$\frac{2}{2} \leq \frac{2x}{2}$

$1 \leq x$

$2x+1 \leq 5$

$\frac{2x}{2} \leq \frac{4}{2}$

$x \leq 2$

26/12

④ a. $x^2-5x+6<0$

$\underset{-}{(x-3)}\,\underset{+}{(x-2)}<0$

$x-3<x-2$

$x-3<0 \rightarrow x<3$

$x-2>0 \rightarrow x>2$

$(2,3)$

The Absolute value (القيمة المطلقة)

تعريف

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$|+3| = 3$

$|0| = 0$

$|-3| = -(-3) = 3$

★ $|x| \leq a \leftrightarrow -a \leq x \leq a$

★ $|x| \geq a \leftrightarrow x \leq -a$ or $x \geq a$

⑥ $\sqrt{x^2} = |x|$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{a^2} = a$

$$|x-y| = \begin{cases} x-y & , x-y \geq 0 \rightarrow x \geq y \\ -(x-y) = -x+y & , x-y<0 \rightarrow x<y \end{cases}$$

★ نحدد النقاط التي نساويها بالصفر ونحلها

$x^2-5x+6=0$

$(x-2)(x-3)=0$

$\rightarrow x-2=0 \ , x-3=0$

$\rightarrow x=2 \ , x=3$

★ نحدد النقطتين على خط الأعداد

$-\infty \leftarrow \ 0 \ \ 2 \ \ 2.5 \ \ 3 \ \ 5 \ \rightarrow \infty$

★ نختبر الإشارات على خط الأعداد

$x=0 \rightarrow 0^2-5(0)+6=6>0$

$x=2.5 \rightarrow (2.5)^2-5(2.5)+6$

$=6.25-12.5+6=12.25-12.5$

$=-0.25<0$

$x=5 \rightarrow (5)^2-5(5)+6=6>0$

★ نحدد فترة الحل عن طريقة المتراجحة الأصلية

حرجة

Sin ⊙ $x^2-5x+6>0$, then

the solution set is $(-\infty,2) \cup (3,\infty)$

③ $|a \pm b| \leq |a|+|b|$

$a=10 \ , b=5$

$|a+b|=15$

$|a-b|=5$

★ $|a \pm b|^2 \leq |a|^2+2|a||b|+|b|^2=(|a|+|b|)^2$

$\leq (|a|+|b|)^2$

⑦ a. $|2x+5|=3$

$\pm(2x+5)=3$

$(2x+5)=\pm 3$

$\underset{-5}{2x+5} = \underset{-5}{+3} \quad , \quad \underset{-5}{2x+5} = \underset{-5}{-3}$

$\frac{2x}{2} = \frac{-2}{2} \quad \quad \frac{2x}{2} = \frac{-8}{2}$

$x=-1 \quad \quad x=-4$

b. $|3x-2| \leq 1 \leftrightarrow -1 \leq 3x-2 \leq 1$

$\rightarrow -1+2 \leq 3x-2+2 \leq 1+2$

$\rightarrow 1 \leq 3x \leq 3$

$\rightarrow \frac{1}{3} \leq x \leq 1$

$[\frac{1}{3}, 1]$

H.W $x^2 < 9$

$x^2 - 2x \le 0$

29/12

$|x-6| \geq 5$

Either $x-6 \geq 5$ or $x-6 \leq -5$

Either $x \geq 5+6$ or $x \leq -5+6$

Either $x \geq 11$ or $x \leq 1$

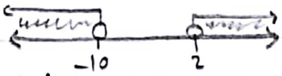
The Solution Set is $(-\infty, 1] \cup [11, \infty)$

$|x+4| > 6$

Either $x+4 > 6$ or $x+4 < -6$

Either $x > 6-4$ or $x < -6-4$

Either $x > 2$ or $x < -10$



The Solution set is $(-\infty, -10) \cup (2, \infty)$

$x^2 - 9 \geq 0$

$x^2 \geq 9$

$\sqrt{x^2} \geq \sqrt{9}$

$|x| \geq 3$

Either $x \geq 3$ or $x \leq -3$

The Solution set is $(-\infty, -3] \cup [3, \infty)$

$\sqrt{x^2} = |x|$

Exer $\frac{1}{16}$ $A(0,3)$, $B(4,0)$ (x_1, y_1; x_2, y_2)

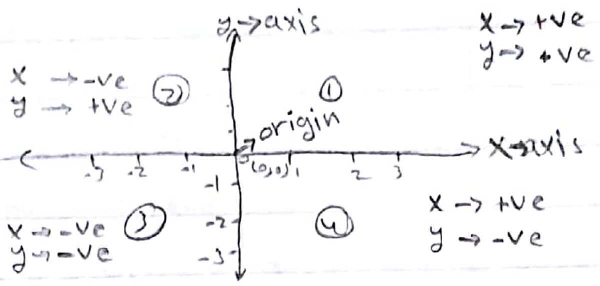
$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(4-0)^2 + (0-3)^2}$

$= \sqrt{(4)^2 + (-3)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

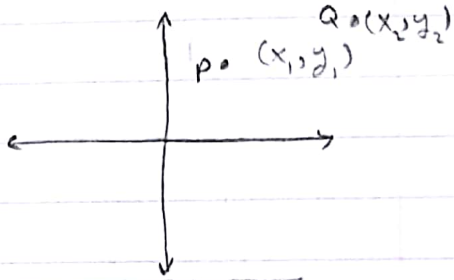
P.2. Cartesion coordinates in the plan



order pair ← (a, b)

x-axis | y-axis

Distance: (المسافة)



$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points in the plane.

The distance $\underline{D}$ between $P(x_1, y_1)$ and $Q(x_2, y_2)$ is: $D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Ex $\frac{2}{12}$ The distance between $A(3, -3)$ (x_1, y_1) and $B(-1, 2)$ (x_2, y_2) is: $D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(-1-3)^2 + (2-(-3))^2}$

$= \sqrt{(-4)^2 + (5)^2}$

$= \sqrt{16+25} = \sqrt{41}$

Ex $\frac{3}{13}$ The distance between from the origin $O(0,0)$ to a point $p(x, y)$ (x_1, y_1) is –

$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(x-0)^2 + (y-0)^2}$

$= \sqrt{x^2 + y^2}$

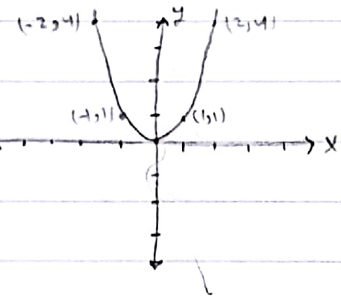
29/12

Graphs:-

Ex $\frac{6}{14}$ $y=x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

(-2,4) (-1,1) (0,0) (1,1) (2,4)



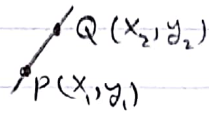
Note: para parallel and perpendicullar

متوازي عامودي

1. Two nonvertical Liens are parallel iff have the same slope
   i.e. If L_1 and L_2 are parallel $\rightarrow m_1=m_2$
2. If two nonvertical Lines L_1 and L_2 are perpendicullar, then
   $m_1 \cdot m_2 = -1$ or $m_1 = \frac{-1}{m_2}$ or $m_2 = \frac{-1}{m_1}$

STraight Lines:-

If $P(x_1,y_1)$ and $Q(x_2,y_2)$ on the line.



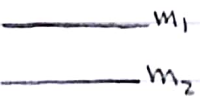
The slope of the line $= m = \frac{y_2-y_1}{x_2-x_1}$

$\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$

Ex $\frac{7}{14}$ The slope of the line joining $A(\underset{x_1}{3}, \underset{y_1}{-3})$ and $B(\underset{x_2}{-1}, \underset{y_2}{2})$ is

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{2-(-3)}{-1-3} = \frac{2+3}{-4} = \frac{5}{-4} = -\frac{5}{4}$$

Mr. Slope

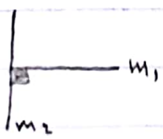


$m_1 = m_2$

$m_1 \cdot m_2 = -1$

$m_1 = \frac{-1}{m_2}$

$m_2 = \frac{-1}{m_1}$



H.W :-

1] $x^2 - 2x \leq 0$

$x(x-2) \leq 0$

$x - 2 \leq x$

$x \geq 0$

$x - 2 \leq 0$

$x \leq 2$

$0 \leq x \leq 2$

$x [0, 2]$

2] $x^2 < 9$

$\sqrt{x^2} < \sqrt{9}$

$|x| < 3$

$|x| < 9 = -\infty < x < \infty$

$x < 3$

$x > -3$

$x^2 - 2x \leq 0$

$x(2 - x) \leq 0$

$x = 0$ $\quad$ $x - 2 = 0$

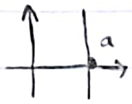
$\quad\quad\quad +2 \quad +2$

$x = 0$ $\quad$ $x = 2$

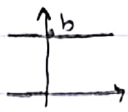
$[0, 2]$

Equations of Lines :-

[1] $x=a$ is equation of the vertical Line

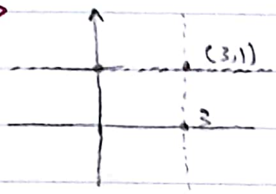


[2] $y=b$ is equation of the horizontal Line



Ex $\frac{8}{15}$ The horizontal and vertical lines passing through the point (3.1) have equation

$y=1$, $x=3$



Exer $\frac{13}{17}$. Find an equation for....

[a] the vertical line , $x=-2$

[b] Horizontal line , $y=\frac{6}{3}$

through the point $(-2,\frac{5}{3})$

Def:-

An equation of the line passing through the point $P_1(x_1,y_1)$ and having slope m is

$$y-y_1=m(x-x_1)$$

or

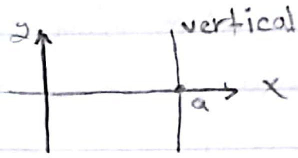
$$y=m(x-x_1)+y_1$$

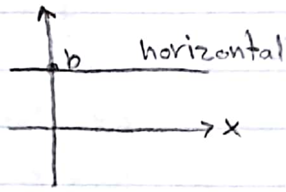
Ex $\frac{9}{15}$ Find an equation of the line of Slope -2 through the point $(1,4)$

$$y-y_1=m(x-x_1)$$
$$y-4=-2(x-1)$$
$$y-4=-2x+2$$
$$y=-2x+2+4$$
$$y=-2x+6$$

① $x=a$

② $y=b$





Exer $\frac{16}{17}$ = Write an equation for the line through $P(-2,2)$ With slope $m=\frac{1}{2}$

$$y-y_1=m(x-x_1)$$
$$y-2=\frac{1}{2}(x-(-2))$$
$$y-2=\frac{1}{2}(x+2)$$
$$y-2=\frac{1}{2}x+1$$
$$y=\frac{1}{2}x+1+2$$
$$y=\frac{1}{2}x+3$$

Ex $\frac{10}{15}$: Find an equation of the line through the points $(1,-1)$ and $(3,5)$

$$y-y_1=m(x-x_1)$$

[1] find m ..

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{5-(-1)}{3-1}=\frac{5+1}{2}=\frac{6}{2}=3$$

$m=3$, $(3,5)$

$$y-5=3(x-3)$$
$$y-5=3x-9$$
$$y=3x-9+5$$
$$y=3x-4$$

③ the slope-point equation (m, (x_1, y_1))

$$y - y_1 = m(x - x_1)$$

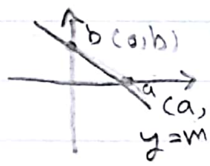
④ 2 points $(x_1, y_1), (x_2, y_2)$

① Find m. $m = \frac{y_2 - y_1}{x_2 - x_1}$

② $y - y_1 = m(x - x_1)$

⑤ $y = mx + b$

⑥ $y = m(x - a)$



$y = mx + b$

$y = m(x - a)$

بدلالة الميل والجزء المقطوع من محور y

$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

$$y - b = m(x - 0)$$

$$y - b = mx$$

$$y = mx + b$$

① The equation $\boxed{y = mx + b}$ is called The slope-y-intercept equation of the line with slope $\underline{m}$ and y-intercept $\underline{b}$

② The equation $y = m(x - a)$ is called the slope-x-intercept $\underline{a}$

Exer $\frac{25}{17}$ Write an equation for the line with slope $m = -2$ and y-intercept $b = \sqrt{2}$

$$y = mx + b$$

$$y = -2x + \sqrt{2}$$

في حالة النسيان

$$y - y_1 = m(x - x_1)$$

$$y - \sqrt{2} = -2(x - 0)$$

$$y - \sqrt{2} = -2x$$

$$y = -2x + \sqrt{2}$$

Ex $\frac{11}{16}$ find the slope and two intercept of the line with equation $8x + 5y = 20$

$$y = mx + b$$

$$8x + 5y = 20$$

$$5y = -8x + 20$$

$$y = -\frac{8}{5}x + \frac{20}{5}$$

$$y = -\frac{8}{5}x + 4$$

$m = -\frac{8}{5}$, $b = 4$ $a = ?$

$y = 0 \rightarrow 8x + 5(0) = 20$

$\rightarrow 8x = 20$

$\rightarrow x = \frac{20^{5\times4}}{8_{2\times4}}$

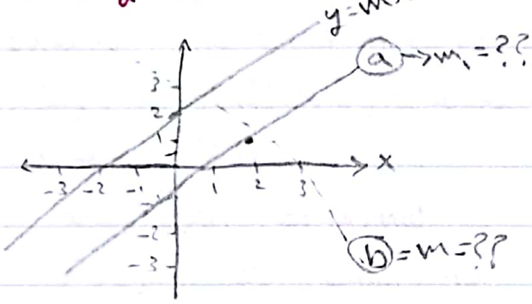
$\rightarrow x = \frac{5}{2}$

$a = \frac{5}{2}$

Exer $\frac{31}{17}$:
Find equations for the Lines through $P(2,1)$ that are
ⓐ parallel to, and
ⓑ perpendicular to

$y = mx + b$

the line $y = x + 2 = m1$ $y = mx + 2$ $m = 1$



$y - y_1 = m(x - x_1)$

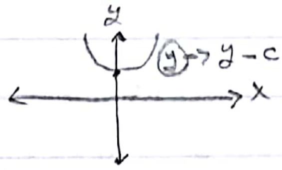
ⓐ Since the lines are parallel المتوازي
then $m_1 = m = 1$ $m_1 = \frac{-1}{m_2}$
$m_1 = 1$, $(2,1)$
$y - 1 = 1(x - 2)$
$y - 1 = x - 2$
$y = x - 2 + 1$
$y = x - 1$

ⓑ Since the Lines are perpendicular متعامد
then $m_2 = \frac{-1}{m_1} = \frac{-1}{\cancel{1}} = -1$ $m_2 = \frac{-1}{m_1}$
$m_2 = -1$, $(2,1)$
$y - 1 = -1(x - 2)$
$y - 1 = -x + 2$
$y = -x + 2 + 1$
$y = -x + 3$
$y = 3 - x$

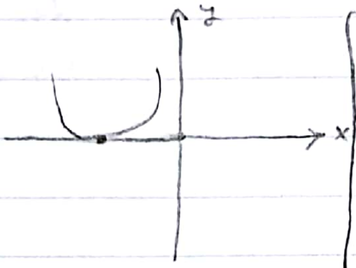
P.3 Graphs of Quadratic Equations.

Shifting Graph :-

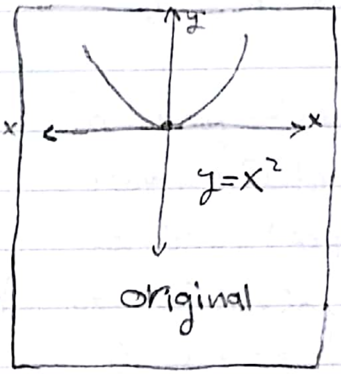
$y - c = x^2$
$y = x^2 + c$

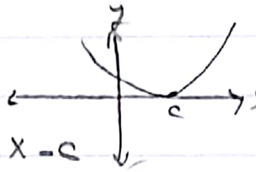


Shift up $\underline{c}$ units

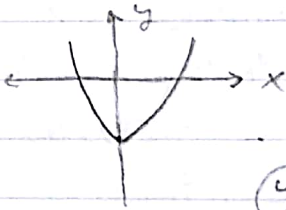


Shifted Left $\underline{c}$ units
x → x + c
$y = (x + c)^2$





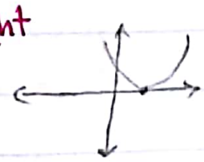
x → x − c
$y = (x - c)^2$
Shift right $\underline{c}$ units



y → y + c
Shifted down $\underline{c}$ units

Ex 8 + Ex 9 $\frac{}{21}$: The equation of the graph of $y = x^2$ shifted by :-

ⓐ $\underline{3}$ units to the right
$y = (x - 3)^2$



ⓑ $\underline{1}$ unit to the Left
$y = (x + 1)^2$

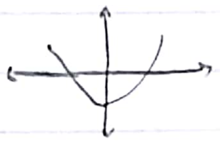
ⓒ $\underline{1}$ unit up
$y - 1 = x^2$
$y = x^2 + 1$

1/6

$\boxed{d}$ 3 unit down

$y+3=x^2$

$y=x^2-3$



Exer $\frac{36}{23}$ The equation of the graph $x^2+y^2=25$ Shifted by 2 units up and 4 units Left is ....

$x^2+y^2=25$ original

$x^2+(y-2)^2=25$ 2 units up

$(x+4)^2+(y-2)^2=25$ 4 units left

Exer $\frac{38}{23}$: The equation of the graph Shifted by 2 units down and 4 units left is ....

$y=\sqrt{x}$

$y=\sqrt{x}$ original

$y+2=\sqrt{x}$ 2 units down

$y+2=\sqrt{x+4}$ 4 units left

$y=\sqrt{x+4}-2$

P.6 polynomial

Def :-

polynomial (كثيرة حدود) : $a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$

Coefficient (معاملات) : $a_n, a_{n-1}, \dots, a_2, a_1, a_0$

Degree : n

Ex :

3 is a polynomial of degree 0

constant : عدد حقيقي

$1-x$ is a polynomial of degree 1

Linear : خطية

$2x^3-17x+1$ is a polynomial of degree 3.

1. دائماً تكون الأسس عدد صحيح موجب
2. المعاملات تكون أعداد حقيقية
3. دائماً (X) تكون موجودة في البسط

Factoring polynomial:

(a) common factor

$ax^2+bx=x(ax+b)$

Ex: $6x^2+3x=3x(2x+1)$

(b) Difference of Squars:

$x^2-a^2=(x-a)(x+a)$

Ex: $x^2-16=x^2-4^2=x^2-4^2=(x-4)(x+4)$

Note: x^2+a^2 لا تحلل تحليله

$(x+a)^2$, $(x-a)^2$

(c) Difference of cubes

$x^3-a^3=(x-a)(x^2+ax+a^2)$

Ex: $x^3-8=x^3-2^3=(x-2)(x^2+2x+4)$

(d) Sum of cubes

$x^3+a^3=(x+a)(x^2-ax+a^2)$

Ex: $x^3+1=x^3+1=(x+1)(x^2-x+1)$

(e) $x^2+5x+6=(x+2)(x+3)$

$x^2-5x+6=(x-2)(x-3)$

$x^2+x-6=(x+3)(x-2)$

$x^2-x-6=(x-3)(x+2)$

1/8

P.4 Function and their Graphs :-

دالة

Def: A function F on a set D into a set S is a rule that assigns a unique element f(x) in S to each element x in D.

قاعدة ، وحيد ، عنصر

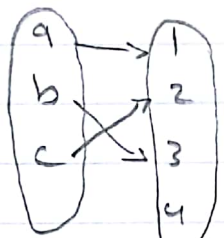
★ In this definition, D = D(f) is the domain of the function F.

مجال

★ The Range R(f) of f is the subset of S consisting of all values f(x) of the function.

$F: D \to S$

$D \longrightarrow S$



domain — co-domin

المجال — المجال المقابل

Domain = {a, b, c}.

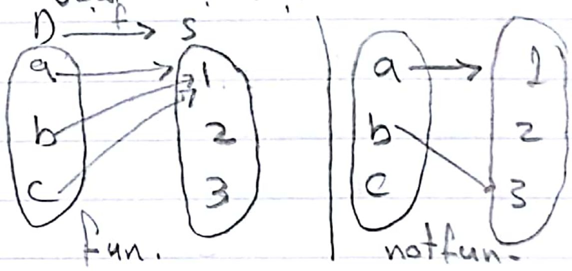
Range = {1, 2, 3}.

f(a) = 1

f(b) = 3

f(c) = 2

[Range = {1} ⊆ S]



fun. — not fun.

Ex $\frac{2}{25}$: F(t) = 2t + 3. Find

F(0) = 2(0) + 3 = 3

F(2) = 2(2) + 3 = 4 + 3 = 7

F(x+2) = 2(x+2) + 3 = 2x + 4 + 3 = 2x + 7

F(F2) = F(7) = 2(7) + 3 = 14 + 3 = 17

$\sqrt[3]{8} = 2$, $\sqrt[3]{-8} = -2$

$\sqrt{4} = 2$, $\sqrt{0} = 0$, $\sqrt{-4}$ = undefined

The Domain convention :-

① polynomial function.

$F(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Domain: R

Exer $\frac{1}{32}$: Find the domain of the function

F(x) = 1 + x² / Domain = R

② Root function :-

$f(x) = \sqrt[n]{P(x)}$, where P(x) is polynomial. (دليل الجذر n)

n: 1, 2, 3, ...

Domain :-

① If n is odd : IR

فردي

② If n is elem : Solution P(x) ≥ 0

زوجي

Ex $\frac{3}{25}$: $f(x) = \sqrt{x}$

$x \geq 0$, Domain = [0, ∞) , $\sqrt{x^2} = |x|$

Ex $\frac{5}{26}$ (St) = $\sqrt{1-t^2}$

$1 - t^2 \geq 0 \to 1 \geq t^2 \to 1$

$\sqrt{t^2} \leq \sqrt{1} \to |t| \leq 1 \to -1 \leq t \leq 1$

Domain: [-1, 1]

③ Rational function :-

الكسرية

$F(x) = \frac{P(x)}{q(x)}$, P, q are polynamials

Domain: IR − {اصفار المقام} Solution q(x) = 0

zero of q(x)

Ex $\frac{4}{25}$: $h(x) = \frac{x}{x^2-4}$

$x^2-4=0 \rightarrow (x-2)(x+2)=0$

$\rightarrow x-2=0 \,, x+2=0$

$\rightarrow x=2 \,, x=-2$

Domain $= \mathbb{R} - \{2,-2\} = (-\infty,-2) \cup (-2,2) \cup (2,\infty)$

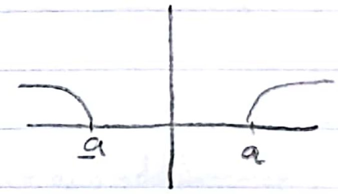
Exer $\frac{5}{32}$: $h(t) = \frac{t}{\sqrt{2-t}}$

$2-t > 0 \rightarrow 2 > t$

Domain $= (-\infty, 2)$.

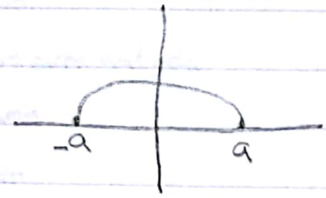
$F(x) = \sqrt{x^2-a^2} \longrightarrow$

Domain $= (-\infty,-a] \cup [a,\infty)$



$F(x) = \sqrt{a^2-x^2} \longrightarrow$

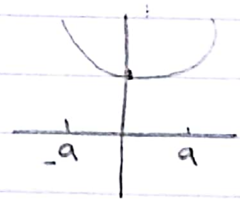
Domain $= [-a,a]$



Ex $\frac{5}{25}$: $S(t) = \sqrt{1-t^2} \rightarrow$ Domian$=[-1,1]$

$F(x) = \sqrt{a^2+x^2} \longrightarrow$

Domain $= \mathbb{R}$



$D = x \geq 0 \leftarrow$ الدالة الجذرية $\rightarrow y = \sqrt{x}$

$D = x \geq 0 \leftarrow$ دالة القطع المكافئ $\rightarrow y = x^2$

$D = x > 0 \leftarrow$ الدالة الكسرية $\rightarrow y = \frac{1}{x}$

(1) $f(x) = \sqrt{x^2-a^2} \rightarrow D = (-\infty,-a] \cup [a,\infty)$

(2) $f(x) = \sqrt{a^2-x^2} \rightarrow D = [-a,a]$

(3) $f(x) = \sqrt{x^2+a^2} \rightarrow D = R$

المجال والمدى الدوال الأم

$f(x) = x^2 \rightarrow D = (-\infty,\infty) \rightarrow R = [0,\infty)$

$f(x) = x^3 \rightarrow D, R \rightarrow (-\infty,\infty)$

$f(x) = \sqrt{x} \rightarrow D, R \rightarrow [0,\infty)$

$f(x) = \frac{1}{x} \rightarrow D, R \rightarrow \mathbb{R} - \{0\}$

$f(x) = |x| \rightarrow D = (-\infty,\infty), R \rightarrow [0,\infty)$

$f(x) = [x] \rightarrow D = (-\infty,\infty) \rightarrow R \rightarrow Z$

$(-\infty,\infty) = R$

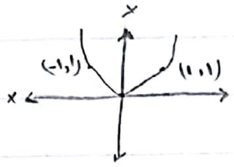
Graphs of functions:-

Ex $\frac{6}{26}$: Graph of the function

$f(x) \leftarrow y = x^2$

Page 26

x	-2	-1	0	1	2
f(x)	4	1	0	1	4

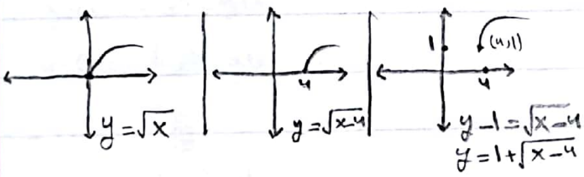


Ex $\frac{7}{26}$: Sketch the graph of

$y = 1 + \sqrt{x-4}$

$y - 1 = \sqrt{x-4}$

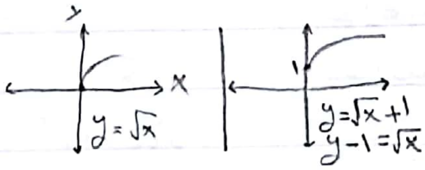
This is the graph $y = \sqrt{x}$ shifted by $\underline{4}$ units to the right and $\underline{1}$ units to the up.



Exer $\frac{29}{33}$: $y = \sqrt{x} + 1$

$y - 1 = \sqrt{x}$

This is the graph $\boxed{y = \sqrt{x}}$ shifted by $\underline{1}$ units to the up.



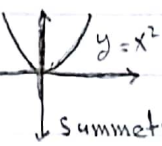
زوجي فردي

Even and odd function:-

Def:

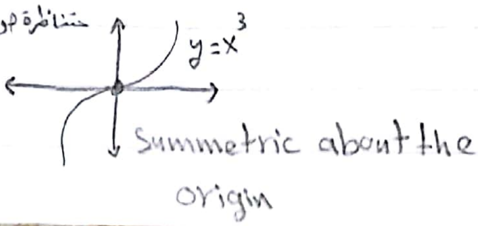
★ (f) is even function if $f(-x) = f(x)$ متناظرة حول y

$\forall x$ in the domain of f.

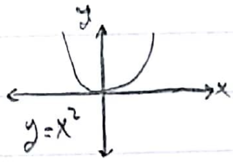


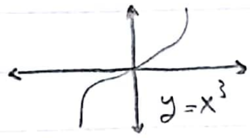
★ (f) is odd function if $f(-x) = -f(x)$

$\forall x$ in the domain of f. متناظرة حول نقطة الأصل



Even function	odd function
$f(-x) = f(x)$	$f(-x) = -f(x)$
Summetric about the y-axis	summetric about the original Point (0,0)
$f(x) = x$ even	$f(x) = x$ odd
$f(x) = \lvert x \rvert$	
$f(x) = $ number	
$\cos(x)$, $\sec(x)$	$\sin(x)$, $\tan(x)$, $\cot(x)$, $\csc(x)$





How to test the even and odd function

① Even ± Even = Even

② odd ± odd = odd

③ Even ± odd = Not even and odd

④ Even × even = even

⑤ odd × odd = even

⑥ Even × odd = odd

Ex :

① $3x^4 - 5x^2 - 1$

even.even - even.even - even

Even fun.

$f(-x) = 3(-x)^4 - 5(-x)^2 - 1 = f(x) = 3x^4 - 5x^2 - 1$

② $4x^3 - \frac{2}{x}$

even.odd $- \frac{\text{even}}{\text{odd}}$ = odd - odd = odd fun.

③ $g(x) = x^2 - 2x$

even - even.odd / even - odd / Not even and odd

Exer : $\frac{11}{33}$, $f(x) = x^2 + 1$

even + even / even function

summetric a bbout the y-axis

$\frac{12}{33}$: $f(x) = x^3 + x$

odd + odd / odd fun.

summetric aboute the original point(0,0)

$\frac{18}{33}$; $f(x) = x^3 - 2$

odd - even / Not even and odd.

P.5 : combining functions to make New fun.

Sums, Differences, Products, Quotents and Multiples.

Def: If f and g are fun. :

$$\left.\begin{aligned} (f+g)(x) &= f(x) + g(x) \\ (f-g)(x) &= f(x) - g(x) \\ (f\cdot g)(x) &= f(x)\cdot g(x) \end{aligned}\right] D_f \cap D_g$$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ $D_f \cap D_g -$ {أصفار المقام}

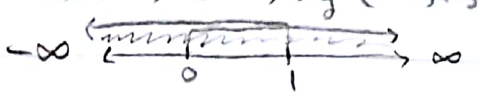
$(cf)(x) = c \cdot f(x)$ D_f

Ex $\frac{3}{34}$: $f(x) = \sqrt{x}$, $g(x) = \sqrt{1-x}$

function	Domain
$f(x) = \sqrt{x}$	$\rightarrow [0, \infty)$
$g(x) = \sqrt{1-x}$	$\rightarrow (-\infty, 1]$
$3f(x) = 3\sqrt{x}$	$\rightarrow [0, \infty)$
$(f+g)(x) = \sqrt{x} + \sqrt{1-x}$	$\rightarrow D_f \cap D_g = [0, 1]$
$(f-g)(x) = \sqrt{x} - \sqrt{1-x}$	$\rightarrow [0, 1]$
$(f \cdot g)(x) = \sqrt{x} \cdot \sqrt{1-x} = \sqrt{x(1-x)}$	$\rightarrow [0, 1]$

(f) $x \geq 0 \rightarrow D_f = [0, \infty)$ /

(g) $1 - x \geq 0 \rightarrow 1 \geq x \rightarrow D_g = (-\infty, 1]$. /



1/15

Ex $\frac{3}{34}$: $f(x) = \sqrt{x}$, $g(x) = \sqrt{1-x}$

Formula

$\frac{f}{g}(x) = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}}$ → Domain $= [0,1)$

$\frac{\sqrt{x}}{\sqrt{1-x}}$ $\quad x \geq 0 \rightarrow [0,\infty)$

$1-x \geq 0 \rightarrow 1 \geq x \rightarrow (-\infty, 1]$

$[0,\infty) \cap (-\infty,1] = [0,1)$

$[0,1] \rightarrow [0,1)$

or → $(D_f \cap D_g) - \{$أصفار المقام$\}$

$1 - x = 0$

$1 = x$

$\frac{g}{f}(x) = \frac{\sqrt{1-x}}{\sqrt{x}} = \sqrt{\frac{1-x}{x}}$ → Domain $= [0,1] - \{0\}$
$= (0,1]$

Composite Function:

Def: If f and g are functions.

$f \circ g(x) = f(g(x))$

The domain of $f \circ g$:

$D_{\text{الناتج}} \cap D_g$

$g \circ f$: $D_{\text{الناتج}} \cap D_f$

Note:- $f \circ g \neq g \circ f$

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[n]{x} = x^{\frac{1}{n}}$

Ex $\frac{4}{35}$: $f(x) = \sqrt{x}$ ($x \geq 0$) and $g(x) = x+1$

$D_f = [0,\infty) \rightarrow D_g = \mathbb{R}$.

Formula	Domain
$f \circ g(x) = f(g(x))$ $= f(x+1) = \sqrt{x+1}$ $x+1 \geq 0 \rightarrow x \geq -1$	$D_{\text{الناتج}} \cap D_g$ $[-1,\infty) \cap \mathbb{R}$ $= [-1,\infty)$.
$g \circ f(x) = g(f(x))$ $= g(\sqrt{x}) = \sqrt{x}+1$ $[0,\infty) \cap \mathbb{R} = [0,\infty)$	$D_{\text{الناتج}} \cap D_f$ $= [0,\infty) \cap [0,\infty)$ $= [0,\infty)$
$f \circ f(x) = f(f(x))$ $= f(\sqrt{x}) = \sqrt{\sqrt{x}}$ $= (x^{\frac{1}{2}})^{\frac{1}{2}} = x^{\frac{1}{4}} = \sqrt[4]{x}$ $x \geq 0$	$D_{\text{الناتج}} \cap D_f$ $= [0,\infty) \cap [0,\infty)$ $= [0,\infty)$
$g \circ g(x) = g(g(x))$ $= g(x+1) = (x+1)+1$ $= x+2$	$D_{\text{الناتج}} \cap D_g$ $\mathbb{R} \cap \mathbb{R}$ $= \mathbb{R}$

Exer $\frac{7}{38}$: $f(x) = x+5$ and $g(x) = x^2-3$

ⓐ $f \circ g(0) = f(g(0)) = f(0^2-3) = f(-3)$
$= -3+5 = 2$

ⓑ $g(g(2)) = g(2^2-3) = g(4-3) = g(1)$
$= 1-3 = -2$

1/15

Exer $\frac{9}{38}$: $f(x) = \frac{1}{1-x}$; $g(x) = \sqrt{x-1}$

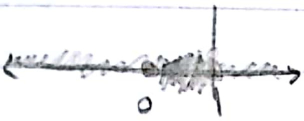
$g\circ f(x) = g(f(x)) = g\left(\frac{1}{1-x}\right)$

$= \sqrt{\frac{1}{1-x} - \frac{1(1-x)}{1-x}} = \sqrt{\frac{1-(1-x)}{1-x}}$

$= \sqrt{\frac{1-1+x}{1-x}} = \sqrt{\frac{x}{1-x}}$

$D_{\text{الناتج}} \cap D_f = [0,1) \cap \mathbb{R} - \{1\}$

$= [0,1)$



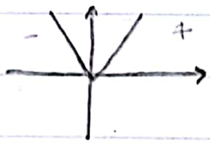
$D_f : \mathbb{R} - \{$اصفار المقام$\}$

$1-x=0 \rightarrow x=1 \rightarrow \mathbb{R}-\{1\}$

Piecewise Defind Function:

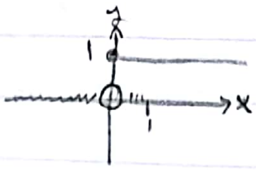
For ex:-

$|x| = \begin{cases} x & \text{if } x>0 \\ -x & \text{if } x<0 \end{cases}$



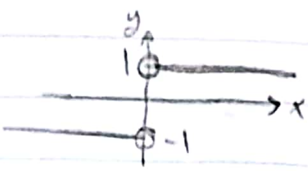
Ex $\frac{6}{36}$: The heaviside fun. (or unit step fun.)

$H(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x<0 \end{cases}$

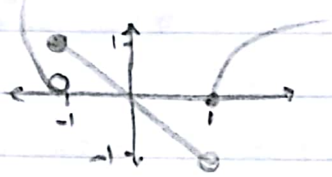


Ex $\frac{7}{36}$: The Signum Fun.

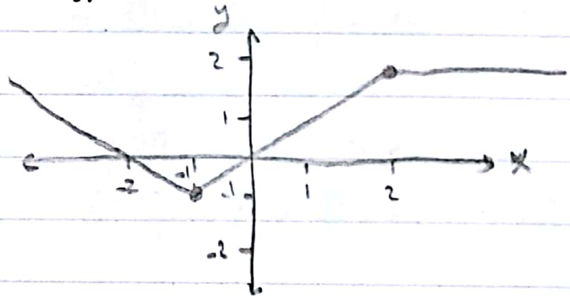
$Sgn(x) = \frac{x}{|x|} = \begin{cases} 1 & \text{if } x>0 \\ -1 & \text{if } x<0 \\ \text{undefined} & \text{if } x=0 \end{cases}$



Ex $\frac{8}{36}$ $f(x) = \begin{cases} (x+2)^2 & \text{if } x<-1 \\ -x & \text{if } -1 \leq x < 1 \\ \sqrt{x-1} & \text{if } x \geq 1 \end{cases}$



Ex $\frac{9}{37}$: find a formula for $g(x)$



Solution Ex9.

$g(x) = \begin{cases} -(x+2) & \text{if } x \leq -1 \\ x & \text{if } -1 \leq x \leq 2 \\ 2 & \text{if } x \geq 2 \end{cases}$

x (f) $g(g(2)) = g(2^2-3) = g(4-3)$

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※ الدورة تكون منغلقة

Ex $\frac{10}{37}$: The greatest integer fun.

※ التقريب لأقل عدد صحيح

$[x]$; $\lfloor x \rfloor$; $[[x]]$

$\lfloor x \rfloor$ = greatest integer Less than or equal to(x).

$\lfloor 2.4 \rfloor = 2$; $\lfloor 1.9 \rfloor = 1$

$\lfloor 0 \rfloor = 0$; $\lfloor -1.2 \rfloor = -2$

$\lfloor 2 \rfloor = 2$; $\lfloor 0.2 \rfloor = 0$

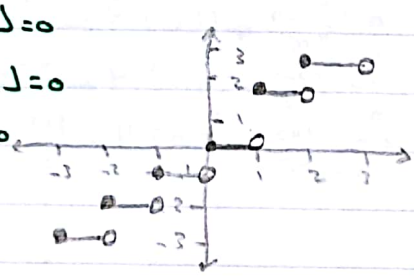
$\lfloor -0.3 \rfloor = -1$; $\lfloor -2 \rfloor = -2$

$\lfloor 0 \rfloor = 0$

$\lfloor 0.5 \rfloor = 0$

$\lfloor 0.8 \rfloor = 0$

$\lfloor 0.9 \rfloor = 0$



P.7 : The Trigonometric fun.

Measuring Angles:

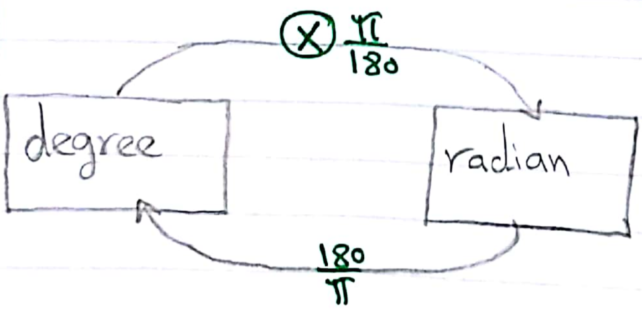
π radian = $180°$ degree

Famous Angles:

0 rad $= 0°$, $\frac{\pi}{4}$ rad $= 45°$

$\frac{\pi}{6}$ rad $= 30°$, $\frac{\pi}{2}$ rad $= 90°$

converting degree $\longleftrightarrow$ radian



Example:

※ convert from degree to radian

a) $45° \to 45 \times \frac{\pi}{180} = \frac{45}{180}\pi = \frac{1}{4}\pi = \frac{\pi}{4}$

b) $120° \to 120 \times \frac{\pi}{180} = \frac{12}{18}\pi = \frac{2}{3}\pi$

c) $12° \to 12 \times \frac{\pi}{180} = \frac{12}{18}\pi = \frac{1}{15}\pi = \frac{\pi}{15}$

d) $270° \to 270 \times \frac{\pi}{180} = \frac{27}{18}\pi = \frac{3}{2}\pi$

convert from radian to degree

a) $\frac{2\pi}{3} \to \frac{2\pi}{3} \times \frac{180}{\pi} = 2(60) = 120°$

b) $\frac{\pi}{3} \to \times \frac{180}{\pi} = 60°$

c) $\frac{5\pi}{6} \to \times \frac{180}{\pi} = 5(30) = 150°$

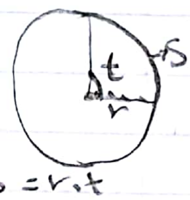
d) $\frac{3\pi}{4} \to \frac{3\pi}{4} \times \frac{180}{\pi} = 3(45) = 135°$

Arc Length

t : angle (radian)

S : arc length

r : radian



$S = r.t$

Example:

※ لازم أحول للراديان

※ If the radians of a circle is $9cm$ (r), What angle (t) is Subtended by an arc (S) of $12cm$?? $S = r.t$, $12 = 9.t$

$\to t = \frac{12}{9} = \frac{4}{3}$ rad.

※ If a circle has radius $4cm$ (r), What is the length (S??) of an arc subtended by a central angle of $\frac{3\pi}{4}$ (t) rad??

$S = r.t$

$S = \cancel{4}.(\frac{3\pi}{\cancel{4}}) = 3\pi$ cm.

1/17

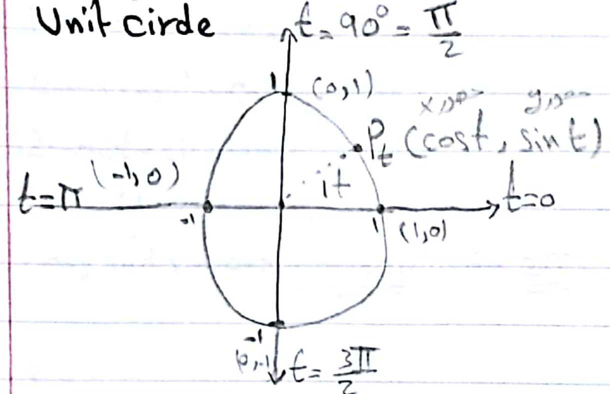
$-1 \le \sin t \le 1$

$-1 \le \cos t \le 1$

Sin +, cos −

Trigonometric Function.

Unit circle



If $t = 0 \rightarrow P(1,0) \rightarrow \cos 0 = 1$, $\sin 0 = 0$

If $t = \frac{\pi}{2} \rightarrow P(0,1) \rightarrow \cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$

If $t = \pi \rightarrow P(-1,0) \rightarrow \cos\pi = -1$, $\sin\pi = 0$

If $t = \frac{3\pi}{2} \rightarrow P(0,-1) \rightarrow \cos\frac{3\pi}{2} = 0$, $\sin\frac{3\pi}{2} = -1$

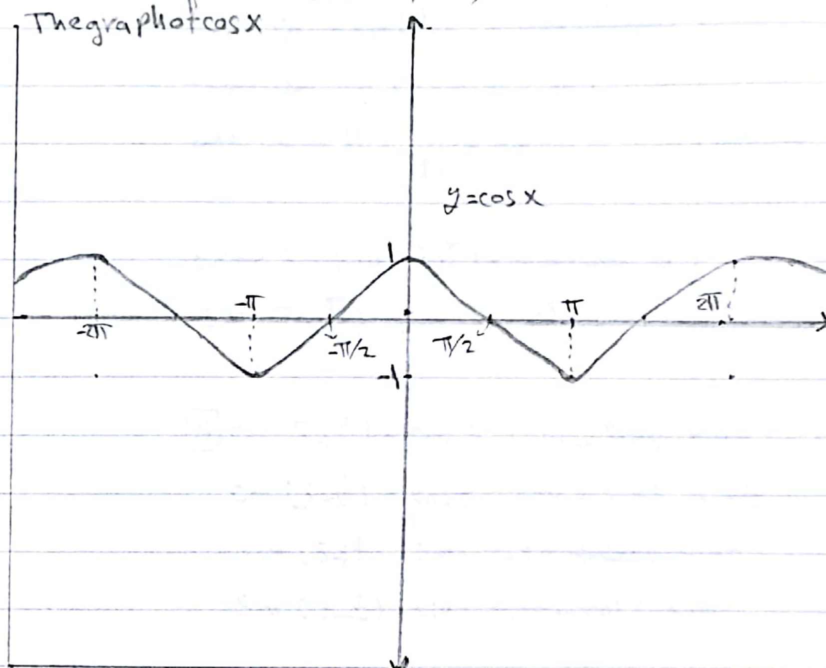
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	
	$0°$	$30°$	$45°$	$60°$	$90°$	
Sin	0	1	2	3	4	
cos	4	3	2	1	0	البسط
			2			المقام

$\sin 30° = \frac{\sqrt{1}}{2} = \frac{1}{2}$

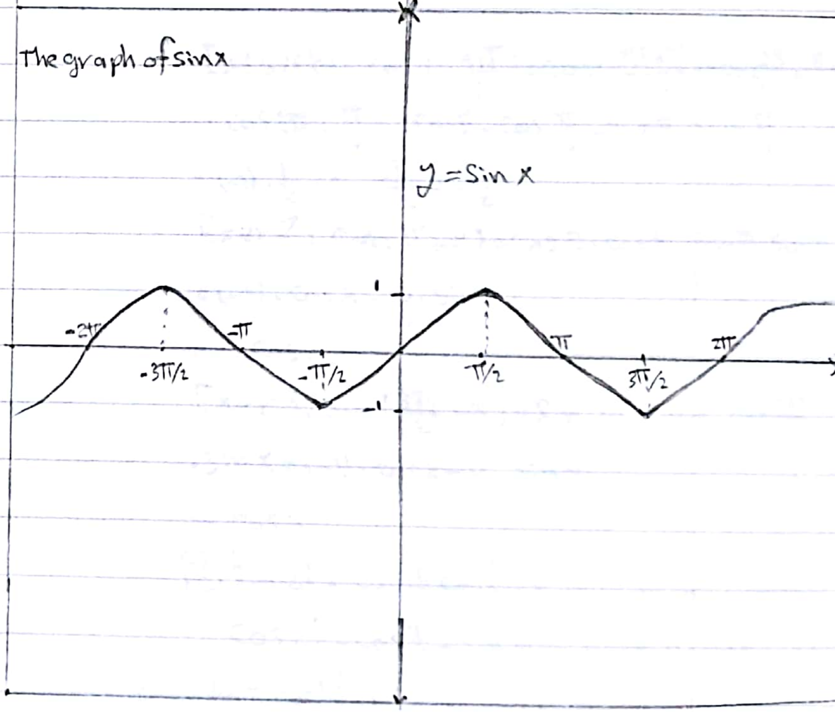
$\cos 30° = \frac{\sqrt{3}}{2}$

$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

The graph of cos x



The graph of sin x



☆ الرسم تتكرر بعد 2π

Degree	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	−1
Sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

20/2

H.w: Exer 2 : $\cos\left(\frac{3\pi}{4}\right) = ??$ $\left(\frac{4\pi - \pi}{}\right)$

H.w: Exer 5 : $\cos\left(\frac{5\pi}{12}\right) = ??$ $\left(3\pi + 2\pi\right)$

Some Useful Identites:

1. $\cos^2 t + \sin^2 t = 1$

Note: $\cos t^2 \neq \cos^2 t = (\cos t)^2$.

2. $\cos(t + 2\pi) = \cos t$.

   $\sin(t + 2\pi) = \sin t$.

(cos and sin are <u>periodic</u> with $\underline{2\pi}$) دورية

Ex:

* $\cos(5\pi) =$

$\cos(3\pi + 2\pi) = \cos(3\pi) = \cos(\pi + 2\pi) = \cos\pi = -1$

* $\sin(4\pi) =$

$\sin(2\pi + 2\pi) = \sin(2\pi) = \sin(0 + 2\pi) = \sin 0 = 0$

3. $\cos(-t) = \cos t$ (even fun.)

   $\sin(-t) = -\sin t$ (odd fun.)

Ex:

* $\cos(-\pi) =$

$\cos\pi = -1$

* $\sin\left(-\frac{\pi}{3}\right) =$

$-\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

4. $\cos\left(\frac{\pi}{2} - t\right) = \sin t$

   $\sin\left(\frac{\pi}{2} - t\right) = \cos t$

5. $\cos(\pi - t) = -\cos t$

   $\sin(\pi - t) = \sin t$

Ex : $\cos(120°) =$

$\cos\left(\frac{2\pi}{3}\right) = \cos\left(\frac{3\pi - \pi}{3}\right) = \cos\left(\frac{3\pi}{3} - \frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right)$

by 5 $-\cos\frac{\pi}{3} = -\frac{1}{2}$

Ex 5 (a) : $\sin\left(\frac{3\pi}{4}\right) = \sin\left(\frac{4\pi - \pi}{4}\right)$

$= \sin\left(\frac{4\pi}{4} - \frac{\pi}{4}\right) = \sin\left(\pi - \frac{\pi}{4}\right)$

by 5 : $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Exer 3 : $\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{3\pi - \pi}{3}\right)$

$= \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

6. $\cos(s + t) = \cos s \cos t - \sin s \sin t$.

   $\cos(s - t) = \cos s \cos t + \sin s \sin t$.

   $\sin(s + t) = \sin s \cos t + \cos s \sin t$.

   $\sin(s - t) = \sin s \cos t - \cos s \sin t$.

Ex (5)(b) : $\cos\left(\frac{4\pi}{3}\right) = \cos\left(\frac{3\pi + \pi}{3}\right) = \cos\left(\frac{3\pi}{3} + \frac{\pi}{3}\right)$

$= \cos\left(\pi + \frac{\pi}{3}\right) = \cos\pi \cdot \cos\frac{\pi}{3} - \sin\pi \cdot \sin\frac{\pi}{3}$

$= (-1) \cdot \frac{1}{2} - 0 \cdot \frac{\sqrt{3}}{2} = -\frac{1}{2}$

Exer 7 : $\cos(\pi + x) = \cos\pi \cdot \cos x - \sin\pi \cdot \sin x$

$= (-1) \cdot \cos x - 0 \cdot \sin x$

$= -\cos x$.

Exer 9 : $\sin\left(\frac{3\pi}{2} - x\right) = \sin\frac{3\pi}{2} \cdot \cos x - \cos\frac{3\pi}{2} \cdot \sin x = (-1) \cdot \cos x - 0 \cdot \sin x$

$= -\cos x$

7. $\sin 2t = 2\sin t \cos t$ $(t + t)$ $\sin t \cos t + \sin t \cos t$

   $\cos 2t = \cos^2 t - \sin^2 t$ $\cos t \cdot \cos t - \sin t \sin t$

$\cos 2t = 2\cos^2 t - 1$

$\cos 2t = 1 - 2\sin^2 t$

Ex : $\sin\left(\frac{14\pi}{5}\right) = \dots ??$

a) $\sin\frac{\pi}{5}$

b) $2\sin\frac{\pi}{5}\cos\frac{\pi}{5}$

$\boxed{c)}$ $2\sin\frac{7\pi}{5} \cdot \cos\frac{7\pi}{5}$

8. $\cos^2 t = \frac{1 + \cos 2t}{2}$

   $\sin^2 t = \frac{1 - \cos 2t}{2}$

22/1

* ال cos, sin تتكرر كل 2π
* tan, cot تتكرر كل π

other Trigonometric Functions :-

$\tan t = \frac{\sin t}{\cos t}$; $\cot t = \frac{1}{\tan t} = \frac{\cos t}{\sin t}$

$\sec t = \frac{1}{\cos t}$

$\csc t = \frac{1}{\sin t}$

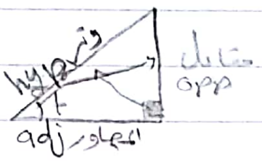
Note: tan and cot have period π (الرديان)

Trigonometric function from a right triangle :-

$\sin t = \frac{opp}{hyp}$

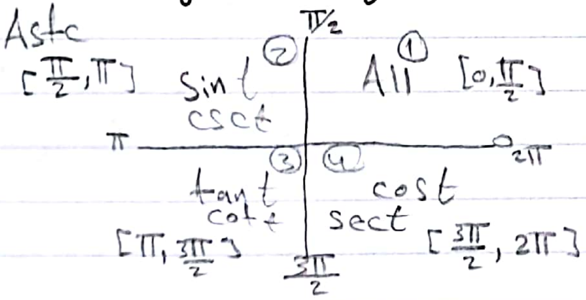
$\cos t = \frac{adj}{hyp}$

$\tan t = \frac{opp}{adj}$



Positive sign of trigonometric fun. :-

Astc



Ex 7 : θ in $[\pi, \frac{3\pi}{2}]$; $\cos\theta = \frac{-1}{3}$

$\boxed{1}$ نحدد الإشارة

θ in $[\pi, \frac{3\pi}{2}] \rightarrow$ quad ③

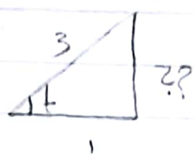
$\tan\theta = +$; $\cot\theta = +$

$\sin\theta = -$; $\csc\theta = -$

$\sec\theta$ $-$

$\boxed{2}$ نحدد القيم

$\cos\theta = -\frac{1}{3} = \frac{adj}{hyp}$



باستخدام نظرية فيثاغورس

$|hyp|^2 = |opp|^2 + |adj|^2$

$9 = |opp|^2 + 1$

$|opp| = \sqrt{8} = 2\sqrt{2}$ (4×2)

$\tan\theta = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$; $\cot\theta = \frac{1}{2\sqrt{2}}$

$\sin\theta = -\frac{2\sqrt{2}}{3}$; $\csc\theta = \frac{-3}{2\sqrt{2}}$

$\sec\theta = -3$

Exer 29 : θ in $[\pi, \frac{3\pi}{2}]$; $\sin\theta = -\frac{1}{2}$

① θ in $[\pi, \frac{3\pi}{2}] \rightarrow$ quad. ③

$\tan\theta = +$; $\cot = +$; $\cos\theta = -$

$\sec\theta = -$; $\csc\theta = -$

② $\sin\theta = \frac{-1}{2} = \frac{opp}{hyp}$

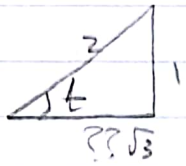
③ $\tan\theta = \frac{1}{\sqrt{3}}$; $\cot\theta = \sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$; $\sec\theta = \frac{-2}{\sqrt{3}}$; $\csc\theta = -2$

$|hyp|^2 = |opp|^2 + |adj|^2$

$4 = 1 + |adj|^2$

$|adj| = \sqrt{3}$



$\frac{0}{5} = 0$ صفر

$\frac{5}{0} = +\infty$

$\frac{-5}{0} = -\infty$

الانتاج بالنسبة لمصدر

$(x-a)(x+a) = x^2 - a^2$

22/1

1.2. limits of function:

نهايات الدوال

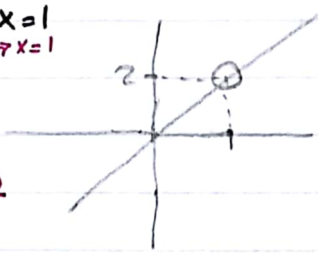
Ex1: $f(x) = \frac{x^2-1}{x-1}$ near $x=1$

$x-1=0 \rightarrow x=1$

① $D_f = \mathbb{R} - \{1\}$.

② $f(1) =$ undefined.

③ $\lim_{x\to1} f(x) = \lim_{x\to1} \frac{x^2-1}{x-1} = 2$



Def: 1:-

function $\underline{f}$ approaches the limit $\underline{L}$ as $\underline{x}$ approaches $\underline{a}$

$\lim_{x\to a} f(x) = L$

Ex 3:

a) $\lim_{x\to3} x = 3$

In general; $\lim_{x\to a} x = a$

b) $\lim_{x\to0} 2 = 2$

in general; $\lim_{x\to a} c = c$

(where c is a constant)

From Ex 1:

$\lim_{x\to1} \frac{x^2-1}{x-1} = \frac{1-1}{1-1} = \frac{0}{0}$

$\lim_{x\to1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$\lim_{x\to1} x+1 = 1+1 = 2.$

Ex 4: a) $\lim_{x\to-2} \frac{x^2+x-2}{x^2+5x+6} = \frac{4-2-2}{4-10+6} = \frac{0}{0}$

$\lim_{x\to-2} \frac{\cancel{(x+2)}(x-1)}{\cancel{(x+2)}(x+3)}$

$= \lim_{x\to-2} \frac{x-1}{x+3} = \frac{-2-1}{-2+3} = \frac{-3}{1} = -3$

c) $\lim_{x\to4} \frac{\sqrt{x}-2}{x^2-16} = \frac{\sqrt{4}-2}{16-16} = \frac{0}{0}$

$\lim_{x\to4} \frac{\sqrt{x}-2}{x^2-16} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

$= \lim_{x\to4} \frac{(\sqrt{x}-2)\cdot(\sqrt{x}+2)}{(x^2-16)\cdot(\sqrt{x}+2)}$

$= \lim_{x\to4} \frac{(\sqrt{x})^2-(2)^2}{(x+4)(x-4)\cdot(\sqrt{x}+2)}$

$= \lim_{x\to4} \frac{(x-4)}{(x+4)(x-4)(\sqrt{x}+2)}$

$= \lim_{x\to4} \frac{1}{(x+4)(\sqrt{x}+2)} = \frac{1}{(8)\cdot(2+2)} = \frac{1}{32}$

2/24

Exer 18: $\lim_{h \to 0} \frac{\sqrt{4+h}-2}{h} = \frac{\sqrt{4+0}-2}{0} = \frac{0}{0}$

$\lim_{h \to 0} \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$

$= \lim_{h \to 0} \frac{(\sqrt{4+h}-2)(\sqrt{4+h}+2)}{h \cdot (\sqrt{4+h}+2)}$

$= \lim_{h \to 0} \frac{\sqrt{4+h}^2-(2)^2}{h \cdot (\sqrt{4+h}+2)}$

$= \lim_{h \to 0} \frac{\cancel{4}+h-\cancel{4}}{h \cdot (\sqrt{4+h}+2)}$

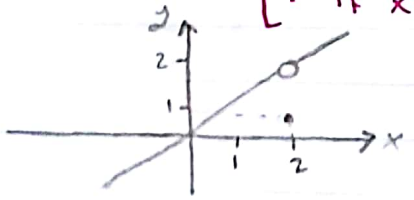
$\lim_{h \to 0} \frac{h}{h \cdot (\sqrt{4+h}+2)}$

$= \lim_{h \to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4+0}+2}$

$= \frac{1}{2+2} = \frac{1}{4}$

Note: $\lim_{x \to a} f(x) \neq f(a)$
ليس شرطاً أساسياً

Ex 5: let $g(x) = \begin{cases} x & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$



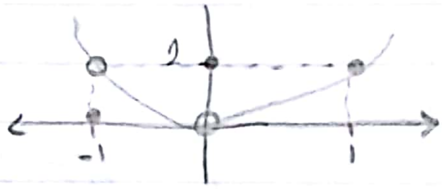
(لما تكون نقطة ترسب)
عند 2= lim ≠ g6

$g(2) = 1$

$\lim_{x \to 2} g(x) = \lim_{x \to 2} x = 2$

* الأسهم تتبع الرسمة
* الرسم يتبع النقطة

Exer 1:



a) $f(-1) = 0$
$\lim_{x \to -1} f(x) = 1$

b) $f(0) = 1$
$\lim_{x \to 0} f(x) = 0$

c) $f(1) = 1$
$\lim_{x \to 1} f(x) = 1$

One Sided Limits:

Note:

Limits are unique:
if $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} f(x) = M$ then $L = M$

For example:

$f(x) = \frac{1}{x}$

$\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to 0} f(x) = D.N.E \to$ does not exist.

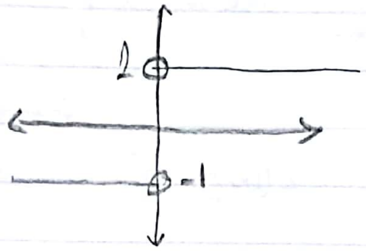
Def:

1. $x \to a$
$\lim_{x \to a} f(x) = L$ is called the left limits
($\underline{x}$ approaches $\underline{a}$ from the left).

2. $a \leftarrow x$
$\lim_{x \to a^+} f(x) = L$ is called the right limit.
(x approaches $\underline{a}$ from the right)

2/24

Ex6: $Sgn(x) = \frac{x}{|x|}$



$\lim_{x\to 0^+} Sgn(x) = 1$

$\lim_{x\to 0^-} sgn(x) = -1$

$\lim_{x\to 0} sgn(x) = D.N.E$
becouse $\lim_{x\to 0^+} sgn(x) \neq \lim_{x\to 0^-} sgn(x)$

Theorem 2: $\lim_{x\to a} f(x) = L$ (Th) نظرية

$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$

$\neq \to D.N.E$

Ex7: If $f(x) = \frac{|x-2|}{x^2+x-6}$
find $\lim_{x\to 2^+} f(x)$; $\lim_{x\to 2^-} f(x)$; $\lim_{x\to 2} f(x)$

$$f(x) = \begin{cases} \frac{-(x-2)}{x^2+x-6} & \text{if } x<2 \\ \frac{+(x-2)}{x^2+x-6} & \text{if } x>2 \end{cases}$$

$\lim_{x\to 2^+} \frac{+(x-2)}{x^2+x-6} = \frac{2-2}{0} = \frac{0}{0}$

$\lim_{x\to 2^+} \frac{(x-2)}{(x-2)(x+3)}$

$= \lim_{x\to 2^+} \frac{1}{x+3} = \frac{1}{2+3} = \frac{1}{5}$

$\lim_{x\to 2^-} = \frac{-(x-2)}{x^2+x-6} = \frac{0}{0}$

$\lim_{x\to 2^-} \frac{-(x-2)}{(x-2)(x+3)}$

$= \lim_{x\to 2^-} \frac{-1}{x+3} = \frac{-1}{5}$

$\therefore \lim_{x\to 2} f(x) = D.N.E$
becouse $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$

Rules for calculating limits:-

Th(2): limits rules:

If $\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M$

☆ $\lim_{x\to a} [f(x) \pm g(x)] = L \pm M$

☆ $\lim_{x\to a} kf = k.L$ (were k is a constant)

☆ $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M}$; if $M \neq 0$

☆ $\lim_{x\to a} [f(x)]^{\frac{m}{n}} = [L]^{\frac{m}{n}}$

☆ $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$ $f(x) \leq g(x)$
$\to L \leq M$

2/27

Exer 66: (1.2) (لكل = ∀)

$\lim_{x\to a} f(x) = 4$ and $\lim_{x\to a} g(x) = -2$

Find:-

a) $\lim_{x\to a} (f(x)+g(x)) = 4+(-2) = 2$

b) $\lim_{x\to a} (f(x)\cdot g(x)) = (4)\cdot(-2) = -8$

c) $\lim_{x\to a} 4\cdot g(x) = 4\cdot(-2) = -8$

d) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{4}{-2} = -2$

Ex9:-

a) $\lim_{x\to a} \frac{x^2+x+4}{x^3-2x^2+7} = \frac{a^2+a+4}{a^3-2a^2+7}$

if $a^3-2a^2+7 \neq 0$

b) $\lim_{x\to 2} \sqrt{2x+1} = \sqrt{2(2)+1}$

$= \sqrt{4+1}$

$= \sqrt{5}$

T.h.3:-

① If $p(x)$ is a polynomial; then..
$\lim_{x\to a} p(x) = P(a)$.

② If $p(x)$ and $Q(x)$ are polynomial, and $Q(a)\neq 0$ then.. $\lim_{x\to a} \frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)}$

Exer9: $\lim_{x\to 3} \frac{x+3}{x+6} = \frac{3+3}{3+6} = \frac{6}{9} = \frac{2}{3}$.

The squeeze theorem:

① If $f(x) \le g(x) \le h(x)$

② $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$.

Then.. $\lim_{x\to a} g(x) = L$

Ex10: $3-x^2 \le u(x) \le 3+x^2$. $\forall x \neq 0$

Find $\lim_{x\to 0} u(x)$

$\lim_{x\to 0} 3-x^2 = 3-0 = 3$

$\lim_{x\to 0} 3+x^2 = 3+0 = 3$

$\lim_{x\to 0} 3-x^2 = 3 = \lim_{x\to 0} 3+x^2$

By Squeeze th.; $\lim_{x\to 0} u(x) = 3$.

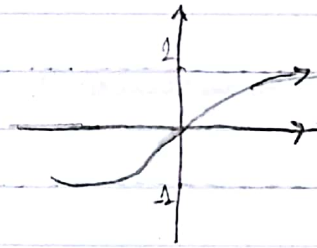
(1.3) limits at infinity and infinit

limits. $\lim_{x\to\pm\infty} =$ $\qquad$ $\lim_{x\to a} = \pm\infty$

Limits at infinity:

forex;

$f(x) = \frac{x}{\sqrt{x^2+1}}$



$\lim_{x\to\infty} f(x) = 1$

$\lim_{x\to\infty} f(x) = -1$

$y=1$ and $y=-1$ are a horizonat asymptote.

Def: If $\lim_{x\to\pm\infty} f(x) = L$, than $y = L$ is a horizontal asymptote

Def.3:-

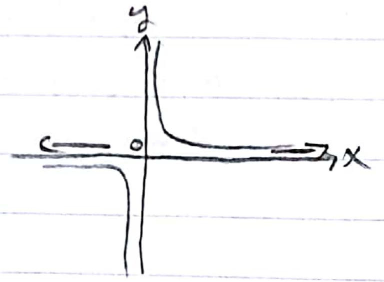
1 $\lim_{x\to\infty} f(x) = L$. ($f$ approaches the limits L as x approaches ∞)

2 $\lim_{x\to-\infty} f(x) = M$. ($f$ approaches the limit M as x approaches $-\infty$)

2/27

Note:

∞ is called infinity, does not represent a real number

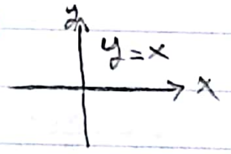


Ex1:

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to-\infty} \frac{1}{x} = 0$

$y=0$ is a H. Asymp. (x - axis)



Note:-

1) $\lim_{x\to\pm\infty} \frac{1}{x^n} = 0$

2) $\frac{\text{عدد}}{\pm\infty} = 0$

$\frac{\text{عدد}}{0} = \pm\infty$ لتحديد الإشارة نعتبره (موجبا)

$\frac{0}{\text{عدد}} = 0$

$\forall n \in Z^+ = 1, 2, 3, \dots$

✱ طريقة إيجاد النهاية $\lim_{x\to\pm\infty} \frac{\text{البسط}}{\text{المقام}}$

1) درجة البسط = درجة المقام ← الناتج $\frac{\text{معامل أكبر أس في البسط}}{\text{معامل أكبر أس في المقام}}$

2) درجة البسط أصغر من درجة المقام ← الناتج zero

3) درجة البسط أكبر من درجة المقام ← الناتج $\pm\infty$

ولتحديد الإشارة نعوض في الحد الذي يحتوي على أكبر أس في البسط و الحد الذي يحتوي على أكبر أس في المقام.

limits at infinty for rational fun.

Ex3: Solution 1

درجة البسط = 2 = درجة المقام

$\lim_{x\to\pm\infty} \frac{2x^2-x+3}{3x^2+5} = \frac{2}{3}$

نقسم جميع الحدود على أكبر أس x^2

$\lim_{x\to\pm\infty} \frac{\frac{2x^2}{x^2} - \frac{x}{x^2} + \frac{3}{x^2}}{\frac{3x^2}{x^2} + \frac{5}{x^2}}$

$\lim_{x\to\pm\infty} \frac{2 - \frac{1}{x} + \frac{3}{x^2}}{3 + \frac{5}{x^2}} = \frac{2 - \frac{1}{\pm\infty} + \frac{3}{(\pm\infty)^2}}{3 + \frac{5}{(\pm\infty)^2}}$

$= \frac{2-0+0}{3+0} = \frac{2}{3}$

Solution 2

درجة البسط = 1، درجة المقام 3

Ex4: $\lim_{x\to\pm\infty} \frac{5x+2}{2x^3-1} = 0$

Solution 2 $\to \lim_{x\to\pm\infty} \frac{\frac{5x}{x^3} + \frac{2}{x^3}}{\frac{2x^3}{x^3} - \frac{1}{x^3}}$

$= \lim_{x\to\pm\infty} \frac{\frac{5}{x^2} + \frac{2}{x^3}}{2 - \frac{1}{x^3}}$

$= \frac{\frac{5}{(\pm\infty)^2} + \frac{2}{(\pm\infty)^3}}{2 - \frac{1}{(\pm\infty)^3}} = \frac{0+0}{2-0} = \frac{0}{2} = 0$

1/29

$\lim_{x\to+\infty} \pm\sqrt{\ } = +\sqrt{\ }$

$\lim_{x\to-\infty} \pm\sqrt{\ } = -\sqrt{\ }$

$\sqrt{x} = |x|$

$(\pm\infty)^{\text{even}} = +\infty$

$(-\infty)^{\text{odd}} = -\infty$

Ex 9: (2.3)

$$\lim_{x\to\infty} \frac{x^3+1}{x^2+1} = \underline{\pm}\infty \to +\infty$$

درجة البسط = 3 > من درجة للمقام 2

لتحديد الإشارة نعوض في أكبر أس موجود في البسط والمقام.

$x\to\infty \; ; \; \frac{x^3}{x^2}$

$\frac{(\infty)^3}{(\infty)^2} = \frac{+}{+} = +$

$$\lim_{x\to-\infty} \frac{3x^3+1}{x^2+1} = \pm\infty \to -\infty$$

$\frac{(-\infty)^3}{(-\infty)^2} = \frac{-}{+} = -$

Ex: $\lim_{x\to\infty} \frac{\sqrt{5x^2+1}}{2x} =$

درجة للمقام = 1 ، درجة البسط = 1

$(\sqrt{x^2} = (x^2)^{\frac{1}{2}} = x^{\frac{2}{2}} = x^1)$

$$\lim_{x\to\infty} \frac{\sqrt{5x^2+1}}{2x} = \frac{\pm\sqrt{5}}{2} = \frac{+\sqrt{5}}{2}.$$

$$\lim_{x\to-\infty} \frac{\sqrt{5x^2+1}}{2x} = \frac{-\sqrt{5}}{2}$$

Ex 2: $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \frac{1}{+\sqrt{1}} = \frac{1}{1} = 1$

(الدرجة = 1 ، الدرجة = 1)

$$\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}} = \frac{1}{-\sqrt{1}} = \frac{1}{-1} = -1$$

Exer 9: $\lim_{x\to-\infty} \frac{2x-1}{\sqrt{3x^2+x+1}} = \frac{2}{-\sqrt{3}}$

(الدرجة 1 ، الدرجة 1)

Ex: $\lim_{x\to\infty} \frac{3x}{\sqrt{4x^2+x}+2x} =$

(الدرجة = 1 ، الدرجة = 1)

$$= \frac{3}{+\sqrt{4}+2} = \frac{3}{2+2} = \frac{3}{4}$$

Ex 5: $\lim_{x\to\infty} \sqrt{x^2+x} - x = \infty - \infty$

$$= \lim_{x\to\infty} \sqrt{x^2+x} - x \cdot \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^2+x}-x)\cdot(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^2+x})^2 - x^2}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x}+x} = \frac{1}{+\sqrt{1}+1}$$

$$= \frac{1}{1+1} = \frac{1}{2}$$

Infinite limits: -

If $p(x)$ is a Polynomial than $\lim_{x\to\pm\infty} p(x) = \pm\infty$.

لتحديد الإشارة نعوض في الحد الذي يحتوي على أكبر أس.

Ex 8:

a) $\lim_{x\to\infty} 3x^3 - x^2 + 2 = \pm\infty = +\infty$

$x\to\infty \; ; \; 3x^3 = 3(\infty)^3 = +\infty$

b) $\lim_{x\to-\infty} 3x^3 - x^2 + 2 = -\infty$

$x\to-\infty \; ; \; 3x^3 = 3(-\infty)^3 = -\infty$

c) $\lim_{x\to\infty} x^4 - 5x^3 - x = \infty$

d) $\lim_{x\to-\infty} x^4 - 5x^3 - x = +\infty$

$(-\infty)^4 = +\infty$

Exer 35 – 37 – 43
47 – 49

2/29

Def:-

✲ If $\lim_{x\to a} f(x) = \pm\infty$

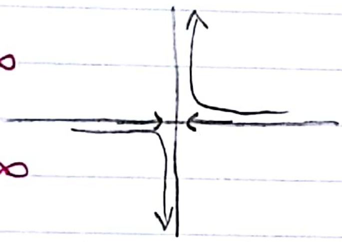
then → $x = a$ is a vertical asymptote خط تقارب عمودي

✲ If $\lim_{x\to\pm\infty} f(x) = L$

then → $y = L$ is a H. asym.

Ex:-

$\lim_{x\to 0^+} \frac{1}{x} = +\infty$

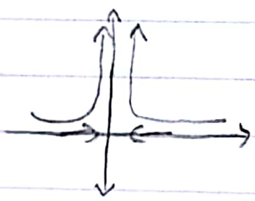


$\lim_{x\to 0^-} \frac{1}{x} = -\infty$

$x = 0$ is a V. asym
(y-axis)

$y = 0$ is a H. asym

$\lim_{x\to 0} \frac{1}{x^2} = +\infty$



$x = 0$ is V. asym

Ex 20:

a) $\lim_{x\to 2} \frac{(x-2)^2}{x^2-4} = \frac{0}{0}$

$\lim_{x\to 2} \frac{\cancel{(x-2)}(x-2)}{\cancel{(x-2)}(x+2)}$

$\lim \frac{x-2}{x+2} = \frac{2-2}{2+2} = \frac{0}{4} = 0$.

b) $\lim_{x\to 2} \frac{x-2}{x^2-4} = \frac{0}{0}$

$\lim_{x\to 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \frac{1}{x+2} = \frac{1}{4}$

c) $\lim_{x\to 2^+} \frac{x-3}{x^2-4} = \frac{2-3}{x^2-4} = \frac{2-3}{4-4} = \frac{-1}{0} = \pm\infty$
$= -\infty$
لتحديد الإشارة

$x \to 2^+$ — 2 2.1

2.1 ; $\frac{x-3}{x^2-4} = \frac{2.1-3}{(2.1)^2-(2)^2} = \frac{-}{+} = -$

d) $\lim_{x\to 2^-} \frac{x-3}{x^2-4} = \frac{2-3}{4-4} = \frac{-1}{0} = \pm\infty$
$= +\infty$

$x \to 2^-$ — 1.9 2

$\frac{1.9-3}{(1.9)^2-(2)^2} = \frac{-}{-} = +$

e) $\lim_{x\to 2} \frac{x-3}{x^2-4} =$ D.N.E.

because $\lim_{x\to 2^+} \frac{x-3}{x^2-4} = -\infty$
$\neq \lim_{x\to 2^-} \frac{x-3}{x^2-4} = +\infty$

Exer 13: $\lim_{x\to 3^-} \frac{1}{3-x} = \frac{1}{3-3} = \frac{1}{0} = \pm\infty$
$= +\infty$

2.9 3 $\frac{1}{3-2.9} = \frac{+}{+} = +$

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2/2

ـه لازم النقطة تكون داخلة في domain

Ex 10,

f) $\lim\limits_{x\to 2} \frac{2-x}{(x-2)^3} = \frac{2-2}{(2-2)^3} = \frac{0}{0}$

$\lim\limits_{x\to 2} \frac{-(x-2)}{(x-2)^{3}{}_{2}} = \lim\limits_{x\to 2} \frac{-1}{(x-2)^2}$

$= \frac{-1}{0} = \pm\infty$
$= -\infty$

$\lim\limits_{x\to 2^+} \frac{2-x}{(x-2)^3} = \pm\infty = \left[\frac{2-2.1}{(2.1-2)^3} = \frac{-}{(+)^3} = -\right.$

$\lim\limits_{x\to 2} \frac{2-x}{(x-2)^3} = \pm\infty = -\infty \left[\frac{2-1.9}{(1.9-2)^3} = \frac{+}{(-)^3} = -\right.$

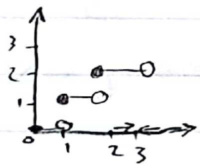
Exer 35 : 1

Exer 37 : 1

Exer 43 : -1

Exer 47 :

$\lim\limits_{x\to 3^+} \lfloor x \rfloor = 3$



Exer 49 :

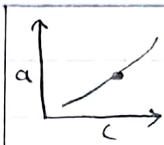
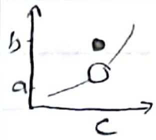
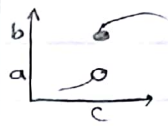
$\lim\limits_{x\to 3} \lfloor x \rfloor = D.N.E$

$\lim\limits_{x\to 3^+} \lfloor x \rfloor = 3$

$\lim\limits_{x\to 3^-} \lfloor x \rfloor = 2 \neq$

1.4 : Continuity --

Continuity at point : -

		
f is cont. at c	f is dis cont at c	f is discont at c
1 $f(c) = a$	1 $f(c) = b$	1 $f(c) = b$
2 $\lim\limits_{x\to c} f(x) = a$	2 $\lim\limits_{x\to c} f(x) = a$	2 $\lim\limits_{x\to c} f(x) = D.N.E$
3 $1 = 2$	3 $1 \neq 2$	$\lim\limits_{x\to c^+} f(x) = b \neq$ $\lim\limits_{x\to c^-} f(x) = a$
f is cont at c	f is discont at c.	☆ f is discont at c. because $\lim\limits_{x\to c} f(x) = D.N.E$

☆ f is right cont. at c because $\lim\limits_{x\to c^+} f(x) = f(c) = b$.

☆ f is not left cont. at c because $\lim\limits_{x\to c^-} f(x) \neq f(c)$

Def: 4 :-

☆ f is continuous at an interior point c of its domain if $\lim\limits_{x\to c} f(x) = f(c)$

داخلي

☆ If either $\lim\limits_{x\to c} f(x) = D.N.E$ or $\lim\limits_{x\to c} f(x) \neq f(c)$, then we will say that f is discontinuous at c.

Note : f is cont. at c if

لواقحة

1 $f(c)$ defined

2 $\lim\limits_{x\to c} f(x)$ exist $\left[\because \lim\limits_{x\to c^+} f = \lim\limits_{x\to c^-} f\right]$

3 $1 = 2$

2/2

Def 5:-

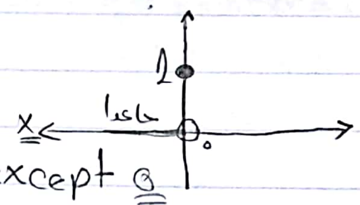
① f is right cont. at c if
$$\lim_{x \to c^+} f(x) = f(c)$$

② f is left cont. at c if
$$\lim_{x \to c^-} f(x) = f(c)$$

Th. 5:
f is cont. at c if f it's both right cont. and left cont. at c.

Ex 1:
The Heaviside function
$$H(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

H is cont. at every number



except 0

① $H(0) = 1$

② $\lim_{x \to 0} H(x) = D.N.E$

$$\lim_{x \to 0^+} H(x) = 1$$
$$\neq$$
$$\lim_{x \to 0^-} H(x) = 0$$

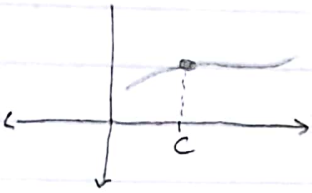
* H is discont. at 0 becauce $\lim_{x \to 0} H(x) = D.N.E$

* H is right cont. at 0 becauce $\lim_{x \to 0^+} H(x) = 1 = H(0)$

* H is not left cont. at 0 becauce $\lim_{x \to 0^-} H(x) \neq H(0)$

2/22

Case (1): Interior point. (1.4)



f is cont. at c because left cont. and right cont at c

Case (2): End point.

a) left end point

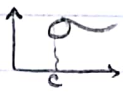
f is cont (right). at c.

f is cont. at c

---

f is not right cont. at c

f is dis cont. at c.



b) right point end.

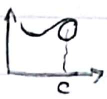
f is left cont at c.

f is cont. at c.

---

f is not left cont at c.

f is dis cont. at. c



Def. 6:

1. f is cont. at a left end point c of it's domian if it's right cont there.
2. f is cont. at right end point c of it's domain if it's left cont. there.

Ex. 2: $f(x) = \sqrt{4-x^2}$, $D_f = [-2,2]$

case 1: If $-2 < c < 2$

f is cont. at c become

$\lim_{x \to c} \sqrt{4-x^2} = \sqrt{4-c^2} = f(c)$

f is cont. at every interior point of it's domain.

case 2: If $c = -2$ (left end point)

1. $f(-2) = \sqrt{4-4} = \sqrt{0} = 0$
2. $\lim_{x \to -2^+} \sqrt{4-x^2} = \sqrt{4-4} = \sqrt{0} = 0$
3. ① = ②

f is cont. at left end point -2.

case 3: If $c = 2$ (right end point)

1. $f(2) = \sqrt{4-4} = 0$
2. $\lim_{x \to 2^-} \sqrt{4-x^2} = \sqrt{4-4} = \sqrt{0} = 0$
3. ① = ②

f is cont. at right end point 2

From case ①, ②, and ③; f is cont at every number of it's domain $[-2,2]$

continuity on an Interior.

Def. 7:

f is cont. on the interior I if it's cont at each point of I. (i.e; f is cont. at every point of it's domain.)

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2/12

Ex.3: $f(x)=\sqrt{x}$

$D_f=[0,\infty)$

① f is cont. at 0 because

$\lim_{x\to 0^+}\sqrt{x}=\sqrt{0}=0=f(0)$

② f is cont at $c>0$

because $\lim_{x\to c}\sqrt{x}=\sqrt{c}=f(c)$

From ①, ②;

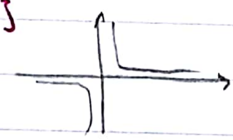
f is cont. at every point of the domain $[0,\infty)$

Ex.4: $g(x)=\frac{1}{x}$

$D_g=(-\infty,0)\cup(0,\infty)=\mathbb{R}-\{0\}$

① g is discont. at 0.

② g is cont. on the domain $\mathbb{R}-\{0\}\to(-\infty,0)\cup(0,\infty)$



Ex.5: The greatest integer fun.

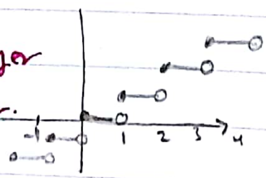
$f(x)=\lfloor x\rfloor$

① f is right cont. on every integer

② f is not left cont on every integer.

From ① and ②:

f is discont. on every integer. $[0,1),[1,2),[2,3)$



* f is cont. on every interval $[n,n+1)$.

Example of cont. fun:

1. polynomial → cont. on the domain $\mathbb{R}$.
2. Rational fun → cont. on the domain $\mathbb{R}-\{\text{بسط},\text{مقام}\}$ → (discont, at بسط, مقام)
3. Rational power ($x^{\frac{m}{n}}=\sqrt[n]{x^m}$
4. Root fun. $\sqrt[n]{f(x)}$

n: odd → cont. on $\mathbb{R}$

n: even → cont. on $f(x)\geq 0$

5. sin, cos, tan, sec, cot, csc cont, on the domain $\mathbb{R}$
6. The absolute value fun. $|x|$ cont. on the domain $\mathbb{R}$

Th. 6+7: If f and g are cont. at c, then

① $f\pm g$ is cont. at c.

② kf is cont. at c. (k is any number)

③ $\frac{f}{g}$ is cont. at c. (if $g(c)\neq 0$)

④ $\sqrt[n]{f(x)}$ is cont. at c.

⑤ $f\circ g$ is cont. at c.

$[\lim_{x\to c}f(g(x))=f(\lim_{x\to c}g(x))=f(g(c))$

Ex. 6:

a) $3x^2-2x$ is cont. on the domain $\mathbb{R}$.

b) $\frac{x-2}{x^2-4}$ is cont. on $\mathbb{R}-\{2,-2\}$. (dis cont at $-2,2$)

c) $|x^2-1|$ is cont. on the domain $\mathbb{R}$.

d) $\sqrt{x}$ is cont. on the domain $[0,\infty)$.

2/14

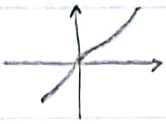
1.4) Exer. 7 : $f(x) = \begin{cases} x & \text{if } x<0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

$\boxed{1}$ $f(0) = 0^2 = 0$

$\boxed{2}$ $\lim_{x \to 0} f(x) =$

$\lim_{x\to 0^+} f(x) = \lim_{x \to 0^+} x^2 = 0^2 = 0$

$\lim_{x \to 0^-} f(x) = \lim_{x\to 0^-} x = 0$



$\boxed{3}$ ① = ②

f is cont. at 0

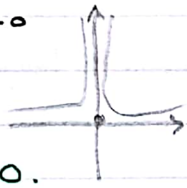
we know that; and x^2 are cont. fun

Therefore; f is cont. on the domain $\mathbb{R}$.

Exer. 9 : $f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

① $f(0) = 0$

② $\lim_{x\to 0} f(x) = \lim_{x \to 0} \frac{1}{x^2} = \frac{1}{0} = \infty$



$\boxed{3}$ ① ≠ ②, f is discont. at 0.

∴ f is cont every were except at $x=0$

Exer. 18 : Find m so that

$g(x) = \begin{cases} x-m & \text{if } x<3 \\ 1-mx & \text{if } x \geq 3 \end{cases}$

is cont g is cont. at 3

$\to \lim_{x\to 3} g(x) = g(3)$

$\to \lim_{x \to 3} g(x)$ exist

$\to \lim_{x\to 3^+} g(x) = \lim_{x \to 3^-} g(x)$

$\to \lim_{x\to 3^+} (1-mx) = \lim_{x\to 3^-} (x-m)$

$\to 1-3m = 3-m$

$\to m - 3m = 3-1$

$\to -2m = 2$

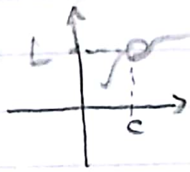
$\to m = -1$

continuous Extension:

If ① $f(c)$ is undefined

② $\lim_{x\to c} f(x) = L$

then f is not cont. at c.



But: we can define a new fun $F(x)$ by

$F(x) = \begin{cases} f(x) & x \in D_f \\ L & x = c \end{cases}$; and F is cont at c

Def: F is called a continuous extension of f to c

Exer. 14: Find the cont. extension of the fun. $f(x) = \frac{1+x^3}{1-x^2}$ at $x=-1$

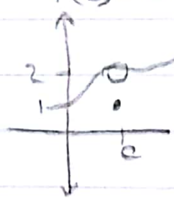
① $f(-1) = \frac{1+(-1)^3}{1-(-1)^2} = \frac{1-1}{1-1} = \frac{0}{0}$

② $\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{1+x^3}{1-x^2}$

$= \lim_{x\to -1} \frac{(1+x)(1-x+x^2)}{(1+x)(1-x)}$

$= \lim_{x\to -1} \frac{1-x+x^2}{1-x} = \frac{1-(-1)+(-1)^2}{1-(-1)} = \frac{3}{2}$

$f(c) = 2$



$\boxed{3}$ The cont. exten is

$F(x) = \begin{cases} \frac{1+x^3}{1-x^2} & \text{if } x \neq -1 \\ \frac{3}{2} & \text{if } x = -1 \end{cases}$

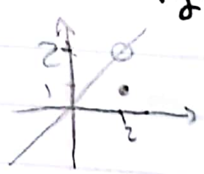
Removable Discontinuities:

If ① $F(x) = \begin{cases} f(x) & \text{if } x \neq c \\ a & \text{if } x = c \end{cases}$

② f(c) can be redfind then we can redefine .. $F(x) = f(x)\ \forall x$

Def..: f has a removeble dis continuity.

Ex. 8 : $g(x) = \begin{cases} x & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$



$g(2) = 1$, $\lim_{x \to 2} g(x) = 2$

g has a removal dis To remove it redefind $g(2) = 2$

Then ; $g(x) = x\ \forall x$.

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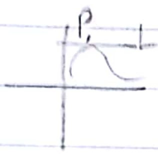
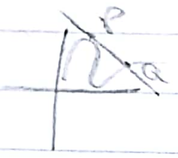
2/14

2.1 Tangent lines and their slopes:

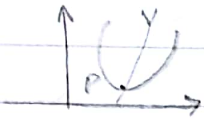
Def: A tangent line is a straight line that touches a function at only one point.

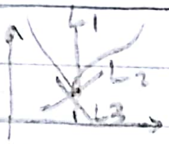
Example:





L is tangent line. | L is not tangent line





L is not tangent line | L_1, L_2, and L_3 are not tanget line.

Def: 1+2: The slop of tangent, through $P(x_0, y_0)$ is given by Newton quotient ← m =

$$\lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$$

m

m = any number	$m = \pm\infty$	m = D.N.E
tangent line	$x = x_0$	tangent line
$y = m(x-x_0)+y_0$	vertical line	D.N.E
Nonvertical.		

Ex. 1: Find an equation of the tangent line to the curve $y = x^2$ at the point $(1,1)$

$$y = m(x - x_0) + y_0$$

$$(1,1) = (x_0, y_0)$$

$$f(x) = x^2$$

$$m = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$$

$$= \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \frac{(1+h)^2-1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{1}+2h+h^2\cancel{-1}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 2+h = 2$$

$$\boxed{m = 2}$$

The eqn. of the tanget line at $(1,1)$ is

$$y = 2(x-1)+1$$

$$y = 2x - 2 + 1$$

$$y = 2x - 1$$

(2.1): Ex.4: Does the graph of $y=|x|$ have tangent line at $x=0$??

$$m=\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$$

$$=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{|h|-|0|}{h}$$

$$=\lim_{h\to 0}\frac{|h|}{h}$$

$$m=\lim_{h\to 0}\frac{|h|}{h}=\begin{cases}\lim_{h\to 0^+}\frac{+h}{h}=\lim_{h\to 0^+}1=1\\ \lim_{h\to 0^-}\frac{-h}{h}=\lim_{h\to 0}-1=-1\end{cases}$$

$m=$ D.N.E.

$y=|x|$ has not tangent line at $x=0$.

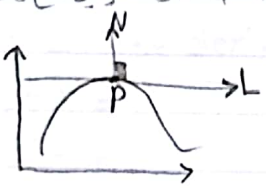
Def.3:

Slope of a curve at a point p

Slope of a tangent line at p.

Normal:

هو الخط العامودي على المماس



Slope of normal $=\dfrac{-1}{\text{Slope of tangent}}$

$$m_N=\frac{-1}{m_t}$$

Ex.6: Find an equation of the normal to $y=x^2$ at $(1,1)$

$$y=m_N(x-x_0)+y_0$$

From Example 1; $m_t=2 \to m_N=\frac{-1}{2}$

The equ. of normal line at $(1,1)$

$$y=-\frac{1}{2}(x-1)+1$$

$$y=-\frac{1}{2}x+\frac{1}{2}+1$$

$$y=-\frac{1}{2}x+\frac{3}{2}$$

Ex7: we will explain at 2.3.

(2.2): The Derivative

Def.:-

The derivative of a fun. f is a fun. f' defined by $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ for all x.

Note:

1) The slope of the tanget line at point = the derivative of $f(x)$ at p.

2) Equation of the tanget line to= $y=f(x)$ at $(x_0, f(x_0))$ is

$$y=m(x-x_0)+y_0$$

$$y=f'(x_0)(x-x_0)+f(x_0)$$

3) If $f'(x_0)$ does not exist (f is not defferent iable at x_0) then x_0 is called singular point.

$f'(x_0)$ D.N.E $=$ m D.N.E

No tangent line.

2/16

$a^{-1} = \frac{1}{a}$

$\boxed{4}$ Right derivative: $f'_{+}(x) = \lim_{h \to 0^+} \frac{f(x+h)-f(x)}{h}$

left derivative: $f'_{-}(x) = \lim_{h \to 0^-} \frac{f(x+h)-f(x)}{h}$

$\boxed{5}$ The value of the derivative of f at x_0

$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$

$= \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$

Ex. 1: Show that if $f(x)=ax+b$ then $f'(x)=a$

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h \to 0} \frac{[a(x+h)+b]-[ax+b]}{h}$

$= \lim_{h \to 0} \frac{\cancel{ax}+ah+\cancel{b}-\cancel{ax}-\cancel{b}}{h}$

$= \lim_{h \to 0} \frac{a\cancel{h}}{\cancel{h}}$

$= \lim_{h \to 0} a = a$

$f'(x) = a$

Ex. 2: The derivative formule of $f(x)$:-

a) $f(x) = x^2$

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

Some Important Derivative

$f(x)$	$f'(x)$
$f(x) = c$, c: constant	0
$f(x) = x$	1
$f(x) = x^2$	$2x$
$f(x) = x^r$	rx^{r-1}
$f(x) = \frac{1}{x} = x^{-1}$	$-x^{-1-1} = -x^{-2} = \frac{-1}{x^2}$
$f(x) = \sqrt{x} = x^{\frac{1}{2}}$	$\frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

Ex. 3:

a) $f(x) = x^{\frac{5}{3}}$

$f'(x) = \frac{5}{3}x^{\frac{5}{3}-1} = \frac{5}{3}x^{\frac{2}{3}}$

b) $g(t) = \frac{1}{\sqrt{t}} = \frac{1}{t^{\frac{1}{2}}} = t^{-\frac{1}{2}}$

$g' = -\frac{1}{2}t^{-\frac{1}{2}-1} = -\frac{1}{2}t^{-\frac{3}{2}} = \frac{-1}{2t^{\frac{3}{2}}}$

$= \frac{-1}{2(\sqrt{t})^3} = \frac{-1}{2\sqrt{t^3}}$, $\frac{3}{2} = 3\frac{1}{2}$

Leibniz Notation:

If $y = f(x)$

$D_x y = y' = \frac{dy}{dx} = \frac{d}{dx} f(x)$

$= f'(x) = D_x f(x) = D f(x)$

Ex: $\frac{d}{dx} x^4 \Big|_{x=-1} = 4x^3 \Big|_{x=-1} = 4(-1)^3 = -4$

2/19

Th.1: If f is differentiable at x Then f is cont. at a. (العكس غير صحيح)

Sums and Constant Multiples:-

Th.2:

$(f \pm g)'(x) = f' \pm g'$

$(cf)'(x) = c.f'(x)$

Note:

In general;

$(f_1+f_2+\dots+f_n)' = f_1'+f_2'+\dots+f_n'$

Ex.1:

a) $\frac{d}{dx}(2x^3-5x^2+4x+7)$

$=2.(3x^2)-5.(2x)+4(1)+0$

$=6x^2-10x+4$

(2.3): Differentiation Rules:

b) $f(x)=5\sqrt{x}+\overbrace{\frac{3}{x}}^{3x^{-1}}-18$

$f'=5.(\frac{1}{2\sqrt{x}})+3.(\frac{-1}{x^2})-0$

$=\frac{5}{2\sqrt{x}}-\frac{3}{x^2}$

c) $y=\frac{1}{7}t^4-3t^{\frac{7}{3}}$

$y'=\frac{4}{7}t^3-3.(\frac{7}{3}t^{\frac{7}{3}-1})$

$=\frac{4}{7}t^3-7t^{\frac{4}{3}}$

$\frac{7}{3}-1=\frac{7-3}{3}=\frac{4}{3}$

Exer.7: $g(t)=t^{\frac{1}{3}}+2t^{\frac{1}{4}}+3t^{\frac{1}{5}}$ (H.W)

Exer.9: $u=\frac{3}{5}x^{\frac{5}{3}}-\frac{5}{3}.x^{\frac{-3}{5}}$

$u'=\frac{3}{5}(\frac{5}{3}.x^{\frac{5}{3}-1})-\frac{5}{3}(-\frac{3}{5}x^{-\frac{3}{5}-1})$

$u'=x^{\frac{2}{3}}+x^{-\frac{8}{5}}$

The product Rule:

Th.3:

$(f.g)'(x)=f'.g+f.g'$

Ex.3: $\frac{d}{dx}[(x^2+1).(x^3+4)]$

$=f'.g+f.g'$

$=(2x).(x^3+4)+(x^2+1).(3x^2)$

$=2x^4+8x+3x^4+3x^2$

$=5x^4+3x^2+8x$

$f=x^2+1;\ f'=2x;\ g=x^3+4;\ g'=3x^2$

Ex.4: $y=(2\sqrt{x}+\frac{3}{x}).(3\sqrt{x}-\frac{2}{x})$.

$y=6(\underbrace{\sqrt{x}}_{x^{\frac12}}.\underbrace{\sqrt{x}}_{x^{\frac12}})-4.x^{\frac{1}{2}}.x^{-1}+9x^{\frac{1}{2}}.x^{-1}-6.x^{-2}$

$y=6.x-4x^{-\frac{1}{2}}+9x^{-\frac{1}{2}}-6x^{-2}$

$y=6x+5x^{-\frac{1}{2}}-6x^{-2}$

$y'=6+5.(-\frac{1}{2}x^{-\frac{1}{2}-1})-6(-2x^{-2-1})$

$=6-\frac{5}{2}x^{-\frac{3}{2}}+12x^{-3}$

Note:

$(f_1 f_2 \dots f_n)'=f_1'.f_2\dots f_n+f_1.f_2'.f_3\dots f_n+\dots+f_1.f_2\dots f_{n-1}.f_n'$.

2/19

The Reciprocal Rule:

Th. 4: $\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}$

Ex. 7:

a) $\frac{d}{dx}\left(\frac{1}{x^2+1}\right)$

$= \frac{-(2x)}{(x^2+1)^2}$

Note: $\frac{d}{dx}(x^{-n}) = -n x^{-n-1}$

Ex. 8: $\frac{d}{dx}\left(\frac{x^2+x+1}{x^3}\right)$

$= \frac{d}{dx}\left(\frac{x^2}{x^3} + \frac{x}{x^3} + \frac{1}{x^3}\right)$

$= \frac{d}{dx}\left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}\right)$

$= \frac{d}{dx}\left(x^{-1} + x^{-2} + x^{-3}\right)$

$= -x^{-2} - 2x^{-3} - 3x^{-4}$

$= \frac{-1}{x^2} - \frac{2}{x^3} - \frac{3}{x^4}$.

The Quotient Rule:

Th. 5: $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{(g^2)}$

★ (مقام البسط ← مقام)

★ (مقام المقام ← بسط)

Ex. 9: (a) $y = \frac{1-x^2}{1+x^2}$ $\begin{matrix} f \\ g \end{matrix}$

$f = 1-x^2$

$f' = -2x$

$g = 1+x^2$

$g' = 2x$

$y' = \frac{f' \cdot g - f \cdot g'}{(g)^2}$

$y' = \frac{(-2x)\cdot(1+x^2) - (1-x^2)\cdot(2x)}{(1+x^2)^2}$

$y' = \frac{-2x-2x^3-[2x-2x^3]}{(1+x^2)^2}$

$y' = \frac{-2x-\cancel{2x^3}-2x+\cancel{2x^3}}{(1+x^2)^2}$

$y' = \frac{-4x}{(1+x^2)^2}$

(c) $f(\theta) = \frac{a+b\theta}{m+n\theta}$

$f_1 = a+b\theta$

$f_1' = b$

$f_2 = m+n\theta$

$f_2' = n$

$f' = \frac{f_1' \cdot f_2 - f_1 \cdot f_2'}{(f_2)^2}$

$f' = \frac{b(m+n\theta) - (a+b\theta)\cdot(n)}{(m+n\theta)^2}$

$f' = \frac{bm + \cancel{bn\theta} - an - \cancel{bn\theta}}{(m+n\theta)^2}$

$f' = \frac{+bm - an}{(m+n\theta)^2}$

b) $\frac{d}{dt}\left(\frac{\sqrt{t}}{3-5t}\right)$

$f = \sqrt{t}$

$f' = \frac{1}{2\sqrt{t}}$

$g = 3-5t$

$g' = -5$

$= \frac{f' \cdot g - f \cdot g'}{g^2}$

$= \frac{\left(\frac{1}{2\sqrt{t}}\right)\cdot(3-5t) - (\sqrt{t})\cdot(-5)}{(3-5t)^2}$

$= \frac{\frac{3-5t}{2\sqrt{t}} + 5\sqrt{t}\,\frac{\times 2\sqrt{t}}{2\sqrt{t}}}{(3-5t)^2}$

$= \frac{\frac{3-5t+10t}{2\sqrt{t}}}{(3-5t)^2}$

$= \frac{3+5t}{2\sqrt{t}(3-5t)^2}$

2/21

Ex.7:

b) $f(t) = \frac{1}{t+\frac{1}{t}}$

$f(t) = \frac{1}{\frac{t^2+1}{t}} = \frac{t}{t^2+1}\ \frac{f_1}{f_2}$

$f_1 = t,\ f_1' = 1,\ f_2 = t^2+1,\ f_2' = 2t$

$f'(t) = \frac{f_1' \cdot f_2 - f_1 \cdot f_2'}{f_2^2} = \frac{(1)\cdot(t^2+1) - t\cdot(2t)}{(t^2+1)^2}$

$f' = \frac{t^2+1-2t^2}{(t^2+1)^2}$

$f' = \frac{-t^2+1}{(t^2+1)^2} = \frac{1-t^2}{(t^2+1)^2}$

Exer.42: Find equ. of the tangent and normal to $y = \frac{x+1}{x-1}\ \frac{f}{g}$ at $x = 2$

$f = x+1,\ f' = 1,\ g = x-1,\ g' = 1$

$y = m(x-x_0) + y_0$

$(x_0, y_0) \to x_0 = 2$

$(2,3) \to y_0 = \frac{2+1}{2-1} = \frac{3}{1} = 3$

$m = y' = \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{x\cancel{-1}-x\cancel{-1}}{(x-1)^2}$

$m = y' = \frac{-2}{(x-1)^2}$

$m\Big|_{x=2} = y'\Big|_{x=2} = \frac{-2}{(2-1)^2} = \frac{-2}{1} = -2 \quad (m_t = -2)$

The equ of tangent line at $x=2$ is

$y = -2(x-2)+3$

$y = -2x+7$

Normal: $m_N = \frac{-1}{m_t} = \frac{-1}{-2} = \frac{1}{2}$

The equ. of normal line at $x=2$ is

$y = \frac{1}{2}(x-2)+3$

$y = \frac{1}{2}x+2$

Ex.7. (in section 2.1)

Find equ. of tangent line and normal line to the curve $y = \sqrt{x}$ at $(4,2)$. $x_0\ y_0$

$y = m(x-x_0)+y_0$

$m\Big|_{x=4} = y'\Big|_{x=4} = \frac{1}{2\sqrt{x}}\Big|_{x=4} = \frac{1}{2(2)} = \frac{1}{4}$

$m_t = \frac{1}{4}$

The equ. of the tangent line at $x=4$

$y = \frac{1}{4}(x-4)+2$

$y = \frac{1}{4}x+1$

Normal: $m_N = \frac{-1}{m_t} = \frac{-1}{\frac{1}{4}} = -4$

The equ. of the normal line at $x=4$

$y = -4(x-4)+2$

$y = -4x+18$

2/21

Ex.2: (2.4). Find the derivative of $y=\sqrt{x^2+1}$

$$y' = \frac{1}{2\sqrt{x^2+1}} \cdot (2x) = \frac{x}{\sqrt{x^2+1}}$$

Ex.1: $\frac{d}{dx}\left(\frac{1}{x^2-4}\right) =$

$$= \frac{-1}{(x^2-4)^2} \cdot (2x)$$

$$= \frac{-2x}{(x^2-4)^2}$$

$$\frac{d}{dx}(x^2-4)^{-1} = -1(x^2-4)^{-1-1} \cdot (2x)$$

$$= (-2x) \cdot (x^2-4)^{-2} = \frac{-2x}{(x^2-4)^2}$$

Note: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

Th: The chain Rule (page 58)

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$

Ex.3: (a) $\frac{d}{dx}(7x-3)^{10} =$

$$= 10(7x-3)^9 \cdot (7)$$
$$= 70(7x-3)^9.$$

Ex.: $\frac{d}{dx}(x^2-3)^{10}\Big|_{x=2} =$

$$= 10(x^2-3)^9 \cdot (2x)\Big|_{x=2}$$

$$= 20x \cdot (x^2-3)^9\Big|_{x=2}$$

$$= 20(2) \cdot (4-3)^9 = 40 \cdot (1) = 40.$$

Ex.5: Find f'. $f(t) = \frac{t^2+1}{\sqrt{t^2+2}}$ $\begin{matrix} h \\ g \end{matrix}$

$h = t^2+1$, $h' = 2t$, $g = \sqrt{t^2+2}$, $g' = \frac{1}{2\sqrt{t^2+2}}$

$$g' = \frac{1}{2\sqrt{t^2+2}} \cdot (2t) = \frac{t}{\sqrt{t^2+2}}$$

$$f' = \frac{h' \cdot g - h \cdot g'}{g^2}$$

$$f' = \frac{(2t) \cdot (\sqrt{t^2+2}) - (t^2+1) \cdot \left(\frac{t}{\sqrt{t^2+2}}\right)}{(\sqrt{t^2+2})^2}$$

$$= \frac{2t \cdot \sqrt{t^2+2} - \frac{t^3+t}{\sqrt{t^2+2}}}{t^2+2}$$

$$f' = \frac{\frac{2t \cdot \sqrt{t^2+2} \cdot \sqrt{t^2+2} - t^3 - t}{\sqrt{t^2+2}}}{t^2+2}$$

$$f' = \frac{2t \cdot (t^2+2) - t^3 - t}{\sqrt{t^2+2}^{\frac{1}{2}} \cdot (t^2+2)^1}$$

$$= \frac{2t^3+4t-t^3-t}{(t^2+2)^{\frac{3}{2}}} = \frac{t^3+3t}{(t^2+2)^{\frac{3}{2}}}$$

$$\frac{d}{dx}(f(x))^n = n(f(x))^{n-1} \cdot f'(x)$$

$$\frac{d}{dx}\left(\frac{1}{f}\right) = \frac{-1}{f^2} \cdot f'$$

$$\frac{d}{dx}(\sqrt{f(x)}) = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$$

$$\frac{d}{dx}|f| = \frac{f}{|f|} \cdot f'$$

Exer.3: $\frac{d}{dx}(\sqrt{3t-7})\Big|_{t=3}$

$$= \frac{3}{2\sqrt{3t-7}}\Bigg|_{t=3} = \frac{3}{2 \cdot \sqrt{2}}$$

2/28   تفاضل ال(x) = صفر ← مثال $\left[\cos(30)^\circ\right] = 0$

Exer 36: Find an equ. of the tangent line to $y=\sqrt{1+2x^2}$ at $x=2$

$y = m(x-x_0)+y_0$

$x_0 = 2 \ ; \ y_0 = \sqrt{1+8} = 3$

$m\Big|_{x=2} = y'\Big|_{x=2} = \dfrac{1}{2\sqrt{1+2x^2}} \cdot \cancel{4}^{2}x \Big|_{x=2}$

$= \dfrac{2x}{\sqrt{1+2x^2}}\Big|_{x=2} = \dfrac{4}{3}$

The equ. of the tangent line at $x=2$

$y = \frac{4}{3}(x-2)+3$

$y = \frac{4}{3}x - \frac{8}{3} + 3$

$y = \frac{4}{3}x \ \frac{-8+9}{3}$

$y = \frac{4}{3}x + \frac{1}{3}$

(2.5) Derivative. of Thrigonoretric fun.

Some special limits

Th.7:

$\lim_{\theta\to 0} \sin\theta = \sin 0 = 0$

$\lim_{\theta\to 0} \cos\theta = \cos 0 = 1$

$\lim_{\theta\to 0} \tan\theta = \tan 0 = 0$

Th.8: $\lim_{\theta\to 0} \dfrac{\sin 2\theta}{3\theta} = \dfrac{2}{3}$

$\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$

Note:

$\boxed{1}$ $\lim_{\theta\to 0} \dfrac{\sin m\theta}{n\theta} = \dfrac{m}{n}$

$\boxed{2}$ $\lim_{\theta\to 0} \dfrac{n\theta}{\sin m\theta} = \dfrac{n}{m}$

$\boxed{3}$ $\lim_{\theta\to 0} \dfrac{\tan m\theta}{n\theta} = \dfrac{m}{n}$

$\boxed{4}$ $\lim_{\theta\to 0} \dfrac{n\theta}{\tan m\theta} = \dfrac{n}{m}$

$\boxed{5}$ $\lim_{\theta\to 0} \dfrac{(\cos\theta)-1}{\theta} = 0$

Ex.: Find

$\lim_{x\to 0} \dfrac{\sin 2x}{x} = \dfrac{2}{1} = 2$

Exer (53):

$\lim_{x\to 0} \dfrac{\tan 2x}{x} = \dfrac{2}{1} = 2$

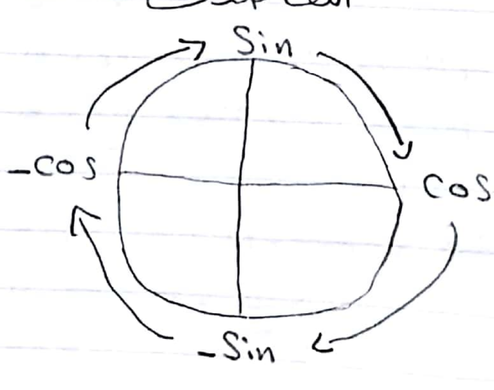
The Derivatives of sin and cos.

The. 9 + Th. 10:

$\dfrac{d}{dx}\sin x = \cos x$

$\dfrac{d}{dx}\cos x = -\sin x$

التفاضلات



2/28

Ex.2:

(a) $\frac{d}{dx}(\sin \pi x + \cos 3x)$

$= \cos(\pi x) \cdot \pi + (-\sin(3x) \cdot 3)$

$= \pi \cdot \cos(\pi x) - 3\sin(3x)$

(b) $\frac{d}{dx}(\underset{f_1}{x^2} \cdot \underset{f_2}{\sin\sqrt{x}})$

$f_1' = 2x \quad , \quad f_2' = \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

$= 2x \cdot \sin\sqrt{x} + x^2 \cdot \frac{1}{2\sqrt{x}} \cos\sqrt{x}$

$= 2x \sin\sqrt{x} + \frac{1}{2} x^{2-\frac{1}{2}} \cdot \cos\sqrt{x}$

$= 2x \sin\sqrt{x} + \frac{1}{2} x^{\frac{3}{2}} \cdot \cos\sqrt{x}$

(c) $\frac{d}{dx}\left(\frac{\cos x}{1-\sin x}\right) =$

$= \frac{-\sin x \cdot (1-\sin x) - \cos \cdot (-\cos x)}{(1-\sin x)^2}$

$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$

$= \frac{-\sin x + 1}{(1-\sin x)^2} = \frac{1-\sin x}{(1-\sin x)^2} = \frac{1}{1-\sin x}$

Ex 3: $f(t) = \sin t \cos t$.

$f' = \cos t \cdot \cos t + \sin t \cdot (-\sin t)$

$= \cos^2 t - \sin^2 t = \cos 2t$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$.

tan	sec	sec

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\csc x) = -\csc x \cdot \cot x$

cot	-csc	csc

Ex5:

(a) $\frac{d}{dx}(3x + \cot(\overset{\frac{1}{2}\cdot x}{\frac{x}{2}})) =$

$= 3 + (-\csc(\frac{x}{2})) \cdot \frac{1}{2}$

$= 3 - \frac{1}{2}\csc^2(\frac{x}{2})$.

(b) $\frac{d}{dx}\left(\frac{3}{\sin 2x}\right) =$

$= \frac{d}{dx}(3 \cdot \frac{1}{\sin 2x})$

$= \frac{d}{dx}(3 \cdot \csc 2x)$

$= 3(-\csc 2x \cdot \cot 2x) \cdot 2$

$= -6 \csc 2x \cdot \cot 2x$.

Exer.17: $y = \sin^3(\frac{\pi}{2}\cancel{x})$

$= (\sin(\frac{\pi}{2}x))^3$

$y' = 3(\sin(\frac{\pi}{2}x))^2 \cdot \cos(\frac{\pi}{2}x) \cdot \frac{\pi}{2}$

$y' = \frac{3\pi}{2} \cdot \sin^2(\frac{\pi}{2}x) \cdot \cos(\frac{\pi}{2}x)$

Exer.26: $y = \tan 3x \cdot \cot 3x$

$y = \tan \cancel{3x} \cdot \frac{1}{\tan \cancel{3x}} = 1$

$y = 0$

2/28

Exer.(35):

$u = \sin(\cos(\tan t))$

$u' = \cos(\cos(\tan)).$

$(-\sin(\tan t)).$

$\sec^2 t$

$= -\cos(\cos(\tan t)).\sin(\tan t).\sec^2 t$

(2.6) $y = f(x)$

$y' = f'$

$y'' = f''$

$y''' = f''' = y^{(3)}$

$y^{(4)} \ldots y^{(n)}$

Ex.2: $y = x^3$

$y' = 3x^2$

$y'' = 6x$

$y^{(3)} = 6$

$y^{(4)} = 0$

$\vdots$

$y^{(n)} = 0$

Ex. Find $y^{(3)}$ if

$y = 3x^4 - x^3 + 2x - 15$

$y' = 12x^3 - 3x^2 + 2$

$y'' = 36x^2 - 6x$

$y^{(3)} = 72x - 6$

$y = x \,.\sin x.$

$y' = (1).\sin x + x.\cos x$

$y' = \sin x + x.\cos x.$

$y'' = \cos x + \cos x + x.(-\sin x)$

$y'' = 2\cos x - x\sin x.$

$y^{(3)} = -2\sin x - [\sin x + x.\cos x]$

$y^{(3)} = -2\sin x - \sin x - x.\cos x$

$y^{(3)} = -3\sin x - x.\cos x.$

Ex8.3: $y = \frac{6}{(x-1)^2}$

$= 6.(x-1)^{-2}$

$y' = 6.(-2(x-1)^{-2-1})$

$y' = -12(x-1)^{-3}$

$y'' = -12(-3(x-1)^{-3-1})$

$= 36(x-1)^{-4}$

$y^{(3)} = 36(-4(x-1)^{-4-1})$

$= -144(x-1)^{-5}$

$= \frac{-144}{(x-1)^5}$

2/30 دائمـا دويدنوا ختوح تفاضل A

Exer. 11 : $y = \cos(x^2)$

$y' = -\sin(x^2) \cdot 2x = -2x \cdot \sin(x^2)$

$y'' = -2 \cdot \sin(x^2) - 2x \cdot \cos(x^2) \cdot 2x$

$y^{(3)} = -2 \cdot \cos(x^2) \cdot 2x - [8x \cdot \cos(x^2) + 4(x^2) \cdot (-\sin(x^2))\, 2x]$

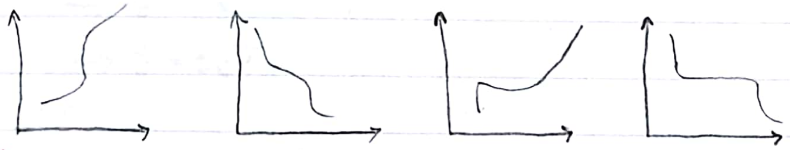
$= -4x\cos(x^2) - 8x\cos(x^2) + 8x^3 \cdot \sin(x^2)$

$= -12x\cos(x)^2 + 8x^3\sin(x^2)$

(2.8): The Mean-Value Theorem

Def. 6 : If $x_1 < x_2$

$f(x_1) < f(x_2)$ | $f(x_1) > f(x_2)$ | $f(x_1) \le f(x_2)$ | $f(x_1) \ge f(x_2)$



f is increasing | f is decreasing | f is nondecreasing | f is nonincreasing

The 12 : Suppose that f is cont. on I, and diff. on J

J: be an open interval ; I: be an interval.

a) If $f'(x) > 0 \to f$ is increasing

b) If $f'(x) < 0 \to f$ is decreasing

c) If $f'(x) \ge 0 \to f$ is nondecreasing

d) If $f'(x) \le 0 \to f$ is non increasing

Ex. 4 : $f(x) = x^3 - 12x + 1$

① $f' = 3x^2 - 12$

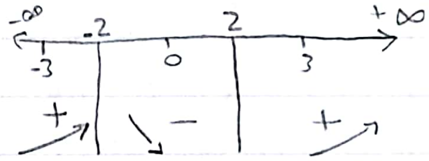
② $f' = 0 \to 3x^2 - 12 = 0$

$\to 3(x^2 - 4) = 0$

$\to 3(x-2)(x+2) = 0$

$\to x - 2 = 0,\ x + 2 = 0$

$\to x = 2,\ x = -2$



$f'(-3) = 3(9) - 12 = 27 - 12 = + > 0$

$f'(0) = 3(0) - 12 = -12 < 0$

$f'(-3) = 3\,9 - 12 = 27 - 12 = + > 0$

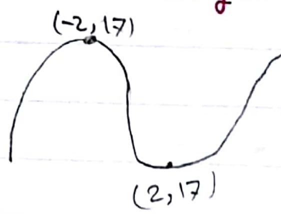
① find f'

② $f' = 0 \to$ نجعل

③ نجمعها على خط الأعداد ← ④ نختبر الإشارات و نحدد التزايدية والتناقصية

f is increasing on $(-\infty, -2) \cup (2, \infty)$

f is decreasing on $(-2, 2)$



2/30

(2.9): Implicit diff.

$y = f(x)$

$y = 2x \to y' = 2$

$xy \qquad y'$

$x^2 + y^2 = 25$

$2x + 2y \cdot y' = 0$

$2y \cdot y' = -2x$

$y' = \frac{-\cancel{2}x}{\cancel{2}y} = \frac{-x}{y}$

Ex 1: Find $\frac{dy}{dx}$

$y^2 = x \to \sqrt{y^2} = \sqrt{x}$

$\to |y| = \sqrt{x}$

$\to y = \pm\sqrt{x}$

$2y \cdot y' = 1$

$y' = \frac{1}{2 \cdot y}$

$y' = \frac{1}{\pm 2\sqrt{x}}$

Ex.3: $y \sin x = x^3 + \cos y$

$f_1' \cdot f_2 + f_1 \cdot f_2'$

$(1)\cdot y' \cdot \sin x + y \cdot \cos x = 3x^2 - \sin y \cdot (1) y'$

$y' \cdot \sin x + y \cos x = 3x^2 - y' \cdot \sin y$

$y' \cdot \sin x + y' \sin y = 3x^2 - y \cos x$

$y'(\sin x + \sin y) = 3x^2 - y \cos x$

$y' = \frac{3x^2 - y \cos x}{\sin x + \sin y}$

Exer. 1: $xy - x + 2y = 1$ (H.W)

Exer.3: $x^2 + xy = y^3$.

$2x + (1)\cdot y + x \cdot (1) = 3y^2 \cdot y'$

$2x + y = 3y^2 \cdot y' - xy'$

$2x + y = y'(3y^2 - x)$

$y' = \frac{2x + y}{3y^2 - x}$

Exer.5: $x^2 \cdot y^3 = 2x - y$.

$2x \cdot y^3 + x^2 \cdot 3y \cdot y' = 2 - (1)\cdot y'$

$y' + 3x^2y^2 \cdot y' = 2 - 2xy^3$

$y'(1 + 3x^2y^2) = 2 - 2xy^3$

$y' = \frac{2 - 2xy^3}{1 + 3x^2y^2}$

(3.1): Inverse Functions

Def. 1: f is one-to-one (1-1) if

1. $x_1 \neq x_2 \to f(x_1) \neq f(x_2)$ or
2. $f(x_1) = f(x_2) \to x_1 = x_2$

s.t. $x_1, x_2 \in D_f$.

اختبار الخط الأفقي

نرسم خط أفقيا

قطع المنحنى في نقطة: f is 1-1

قطع المنحنى في نقطتين: f is not 1-1

Ex: $f(x) = x^2$

f is not 1-1

$2 \neq -2$

$(2)^2 = 4 = (-2)^2$

f is not 1-1

$f(x) = x^3$

f is 1-1

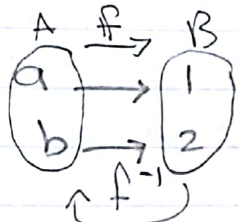
$f(x_1) = f(x_2)$

$\sqrt[3]{x_1^3} = \sqrt[3]{x_2^3}$

$x_1 = x_2 \to f$ is 1-1

* كل دالة تلغي معكوسها

(3.1)



$f(a)=1$

$f^{-1}(1)=a$

$f^{-1}(2)=b$

$D_f=A$, $R_f=B$

$D_{f^{-1}}=B$, $R_{f^{-1}}=A$

Def.2:

1) If f is $\underline{1-1}$ → it has an invere fun. f^{-1}

2) The value of $f^{-1}(x)$ is the $\underline{\text{unique}}$ number y in D_f

Thus; $x=f(y) \longleftrightarrow f^{-1}(x)=y$

Note:

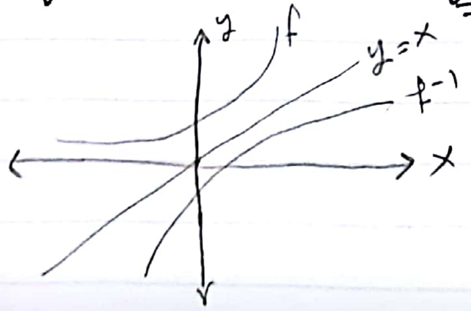
1) $x=f(y) \longleftrightarrow f^{-1}(x)=f^{-1}(f(y))$
$f^{-1}(x)=y$

2) cancellation Law.
$f(f^{-1}(x))=x$
$f^{-1}(f(x))=x$

3) The domain of f^{-1} is the range of f. the range of f^{-1} is the domain of f.

4) $(f^{-1})^{-1}(x)=f(x)$

5) the graph of f^{-1} is the reflection of the graph of f in the line $y=x$



Ex.1: Show that $f(x)=2x-1$ is 1-1, and find $f^{-1}(x)$.

we want to show that f is 1-1

$f(x_1)=f(x_2)$

$2x_1-1^{+1}=2x_2-1^{+1}$

$\frac{2x_1}{2}=\frac{2x_2}{2}$

$x_1=x_2$, f is 1-1

$\underline{\text{find } f^{-1}}$;

1) $y=2x-1$

2) $y+1=2x$

$\rightarrow x=\frac{y+1}{2}$

3) أبدل بين x و y

$\rightarrow y=\frac{x+1}{2}$

4) نكتب f^{-1} بدل y

$f^{-1}(x)=\frac{x+1}{2}$

Ex.2: Show that. $f(x)=\sqrt{2x+1}$ is invetiable, and find f^{-1}.

f is invetiable if f is 1-1

we want to show that f is 1-1, $f(x_1)=f(x_2)$

$(\sqrt{2x_1+1})^2=(\sqrt{2x_2+1})^2$

$2x_1+1^{-1}=2x_2+1^{-1}$

$\frac{2x_1}{2}=\frac{2x_2}{2}$, $x_1=x_2$

f is 1-1 → f is inver, $\underline{\text{find } f^{-1}}$.

1) $y=\sqrt{2x+1}$, 2) $y^2=2x+1 \rightarrow y^2-1=2x$

$\rightarrow x=\frac{y^2-1}{2}$

3) $y=\frac{x^2-1}{2}$, 4) $f^{-1}(x)=\frac{x^2-1}{2}$

3/3

Exer. f is 1-1 ??
find f^{-1} ??
D_f=?? R_f=??
D_f^{-1}=?? R_f^{-1}=??

[6] $f(x) = 1+\sqrt[3]{x}$
we to show that f is 1-1
$f(x_1) = f(x_2)$
$\cancel{1}+\sqrt[3]{x_1} = \cancel{1}+\sqrt[3]{x_2}$
$(\sqrt[3]{x_1})^3 = (\sqrt[3]{x_2})^3$
$x_1 = x_2$
f is 1-1
find f^{-1}
[1] $y = 1+\sqrt[3]{x}$
[2] $y-1 = \sqrt[3]{x} \rightarrow (y-1)^3 = x$
[3] $y = (x-1)^3$
[4] $f^{-1}(x) = (x-1)^3$
$D_f = \mathbb{R}$; $R_f = \mathbb{R}$
$D_{f^{-1}} = \mathbb{R}$; $R_{f^{-1}} = \mathbb{R}$

[9] $f(x) = \frac{1}{x+1}$
① 1-1 ??
$f(x_1) = f(x_2)$

$\frac{1}{x_1+1} = \frac{1}{x_2+1}$

$x_1+1^{-1} = x_2+1^{-1}$, $x_1 = x_2$
f is 1-1
② find f^{-1}
① $\frac{y}{1} = \frac{1}{x+1}$

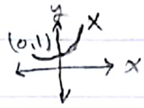
4 إذا كان الأساس متساوي إذن الأسس متساوية

② $y(x+1) = 1$
$x+1 = \frac{1}{y} \rightarrow x = \frac{1}{y} - 1$
③ $y = \frac{1}{x} - 1 = \frac{1-x}{x}$
④ $f^{-1}(x) = \frac{1-x}{x}$ $\boxed{x = 0}$
$D_f = \mathbb{R}-\{-1\}$; $R_f = \mathbb{R}-\{0\}$
$D_{f^{-1}} = \mathbb{R}-\{0\}$; $R_{f^{-1}} = \mathbb{R}-\{-1\}$

مجهول أس ← x ← عند a ← أساس

(3.2): Exponentials:

An exponential fun. is a fun. of the from
$f(x) = a^x$



where
the base a is positive constant
exponent x is the veriable
Ex: 2^x, 3^x, $(\frac{1}{5})^x$

Note:

power fun.	Expo. fun.
$a \rightarrow$ عند	$x \rightarrow$ متغير
$x \rightarrow$ متغير	$a \rightarrow$ عند
Ex: x^2	Ex: 2^x

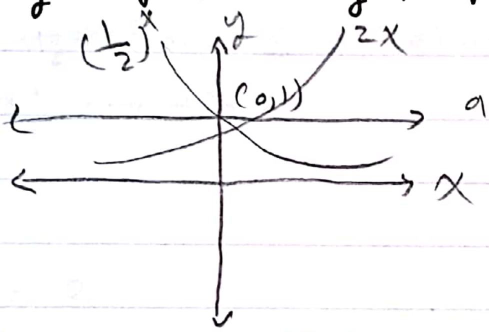
Def. 4 + Law of exponent:
If $a>0$ and $b>0$; and x and y are any real number, then
[1] $a^0 = 1$
[2] $a^n = \underbrace{a.a \ldots a}_{n\text{-times}}$ if $n = 1, 2, \ldots$
[3] $a^{-x} = \frac{1}{a^x}$
[4] $a^{\frac{m}{n}} = a^{m \cdot \frac{1}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ if $n=2,\ldots$ $m=\pm1,\pm2,\ldots$
[5] $a^x . a^y = a^{x+y}$
[6] $\frac{a^x}{a^y} = a^{x-y}$
[7] $(a^x)^y = a^{xy}$
[8] $(ab)^x = a^x . b^x$
[9] $(\frac{a}{b})^x = \frac{a^x}{b^x}$
[10] $(a \pm b)^x =$ لايمكن توزيع
$(a+b)^2 \neq a^2 + b^2$

3/3

Note:

1. If $a=1 \to a^x = 1^x = 1$
2. If $0<a<1$
   a. a^x is decreasing
   b. $\lim_{x\to\infty} a^x = 0$; $\lim_{x\to -\infty} a^x = \infty$
3. If $a>1$
   a. a^x is increasing
   b. $\lim_{x\to\infty} a^x = \infty$; $\lim_{x\to -\infty} a^x = 0$
4. They all pass through the point $(0,1)$

$F(x) = (2^2)^{\frac{1}{4}}$
$= a^{2(\frac{1}{4})}$
$= a^{\frac{1}{2}} = \sqrt{a}$



$D = \mathbb{R} = (-\infty, \infty)$; $R = (0, \infty)$

Exer. 1 $\dfrac{3^3}{\sqrt{3^5}}$

$= \dfrac{3^3}{(3^5)^{\frac{1}{2}}} = \dfrac{3^3}{3^{\frac{5}{2}}}$

$= 3^{\frac{6-5}{2}} = 3^{\frac{1}{2}} = \sqrt{3}$

Example: Solve the following equation:

1. $2^x = 4 \to 2^x = 2^2 \to \boxed{x=2}$
2. $2^{x-2} = 8 \to 2^{x-2} = 2^3 \to x-2=3 \to \boxed{x=5}$
3. $3^{2x-4} = 9 \to 3^{2x-4} = 3^2 \to 2x-4=2$
   $\to 2x = 6 \to \boxed{x=3}$

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3/5

Example:

(4) $4^{x-1} = 8 \rightarrow (2^2)^{x-1} = 2^3 = 2^{2x-2} = 2^3 \rightarrow 2x-2 = 3 \rightarrow 2x = 5 \rightarrow x = \frac{5}{2}$

(5) $9^{x+1} = 27 \rightarrow (3^2)^{x+1} = 3^3 \rightarrow 3^{2x+2} = 3^3 \rightarrow 2x+2 = 3 \rightarrow 2x = 1 \rightarrow x = \frac{1}{2}$ / $(2^2) = 4$, $(2^3) = 8$

logaritms:

If $a > 0$, $a \neq 1$;

$f(x) = a^x$ is 1-1 $\rightarrow$ f has an inverse (Logarithmic fun).

Def .5:

If $a > 0$ and $a \neq 1$,

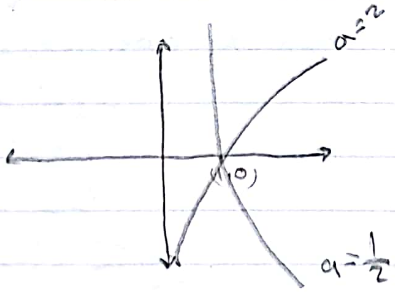
أس $\boxed{\log_a^x}$ Logarithm of $\underline{x}$ to the base $\underline{a}$

$y = \boxed{\log_a^x} \leftrightarrow a^y = x$

Ex:

$\log_2^4 = 2$

$\log_2^8 = 3$



Note:

(1) Domian = $(0, \infty)$

Range = $\mathbb{R}$

(2) If $a > 1$;

$\lim_{x \to 0^+} \log_a^x = -\infty$

$\lim_{x \to \infty} \log_a^x = \infty$

(3) If $\boxed{0 < a < 1}$

$\lim_{x \to 0^+} \log_a^x = \infty$

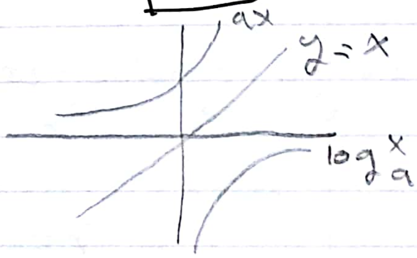
$\lim_{x \to \infty} \log_a^x = -\infty$

(4) $\log_a a^x = x$

$(a)^{\log_a^x} = x$

(5) They all pass through the point $\underline{(1,0)}$.

(6) Reflection in the line $\boxed{y = x}$



Laws of Logarthems:

If $(x, y, a, b) > 0$

$a \neq 1$ and $b \neq 1$

Then.

(1) $\log_a^1 = 0$ $[a^0 = 1]$

(2) $\log_a^a = 1$ $[a^1 = a]$

(3) $\log_a(xy) = \log_a^x + \log_a y$

(4) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

(5) $\log_a (x^y) = y \cdot \log_a^x$

(6) $\log_a\left(\frac{1}{x}\right) = \log_a x^{-1} = -\log_a x$

(7) $\log_a^x = \frac{\log_b^x}{\log_b^a}$

Note: $\log_a(x \pm y) =$ لا يوجد قاعدة

Ex.

(1) $\log_4^2 = \frac{1}{2} \rightarrow \log_{2^2}^2 = \frac{1}{2}$ $(4)^{\frac{1}{2}} = 2$

(2) $\log_8^2 = \frac{1}{3} \rightarrow (8^{\frac{1}{3}}) = 2$

3/5

Ex.3: Simplify

(a) $\log_2 10 + \log_2 12 - \log_2 15$

$= \log_2(10\times 12) - \log_2 15$

$= \log_2\left(\frac{10\times 12}{15}\right) \log_2 8 = 3$

(b) $\log_{a^2} a^3 = 3 \cdot \log_{a^2} a = 3\left(\frac{1}{2}\right) = \frac{3}{2}$ $\quad a^2 = a$

(c) $\log_3 4 \cdot \log_4 9$

$= (3) \cdot \frac{\log_3 4}{\log_3 9} = \left((3)^{\log_3 4}\right)^{\frac{1}{\log_2 3^2}}$

$= \frac{1}{(4)(2)\cdot \log_3 3} = (4) = \frac{1}{2(1)} = \sqrt{4} = 2$

Ex.4: Solve: $3^{x-1} = 2^x$

$\log 3^{x-1} = \log 2^x = (x-1)\log 3 = x.\log 2$

$x\log 3 - \log 3 = x.\log 2$

$x\log 3 - x\log 2 = \log 3$

$x[\log 3 - \log 2] = \log 3$

$x\log\left(\frac{3}{2}\right) = \log(3) = \frac{\log 3}{\log\left(\frac{3}{2}\right)}$

Ex. Simplify

(1) $\log_5 125 = \log_5 5^3 = 3\log_5 5$

(2) $\log_{\frac{1}{3}} 3^{2x} = (3^{-1})^{-1} = 3 \leftarrow 2x.\log_{3^{-1}} 3 = 2x.(-1) = -2x$

(3) $\log 25 + \log 4 = \log(100) = 2$

(4) $\log_2 64 - \log_2 32 + \log_2 2 = \log 2^6 - \log_2 2^5 + 1$

$= 6.\log 2 - 5\log 2 + 1 = 6(1) - 5(1) + 1 = 2$

(5) $\log_3 27 - \log_3 81 + 5\log_3 3 = \log_3 3^3 - \log_3 3^4 + 5(1)$

$= 3 - 4 + 5 = 4.$

(6) $5^{2\log_5 2} = 5^{\log_5 2^2} = 4$

(7) $(\log_2 16)\cdot(\log_4 2) =$

$= (\log_2 4^2)\cdot(\log_{2^2} 2)$

$= (2)\cdot\left(\frac{1}{2}\right) = 1$

Exer.9: Simplify:

$10^{-\log_{10}\left(\frac{1}{x}\right)} = 10^{\log_{10}(x^{-1})^{-1}}$

$= (x^{-1})^{-1} = x^{-1} = x.$

Exer 23: Solve: $\log_x 3 = 5$

$\log_x 3 = x^5$

$\rightarrow 3 = x^5$

$\rightarrow x = \sqrt[5]{3}$

$x^5 = 3$

$x = \sqrt[5]{3}$

(3.3): The Natural Exponen.

$f(x) = a^x$ (general expo.) $a > 0$

$f(x) = e^x$ (natural expo.) $e = 2.71\ldots$

Properties of e^x :-

(1) $e^1 = e$; $e^0 = 1$

(2) $(e^x)^y = e^{xy}$

(3) $e^x . e^y = e^{x+y}$

(4) $\frac{e^x}{e^y} = e^{x-y}$

(5) $e^{-x} = \frac{1}{e^x}$

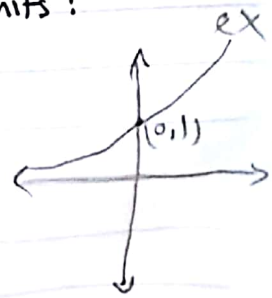
Graph and Limits:

Domain $= \mathbb{R}$

Range $= (0, \infty)$

$\lim_{x\to\infty} e^x = \infty$

$\lim_{x\to-\infty} e^x = 0$



3/5

Derivative of e^x:

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot f'(x).$$

<u>Ex. 3</u>

(a) $\frac{d}{dx} e^{x^2-3x} = e^{x^2-3x} \cdot (2x-3)$

(b) $\frac{d}{dx}\left(\sqrt{1+e^{2x}}\right) = \frac{1}{2\sqrt{1+e^{2x}}} \cdot (e^{2x}) \cdot \cancel{2} = \frac{e^{2x}}{\sqrt{1+e^{2x}}}$

$f' = e^x - e^{-x} \cdot (-1) = e^x + e^{-x}$

$g' = e^x + e^{-x} \cdot (-1) = e^x - e^{-x}$

(c) $\frac{d}{dx}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)\begin{matrix} f \\ g \end{matrix} =$

$$= \frac{\overbrace{(e^x+e^{-x}) \cdot (e^x+e^{-x})}^{(e^x+e^{-x})^2} - \overbrace{(e^x-e^{-x}) \cdot (e^x-e^{-x})}^{(e^x-e^{-x})^2}}{(e^x+e^{-x})^2}$$

$$= \frac{(\cancel{e^{2x}} + 2e^x \cdot e^{-x} + \cancel{e^{-2x}}) - [\cancel{e^{2x}} - 2e^x \cdot e^{-x} + \cancel{e^{-2x}}]}{(e^x+e^{-x})^2}$$

$$= \frac{4e^{x-x}}{(e^x+e^{-x})^2} = \frac{4e^0}{(e^x+e^{-x})^2} = \frac{4}{(e^x+e^{-x})^2}.$$

3/19

(3.3): The Natural Logarithmic Fun.:

$f(x) = \log_a^x$ (general Logarithm function).

$f(x) = \text{Log}_e^x = \text{lin}\, x$ (Natural Logarithm function).

Def:

$y = e^x \longleftrightarrow \text{lin}\, y = x$

Note: $\boxed{1}$ The inverse of lin x is e^x (lin x is 1-1)

$\boxed{2}$ $\text{lin}\, e^x = x$; $e^{\text{lin}\, x} = x$

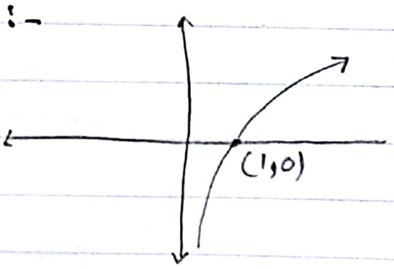
$[y = e^x \longleftrightarrow \text{lin}\, y = \text{lin}\, e^x \longleftrightarrow \text{lin}\, y = x]$

$\boxed{3}$ $D_{e^x} = \mathbb{R}$; $R_{e^x} = (0, \infty)$

$D_{\text{lin}\, x} = (0, \infty)$; $R_{\text{lin}\, x} = \mathbb{R}$

Graph and limits:-

$\lim\limits_{x \to 0^+} \text{lin}\, x = -\infty$



$\lim\limits_{x \to \infty} \text{lin}\, x = \infty$

Properties of lin x ; $e^0 = 1$

1. $\text{lin}\, 1 = 0$
2. $\text{lin}(x \cdot y) = \text{lin}\, x + \text{lin}\, y$
3. $\text{lin}\left(\frac{x}{y}\right) = \text{lin}\, x - \text{lin}\, y$
4. $\text{lin}\, x^r = r \cdot \text{lin}\, x$
5. $\text{lin}\left(\frac{1}{x}\right) = \text{lin}\, x^{-1} = -\text{lin}\, x$

Exer. Simplify

$\boxed{3}$ $e^{5\,\text{lin}\, x} = e^{\text{lin}\, x^5} = x^5$

$\boxed{5}$ $\text{lin}\left(\frac{1}{e^{3x}}\right) = \text{lin}\, e^{-3x} = -3x$

Ex.1 Solve

2. $\text{lin}(3x-2) = 8 \quad \div 2$

$\to \text{lin}(3x-2) = 4$

$\to e^{\text{lin}(3x-2)} = e^4$

$\to 3x - 2 = e^4$

$\to 3x = e^4 + 2$

$\to x = \dfrac{e^4+2}{3}$

Derivative of lin x:-

$\frac{d}{dx} \text{lin}\, x = \frac{1}{x} \quad \left(\frac{1}{\cancel{1+2}}\right)\left(\frac{\cancel{2}+1}{4}\right)$

$\frac{d}{dx} |\text{lin}\, x| = \frac{1}{x}$

$\frac{d}{dx} \text{lin}(f(x)) = \frac{1}{f(x)} \cdot f'(x)$

Ex2: Find the derivative

ⓐ $(\text{lin}|\cos x|)' = \frac{1}{\cos x} \cdot -\sin x$

$= \frac{-\sin x}{\cos x} = -\tan x$

$\boxed{b}$ $\frac{d}{dx} \text{lin}(x + \sqrt{x^2+1}) =$

$= \left(\frac{1}{x+\sqrt{x^2+1}}\right)\left(1 + \frac{1}{\cancel{2}\sqrt{x^2+1}} \cdot \cancel{2}x\right)$

$= \left(\frac{1}{x+\sqrt{x^2+1}}\right) \cdot \left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)$

$= \frac{1}{\sqrt{x^2+1}}$

The General Exponentials and logarithms:

$$\frac{d}{dx} a^x = a^x \cdot \ln a$$

$$\frac{d}{dx} a^{f(x)} = a^{f(x)} \cdot \ln a \cdot f'(x).$$

$$\frac{d}{dx} \log_a^x = \frac{1}{x \cdot \ln a}$$

$$\frac{d}{dx} \log_a^{f(x)} = \frac{1}{f(x) \cdot \ln a} \cdot f'(x)$$

$$* \log_a^x = \frac{\ln x}{\ln a}$$

Ex.:

(a) $y = 2^{3x^5+1}$

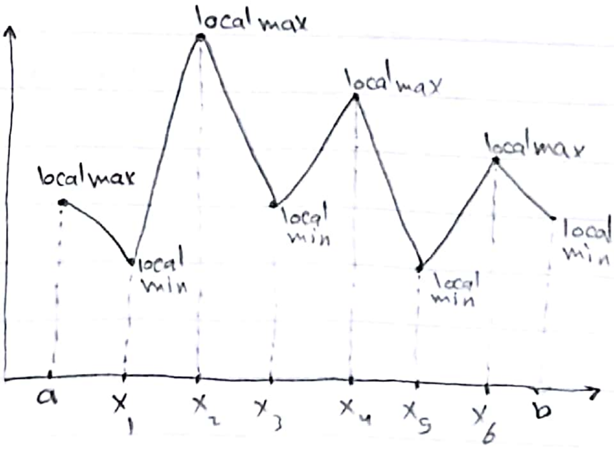
$y' = 2^{3x^5+1} \cdot \ln 2 \cdot (15x^4)$

(b) $y = \log_3 x^2$

$$y' = \frac{1}{x^2 \ln 3} \cdot 2x = \frac{2}{x \cdot \ln 3}$$

(4.4). Extrem values.

Maximum and minimum values:



Def. 1: Absolute extren value:-

(1) A fun. f has an absolute maximmum value $f(x_0)$ at x_0 ($x_0 \in D_f$)
if $f(x) \leq f(x_0)$
$\forall x \in D_f$

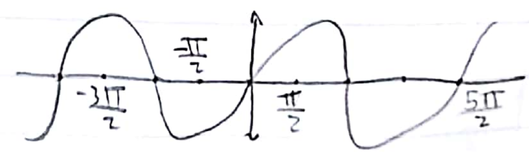
(2) A fun. f has an absolute minmum value $f(x_1)$ at (x_1) ($x_1 \in D_f$)
if $f(x) \geq f(x_1)$
$\forall x \in D_f$

Note:

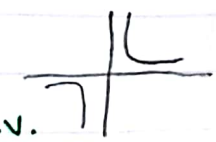
(1) The absolute maximum is the highest of the local maximum

(2) The absolute minumum is the lowest of the local minmum

Ex: $f(x) = \sin x$
f has A.M_x.v.1 at $x = \frac{\pi}{2} + 2n\pi$
; $n = 0, \pm 1, \pm 2, \ldots$
f has A.M_n.v -1 at $x = -\frac{\pi}{2} + 2\pi n$
; $n = 0, \pm 1, \ldots$



Ex: $f(x) = \frac{1}{x}$
f has not
A. max and A. min. v.



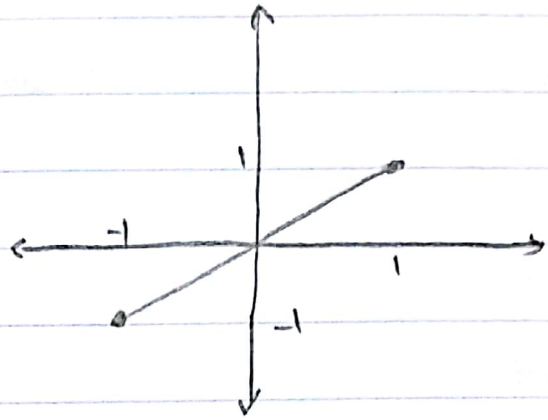
3/19

Ex: $f(x) = x$

$D = [-1, 1]$

A.max: 1^y at 1^x $(1,1)$

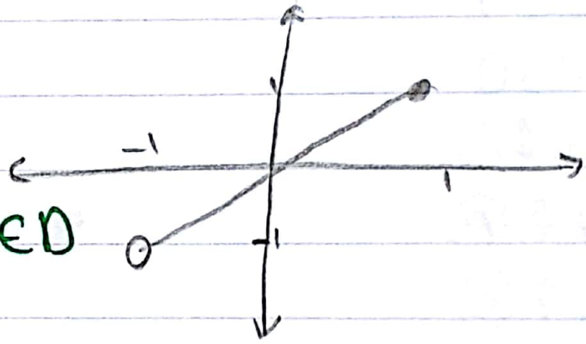
A.min: -1 at -1 $(-1,-1)$



$D = (-1, -1]$

A.max: $(1,1)$

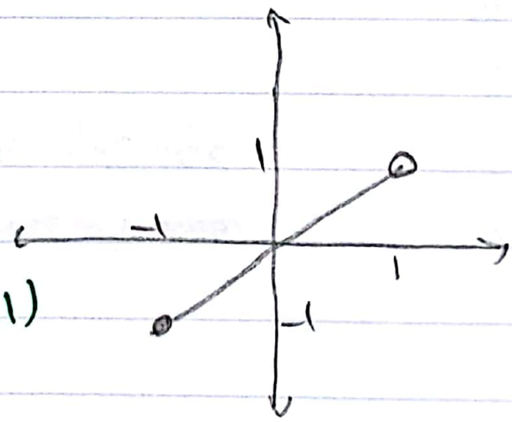
A.min: No, because $-1 \in D$



$D = [-1, 1)$

A.max: No, $1 \in D$

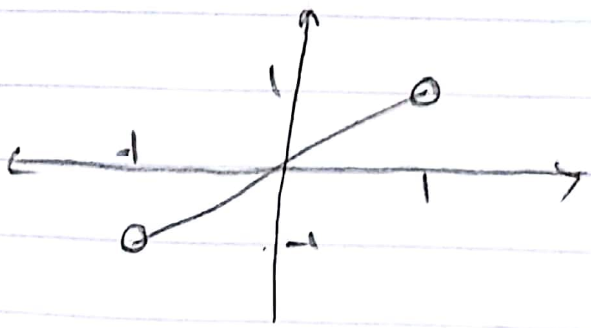
A.min: -1 at -1 $(-1,-1)$



$D = (-1, 1)$

A.max = No

A.min = No



3/21

(4.4)

Def. :

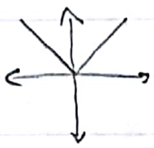
① Critical point : النقاط الحرجة

$f'(x) = 0$

② Singular point.

$f'(x)$ is not defined

Ex : $|x|$



غير قابلة التفاضل عند 0

③ end points : هي حدود الفترة

$[a,b] \rightarrow a,b \in D$

$(a,b) \rightarrow a,b \notin D$

Ex.: Find critical point, the absolute maximum point, the absolute minimum point of the function :

$f(x) = 3x^2 - 12x + 1$ in $[0,3]$.

① critical point

$f'(x) = 6x - 12$

c.p : $f' = 0$

$\rightarrow 6x - 12 = 0 \rightarrow \frac{6x}{6} = \frac{12}{6}$

$\rightarrow x = 2 \in [0,3]$

② To find the A.max. point and A.min. point we must find the value of f at the critical point ($x=2$) and end point ($x=0$, $x=3$)

$f(2) = 12 - 24 + 1 = -11 \rightarrow$ A.min -11 at 2.

$f(0) = 1 \rightarrow$ A.max. 1 at 0

$f(3) = 27 - 36 + 1 = -8$

The A.max. point is : (0,1), The A.min point is : (2,-11)

Exer. 5 : $f(x) = x^2 - 1$, $[-2,3]$

① c.p $f' = 0$

$\rightarrow 2x = 0$

$\rightarrow x = 0 \in [-2,3]$

② $f(0) = -1 \rightarrow$ A.min -1 at 0

$f(-2) = 3$

$f(3) = 8 \rightarrow$ A.max 8 at 3

The A.max. point is (3,8), The A.min point is (0,-1)

(4.5)

Def. 3 :

① f is concave up on an open intraval I if

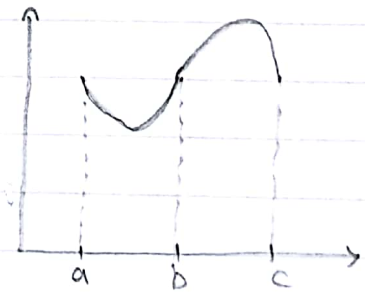
ⓐ f is differentiable

ⓑ f' is increasing fun. on I

② f is concave down on open interaval I if

ⓐ f is differentiable

ⓑ f' is decreasing on I.



f is concave up on (a,b)

f is concave down on (b,c)

مجموعتان

$(b, f(b))$ is inflection point.

نقطة

3/21

Th 9:

① If $f''(x) > 0$ on an interval I
$\to$ f is concave up on I
② If $f''(x) < 0$ on I
$\to$ f is concave down on I
③ $f''(x_0) = 0 \to$ f has an inflection poin at $x_0 \to (x_0, f(x_0))$

Ex.: If $f(x) = x^3 - 3x^2 - 9x + 2$
find the following:

ⓐ The critical point.

$f'(x) = 3x^2 - 6x - 9$

C.P: $f'(x) = 0$

$\to 3x^2 - 6x - 9 = 0$

$\to 3(x^2 - 2x - 3) = 0$

$\to x^2 - 2x - 3 = 0$

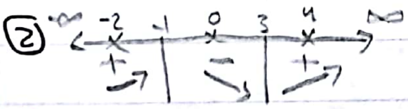
$\to (x-3)(x+1) = 0$

$\to x = 3, x = -1$

ⓑ Increasing interval and Decreasing interval

① $f' = 0 \to x = 3, x = -1$

②



$f'(-2) = 12 + 12 - 9 = + > 0$

$f'(0) = -9 < 0$

$f'(4) = + > 0$

f is increasing on $(-\infty, -1) \cup (3, \infty)$

f is decreasing on $(-1, 3)$

ⓒ local maximum value and local minimum value.

① C.P. $f' = 0 \to x = 3, x = -1$

$f(3) = 27 - 27 - 27 + 2 = -25$

$f(-1) = -1 - 3 + 9 + 2 = 7$

The local max. point is $(-1, 7)$, The local min. poin is $(3, -25)$

ⓓ concave upward and concave downward and inflection point.

① $f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

inf: $f'' = 0$

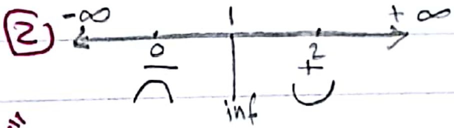
$\to 6x - 6 = 0$

$\to 6x = 6$

$\to \boxed{x = 1}$ inf. point $(1, -9)$

$f(1) = 1 - 3 - 9 + 2 = -9$

②



$f''(0) = -6 < 0$

$f''(2) = 6 > 0$

f is concave up on $(1, \infty)$

f is concave down on $(-\infty, 1)$

The inflection point is $(1, -9)$

3/21 ملخص شابتر ٤

A.max L.max	and	A.min L.min

① find c.p. $f'=0$

② نوجد قيم (c.p) و (E.P) بالتعويض في الدالة الأصلية f

③ أعلى قيمة تكون $\boxed{A.max \text{ or } L.M_x}$

أقل قيمة تكون $\boxed{A.min \text{ or } L.M_{in}}$

Inc. and Dec. interval

① find c.p. $f'=0$

② نحدد نقاط الـ c.p على خط الأعداد

③ نختار أعداد على خط الأعداد ونعوضها في f' لتحديد الإشارة

④ نحدد الـ $\boxed{\text{Inc. and Dec interval}}$

C.U and C.D. interaval.

① find inflection point. $f''=0$

② نحدد نقاط الـ inf على خط الأعداد

③ نختار أعداد على خط الأعداد ونعوضها في f'' لتحديد الإشارة

④ نحدد $\boxed{\text{C.U and C.D interaval}}$