

وبين الحفني

محاضرات في ريفس ١٠١ جامعة الملك سعود  
مع الحفني جوال: ٥٨٣٤٢٢٢٠٠

حل الواجب الاول math (101)



كورس ريفس ١٠١ عبدالله الحفني ٥٨٣٤٢٢٢٠٠

عبدالله الحفني جوال ٥٨٣٤٢٢٢٠٠

كل الشكر للرائع مشرف قروب القيمة

أ/ عبدالباسط سمير جوال: ٥٨٢١٢٨٢٢١

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Question 1

(2 marks for each )

A. Classify the following numbers into rational or irrational

$$\left\{ \sqrt[3]{27}, \frac{\sqrt{8}}{\sqrt{2}}, \sqrt{9} \cdot \pi, \sqrt{\sqrt{25} + \sqrt{16}}, 7.\bar{5}, \sqrt[3]{2}, 4.952 + \frac{1}{3}, 2.45971\dots \right\}$$

خطوات الحل

$\sqrt[3]{27} = 3 \in \mathbb{Q}$	$\sqrt{9}\pi = 3\pi \in I$	$\sqrt{\sqrt{25} + \sqrt{16}} = 3 \in \mathbb{Q}$	
$7.\bar{5} \in \mathbb{Q}$	$\sqrt[3]{2} \in I$	$4.95 + \frac{1}{3} \in \mathbb{Q}$	$2.45571\dots \in I$

B. Solve the following inequalities and write the solution in interval notation

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1.  $4x - 2 \leq 3$

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خطوات الحل

$$4x - \cancel{2} + \cancel{2} \leq 3 + 2 \quad (+ 2)$$

$$4x \leq 5 \Rightarrow x \leq \frac{5}{4}$$

$$s.s = x \in (-\infty, \frac{5}{4}] \quad (\div 4)$$

3.  $\sqrt{\left(\frac{2x-1}{3}\right)^2} + 4 \leq 5$

خطوات الحل

$$\sqrt{\left(\frac{2x-1}{3}\right)^2} \leq 5 - 4 \Rightarrow \left|\frac{2x-1}{3}\right| \leq 1$$

$$\Rightarrow -3 \leq 2x - 1 \leq 3 \Rightarrow -2 \leq 2x \leq 4$$

$$\Rightarrow \frac{-2}{2} \leq x \leq \frac{4}{2} \Rightarrow -1 \leq x \leq 2$$

$$s.s = [-1, 2]$$

2.  $5 - (2x - 4) \leq \frac{6x + 1}{3}$

خطوات الحل

$$5 - 2x + 4 \leq \frac{6x + 1}{3} \Rightarrow 9 - 2x \leq \frac{6x + 1}{3}$$

$$27 - 6x \leq 6x + 1 \Rightarrow -12x \leq -26$$

$$x \geq \frac{13}{6}$$

$$s.s = x \in \left[\frac{13}{6}, \infty\right)$$

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4.  $\left|\frac{1}{x-1}\right| < \left|\frac{1}{x-2}\right|$

خطوات الحل

$|X| < |a|$   
تربيع الطرفين

$$\frac{1}{|x-1|} < \frac{1}{|x-2|} \Rightarrow |x-2| < |x-1| \text{ to } ^2$$

$$\Rightarrow (x-2)^2 < (x-1)^2 \Rightarrow$$

$$\Rightarrow \cancel{x^2} - 4x + 4 < \cancel{x^2} - 2x + 1$$

$$-2x < -3 \Rightarrow x > \frac{3}{2}$$

$$s.s = \left(\frac{3}{2}, 2\right) \cup (2, \infty)$$

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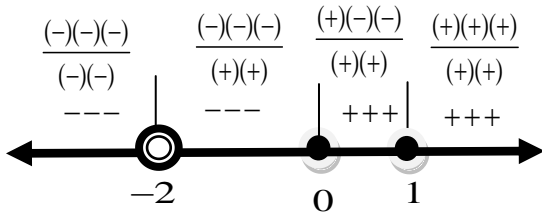
$$5. \frac{(x^2 - 2x + 1) \cdot x}{x^2 + 4x + 4} \geq 0$$

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خطوات الحل

$$\frac{x(x-1)(x-1)}{(x+2)(x+2)} \geq 0 \quad ; x \neq -2$$

$$x = 0, x = 1$$



$$s.s = [0, \infty)$$

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$$6. ||x| - 1| \geq 3$$

خطوات الحل

$$||x| - 1| \geq 3$$

$$|x| - 1 \geq 3$$

$$|x| \geq 4$$

$$x \geq 4 \text{ or } x \leq -4$$

$$|x| - 1 \leq -3$$

$$|x| \leq -2$$

$$s.s = \emptyset$$

$$s.s = (-\infty, -4] \cup [4, \infty)$$

## Question 2

A- Find the domain of the following functions

$$1. f(x) = 9 - (x - 1)^2$$

خطوات الحل

$$D_f = (-\infty, \infty) \text{ because (pol)}$$

$$2. f(x) = \frac{1}{2 + \cos x}$$

خطوات الحل

$$2 + \cos x = 0$$

$$x = \cos^{-1}(-2) \Rightarrow -2 \notin [-1, 1]$$

$$\text{no } s.s \forall x \in \mathbb{R}$$

$$D_f = \mathbb{R} = (-\infty, \infty)$$

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$$3. f(x) = \frac{x+1}{1 - \sqrt{1-2x}}$$

خطوات الحل

$$1 - 2x \geq 0 \quad | \quad 1 - \sqrt{1-2x} = 0 \Rightarrow \sqrt{1-2x} = 1$$

$$x \leq \frac{1}{2}$$

$$1 - 2x = 1 \Rightarrow x = 0$$

$$s.s = (-\infty, 0) \cup (0, \frac{1}{2}]$$

$$4. f(x) = \sqrt{1+|x|} + \sqrt[3]{x^2 - 4}$$

خطوات الحل

$$1 + |x| \geq 0 \Rightarrow |x| \geq -1$$

$$s.s = \mathbb{R}$$

$$D_f = \mathbb{R}$$

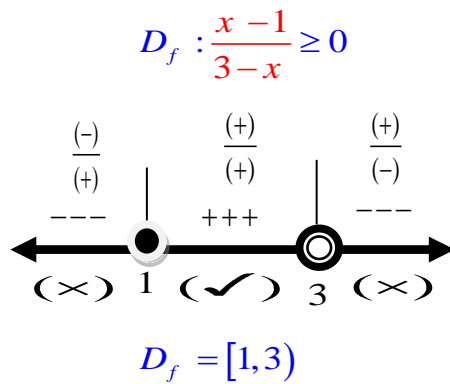
B- Determine whether the functions

$$f(x) = \sqrt{\frac{x-1}{3-x}}, g(x) = \frac{\sqrt{x-1}}{\sqrt{3-x}}$$

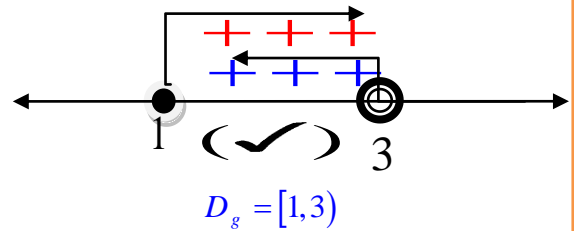
are the same or not.

خطوات الحل

(1) نوجد Domain



$D_g : x-1 \geq 0 \quad 3-x > 0$   
 $\Rightarrow x \geq 1 \quad x < 3$



(2) التبسيط simplify:

$$g(x) = \frac{\sqrt{x-1}}{\sqrt{3-x}} = \sqrt{\frac{x-1}{3-x}} = f(x); \forall x \in D_f \cap D_g$$



(1)  $D_f = D_g$

(2)  $f = g$

(\* F and g are the same

- لحجز ودراسة كورس رياضيات ١٠١
- (١) لدينا اقوي مراجعات للميد الاول للعام
  - (٢) لدينا مذكرات تحتوي علي شرح كامل للكورس (نحل EXAMPLE المهمة نحل EXERCISES)
  - (٣) لدينا حلول جميع الاختبارات السابقة
  - (٤) لدينا مذكرة ليلة الامتحان (A+)
  - (٥) نظام فردي وقروبات

Question 3

Let  $f(x) = \frac{x+4}{x-5}$ .

1. Find  $D_f$ .
2. Show that  $f$  is one-to-one.
3. Find  $f^{-1}$ .
4. Find the range of  $f$ .

1. Find  $D_f$ .

خطوات الحل

$$D_f : x - 5 = 0 \Rightarrow x = 5$$

$$D_f = \mathbb{R} - \{5\}$$

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3. Find  $f^{-1}$ .

خطوات الحل

$\because f(x)$  is (1-1)

put  $f(x) = x, x = f^{-1}(x)$

$$x = \frac{f^{-1}(x)+4}{f^{-1}(x)-5} \Rightarrow x f^{-1}(x) - 5x = f^{-1}(x) + 4$$

$$(x-1)f^{-1}(x) = 5x + 4$$

$$f^{-1}(x) = \frac{5x+4}{x-1}; \forall x \in \mathbb{R} - \{1\}$$



2. Show that  $f$  is one-to-one.

خطوات الحل

let  $f(x_1) = f(x_2) \forall x_1, x_2 \in \mathbb{R} - \{5\}$

$$\frac{x_1+4}{x_1-5} = \frac{x_2+4}{x_2-5}$$

$$\frac{x_1-5+9}{x_1-5} = \frac{x_2-5+9}{x_2-5} \Rightarrow x_1 + \frac{9}{x_1-5} = x_2 + \frac{9}{x_2-5}$$

$$\cancel{x_1} + \frac{9}{\cancel{x_1}-5} = \cancel{x_2} + \frac{9}{\cancel{x_2}-5} \Rightarrow x_1 \cancel{-5} = x_2 \cancel{-5}$$

$$x_1 = x_2$$

$f$  is (1-1)

4. Find the range of  $f$ .

خطوات الحل



لايجاد مدى  $f(x)$

(١) نستخدم الدالة العكسية

$$R_f = D_{f^{-1}} \quad (٢)$$

$$f^{-1}(x) = \frac{5x+4}{x-1}$$

$$x-1=0 \Rightarrow x=1$$

$$R_f = D_{f^{-1}} = \mathbb{R} - \{1\}$$

Question 5

Let  $f(x) = \sqrt{x^2 - 1}$ ,  $g(x) = \frac{1}{x-2}$ .

1. Find  $(f \cdot g)(x)$  and its domain.
2. Find  $\left(\frac{f}{g}\right)(x)$  and its domain.
3. Find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .

1. Find  $(f \cdot g)(x)$  and its domain.

خطوات الحل

$$D_f : x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow \sqrt{x^2} \geq \sqrt{1}$$

$$|x| \geq 1 \Rightarrow x \geq 1 \text{ or } x \leq -1$$

$$D_f = (-\infty, -1] \cup [1, \infty)$$

$$D_g : x - 1 = 0 \Rightarrow x = 1$$

$$D_g = \mathbb{R} - \{2\}$$

$$D_{(f \cdot g)} = D_f \cap D_g = (-\infty, -1] \cup [1, 2) \cup (2, \infty)$$



2. Find  $\left(\frac{f}{g}\right)(x)$  and its domain.

خطوات الحل

$$D_{\left(\frac{f}{g}\right)} = D_f \cap D_g - \{g(x) = 0\} = (-\infty, -1] \cup [1, 2) \cup (2, \infty)$$

3. Find  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .

خطوات الحل

$$\begin{aligned} (f \circ g)(x) &= f\left(\frac{1}{x-2}\right) \\ &= \sqrt{\left(\frac{1}{x-2}\right)^2 - 1} \end{aligned}$$

$$\begin{aligned} (g \circ f)(x) &= g(\sqrt{x^2 - 1}) \\ &= \frac{1}{\sqrt{x^2 - 1} - 2} \end{aligned}$$

Question 6

A. Let  $\phi$  be an angle in standard position, its arc length 110 cm, and the diameter of the circle is 40 cm. Determine the angle in  $\phi$  degree, if the rotation is clockwise.

خطوات الحل

$$\phi = s \cdot \frac{360}{2\pi r} = -110 \cdot \frac{360}{40\pi}$$

$$= -315^\circ 7' 43''$$



$$s = 110$$

$$d = 40$$

B. Use reference angles to find the exact value of the following:

1.  $\cos(210^\circ)$
2.  $\sin(-\frac{3\pi}{4})$

خطوات الحل

1.  $\cos(210^\circ)$

حل اول

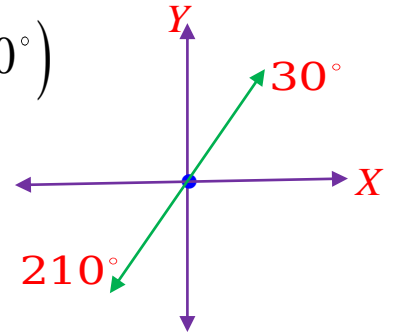
$$\theta' = 210^\circ - 180^\circ = 30^\circ$$

$$\cos 210^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

حل ثانى

$$\cos 210^\circ = \cos(180^\circ + 30^\circ)$$

$$= -\cos(30^\circ) = -\frac{\sqrt{3}}{2}$$



2.  $\sin(-\frac{3\pi}{4})$

حل اول

$$\theta = 2\pi - \frac{3\pi}{4} = \frac{5\pi}{4}$$

$$\theta' = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$$

$$\sin(-\frac{3\pi}{4}) = -\sin(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$$

حل ثانى

$$\sin(-\frac{3\pi}{4}) = -\sin(\frac{3\pi}{4})$$

$$= -\sin(\frac{\pi}{4})$$

$$= -\frac{\sqrt{2}}{2}$$

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Question 7

Find the exact value of the following, without using calculator:

- $\sin^{-1}(\sin(\frac{5\pi}{4}))$ .
- $\cos(\sin^{-1}(\frac{2}{3}) + \tan^{-1}(\frac{-1}{3}))$

خطوات الحل

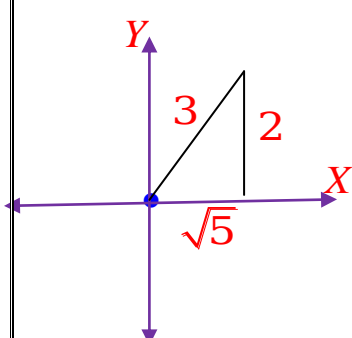


1.  $\sin^{-1}(\sin(\frac{5\pi}{4}))$ .

$$\begin{aligned} \sin^{-1} \sin(\frac{5\pi}{4}) &= \sin^{-1} \sin(\cancel{\frac{5\pi}{4}} + \frac{\pi}{4}) \\ &= \sin^{-1} [-\sin(\frac{\pi}{4})] = \sin^{-1} (-\frac{\sqrt{2}}{2}); -\frac{\sqrt{2}}{2} \in [-1, 1] \\ &= -\frac{\pi}{4} \in [-\frac{\pi}{2}, \frac{\pi}{2}] \end{aligned}$$

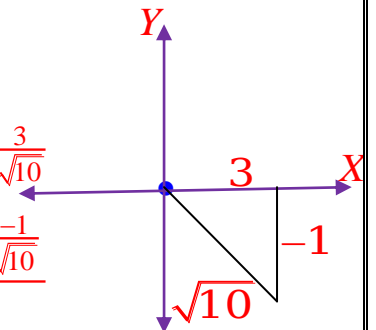
2.  $\cos(\sin^{-1}(\frac{2}{3}) + \tan^{-1}(\frac{-1}{3}))$

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$$\begin{aligned} \cos(\sin^{-1}(\frac{2}{3})) &= \frac{\sqrt{5}}{3} \\ \sin(\sin^{-1}(\frac{2}{3})) &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \cos \tan^{-1}(-\frac{1}{3}) &= \frac{3}{\sqrt{10}} \\ \sin \tan^{-1}(-\frac{1}{3}) &= \frac{-1}{\sqrt{10}} \end{aligned}$$



$$\cos[\sin^{-1}(\frac{2}{3}) + \tan^{-1}(-\frac{1}{3})] = \cos(A+B)$$

$$= \cos(\sin^{-1}(\frac{2}{3})) \cos(\tan^{-1}(-\frac{1}{3})) - \cancel{\sin(\sin^{-1}(\frac{2}{3}))} \sin(\tan^{-1}(-\frac{1}{3}))$$

$$= \frac{\sqrt{5}}{3} \cdot \frac{3}{\sqrt{10}} - \frac{2}{3} \cdot \frac{-1}{\sqrt{10}} = \frac{3\sqrt{5}+2}{3\sqrt{10}} = \frac{15\sqrt{2}+2\sqrt{10}}{30}$$

يوجد طرق اخرى للحل



Question 8

Solve the trigonometric equation

$$\cos(2x) = \sin x, \quad x \in [0, 4\pi]$$

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$$1 - 2\sin^2 x = \sin x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \sin^{-1}\left(\frac{1}{2}\right)$$

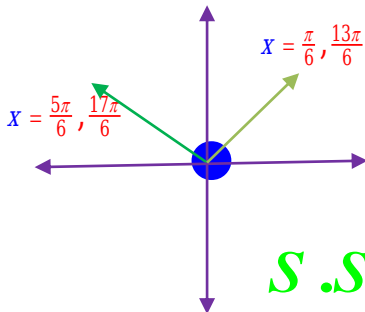
$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$



$$\sin x = -1$$

$$x = \sin^{-1}(-1)$$

$$x = \frac{3\pi}{2}, \frac{7\pi}{2}$$



$$S.S = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{3\pi}{2}, \frac{7\pi}{2} \right\}$$

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