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BMT223 Assignment #6

Find the area between f(x) & g(x) where:

1- $f(x) = 2x^3 - x^2 - x$ and $g(x) = x^3 + x^2 + 2x$

Equalizing both equations to find all x roots "boundaries":

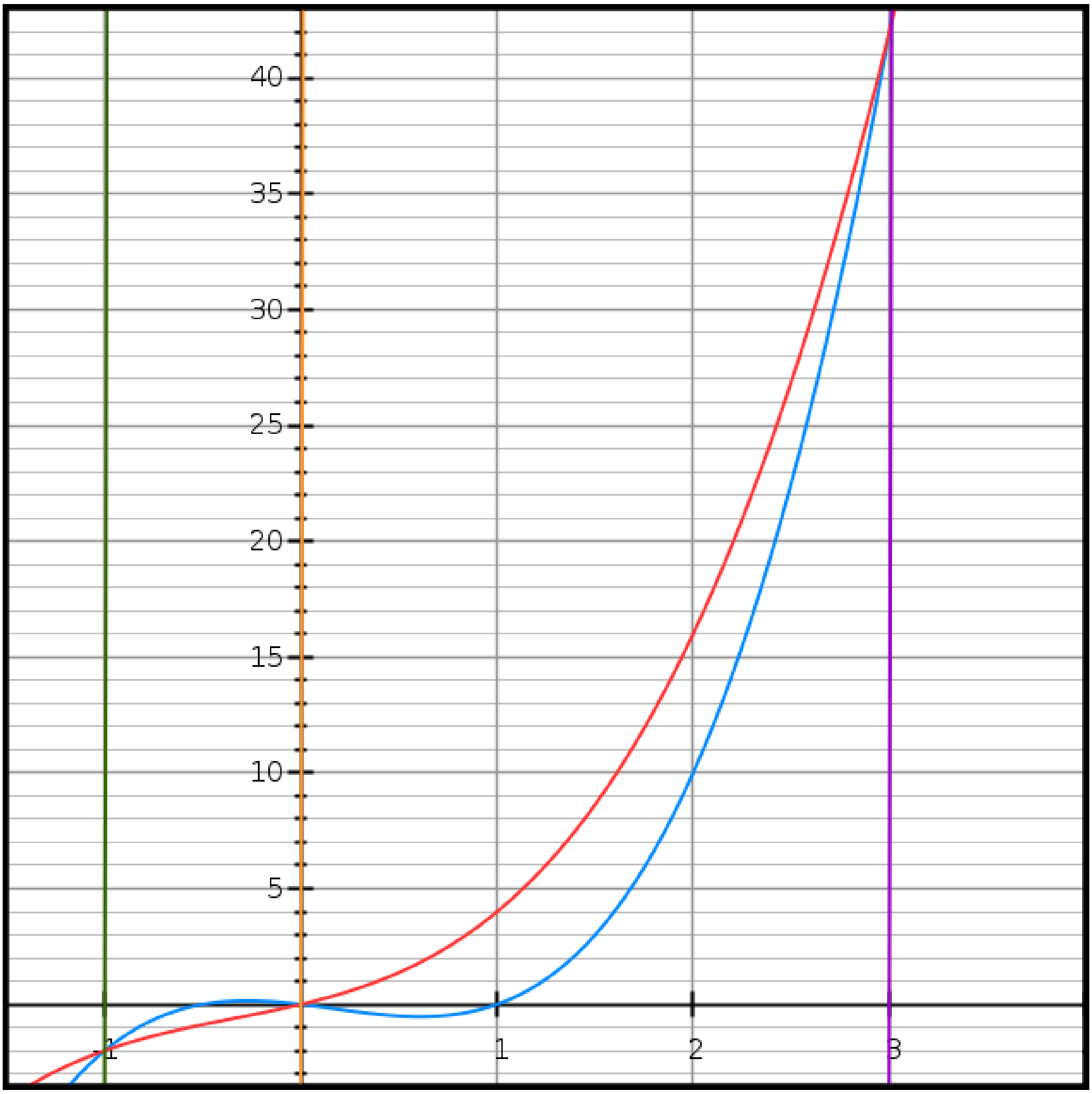
$$2x^3 - x^2 - x = x^3 + x^2 + 2x$$

$$x^3 - 2x^2 - 3x = 0 \quad , \quad \text{So: } x_1 = -1 \quad , \quad x_2 = 0 \quad , \quad x_3 = 3$$

Checking which function is bigger between these values:

Between [x=-1] & [x=0]	$f\left(-\frac{1}{2}\right) = 0$	$f(x) > g(x)$
	$g(-\frac{1}{2}) = -\frac{7}{8}$	
Between [x=0] & [x=3]	$f(1) = 0$	$f(x) < g(x)$
	$g(1) = 4$	

Graphical explanation:



	$f(x) = 2x^3 - x^2 - x$
	$g(x) = x^3 + x^2 + 2x$
	$x = -1$
	$x = 0$
	$x = 3$

$$A = \int_{-1}^{0} f(x) - g(x)\,dx + \int_{0}^{3} g(x) - f(x)\,dx$$

$$A = \int_{-1}^{0} (2x^3 - x^2 - x) - (x^3 + x^2 + 2x)\,dx$$

$$+ \int_{0}^{3} (x^3 + x^2 + 2x) - (2x^3 - x^2 - x)\,dx$$

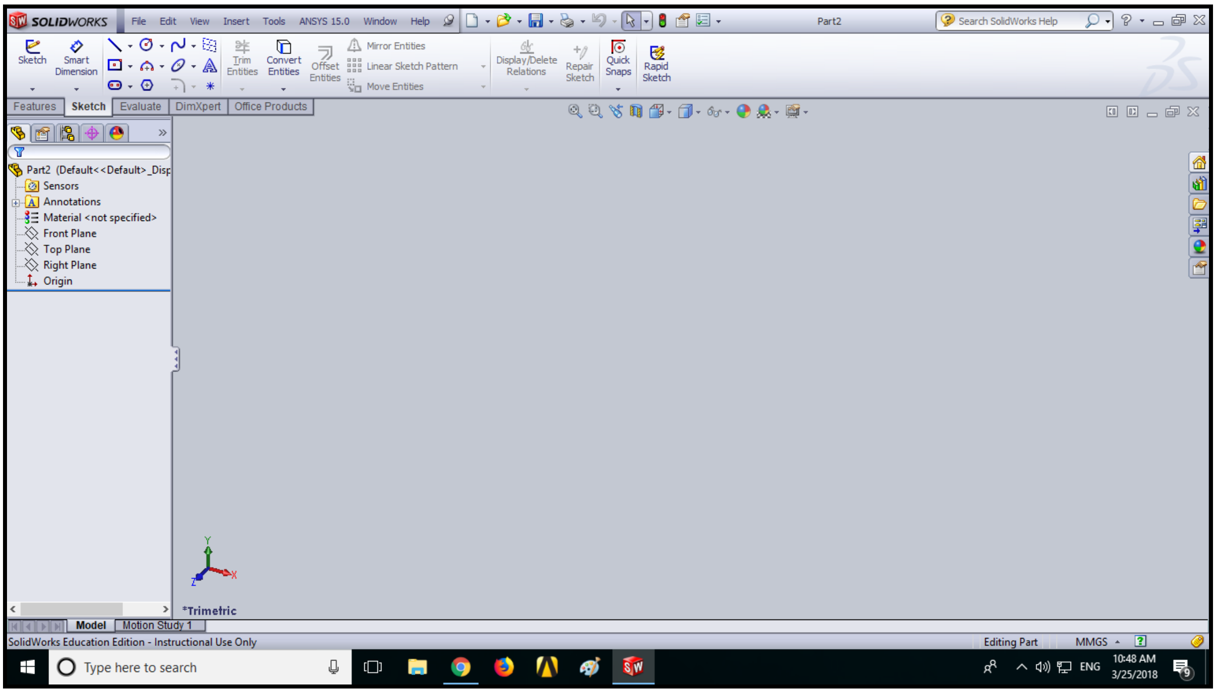
$$A = \int_{-1}^{0} x^3 - 2x^2 - 3x\,dx + \int_{0}^{3} -x^3 + 2x^2 + 3x\,dx$$

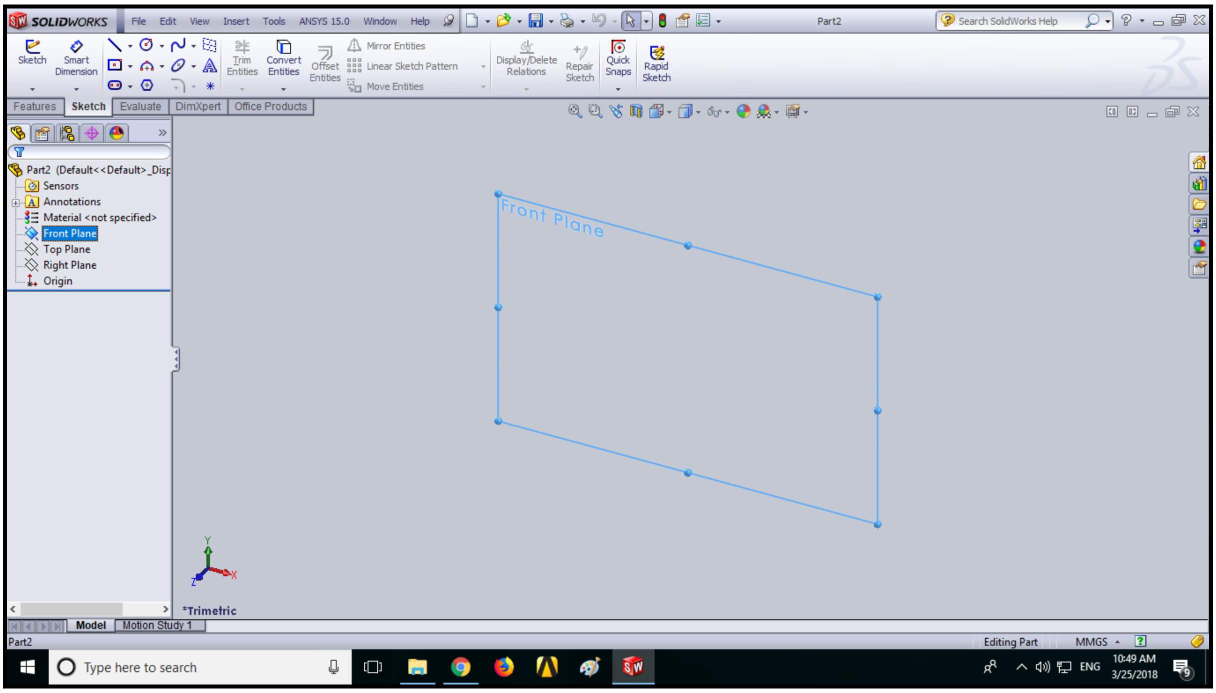
$$= \left(\frac{x^4}{4} - \frac{2x^3}{3} - \frac{3x^2}{2}\right)_{-1}^{0} + \left(\frac{-x^4}{4} + \frac{2x^3}{3} + \frac{3x^2}{2}\right)_{0}^{3} = \frac{7}{12} + \frac{45}{4}$$

$$= \frac{71}{6}\ Sq.Unit$$

Let’s confirm our solution by using “SolidWorks” software:

First, let’s perform area calculation between [x=-1] & [x=0]:

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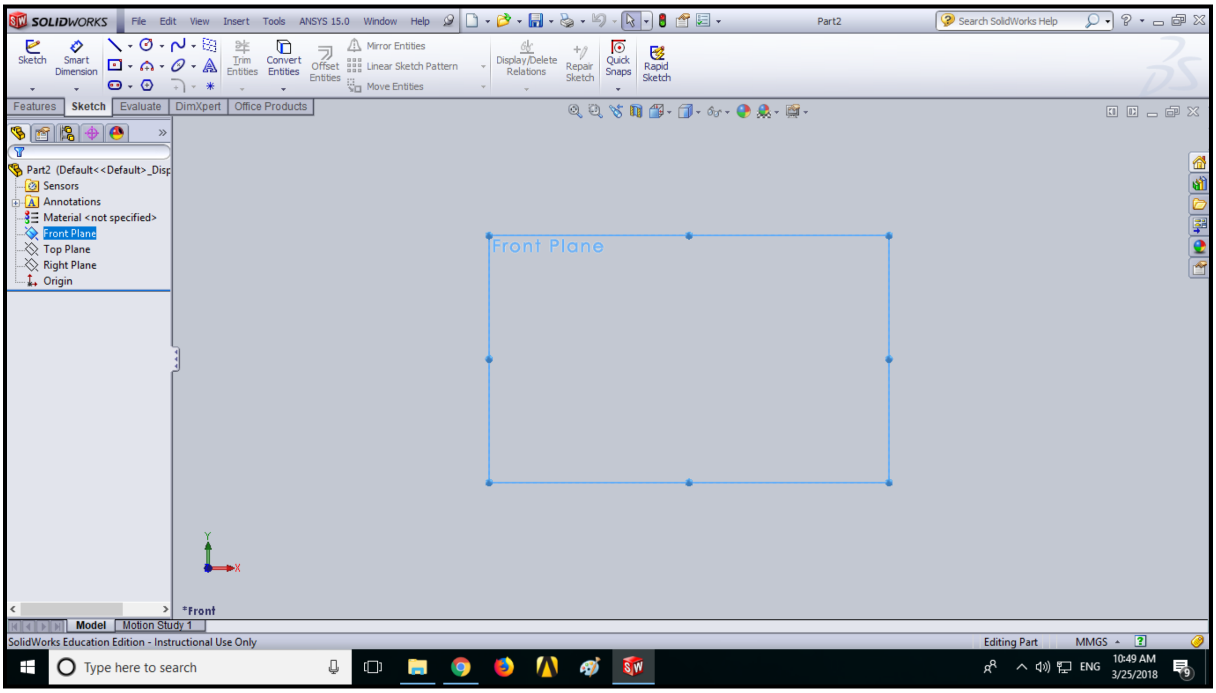
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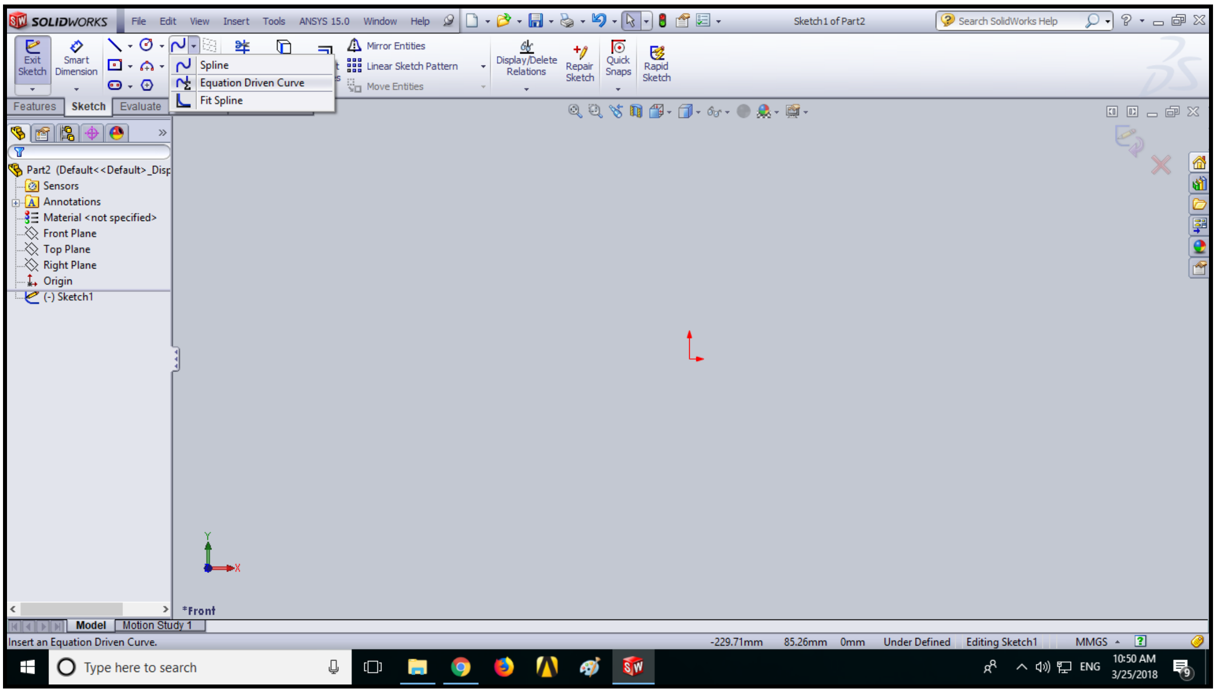
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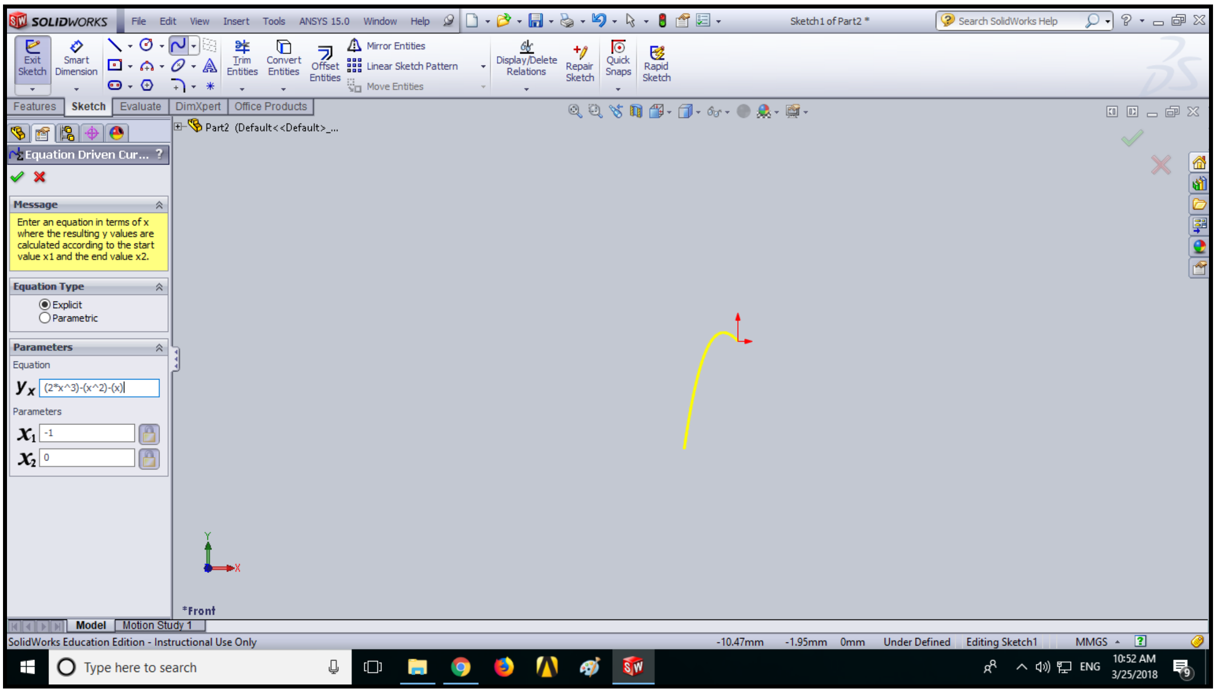
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Equation Type
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Parametric
Parameters
Equation
(2*x^3)-(x^2)-(x)
Parameters
-1
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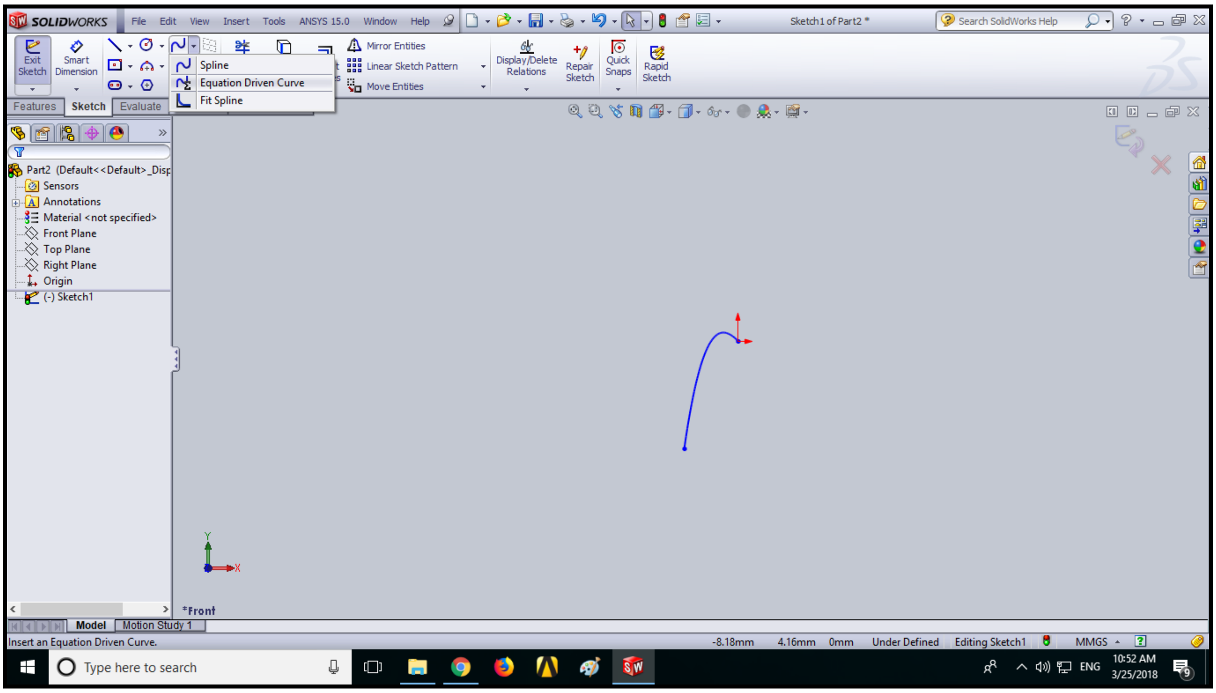
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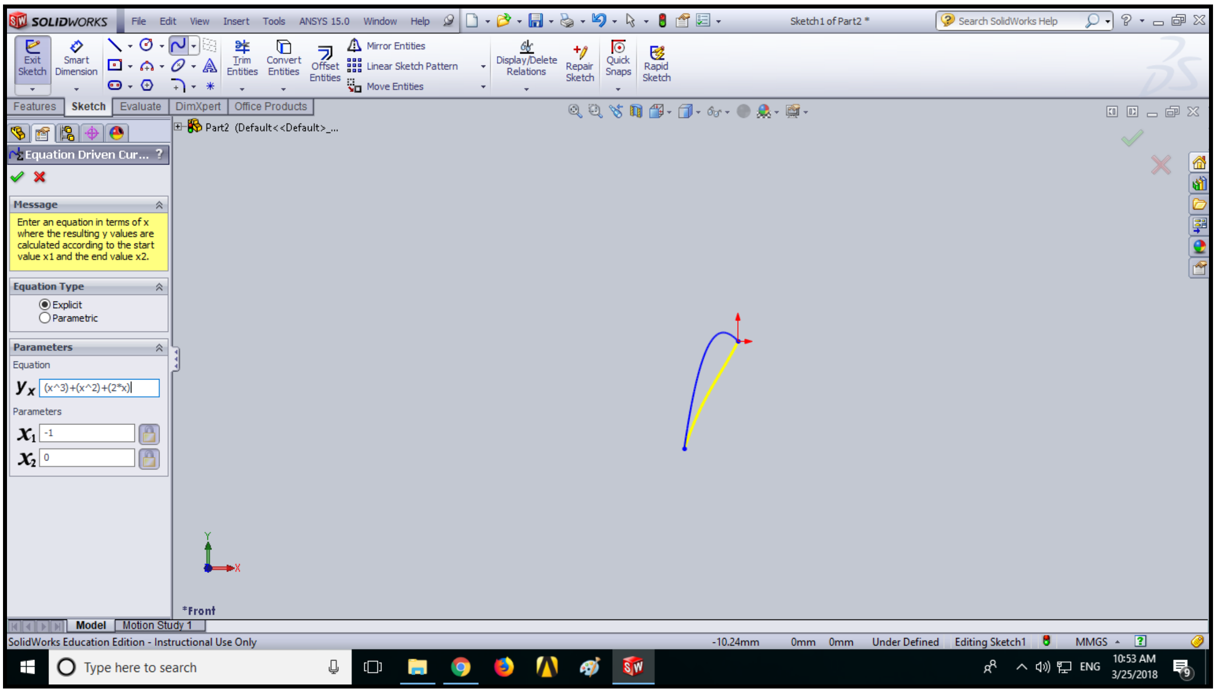
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Enter an equation in terms of x where the resulting y values are calculated according to the start value x1 and the end value x2.
Equation Type
Explicit
Parametric
Parameters
Equation
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Parameters
-1
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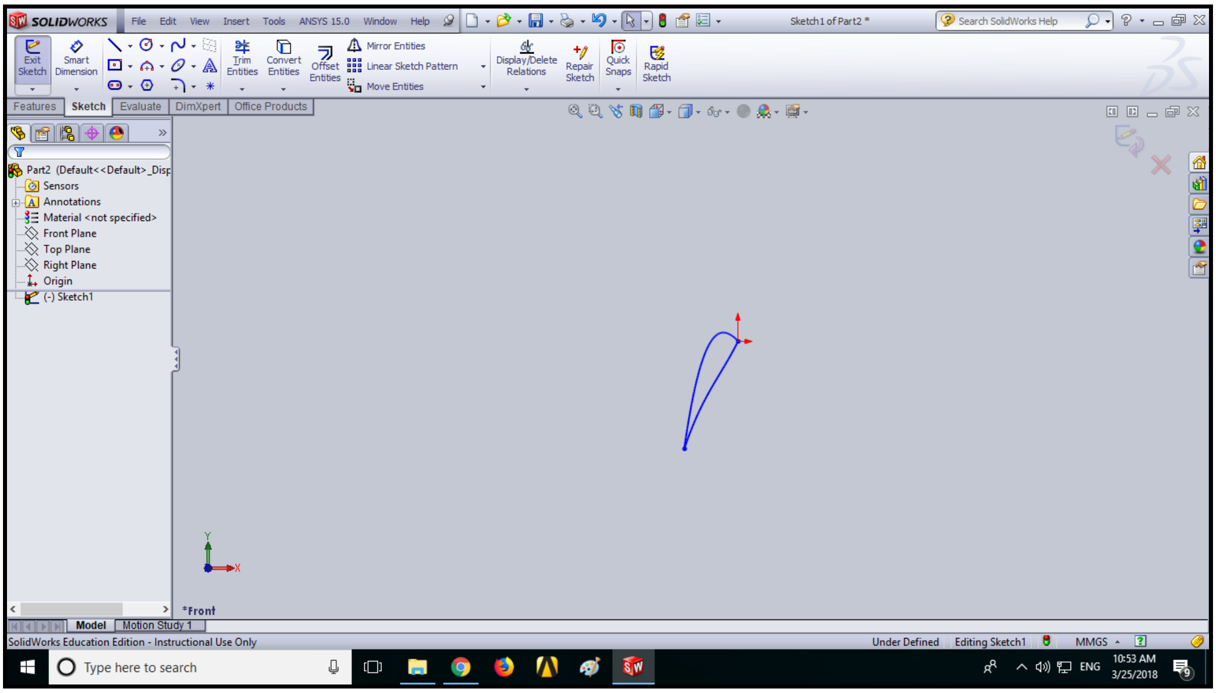

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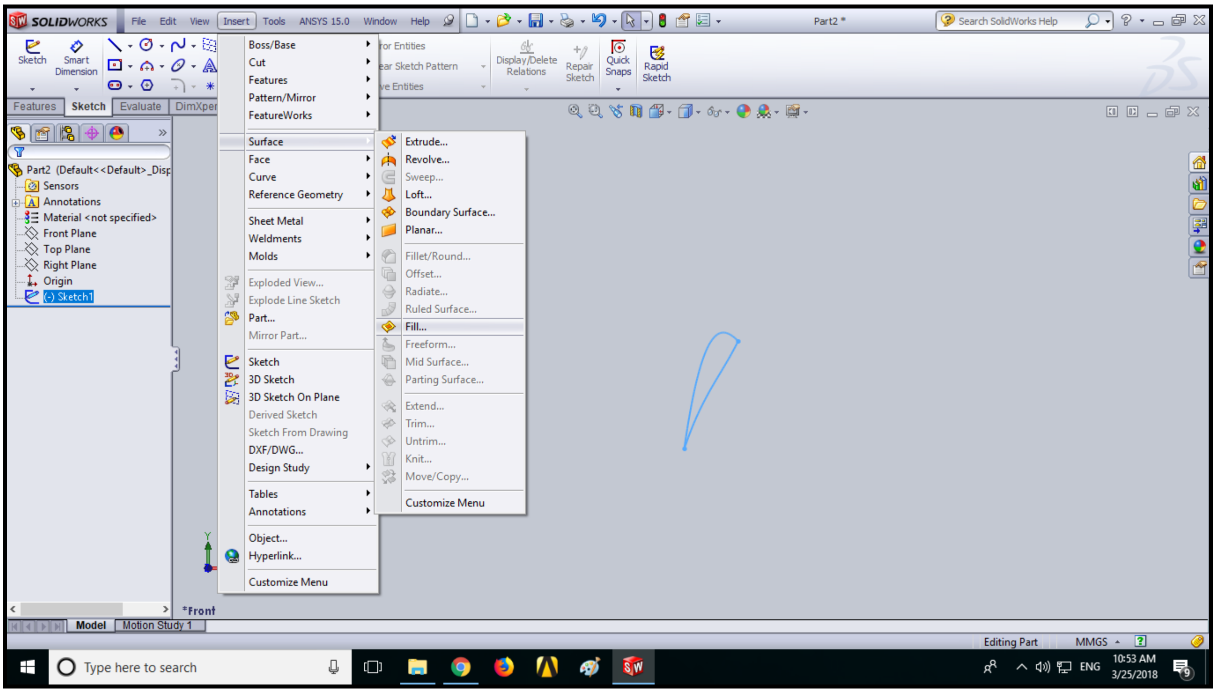

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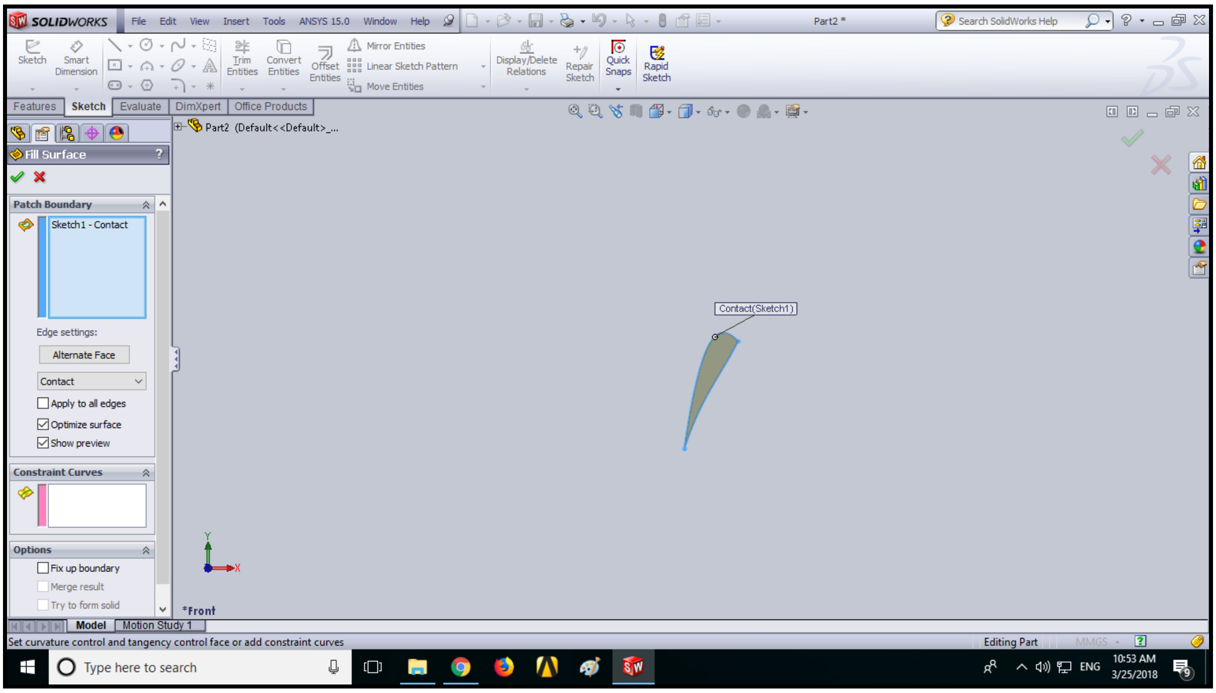
Fill Surface
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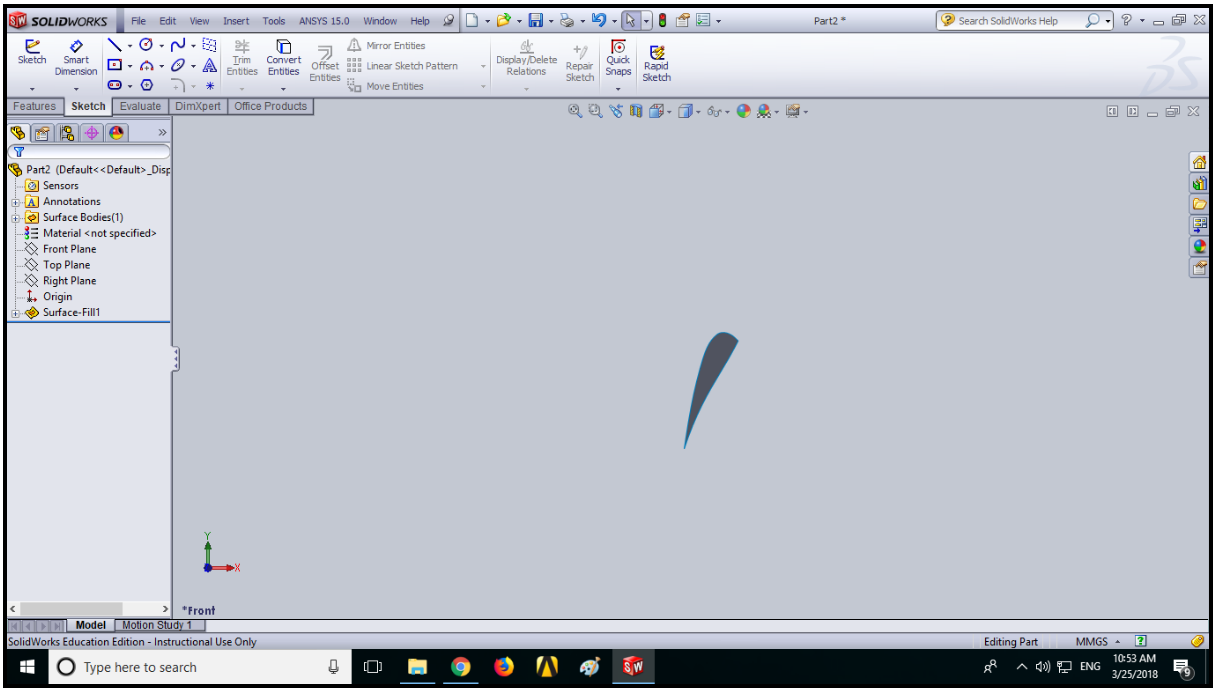
Edge settings:
Alternate Face
Contact
Apply to all edges
Optimize surface
Show preview
Constraint Curves
Options
Fix up boundary
Merge result
Try to form solid
Contact(Sketch1)
*Front
Set curvature control and tangency control face or add constraint curves
Editing Part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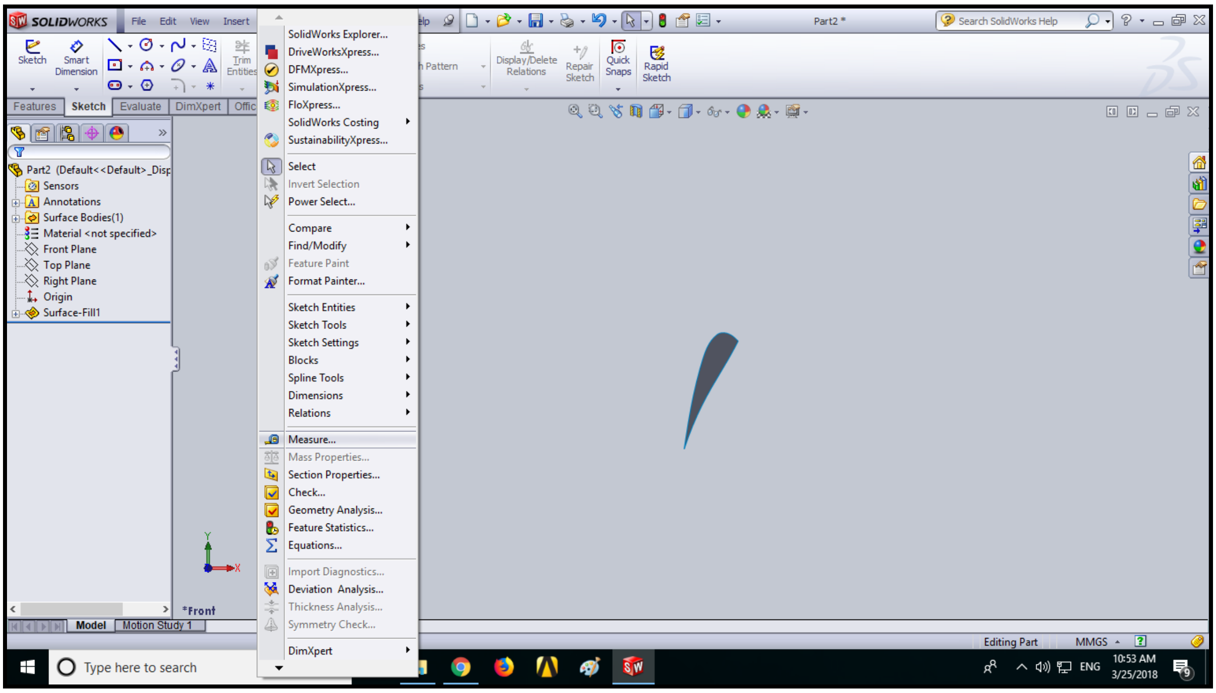
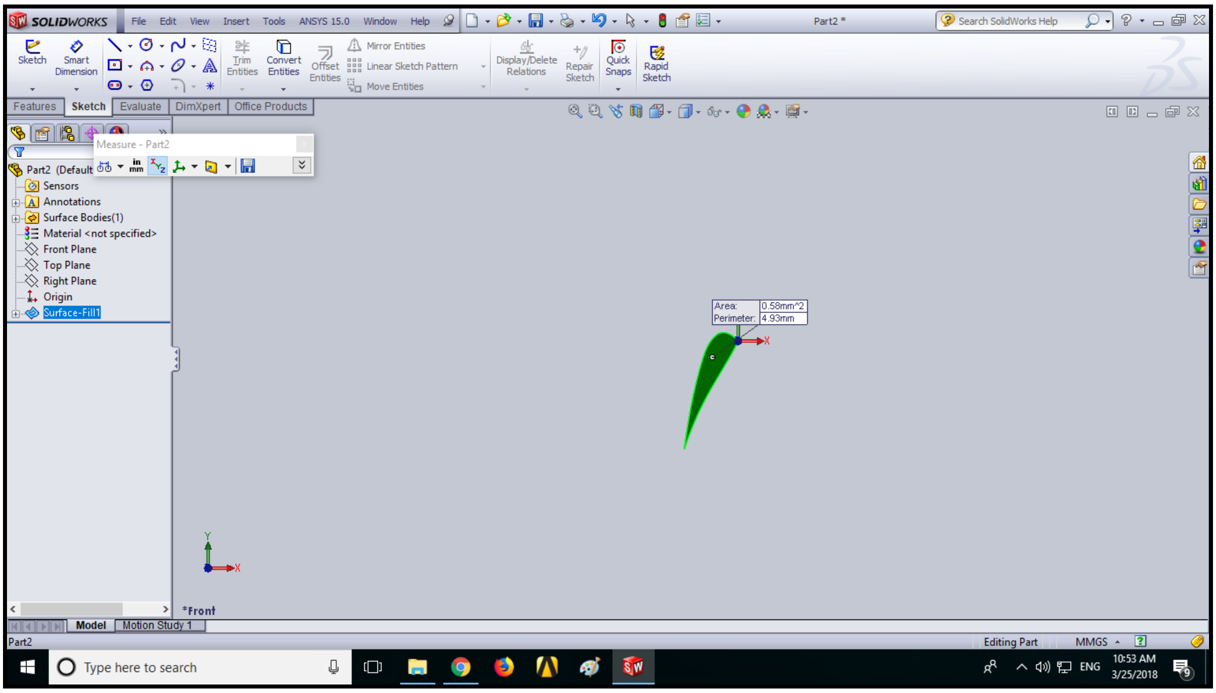
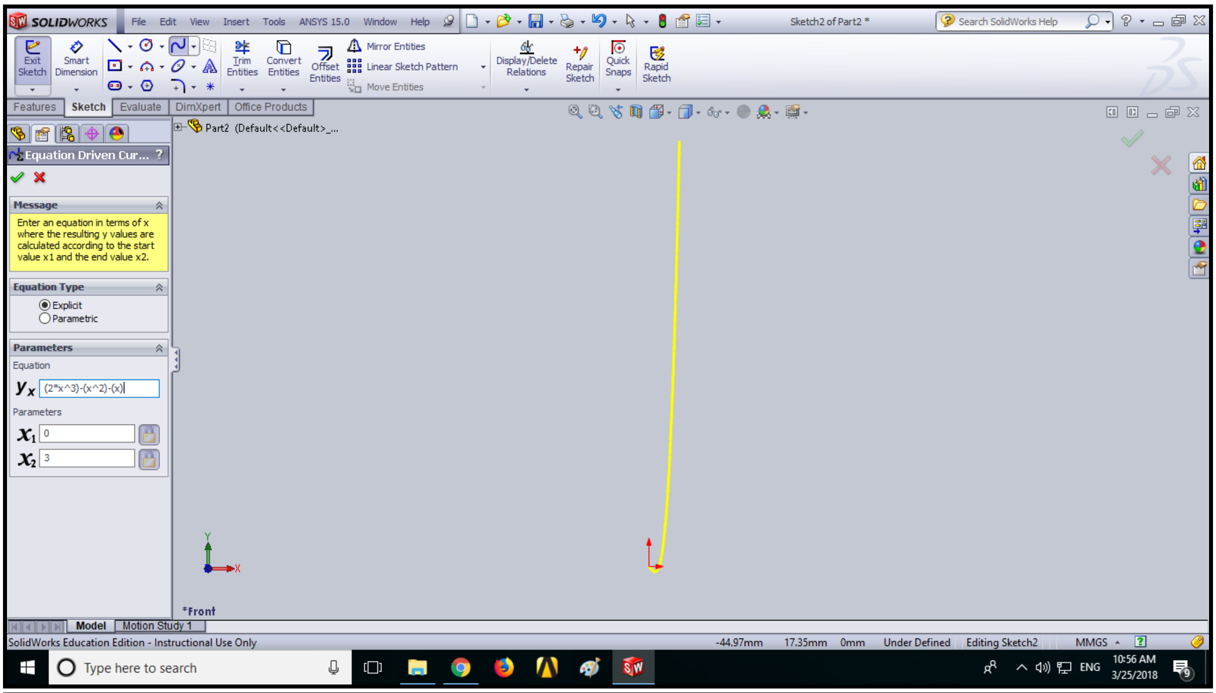
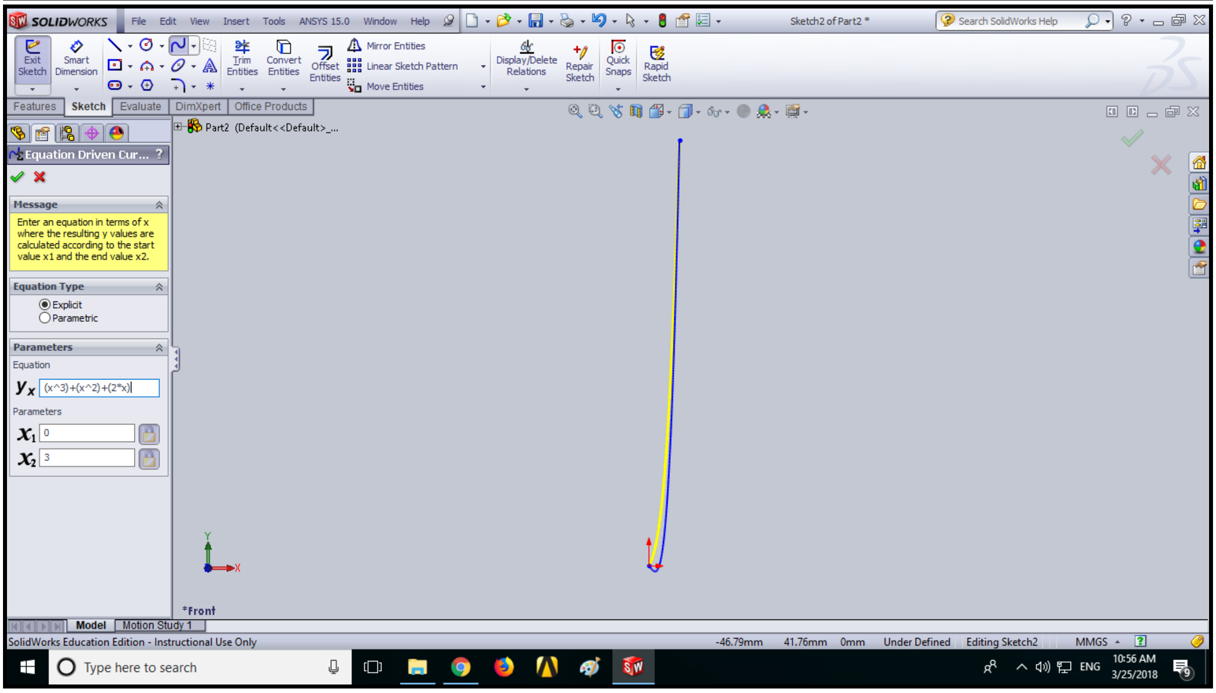


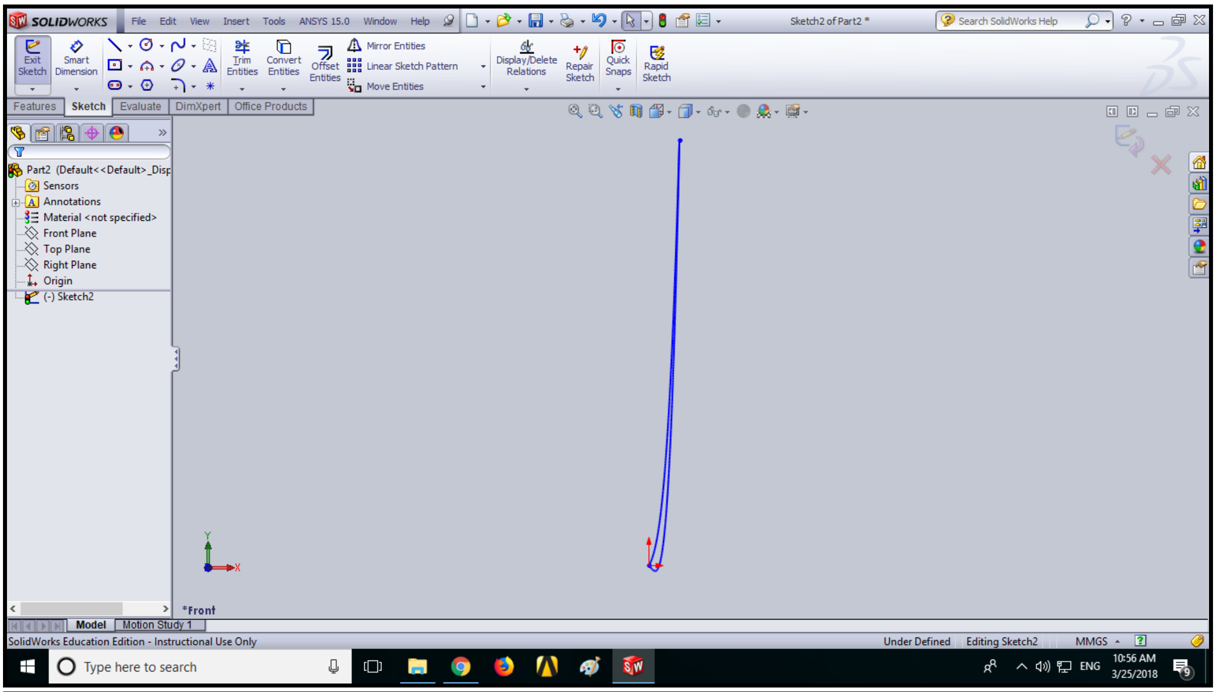


We can see that the area is approximately equal to $\frac{7}{12}$ “0.58 mm^2 as shown in the figure”

Now let's find the area between [x=0] & [x=3]:



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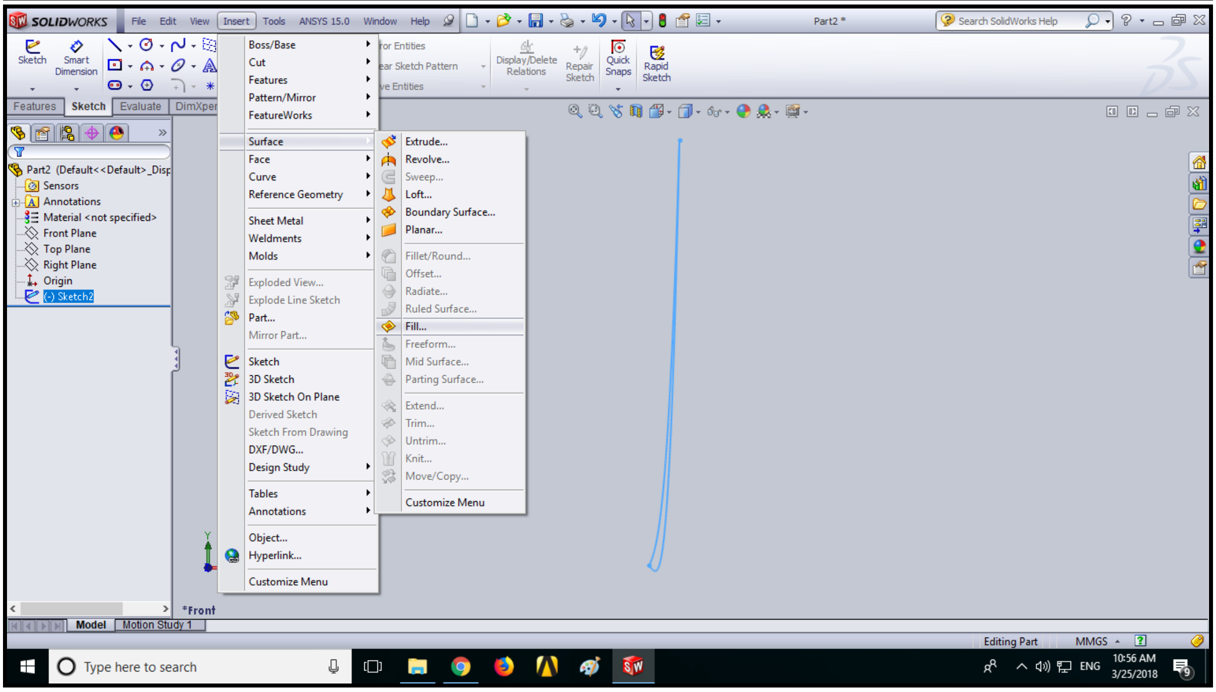
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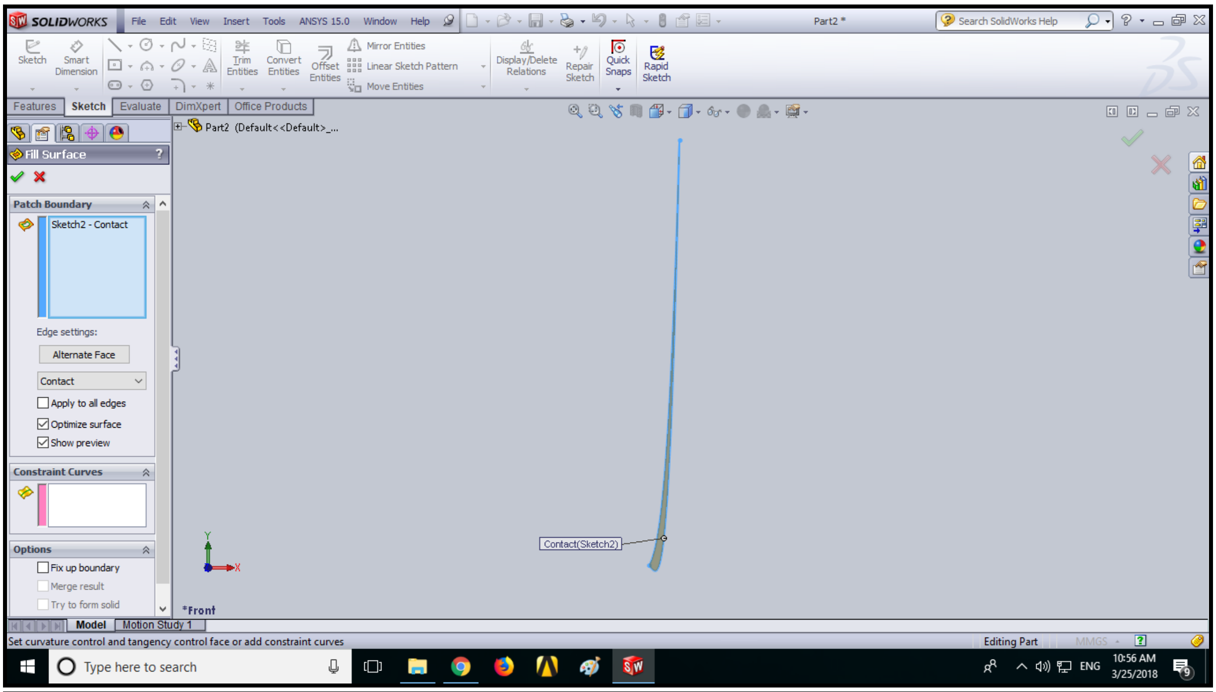
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Edge settings:
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Try to form solid
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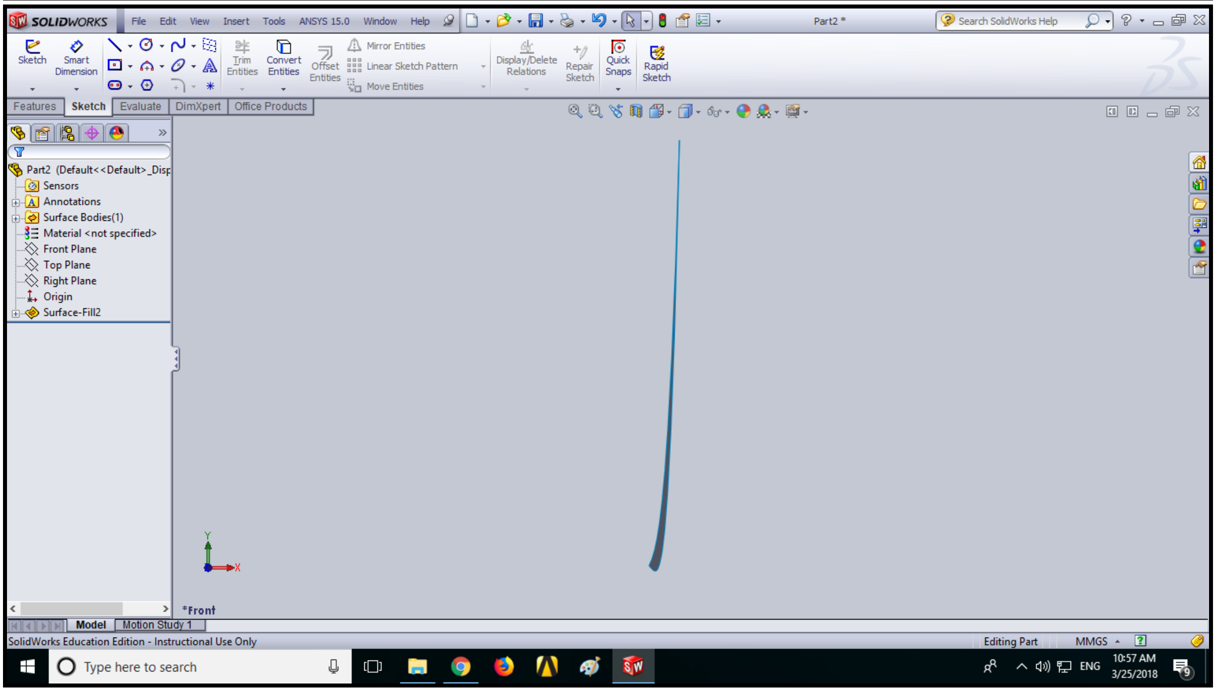
Set curvature control and tangency control face or add constraint curves
Editing Part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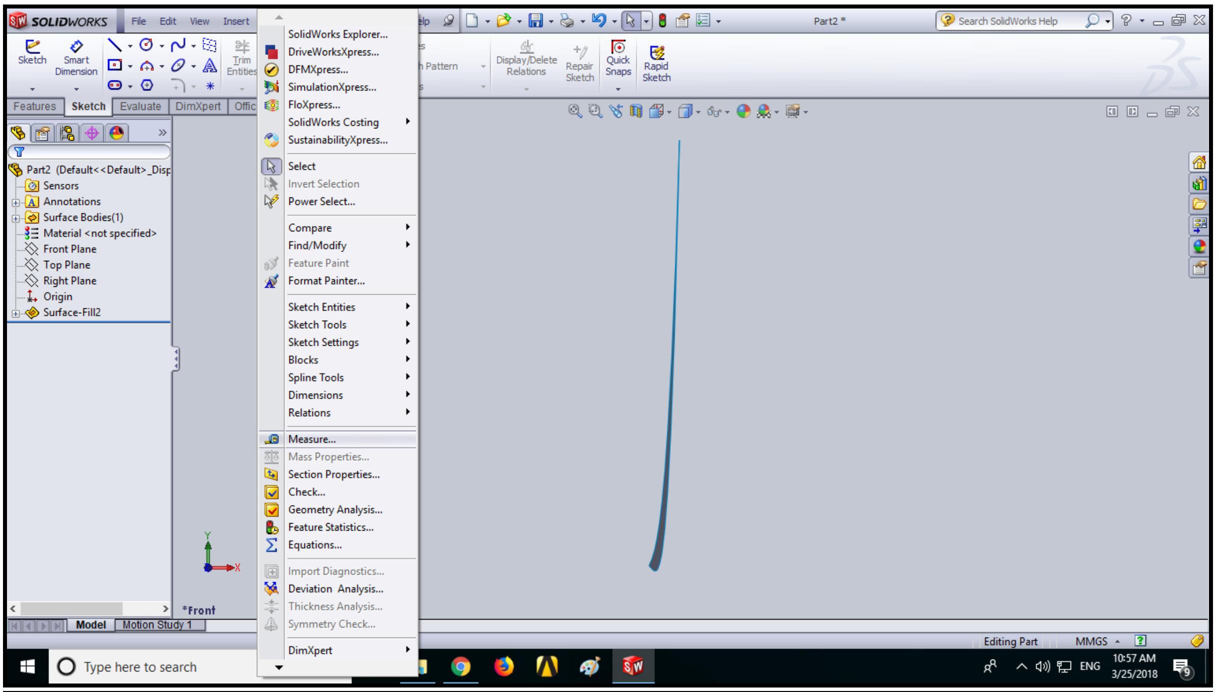
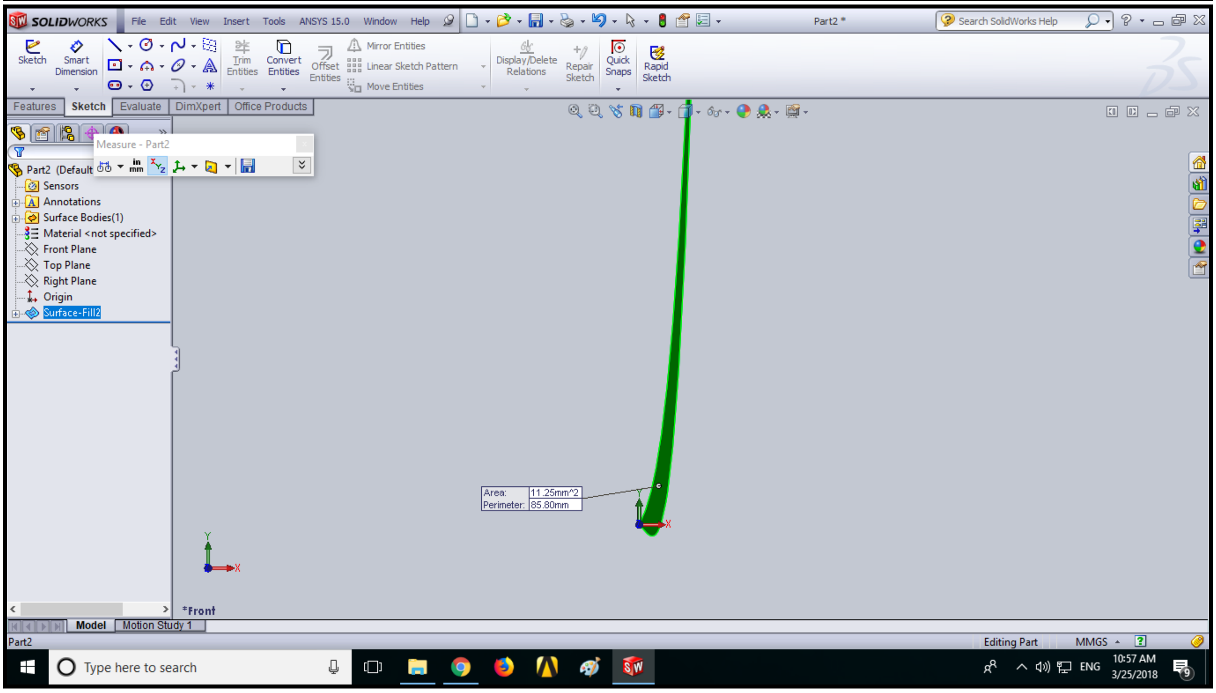




We can see that the area is equal to $\frac{45}{4}$ "11.25 mm^2 as shown in the figure"

Finally, by summing up those 2 values, we get: 11.83 mm^2

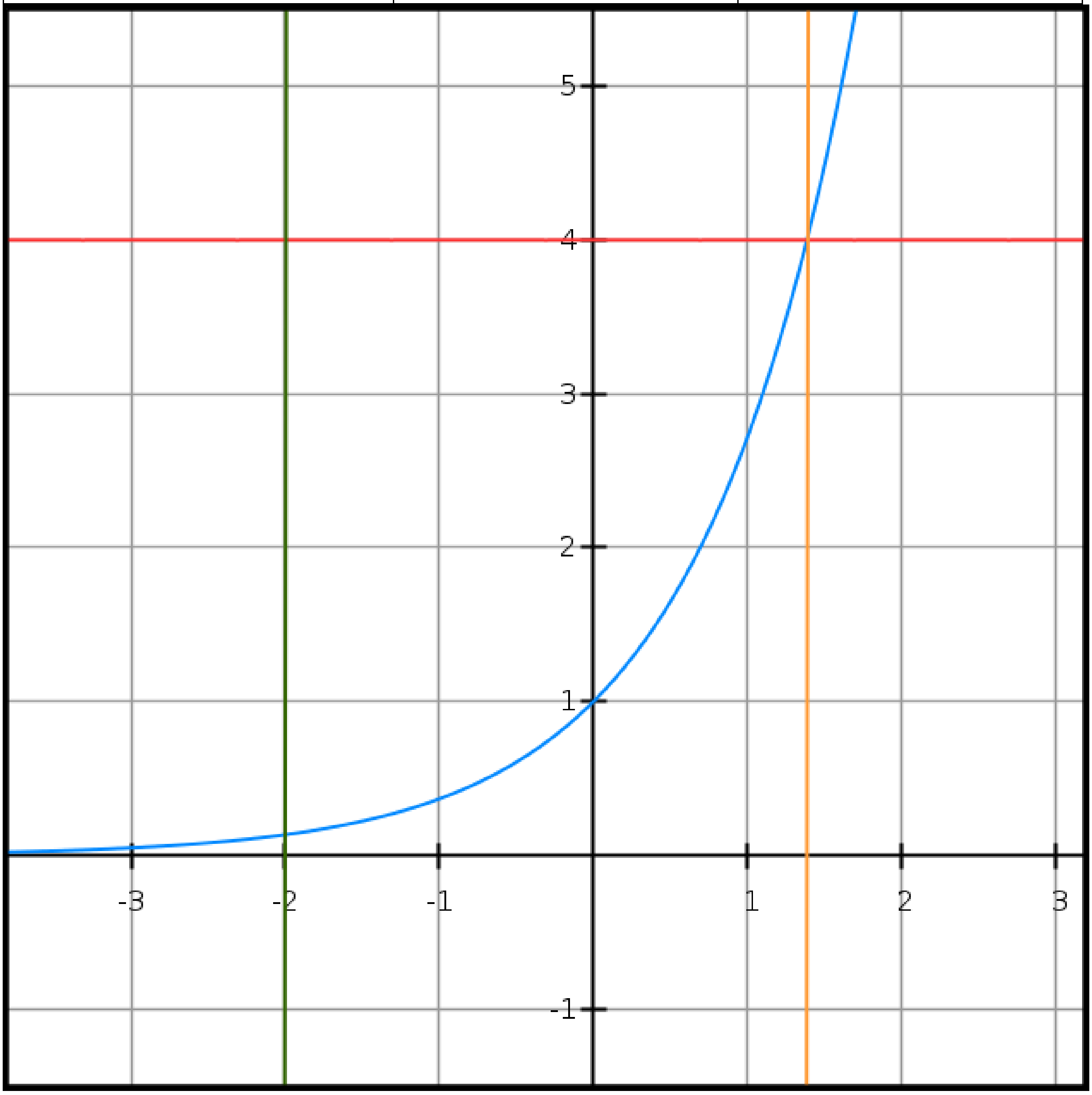
2- $f(x) = e^x$ and $g(x) = 4$ and $x = -2$

Equalizing both equations to find any x roots “boundaries”:

$$e^x = 4 \quad , \quad \ln(e^x) = \ln(4) \; , \; x = \ln(4) = 1.386$$

Checking which function is bigger between these values:

Between [x=-2] & [x=1.386]	$f(0) = 1$ $g(0) = 4$	$f(x) < g(x)$



	$f(x) = e^x$
	$g(x) = 4$
	$x = -2$
	$x = 1.386 \cong \ln(4)$

Now, let's solve by finding the area under g(x), and then the area under f(x) "between the x boundaries", and finally we just subtract them to get the area between both curves:

$$A_{g(x)} = \int_{-2}^{1.386} 4 \; dx = (4x)_{-2}^{1.386} = 4(1.386) - 4(-2) = 13.544$$

$$A_{f(x)} = \int_{-2}^{1.386} e^{x} \; dx = (e^{x})_{-2}^{1.386} = e^{(1.386)} - e^{(-2)} = 3.866$$

$$Area\ between\ f(x)\ \&\ g(x) = A = A_{g(x)} - A_{f(x)} = 13.544 - 3.866$$
$$= 9.678\ Sq.Unit$$