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BMT223 Assignment #4

1- Approximate the following using the derivative formula:

(a) $\tan(44°)$

$f(x) = \tan(x)$
$f'(x) = \sec^2(x)$
$x_o = 45° = \frac{\pi}{4}$
$X = 44° = \frac{11\pi}{45}$
$\Delta x = X - x_0 = \frac{11\pi}{45} - \frac{\pi}{4} = -\frac{\pi}{180}$

$$f(X) = f(x_o + \Delta x) \cong f(x_o) + \Delta x\, f'(x_o)$$

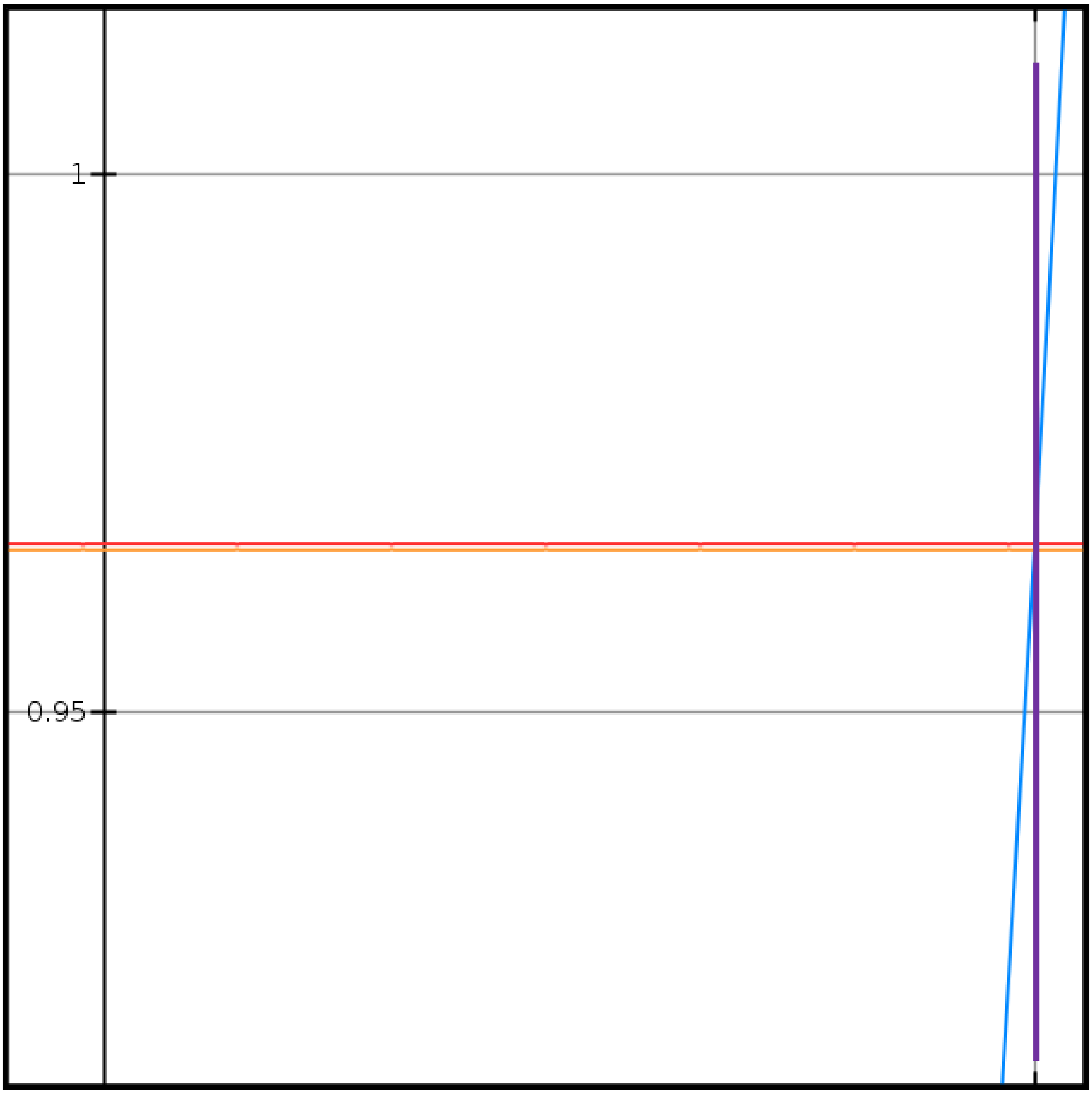
$$\tan\left(\frac{11\pi}{45}\right) \cong \tan\left(\frac{\pi}{4}\right) - \left(\frac{\pi}{180}\right)\left(\sec^2\left(\frac{\pi}{4}\right)\right)$$

$$\tan\left(\frac{11\pi}{45}\right) \cong 1 - \left(\frac{\pi}{180}\right)(2) \cong 1 - \frac{\pi}{90} \cong 0.96509 = Approximated\ value$$

$$Exact\ value = \tan\left(\frac{11\pi}{45}\right) = 0.96569$$

$$\varepsilon_{error\%} = \left|\frac{Approximated\ value - Exact\ value}{Exact\ value}\right| x100\% = 0.062\%$$

Graphical explanation:



	Main Function: [$f(x) = \tan(\mathrm{x})$]
	$x = \frac{11\pi}{45}$
	Approximated Value = 0.96509
	Exact value = 0.96569

(b) $\sqrt{24}$

[Use these 2 values: “16” & “25”, and then compare the results and write your conclusion]

Let’s start with 16:

$f(x) = \sqrt{x}$
$f'(x) = \dfrac{1}{2\sqrt{x}}$
$x_o = 16$
$X = 24$
$\Delta x = X - x_0 = 24 - 16 = 8$

$$f(X) = f(x_o + \Delta x) \cong f(x_o) + \Delta x\, f'(x_o)$$

$$\sqrt{24} \cong \sqrt{16} + (8)\left(\frac{1}{2\sqrt{16}}\right)$$

$$\sqrt{24} \cong 4 + (8)\left(\frac{1}{8}\right) \cong 4 + 1 \ \cong 5 = Approximated\ value$$

$$Exact\ value = \sqrt{24} = 4.89898$$

$$\varepsilon_{error\%} = \left|\frac{Approximated\ value - Exact\ value}{Exact\ value}\right| x100\% = 2.062\%$$

Now let’s try 25:

$f(x) = \sqrt{x}$
$f'(x) = \dfrac{1}{2\sqrt{x}}$
$x_o = 25$
$X = 24$
$\Delta x = X - x_0 = 24 - 25 = -1$

$$f(X) = f(x_o + \Delta x) \cong f(x_o) + \Delta x\, f'(x_o)$$

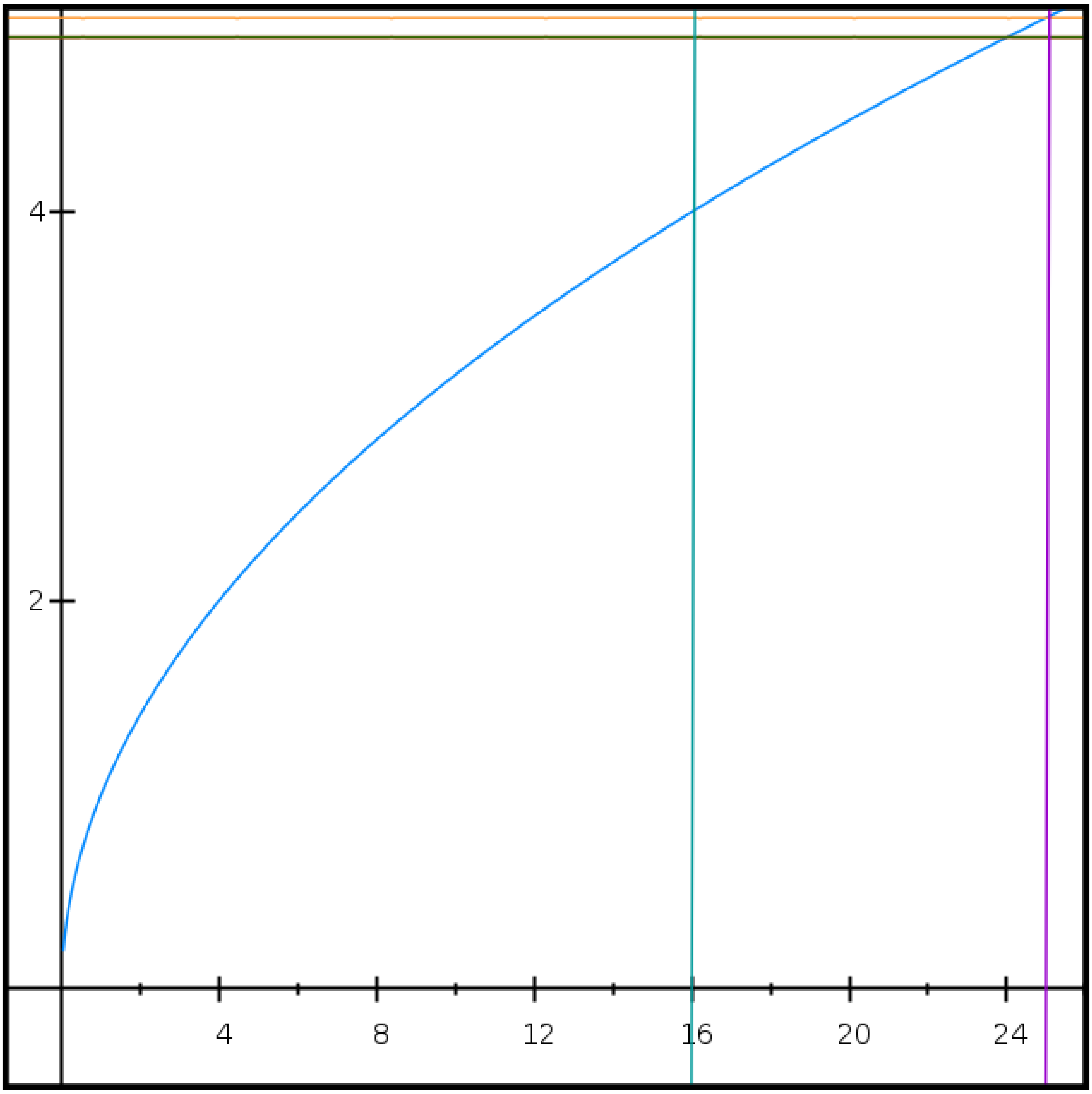
$$\sqrt{24} \cong \sqrt{25} - (1)\left(\frac{1}{2\sqrt{25}}\right)$$

$$\sqrt{24} \cong 5 - \left(\frac{1}{10}\right) \cong 5 - 0.1 \;\cong 4.9 = Approximated\ value$$

$$Exact\ value = \sqrt{24} = 4.89898$$

$$\varepsilon_{error\%} = \left|\frac{Approximated\ value - Exact\ value}{Exact\ value}\right| x100\% = 0.021\%$$

Comparison:



	Main Function: [$f(x) = \sqrt{x}$]
	$x = 25$
	$x = 16$
	Approximated Value = 5 [using x=16]
	Approximated Value = 4.9 [using x=25]
	Exact value = 4.89898

You can see that approximating by using [x=25] is much more accurate than by using [x=16].

By observing the graph above, you can see that the approximated value by using [x=25] is almost overlapping with the exact value.

Conclusion:

The reasons behind the high accuracy when using [x=25] can be explained by observing both the distance between x=24 and x=25, and the relatively high radius of curvature of the function’s curve at that specific location, which can lead to more accurate results when performing approximation.

(c) $\ln(1.05)$

$f(x) = \ln(x)$
$f'(x) = \frac{1}{\mathrm{x}}$
$x_o = 1$
$X = 1.05$
$\Delta x = X - x_0 = 1.05 - 1 = 0.05$

$$f(X) = f(x_o + \Delta x) \cong f(x_o) + \Delta x\, f'(x_o)$$

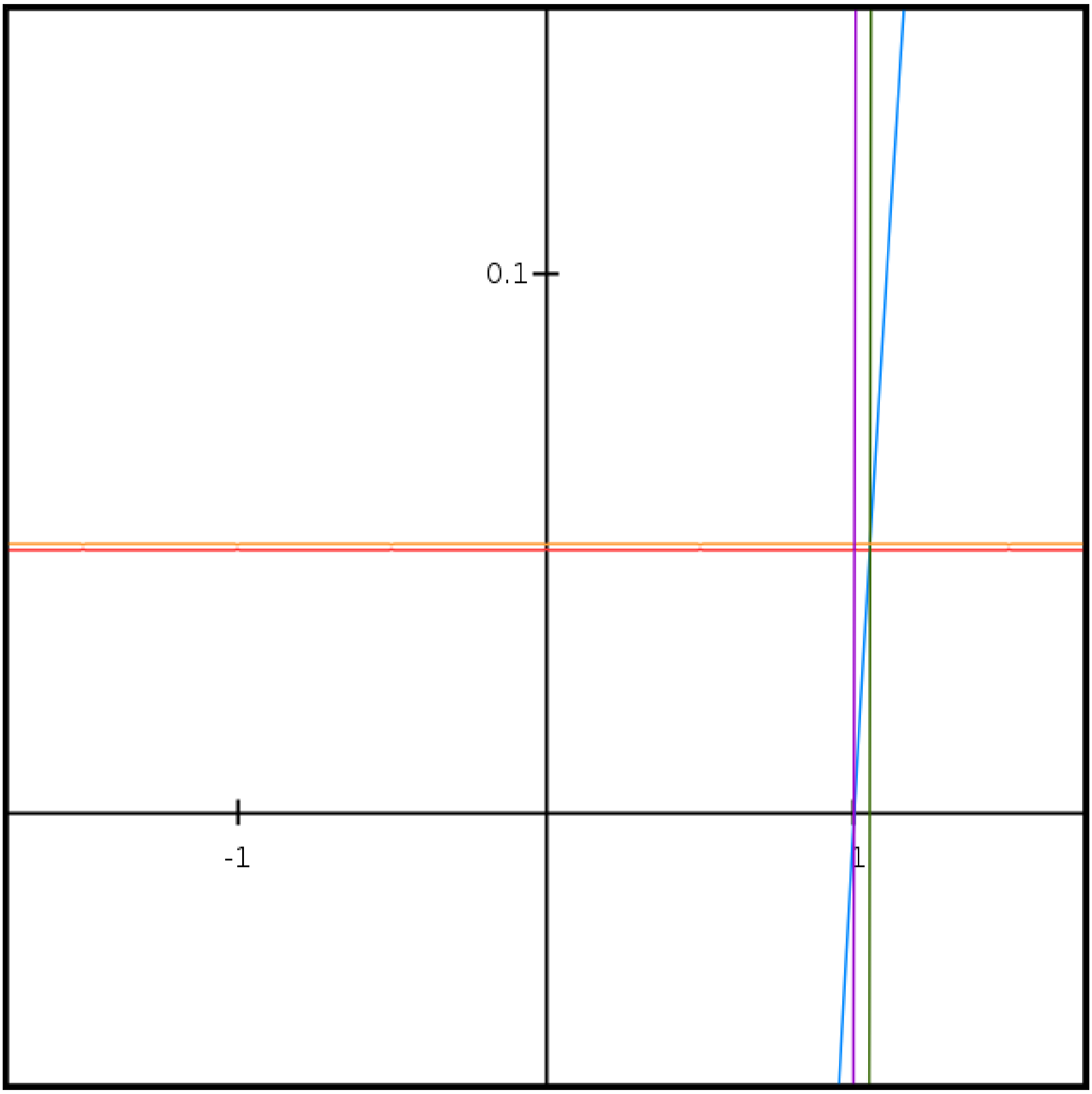
$$\ln(1.05) \cong \ln(1) + (0.05)\left(\frac{1}{1}\right)$$

$$\ln(1.05) \cong 0 + 0.05 \cong 0.05 = Approximated\ value$$

$$Exact\ value = \ln(1.05) = 0.04879$$

$$\varepsilon_{error\%} = \left|\frac{Approximated\ value - Exact\ value}{Exact\ value}\right| x100\% = 2.48\%$$

Graphical explanation:



	Main Function: [$f(x) = \ln(x)$]
	$x = 1$
	Approximated Value = 0.05
	$x = 1.05$
	Exact value = 0.04879

2- Using the first derivative and second derivative tests, determine the location of all maximum, minimum and inflexion points for:

a. $f(x) = \frac{2x^3}{3} - \frac{5x^2}{2} + 1$

$$f'(x) = 2x^2 - 5x$$

$$f''(x) = 4x - 5$$

$$f'(x) = 2x^2 - 5x = 0 \quad , \qquad \text{so: } x_1 = 0 \quad , \quad x_2 = \frac{5}{2}$$

1st Derivative test for point A (0,1):

$f'(-1) = 7$	> 0	So: Point A (0,1) is Relative Maximum
$f'(0) = 0$	$= 0$	
$f'\left(\frac{1}{2}\right) = -2$	< 0	

1st Derivative test for point B ($\frac{5}{2}, \frac{-101}{24}$):

$f'(2) = -2$	< 0	So: Point B ($\frac{5}{2}, \frac{-101}{24}$) is Relative Minimum
$f'\left(\frac{5}{2}\right)$ =0	$= 0$	
$f'(3) = 3$	> 0	

2nd Derivative test for point A (0,1):

$f''(0) = -5$	< 0	So: Point A (0,1) is Relative Maximum

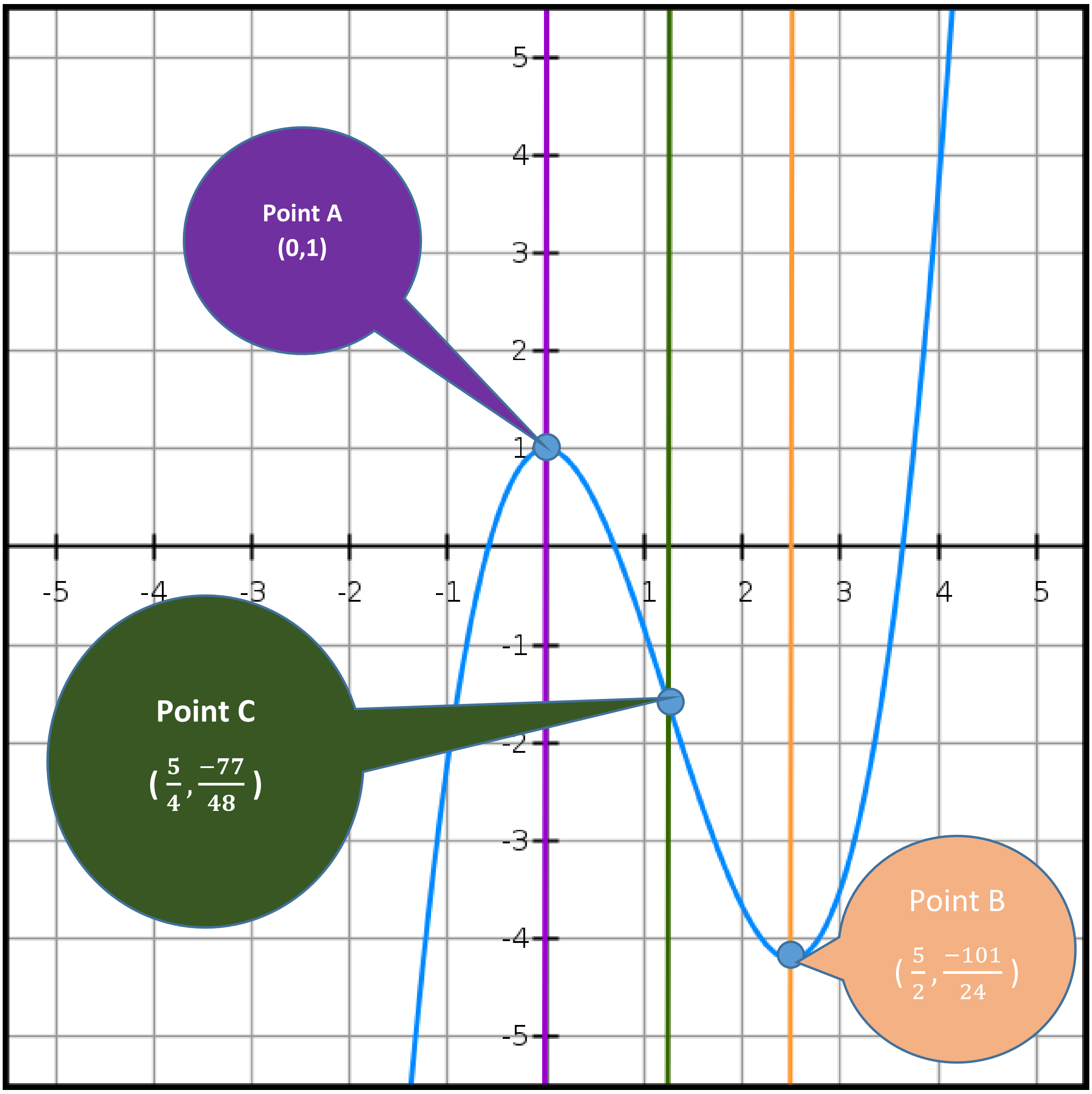
2nd Derivative test for point B ($\frac{5}{2}, \frac{-101}{24}$):

$f''\left(\frac{5}{2}\right) = 5$	> 0	So: Point B ($\frac{5}{2}, \frac{-101}{24}$) is Relative Minimum

$$f''(x) = 4x - 5 = 0 \qquad , \qquad \text{so: } x = \frac{5}{4}$$

So: point C ($\frac{5}{4}, \frac{-77}{48}$) is an inflexion point

Graphical explanation:



	Main Function: [$f(x) = \frac{2x^3}{3} - \frac{5x^2}{2} + 1$]
	$x = 0$
	$x = \frac{5}{4}$
	$x = \frac{5}{2}$

b. $f(x) = \frac{x^4}{4} - \frac{x^2}{2} + 2$

$$f'(x) = x^3 - x$$
$$f''^{(x)} = 3x^2 - 1$$

$f'(x) = x^3 - x = 0$, so: $x_1 = -1$, $x_2 = 0,$ $x_3 = 1$

1st Derivative test for point D $(-1, \frac{7}{4})$:

$f'(-2) = -6$	< 0	So: Point D $(-1, \frac{7}{4})$ is Relative Minimum
$f'(-1) = 0$	$= 0$	
$f'\left(-\frac{1}{2}\right) = \frac{3}{8}$	> 0	

1st Derivative test for point E (0 ,2):

$f'\left(-\frac{1}{4}\right) = \frac{15}{64}$	> 0	So: Point E (0 ,2) is Relative Maximum
$f'(0) = 0$	$= 0$	
$f'\left(\frac{1}{4}\right) = -\frac{15}{64}$	< 0	

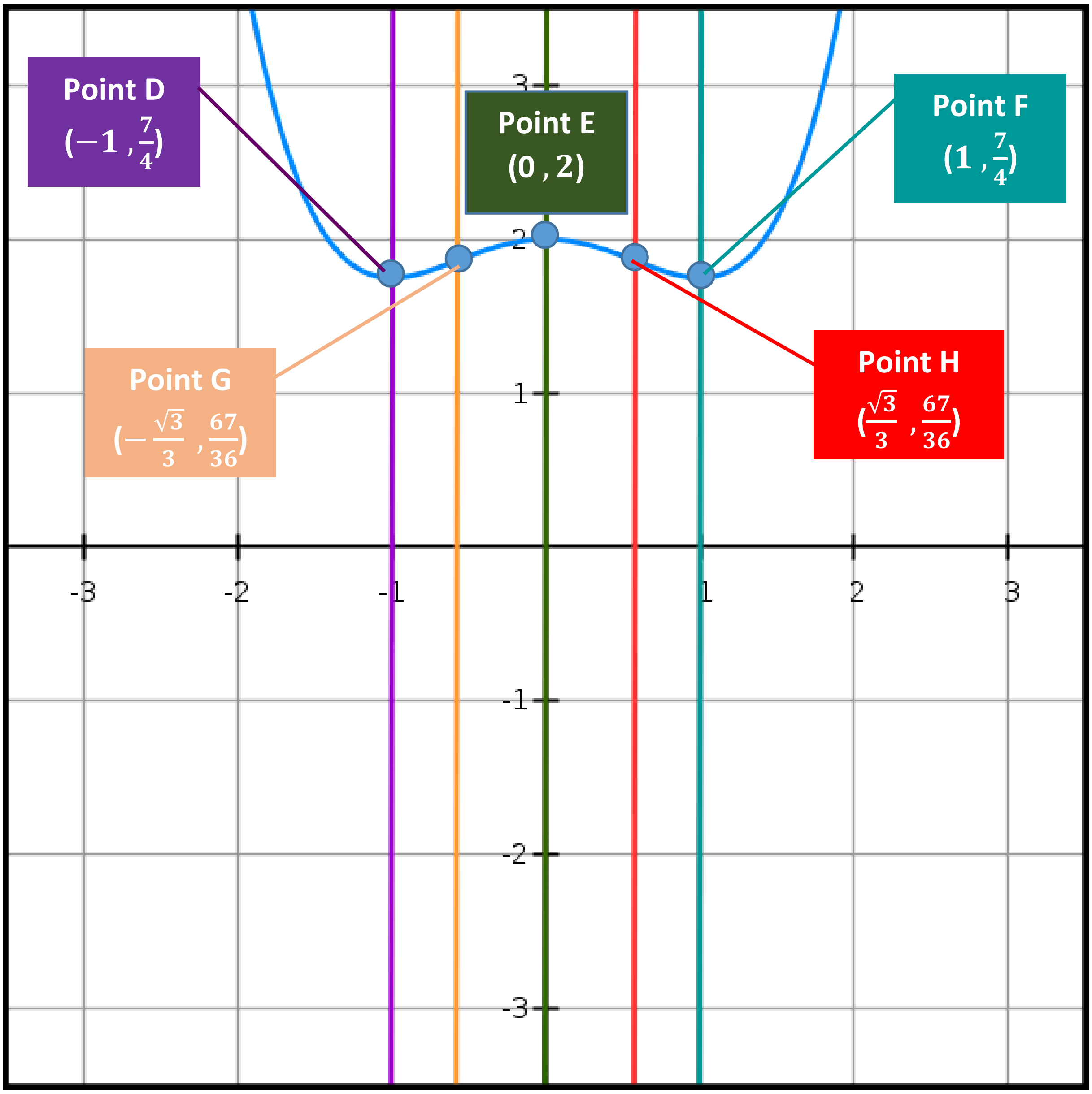
1st Derivative test for point F $(1, \frac{7}{4})$:

$f'\left(\frac{1}{2}\right) = -\frac{3}{8}$	< 0	So: Point F $(1, \frac{7}{4})$ is Relative Minimum
$f'(1) = 0$	$= 0$	
$f'\left(\frac{3}{2}\right) = \frac{15}{8}$	> 0	

$f''^{(x)} = 3x^2 - 1$, so: $x_1 = -\frac{\sqrt{3}}{3}$, $x_2 = \frac{\sqrt{3}}{3}$

So: Point G $(-\frac{\sqrt{3}}{3}, \frac{67}{36})$ and Point H $(\frac{\sqrt{3}}{3}, \frac{67}{36})$ are inflexion points

Graphical explanation:



	Main Function: [$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + 2$]
	$x = -1$
	$x = -\frac{\sqrt{3}}{3}$
	$x = 0$
	$x = \frac{\sqrt{3}}{3}$
	$x = 1$

c. $f(x) = x^3$

$$f'(x) = 3x^2$$
$$f''(x) = 6x$$
$$f'(x) = 3x^2 = 0 \quad , \qquad \text{so: } x = 0$$

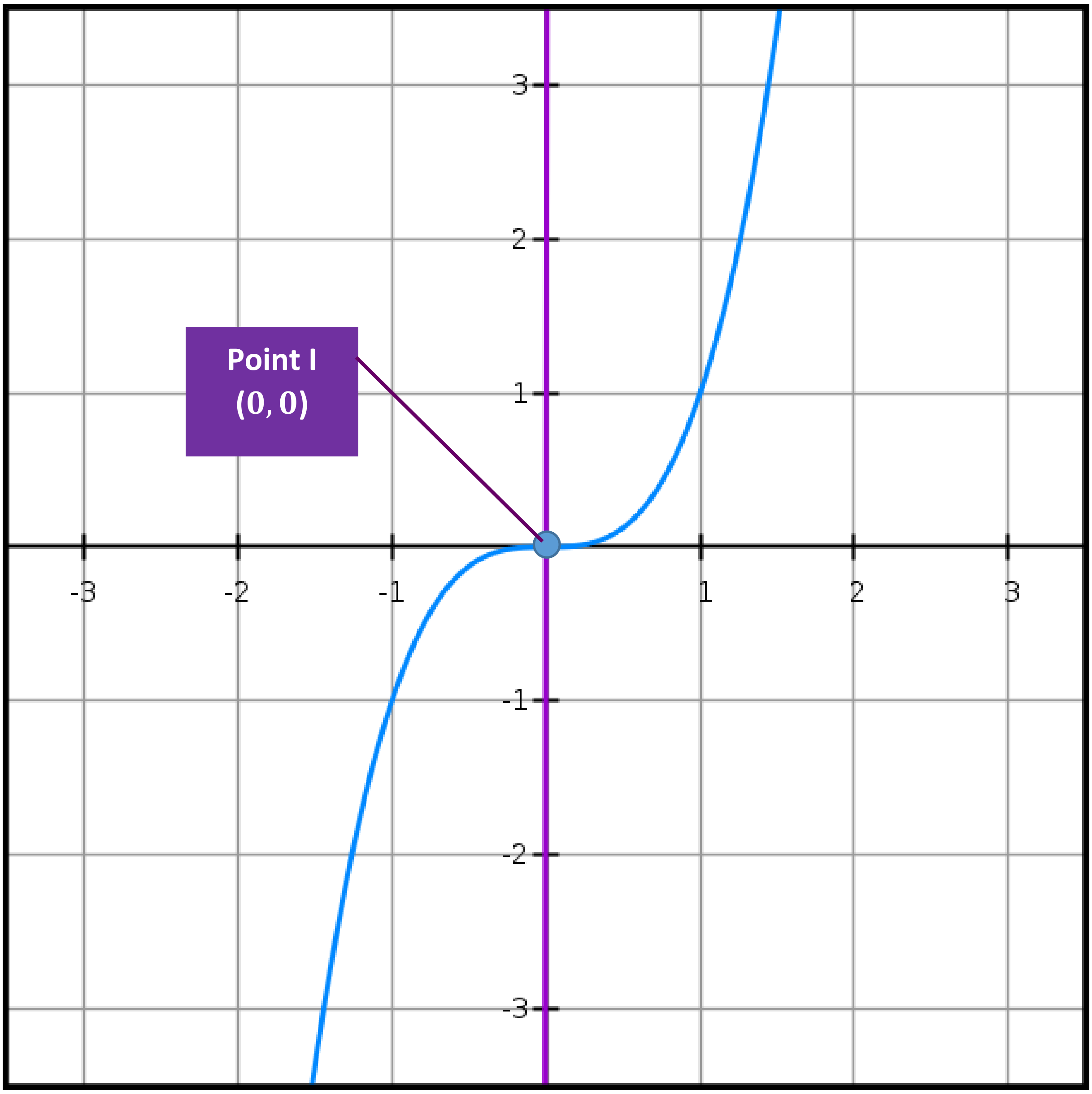
1st Derivative test for point I (0,0):

$f'(-1) = 3$	> 0	So: Point I (0,0) neither can be maximum nor minimum
$f'(0) = 0$	$= 0$	
$f'(1) = 3$	> 0	

$$f''^{(x)} = 6x \quad , \qquad \text{so: } x = 0$$

So: Point I (0,0) is an inflexion point

Graphical explanation:



	Main Function: [$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + 2$]
	$x = 0$