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BMT223 Assignment #1

By using the general law of derivatives, prove that:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Answer:

$$f(x) = x^n$$

$$f'(x) = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right) = \lim_{\Delta x \to 0}\left(\frac{(x+\Delta x)^n - (x)^n}{\Delta x}\right)$$

By using Binomial theorem for the expansion of $(x+\Delta x)^n$:

$$(x+\Delta x)^n = \binom{n}{0} x^n \Delta x^0 + \binom{n}{1} x^{n-1} \Delta x^1 + \binom{n}{2} x^{n-2} \Delta x^2 + \cdots + \binom{n}{n-1} x^1 \Delta x^{n-1} + \binom{n}{n} x^0 \Delta x^n$$

$$f'(x) = \lim_{\Delta x \to 0}\left(\frac{\binom{n}{0}x^n\Delta x^0 + \binom{n}{1}x^{n-1}\Delta x^1 + \binom{n}{2}x^{n-2}\Delta x^2 + \cdots + \binom{n}{n-1}x^1\Delta x^{n-1} + \binom{n}{n}x^0\Delta x^n - x^n}{\Delta x}\right)$$

$$f'(x) = \binom{n}{1} x^{n-1} = \frac{n!}{1!\,(n-1)!}\, x^{n-1} = nx^{n-1}$$