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BMT223 Assignment #3

Question 2 [Page 673]:

Calculate the equations of the normal to:

(a) $y = xe^x$ Where: $x = 1$

(e) $y = \frac{e^x+1}{x}$ Where: $x = 1$

Answers:

-- (a) --

Using product rule to find the derivative "slope of the tangent line to the curve":

$y = uv \rightarrow y' = u'v + uv' \rightarrow u = x\ ,\ v = e^x \rightarrow u' = 1\ ,\ v' = e^x$

$$y' = e^x + xe^x = e^x(1 + x)$$

Inserting the value of [x=1] to find the slope at this point (m):

$$y'(1) = e^1(2) = 5.437 = m$$

Finding the value of the slope of the normal line at this point (m'):

$m \times m' = -1 \rightarrow m' = \frac{-1}{m} = \frac{-1}{5.437} = -0.184$

The main form of the equation of the normal line is: [Y=m'x+b]

Inserting the value of [x=1] into the main function to obtain the y-coordinate at this point:

$$y(1) = e^1 = 2.718 = Y$$

Inserting the value of [x=1] to find the intercept "b":

$Y = y(1) = 2.718 = (-0.184)(1) + b \rightarrow b = 2.718 + 0.184 = 2.902$

So: The equation of the normal line is: $Y = -0.184x + 2.902$

Graphical explanation:

	y “Main function”
	Normal line
	Tangent line

-- (b) --

Using quotient rule to find the derivative “slope of the tangent line to the curve”:

$y = \frac{u}{v} \rightarrow y' = \frac{u'v - uv'}{v^2} \rightarrow u = e^x + 1\ ,\ v = x \rightarrow u' = e^x\ ,\ v' = 1\ ,\ v^2 = x^2$

$$y' = \frac{xe^x - e^x - 1}{x^2} = \frac{e^x(x-1) - 1}{x^2}$$

Inserting the value of [x=1] to find the slope at this point (m):

$$y'(1) = \frac{0 - 1}{1} = -1 = m$$

Finding the value of the slope of the normal line at this point (m’):

$m \times m' = -1 \rightarrow m' = \frac{-1}{m} = \frac{-1}{-1} = 1$

The main form of the equation of the normal line is: [Y=m’x+b]

Inserting the value of [x=1] into the main function to obtain the y-coordinate at this point:

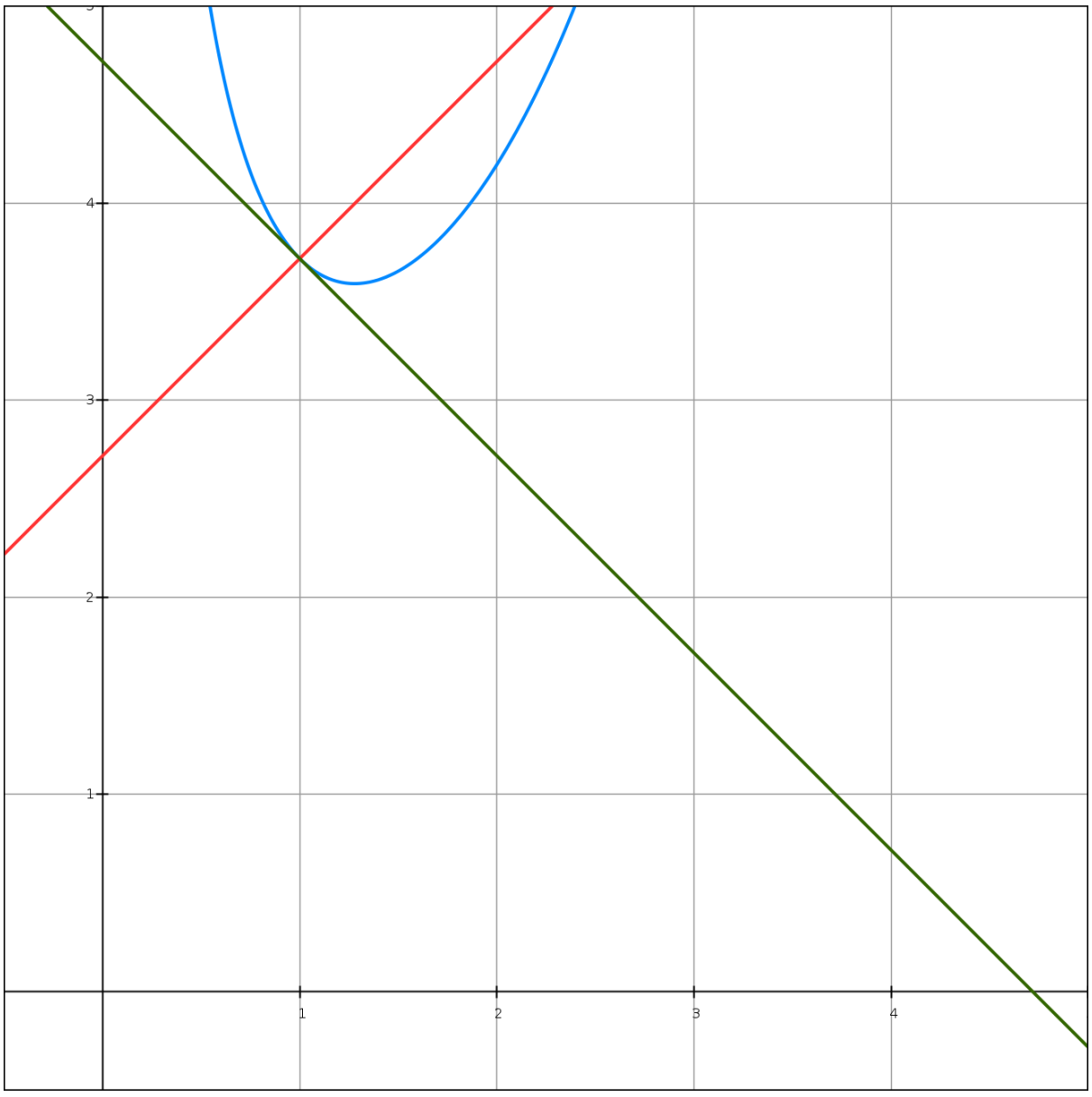
$$y(1) = 3.718 = Y$$

Inserting the value of [x=1] to find the intercept “b”:

$Y = y(1) = 3.718 = (1)(1) + b \rightarrow b = 3.718 - 1 = 2.718$

So: The equation of the normal line is: $Y = x + 2.718$

Graphical explanation:



	y “Main function”
	Normal line
	Tangent line