King Saud University
Department of Mathematics

244
Second Midterm, April 2016

NAME:

Group Number/Instructor name:

ID:

• Duration of the exam: 90 minutes
• Simple calculators are allowed

Question	Grade
I	
II	
III	
IV	
Total	

Question	1	2	3	4	5
Answer					

I) Choose the correct answer (write it on the table above):

1) If A, B and C are square matrices, such that $\det(AC) = 2$ and $\det B = 3$, then $\det(AB^TC)$ is

(A) 6	(B) 5	(C) $\frac{2}{3}$	(D) None

2) Let $v = (4, 17, -8, 3)$ and $w = (11, -2, 8, -18)$ be vectors in $\mathbb{R}^4$. The vector x that satisfies $5x - 2v = 2(w - 5x)$ is

(A) $x = (0, 2, 0, -2)$	(B) $x = (2, 0, 2, -2)$	(C) $x = (2, 2, 0, -2)$	(D) None

3) If $u = (4, 7, -3, 2)$ and $v = (1, -1, -3, k)$ are vectors in $\mathbb{R}^4$, then the value of k such that vectors u and v are orthogonal is

(A) $k = 0$	(B) $k = -3$	(C) $k = 3$	(D) None

4) If u and v are vectors in $\mathbb{R}^n$, such that $\|u\| = 5$, $\|v\| = 12$ and $\|u + v\| = 13$, and θ is the angle determined by u and v, then

(A) $\theta < 90°$	(B) $\theta = 90°$	(C) $\theta > 90°$	(D) None

5) The vector w, which is a linear combination of vectors $v_1 = (1, 2, 0)$, $v_2 = (-2, 0, 1)$ and $v_3 = (0, 1, -1)$, with coefficients $c_1 = 2$, $c_2 = 1$ and $c_3 = -1$ is

(A) $w = (0, 2, 3)$	(B) $w = (0, 3, 2)$	(C) $w = (3, 0, 2)$	(D) None

II) Determine whether the following is **True** or **False**. **Justify your answer**.

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(1) For all $u, v \in \mathbb{R}^n$, $||u+v|| = ||u|| + ||v||$. ( )

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(2) If $||u+v|| = 5$ and $||u-v|| = 3$, then $u \cdot v = 6$. ( )

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(3) If u and v are vectors in $\mathbb{R}^n$, then the distance between the vectors $2u+3v$ and $3u+2v$ is $5\|u+v\|$. ( )

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(4) There exist vectors u, v and w in $\mathbb{R}^3$, such that

$$(u \cdot v) \cdot w = u \cdot (v \cdot w).$$

( )

III) A) Consider the linear system of equations

$$\begin{cases} 2x + y - 3z = -2 \\ x + 2y + z = -3 \\ -3x - y + 3z = 0 \end{cases}$$

Solve for x, using **Cramer's rule**.

B) Find the values of a, such that

$$\begin{vmatrix} a-1 & 1 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & a & a+1 \\ 4 & a^2 & (a+1)^2 \end{vmatrix}$$

IV) A) Let $V = \mathbb{R}$. Prove that V is not a vector space, when endowed with the the usual addition of numbers and the scalar multiplication given by $k \cdot x = k^2 x$, $\forall\ k \in \mathbb{R}$ and $x \in V$.

B) Prove that the set $W = \{(a, b, c) \in \mathbb{R}^3 : a + b + c = 0\}$ is a subspace of $\mathbb{R}^3$.

Scrap paper. This page will not be graded.