KINGDOM OF SAUDI ARABIA
MINSTRY OF HIGHER EDUCATION
JAZAN UNIVERSITY
Preparatory Year Deanship
Department of *Science-mathematics school*

المملكـة العربيـة السـعـودية
وزاره التعـليم العـالـــي
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Questions Bank for Faculty of Computer Science Students
102 Math. Level II

Chapter 1
Systems of Linear Equations and Matrices

Solve each of the following systems by Gauss–Jordan elimination.

(a)
$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$$

(b)
$$\begin{aligned} 2x_1 + 2x_2 + 2x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1 \end{aligned}$$

(c)
$$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x - 3w &= -3 \end{aligned}$$

Solve each of the following systems by Gauss–Jordan elimination.

(a)
$$\begin{aligned} 2x_1 - 3x_2 &= -2 \\ 2x_1 + x_2 &= 1 \\ 3x_1 + 2x_2 &= 1 \end{aligned}$$

(b)
$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= -15 \\ 5x_1 + 3x_2 + 2x_3 &= 0 \\ 3x_1 + x_2 + 3x_3 &= 11 \\ -6x_1 - 4x_2 + 2x_3 &= 30 \end{aligned}$$

(c)
$$\begin{aligned} 4x_1 - 8x_2 &= 12 \\ 3x_1 - 6x_2 &= 9 \\ -2x_1 + 4x_2 &= -6 \end{aligned}$$

MATRICES AND MATRIX OPERATIONS

Q1)

Solve the following matrix equation for a, b, c, and d.

$$\begin{bmatrix} a-b & b+c \\ 3d+c & 2a-4d \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}$$

Q2)

Consider the matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

Compute the following (where possible).

(a) $D+E$	(a) $2A^T + C$	(a) AB
(b) $D-E$	(b) $D^T - E^T$	(b) BA
(c) $5A$	(c) $(D-E)^T$	(c) $(3E)D$
(d) $-7C$	(d) $B^T + 5C^T$	(d) $(AB)C$
(e) $2B - C$	(e) $\frac{1}{2}C^T - \frac{1}{4}A$	(e) $A(BC)$
(f) $4E - 2D$	(f) $B - B^T$	(f) CC^T
(g) $-3(D+2E)$	(g) $2E^T - 3D^T$	(g) $(DA)^T$
(h) $A - A$	(h) $(2E^T - 3D^T)^T$	(h) $(C^T B)A^T$
(i) $\text{tr}(D)$		(i) $\text{tr}(DD^T)$

Q) Let

$$A=\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix}, \quad B=\begin{bmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{bmatrix},$$

$$C=\begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix}, \quad a=4, \quad b=-7$$

Show that

(a) $A+(B+C)=(A+B)+C$	(a) $a(BC)=(aB)C=B(aC)$	(a) $(A^T)^T=A$
(b) $(AB)C=A(BC)$	(b) $A(B-C)=AB-AC$	(b) $(A+B)^T=A^T+B^T$
(c) $(a+b)C=aC+bC$	(c) $(B+C)A=BA+CA$	(c) $(aC)^T=aC^T$
(d) $a(B-C)=aB-aC$	(d) $a(bC)=(ab)C$	(d) $(AB)^T=B^TA^T$

Q) Compute the inverses of the following matrices.

(a) $A=\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

(b) $B=\begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$

(c) $C=\begin{bmatrix} 6 & 4 \\ -2 & -1 \end{bmatrix}$

(d) $D=\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

Q)

Solve the systems

(a)
$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ 2x_1 + 5x_2 + 3x_3 &= 5 \\ x_1 \qquad\quad + 8x_3 &= 9 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ 2x_1 + 5x_2 + 3x_3 &= 6 \\ x_1 \qquad\quad + 8x_3 &= -6 \end{aligned}$$

Q)

Determine whether the matrix is invertible; if so, find the inverse by inspection.

(a) $\begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

Q)

Find A^2, A^{-2}, and A^{-k} by inspection.

(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

(b) $A = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

By inspection, determine whether the given triangular matrix is invertible.

(a) $\begin{bmatrix} -1 & 2 & 4 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & -2 & 5 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

Find all values of a, b, and c for which A is symmetric.

$$A = \begin{bmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{bmatrix}$$

Chapter 2
Determinants

1. Let

$$A=\begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

(a) Find all the minors of A.

(b) Find all the cofactors.

2. Let

$$A=\begin{bmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{bmatrix}$$

Find

(a) M_{13} and C_{13}

(b) M_{23} and C_{23}

(c) M_{22} and C_{22}

(d) M_{21} and C_{21}

3. Find A^{-1}

$$A=\begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$$

$$A=\begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix}$$

$$A=\begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A=\begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{bmatrix}$$

4. solve by Cramer's rule,

16. $\begin{aligned} 7x_1 - 2x_2 &= 3 \\ 3x_1 + x_2 &= 5 \end{aligned}$

17. $\begin{aligned} 4x + 5y \quad &= 2 \\ 11x + y + 2z &= 3 \\ x + 5y + 2z &= 1 \end{aligned}$

18. $\begin{aligned} x - 4y + z &= 6 \\ 4x - y + 2z &= -1 \\ 2x + 2y - 3z &= -20 \end{aligned}$

1. Verify that $\det(A) = \det(A^T)$ for

(a) $A = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & -3 & 6 \end{bmatrix}$

2. Evaluate the following determinants by inspection.

(a) $\begin{vmatrix} 3 & -17 & 4 \\ 0 & 5 & 1 \\ 0 & 0 & -2 \end{vmatrix}$

(b) $\begin{vmatrix} \sqrt{2} & 0 & 0 & 0 \\ -8 & \sqrt{2} & 0 & 0 \\ 7 & 0 & -1 & 0 \\ 9 & 5 & 6 & 1 \end{vmatrix}$

(c) $\begin{vmatrix} -2 & 1 & 3 \\ 1 & -7 & 4 \\ -2 & 1 & 3 \end{vmatrix}$

1. Verify that $\det(kA) = k^n \det(A)$ for

(a) $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$; $k = 2$

(b) $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & 4 & 5 \end{bmatrix}$; $k = -2$

2. Verify that $\det(AB) = \det(A)\det(B)$ for

$A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}$

Is $\det(A + B) = \det(A) + \det(B)$?

3. By inspection, explain why $\det(A) = 0$.

$$A = \begin{bmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{bmatrix}$$

5. Let

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Assuming that $\det(A) = -7$, find

(a) $\det(3A)$

(b) $\det(A^{-1})$

(c) $\det(2A^{-1})$

(d) $\det((2A)^{-1})$

(e) $\det\begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix}$

Express the following linear systems in the form $(\lambda I - A)\mathbf{x} = \mathbf{0}$.

(a)
$$\begin{aligned} x_1 + 2x_2 &= \lambda x_1 \\ 2x_1 + x_2 &= \lambda x_2 \end{aligned}$$

(b)
$$\begin{aligned} 2x_1 + 3x_2 &= \lambda x_1 \\ 4x_1 + 3x_2 &= \lambda x_2 \end{aligned}$$

(c)
$$\begin{aligned} 3x_1 + x_2 &= \lambda x_1 \\ -5x_1 - 3x_2 &= \lambda x_2 \end{aligned}$$

evaluate the determinant

3. $\begin{vmatrix} 3 & 5 \\ -2 & 4 \end{vmatrix}$

4. $\begin{vmatrix} 4 & 1 \\ 8 & 2 \end{vmatrix}$

5. $\begin{vmatrix} -5 & 6 \\ -7 & -2 \end{vmatrix}$

6. $\begin{vmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{vmatrix}$

7. $\begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix}$

8. $\begin{vmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{vmatrix}$

9. $\begin{vmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{vmatrix}$

10. $\begin{vmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{vmatrix}$

11. $\begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix}$

Find all values of λ for which $\det(A) = 0$

(a) $\begin{bmatrix} \lambda - 2 & 1 \\ -5 & \lambda + 4 \end{bmatrix}$

(b) $\begin{bmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{bmatrix}$

Chapter 3
Vectors in 2-Space and 3-Space

Let $\mathbf{u} = (-3, 1, 2)$, $\mathbf{v} = (4, 0, -8)$, and $\mathbf{w} = (6, -1, -4)$. Find the components of

(a) $\mathbf{v} - \mathbf{w}$

(b) $6\mathbf{u} + 2\mathbf{v}$

(c) $-\mathbf{v} + \mathbf{u}$

(d) $5(\mathbf{v} - 4\mathbf{u})$

(e) $-3(\mathbf{v} - 8\mathbf{w})$

(f) $(2\mathbf{u} - 7\mathbf{w}) - (8\mathbf{v} + \mathbf{u})$

Find the norm of $\mathbf{v}$.

(a) $\mathbf{v} = (4, -3)$

(b) $\mathbf{v} = (2, 3)$

(c) $\mathbf{v} = (-5, 0)$

(d) $\mathbf{v} = (2, 2, 2)$

(e) $\mathbf{v} = (-7, 2, -1)$

(f) $\mathbf{v} = (0, 6, 0)$

Final Exams Sheets

Sheet (1)

<u>Answer the following questions :</u>

Q1: Let $A=\begin{bmatrix}1 & 2\\0 & 4\end{bmatrix}$, $B=\begin{bmatrix}4 & 0\\-3 & 2\\1 & 3\end{bmatrix}$. **Compute** the following (where possible)

1) $-2B^T$ 2) AB 3) BA 4) $tr(3A)$ 5) A^2 6) A^{-2}

Q2: Let $A=\begin{bmatrix}1 & -1\\0 & 3\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\-3 & 1\end{bmatrix}$, $C=\begin{bmatrix}5 & -1\\4 & 0\end{bmatrix}$.

Show that $(A+B)+C=A+(B+C)$

Q3: **Compute the *inverses*** of the following matrices (where possible)

1) $D=\begin{bmatrix}2 & 0 & 0 & 0\\0 & 2 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 4\end{bmatrix}$, 2) $C=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\3 & 0 & 1\end{bmatrix}$, 3) $B=\begin{bmatrix}3 & 6\\2 & 4\end{bmatrix}$, 4) $A=\begin{bmatrix}4 & 4\\1 & 2\end{bmatrix}$

Q1: Let $A=\begin{bmatrix}2 & a & b\\2 & 6 & c\\1 & 9 & 4\end{bmatrix}$

1) If A is **symmetric**, then $a=---$, $b=---$, $c=---$

2) If A is **triangular**, then $a=---$, $b=---$, $c=---$

Q2: If $A=\begin{bmatrix}a & b\\c & d\end{bmatrix}$ **verify that** $\det(3A)=3^2\det(A)$

Q3: Let $A=\begin{bmatrix}1&1&0\\1&2&4\\1&0&1\end{bmatrix}$.

1) Find $\det(A)$

2) Find $adj(A)$

3) Find A^{-1} **[**using $A^{-1}=\frac{1}{\det(A)}adj(A)$ **]**

4) Solve $\begin{bmatrix}1&1&0\\1&2&4\\1&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\7\\3\end{bmatrix}$

Q1: **Let** $A=\begin{bmatrix}1&1&0\\0&-1&2\end{bmatrix}$, $B=\begin{bmatrix}0&1\\1&0\\1&1\end{bmatrix}$

Find 1) $2B-A^T$ 2) $tr(AB)$ 3) BB^T

Q2: **Given** $\begin{bmatrix}b-a&b\\3&d-a\end{bmatrix}=\begin{bmatrix}2&1\\b+c&3\end{bmatrix}$ **, find a,b,c and d**

Q3: **Let** $A=\begin{bmatrix}-1&2\\0&3\end{bmatrix}$

1) find the $inverse\ (A^{-1})$

2) find $P(A)$ **for** $p(x)=x^2-2x-3$

Q1: **Choose the correct answer**

1- The linear system $\begin{matrix}x+y=2,\\3x+3y=6\end{matrix}$ **has (one solution, many solutions, no solutions)**

2- **If** $\begin{bmatrix}2&3\\x&4\end{bmatrix}$ **is symmetric matrix, then** $x=\cdots$ $(2 \quad , \quad 3 \quad , \quad 4)$

3 - $Adj\left(\begin{bmatrix}1&2\\3&4\end{bmatrix}\right)$ **is** **(** $\begin{bmatrix}1&3\\2&4\end{bmatrix}$, $\begin{bmatrix}-1&3\\2&-4\end{bmatrix}$, $\begin{bmatrix}4&-2\\-3&1\end{bmatrix}$ **)**

4- If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $tr(A^{-1}) = \cdots$ (5 , −5 , 2)

5- **The matrix** $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 7 & 6 & 2 \end{bmatrix}$ **is** **(diagonal , symmetric , triangular)**

Q2: **Let** $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$. **Find** 1) $det(A)$ 2) A^{-1}

Q3 : **Solve the following system:**

$$\begin{aligned} x + 2y - z &= 4 \\ 2x - y \quad &= 0 \\ 3x \quad + z &= 4 \end{aligned}$$

Q1: Choose the correct answer

1-_ The linear system $\begin{matrix} x + y = 2, \\ 3x + 3y = 6 \end{matrix}$ **has (one solution, many solutions, no solutions)**

2- **If** $\begin{bmatrix} 2 & 3 \\ x & 4 \end{bmatrix}$ **is symmetric matrix, then** $x = \cdots$ (2 , 3 , 4)

3 - $Adj\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right)$ **is** ($\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $\begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix}$, $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$)

4 - If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $tr(A^{-1}) = \cdots$ (5 , −5 , 2)

5 **- The matrix** $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 7 & 6 & 2 \end{bmatrix}$ **is** **(diagonal , symmetric , triangular)**

Q2: **Let** $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$. **Find** 1) $det(A)$ 2) A^{-1}

Q3 : **Solve the following system:**

$$\begin{aligned} x + 2y - z &= 4 \\ 2x - y \quad &= 0 \\ 3x \quad + z &= 4 \end{aligned}$$

Sheet (2)

Answer the following questions

Question 1 :

Let A , B and C be (n×n) *invertible square matrices*.

Determine whether the statement true (√) or false (×) :

1) $AB \neq BA$ ( )

2) $A^T B^T = (BA)^T$ ( )

3) $\det(A^{-1}) \neq [\det(A)]^{-1}$ ( )

4) $\det(A.B) = \det(A).\det(B)$ ( )

5) $A(B + C) \neq AB + AC$ ( )

6) If $AC = AB$, then $B = C$ ( )

7) $A^{-1}A = AA^{-1} = I$ ( )

8) If $AB = 5I$, then $A^{-1} = 5B$ ( )

9) $A^{-1} = \frac{1}{\det(A)}.adj(A)$ ( )

10) $(A^T)^{-1} = (A^{-1})^T$ ( )

Question 2 :

Let $A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \\ 1 & 0 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & -1 \end{bmatrix}$

1) **Calculate** $A-4B$ **and** $B+3A$
2) **Calculate** $\text{tr}(A)$ **and** A^T
3) **Calculate** $\det(A)$

Question 3 :

(i) Find a **and** b **for which** A **is symmetric,** $A = \begin{bmatrix} 6 & a-3 & b-a \\ 1 & 2 & 5 \\ 3 & 5 & -3 \end{bmatrix}$

(ii) Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, **calculate** A^{-3}, A^3

(C) Let $B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ **calculate** B^{-1}

Question 4 : **Let** $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 1 & 4 \\ 6 & 0 & 5 \end{bmatrix}$

1) **Calculate the <u>characteristic polynomial</u> of the matrix** A.
2) **Calculate the <u>Eigen-values</u>** of A.
3) **Calculate** A^{-1}.

Question 5 : **Solve the following system :**

$$x+y+2z=9$$
$$2x+4y-3z=1$$
$$3x+6y-5z=0$$

Sheet (3)

Answer the following questions

Question 1 :

Let A , B and C be (n×n) *invertible square matrices*.

Determine whether the statement true (√) or false (×) :

1) $A(B + C) = AB + AC$ ( )

2) $(A^T)^{-1} \neq (A^{-1})^T$ ( )

3) $(AB)^T = B^T A^T$ ( )

4) $\det(A+B) \neq \det(A) + \det(B)$ ( )

5) $AB = BA$ ( )

6) $A^{-1}A = AA^{-1} = I$ ( )

7) If $AC = AB$, then $B \neq C$ ( )

8) If $AB = 8I$, then $A^{-1} = \frac{1}{8}B$ ( )

9) $\text{tr}(AB) = \text{tr}(A) \cdot \text{tr}(B)$ ( )

10) $\det(A^{-1}) = (\det(A))^{-1}$ ( )

Question 2 :

Let $A = \begin{bmatrix} 1 & -3 & 4 \\ 2 & 4 & -2 \\ 0 & -1 & 5 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 6 & 3 \\ 3 & 1 & -3 \end{bmatrix}$

1) **Calculate** $2A-B$ **and** $2B+3A$.
2) **Calculate** $\mathrm{tr}(B)$, B^T.
3) **Find** $\det(B)$.

Question 3:

(A) Find a **and** b **for which** A **is *symmetric*,** $A = \begin{bmatrix} 1 & a+2b & a \\ -6 & 3 & -4 \\ -2 & -4 & -3 \end{bmatrix}$

(B) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ **find** A^{-3}, A^3.

(C) Let $B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$, **calculate** B^{-1} .

Question 4: **Solve the following system :**

$$\begin{aligned} 2x+y-z &= 1 \\ 2x+3y-2z &= 2 \\ 4x-2y+3z &= 9 \end{aligned}$$

Question 5: **Let** $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 5 \\ 4 & 0 & 3 \end{bmatrix}$

1) **Calculate the <u>characteristic polynomial</u> of the matrix** A.
2) **Calculate the <u>Eigen-values</u>** of A.
3) **Calculate** A^{-1}.

Sheet (4)

<u>Answer the following questions :</u>

Q1: Let $A = \begin{bmatrix} 5 & 0 \\ 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 & 3 \\ 2 & 0 & 5 \end{bmatrix}$. **Compute** the following (where possible)

1) AB 2) BA 3) $tr(A^2)$ 4) $2B^T$ 5) $adj\,(A)$

Q2: Let $A = \begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 3 \\ 5 & 1 \end{bmatrix}$.

Show that 1) $\det(3A) = 3^2 \det(A)$

2) $A(B + C) = AB + AC$

Q3: **Compute the *inverses*** of the following matrices (where possible)

1) $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, 2) $D = \begin{bmatrix} -4 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ 3) $T = \begin{bmatrix} -4 & 7 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

Q4: 1) Find the value of k if the matrix $B = \begin{bmatrix} 1 & 5 \\ k & 3 \end{bmatrix}$ is **symmetric**

2) Find the **norm** $\|v\|$ of the vector $v = (4, -5, \sqrt{2})$

3) Find the **eigen-values** of the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

Q5: **Solve** the following system:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 10 \\ 2x_1 - x_2 &= 4 \\ x_1 + x_3 &= 4 \end{aligned}$$

Sheet (5)

Question1: Choose the correct answer

1) The matrix $\begin{bmatrix} 1 & 7 \\ 2 & 6 \end{bmatrix}$ is --------

Column matrix	Row matrix	Square matrix

2) If $2\begin{bmatrix} 1 & 2 \\ x & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$, then x =---------

1	2	6

3) $Adj(\begin{bmatrix} a & b \\ c & d \end{bmatrix})$ is ----

$\begin{bmatrix} d & b \\ c & a \end{bmatrix}$	$\begin{bmatrix} -d & b \\ c & -a \end{bmatrix}$	$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

4) The matrix $\begin{bmatrix} 1 & 0 \\ 5 & 3 \end{bmatrix}$ is --------

diagonal	symmetric	triangular

5) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, then $tr(A^3)$ =------

5	-5	9

6) The $size$ of the matrix $\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$ is -------

2×3	3×2	2×2

7) The matrix A is $symmetric$ if and only if -----------

$A = -A$	$A = A^T$	$A = -A^T$

8) If $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $B = [1 \quad 2]$, then AB =---------

$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$	$[5]$	$\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

9) The linear system $\begin{matrix} x + y = 2, \\ 3x + 3y = 9 \end{matrix}$ has -----

one solution	many solutions	no solutions

10) If A is $diagonal$ matrix, then A^T =-------

A^2	A	A^{-1}

Question2: Let $A = \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ Find

1- $2B - A^T =$

2- $AB =$

3- $det(B) =$

4- $B^{-1} =$

5- The $eigenvalues$ of A

Question3: Find the *inverse* A^{-1} of $A = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Question4: *Solve* the following system:

$$\begin{aligned} x + 2y + z &= 0 \\ 2x - y \quad\quad &= 2 \\ 3x \quad\; - 2z &= 5 \end{aligned}$$

Question5: Given $u = (4,0)$ and $v = (2,1)$, find

1) $u + v =$

2) $||u|| =$

3) $d(u,v) =$

4) The *unit* vector that has the same direction of $u =$

5) Determine whether the vectors $u = (4,0)$ and $v = (2,1)$ are linearly independent or linearly dependent in R^2

Sheet (6)

Answer the following questions

Q1: If $A=\begin{bmatrix}-4 & 1 & 8\\ 3 & 0 & 2\\ 4 & -6 & 3\end{bmatrix}$ and $B=\begin{bmatrix}0 & 3 & 0\\ -5 & 1 & 2\\ 1 & 6 & 4\end{bmatrix}$ are two matrices, then find

1) $(2A-B)^T$ 2) AB 3) $tr(A+B)$

Q2: If $v_1=(-1,2,3)$ and $v_2=(2,-4,-6)$ are two vectors

1) Find v_1+v_2

2) Find the **norm** of v_2 ; $\|v_2\|$

3) Find the **distance** between v_1, v_2 ; $d(v_1,v_2)$

4) Find the **unit vector** in the direction of v_2

5) Is v_1, v_2 linearly **independent** or linearly **dependent**?

Q3: (1) Find a, b and c for which A is **symmetric** matrix, $A=\begin{bmatrix}6 & a-b & a+c\\ -1 & 2 & 2\\ 3 & a & -3\end{bmatrix}$

(2) If $D=\begin{bmatrix}5 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 4\end{bmatrix}$ then find

1) D^2 2) D^{-1} 3) D^{-3}

Q4: 1) Find the **Inverse** of the following matrices:

$A=\begin{bmatrix}5 & -1\\ 4 & 2\end{bmatrix}$, $B=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 4 & 0 & 4\end{bmatrix}$

(2) Find the **eigenvalues** of the matrix $A=\begin{bmatrix}2 & -1\\ 3 & 1\end{bmatrix}$

Q5 Let $A=\begin{bmatrix}1 & 2 & -1\\ 1 & -2 & 1\\ 2 & 0 & 1\end{bmatrix}$

1) Find $\det(A)$

2) Find $adj(A)$

3) Solve $\begin{bmatrix}1 & 2 & -1\\ 1 & -2 & 1\\ 2 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}2\\ 0\\ 5\end{bmatrix}$